

Natural Realizations of the Seesaw Mechanism in Miniwarped
Minimal SO(10)

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Natural realizations of the seesaw mechanism in miniwarped minimal SO(10)R. N. Mohapatra,^{*} Nobuchika Okada,[†] and Hai-Bo Yu[‡]*Department of Physics, University of Maryland, College Park, Maryland 20742, USA*

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The minimal supersymmetric SO(10) grand unified theory (GUT) models with **10**, **126**, and **210** Higgs and only renormalizable couplings have been shown to provide a simple way to understand the neutrino mixings as well as the ratio $\Delta m_{\odot}^2/\Delta m_A^2$ in terms of quark mixing parameter θ_{Cabibbo} , provided neutrino masses are described by a type II seesaw formula. However, in this minimal picture, it is impossible to realize type II dominance with renormalizable couplings in 4 dimensions. We show that this problem can be cured by embedding this model into a warped five dimensional space-time with warping between the Planck and the GUT scale, where both type II as well as mixed seesaw formulas can be realized in a natural manner without expanding the Higgs sector. These models also avoid the possible problem of threshold effects associated with large Higgs representations since the theory above the GUT scale is now strongly coupled.

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I. INTRODUCTION

Understanding neutrino masses and mixings has been a major challenge to particle theorists. Many approaches have been proposed [1]. While there is no consensus on the right final solution, some important clues are emerging on which there appears to be a large degree of agreement among theorists. If the neutrino is a Majorana particle, then a seesaw mechanism [2] for understanding the origin of its mass seems to have a strong appeal. The ingredients of this mechanism are as follows: (i) m_ν is related to $B - L$ symmetry breaking, implying that physics beyond the standard model must have this symmetry, and $B - L$ most likely is a local symmetry; (ii) second, it is also possible that the breaking of this symmetry takes place at a high scale by the Majorana mass of the right-handed neutrinos which then provides a natural way to understand the smallness of the neutrino masses for natural values of parameters in the theory. A theoretical support for this kind of scenario comes from the observation that grand unified theories based on the SO(10) group [3] automatically incorporate both the right-handed neutrinos into its spinor multiplets as well as the local $B - L$ symmetry as part of the gauge group, and in most minimal ways of symmetry breaking the coupling constant unification requirement puts the $B - L$ symmetry breaking scale (and hence the right-handed neutrino mass) close to the grand unified theory (GUT) scale of 10^{16} GeV, so that a high seesaw scale close to GUT scale required for understanding atmospheric neutrino observations becomes easier to understand.

The present paper addresses an important aspect of embedding the seesaw mechanism in a minimal super-

symmetric (SUSY) SO(10) model. We focus on SO(10) models with the **126** Higgs field breaking $B - L$ gauge symmetry [4–8] rather than the **16** Higgs [9] since in the first case both R -parity symmetry of the minimal supersymmetric standard model (MSSM) and predictivity for neutrinos arise without imposing any extra symmetries. We will discuss the class of models which we call minimal SO(10) models because of the Higgs content of **10**, **126** \oplus **126**, and **210** and the matter content in three **16** spinors [10]. In [4] and several subsequent papers [5], the neutrino mass discussion in this model was carried out using only the type I seesaw formula. But as is now well known, there are two contributions to the seesaw formula [11] in left-right symmetric as well as SO(10) models, i.e.,

$$\mathcal{M}_\nu = f\nu_L - M_D^T(f\nu)^{-1}M_D. \quad (1.1)$$

When the second term dominates, it is called type I seesaw whereas when the first one dominates, it is called type II seesaw. The advantage of the type II seesaw formula in understanding large atmospheric neutrino mixings in a two generations minimal SO(10) model was first observed in Ref. [6]. It was subsequently shown [7] that the same scenario can help to explain the large solar as well as small reactor mixing angle θ_{13} , bringing these models to the mainstream of neutrino phenomenology. Other detailed questions in the model such as CP violation [7,12] and proton decay [13] as well as symmetry breaking [14] have since been discussed. Because of predictivity in the neutrino sector while keeping the rest of fermion mass phenomenology in agreement with observations as well as general economy of the Higgs sector, these minimal models have become very attractive and are in fact in a better footing than SU(5) models were in the early 1980s, with serious attention being paid to them. One must therefore examine to what extent the model parameters needed for the neutrino predictions can be naturally obtained. It is this aspect of the models that we address in this paper.

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Since in the minimal SO(10) model, GUT symmetry relates the Dirac masses of the neutrinos to the up quark masses, one can ask for a more quantitative understanding of the seesaw formula. For example, the atmospheric neutrino mass difference square $\Delta m_A^2 \sim 0.0025 \text{ eV}^2$ requires that at least one of the right-handed neutrinos has a mass around 10^{14} GeV , if one uses the type I seesaw formula for neutrino masses. This is much less than the GUT scale which determines the $B - L$ breaking and therefore implies a fine-tuning of some Yukawa couplings. In the context of minimal SO(10) models, it in fact turns out that fitting charged fermion masses also requires a Yukawa coupling suppressed to that level [7]. Therefore they go together and clearly, it will be important to understand this mini-fine-tuning from a more fundamental point of view [15].

In this paper we concern ourselves with minimal SUSY SO(10) models that use type II seesaw where a different fine-tuning becomes essential. The magnitude of the type II seesaw contribution to neutrino masses is given by $f \frac{v_{wk}^2}{M_T}$, where M_T is the $B - L = 2$, $SU(2)_L$ triplet mass and for $f \sim 1$, one needs $M_T \sim 10^{14} \text{ GeV}$, whereas for $f \sim 0.01$ as may be required by charged fermion fitting, we need $M_T \sim 10^{12} \text{ GeV}$ [16]. Since M_T is related to M_{GUT} , the discrepancy between them must be explained. An additional challenge for this class of models is that for the type II term to dominate, one must not only have the first term dominate in Eq. (1.1) but the second term must also be simultaneously smaller. In the language of SU(5) submultiplets in the **126** field, M_T must be the mass of the **15** submultiplet.

The problem in understanding type II dominance was discussed in Ref. [18], where it was shown that the requirements given above for type II dominance cannot be satisfied in the minimal four dimensional SUSY SO(10) model with **10** \oplus **126** \oplus **210** Higgs fields. The reason is that at high scale there are only four parameters in the superpotential and constraints of supersymmetry imply that the triplet mass must be at the GUT scale, making the type II term subdominant. This calls into question the viability of the minimal models. The solution to this suggested in [18] was that the model be extended to include a **54** dimensional Higgs field, in which case one can fine-tune parameters to get a lower triplet mass while at the same time suppressing the type I term. Since **54** Higgs does not couple to matter fields, it does not affect the discussion of fermion masses and mixings.

In this paper, we propose a different way to solve these fine-tuning problems without adding extra Higgs fields but rather by embedding the minimal model into a warped five dimensional space-time with warping between the Planck scale and the GUT scale and with all fields of the model in the bulk. We call this “miniwarping” since the warp factor required here is $\omega \equiv M_{\text{GUT}}/M_P \sim 10^{-2}$ rather than the usual m_W/M_P as in canonical Randall-Sundrum (RS) models. Two things happen in such models if the gauge group

and other fields are in the bulk: (i) all mass parameters in the IR brane are suppressed by ω and (ii) depending on bulk mass and the gauge charge, there may be additional suppression factors [19]. A combination of these two factors provides a new way to resolve some of the fine-tuning problems in these models.

An initial application of this idea to understand type I seesaw in minimal SO(10) has recently been discussed by Fukuyama, Kikuchi, and Okada [20], where it was shown how the smallness of the right-handed neutrino mass can be understood as a consequence of miniwarping. In the present paper, we show that miniwarping can also help to explain type II dominance of the seesaw formula. Unlike the case of type I seesaw dominance, the type II case involves a lot of subtle issues such as the magnitude of the GUT scale, structure of the MSSM doublets in terms of the GUT Higgs multiplets, etc., and is highly nontrivial due to interconnections between various terms in the superpotential. We have, however, succeeded in finding an example where this happens. This is the subject of this paper. The significance of our result is that it restores the type II dominated minimal SUSY SO(10) into a viable model.

The paper is organized as follows: In Sec. II we discuss the basic ingredients of the approach; in Sec. III, we discuss the minimal SO(10) and show how type II seesaw arises naturally without extra Higgs fields; we discuss some implications of the model in Sec. IV.

II. BASIC INGREDIENTS OF A MINIWARPED MODEL

Our basic approach consists of embedding the minimal SO(10) model in the warped five dimensional brane world scenario [21] with warping between the Planck scale to the GUT scale. The fifth dimension is compactified on the orbifold S^1/Z_2 with two branes, UV and IR, located on the two orbifold fixed points. As in the RS model, we use the warped metric [21],

$$ds^2 = e^{-2kr_c|y|} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 dy^2, \quad (2.1)$$

with $-\pi \leq y \leq \pi$ and $\eta_{\mu\nu} = (+, -, -, -)$. In the above expression, k is the anti-de Sitter curvature, and r_c and y are the radius and the angle of S^1 , respectively. As is well known, five dimensional $N = 1$ SUSY corresponds to $N = 2$ SUSY in four dimensions. We can therefore write the 5D superfields in terms of $N = 2$ 4D multiplets. The process of compactification leads to $N = 1$ SUSY on the brane as well as in 4D.

The Lagrangian for a generic U(1) gauge theory with matter and Higgs fields in the bulk can be written in terms of 4D $N = 1$ superfields as [22]

$$\begin{aligned} \mathcal{L} = & \int dy \left\{ \int d^4 \theta r_c e^{-2kr_c |y|} (H_i^\dagger e^{-Q_i V} H_i + H_i^c e^{Q_i V} H_i^{c\dagger}) \right. \\ & + \int d^2 \theta e^{-3kr_c |y|} H_i^c \left[\partial_y - (1 + C_i) kr_c \epsilon(y) - Q_i \frac{\chi}{\sqrt{2}} \right] \\ & \left. \times H_i + \text{h.c.} \right\}, \end{aligned} \quad (2.2)$$

where C_i is a dimensionless (bulk mass) parameter, $\epsilon(y) = y/|y|$ is the step function, H_i, H_i^c is the hypermultiplet with the charge Q_i under the gauge group, and

$$\begin{aligned} V = & -\theta \sigma^\mu \bar{\theta} A_\mu - i \bar{\theta}^2 \theta \lambda_1 + i \theta^2 \bar{\theta} \bar{\lambda}_1 + \frac{1}{2} \theta^2 \bar{\theta}^2 D, \\ \chi = & \frac{1}{\sqrt{2}} (\Sigma + i A_5) + \sqrt{2} \theta \lambda_2 + \theta^2 F, \end{aligned} \quad (2.3)$$

are the vector multiplet and the adjoint chiral multiplets, which form an $N = 2$ SUSY gauge multiplet. Z_2 parity for H_i and V is assigned as even, while it is odd for H_i^c and χ . This technique is easily generalized to the case of the SO(10) model. The point to emphasize is that in RS models, the mass scale of the IR brane is warped down by the warp factor [21], $\omega = e^{-kr_c \pi}$, in effective four dimensional theory. If we take the cutoff of the original five dimensional theory and the anti-de Sitter curvature as $M_5 \simeq k \simeq M_P$, the four dimensional (reduced) Planck mass, the cutoff scale in the IR brane is $\Lambda_{\text{IR}} = \omega M_P$. In our case, we choose the warp factor to be such that $M_{\text{GUT}} = \Lambda_{\text{IR}} = \omega M_P$. In the IR brane, the theory becomes nonperturbative above this scale so that the question of large threshold corrections becomes moot.

Let us now assume that the gauge symmetry is broken down and the adjoint chiral multiplet χ develops a vacuum expectation value (VEV). Since its Z_2 parity is odd, the VEV has to take the form

$$\langle \Sigma \rangle = 2\alpha kr_c \epsilon(y). \quad (2.4)$$

In this case, the zero mode wave function of H_i satisfies the following equation of motion:

$$[\partial_y - (1 + C_i + Q_i \alpha) kr_c \epsilon(y)] H_i = 0, \quad (2.5)$$

which yields

$$H_i = \frac{1}{\sqrt{N_i}} e^{(1+C_i+Q_i\alpha)kr_c|y|} h_i(x^\mu), \quad (2.6)$$

where $h_i(x^\mu)$ is the chiral multiplet in four dimensions. Here, N_i is a normalization constant which ensures that the kinetic term is canonically normalized. We have

$$\frac{1}{N_i} = \frac{2(C_i + Q_i \alpha) k}{e^{2(C_i+Q_i\alpha)kr_c\pi} - 1}. \quad (2.7)$$

There are now two typical cases to consider:

- (i) (i) If $e^{(C_i+Q_i\alpha)kr_c\pi} \gg 1$, the wave functions at $y = 0$ and $y = \pi$ are, respectively, given by

$$\begin{aligned} H_i(y = 0) & \simeq \sqrt{2(C_i + Q_i \alpha) k} \omega^{C_i+Q_i\alpha} h(x^\mu), \\ H_i(y = \pi) & \simeq \sqrt{2(C_i + Q_i \alpha) k} \omega^{-1} h(x^\mu). \end{aligned} \quad (2.8)$$

- (ii) (ii) Whereas for $e^{(C_i+Q_i\alpha)kr_c\pi} \ll 1$, the wave functions are

$$\begin{aligned} H_i(y = 0) & \simeq \sqrt{-2(C_i + Q_i \alpha) k} h(x^\mu), \\ H_i(y = \pi) & \simeq \sqrt{-2(C_i + Q_i \alpha) k} \omega^{-(C_i+Q_i\alpha)} \omega^{-1} h(x^\mu). \end{aligned} \quad (2.9)$$

In case (i), the wave function is localized around the IR brane while it is localized around the UV brane in case (ii). These nontrivial wave function profiles lead to important effects, namely, suppression of couplings and masses, in effective four dimensional theory.

To see this, let us consider Yukawa couplings on the IR and UV branes for three bulk hypermultiplets:

$$\begin{aligned} \mathcal{L}_Y = & \int d^2 \theta \omega^3 \frac{Y_1}{M_5^{3/2}} H_i(y = \pi) H_j(y = \pi) H_k(y = \pi) \\ & + \int d^2 \theta \frac{Y_2}{M_5^{3/2}} H_i(y = 0) H_j(y = 0) H_k(y = 0) \\ & + \text{h.c.}, \end{aligned} \quad (2.10)$$

where $Q_i + Q_j + Q_k = 0$ has been assumed for the U(1) gauge invariance, and Y_1 and Y_2 are independent Yukawa coupling constants on the IR and UV branes, respectively. When all the bulk fields are localized around the IR brane ($C_{i,j,k} + Q_{i,j,k} \alpha > 0$), we obtain the Yukawa coupling constant in effective four dimensional theory as

$$Y_{4\text{D}} \sim Y_1 + Y_2 \omega^{C_i+Q_i\alpha} \omega^{C_j+Q_j\alpha} \omega^{C_k+Q_k\alpha} \sim Y_1. \quad (2.11)$$

There is no suppression for the Yukawa coupling constant on the IR brane, while the Yukawa coupling constant on the UV brane is very much suppressed by the small wave function overlapping. A more nontrivial example is to assume H_i is localized around the UV brane ($C_i + Q_i \alpha < 0$) and the others are localized around the IR brane ($C_{j,k} + Q_{j,k} \alpha > 0$). This case leads to the effective Yukawa coupling constant as

$$Y_{4\text{D}} \sim Y_1 \omega^{-(C_i+Q_i\alpha)} + Y_2 \omega^{C_j+Q_j\alpha} \omega^{C_k+Q_k\alpha}. \quad (2.12)$$

Both of the coupling constants are suppressed according to the wave function overlapping between each field. Other cases are completely analogous and the effective Yukawa coupling constants are suppressed or not suppressed according to the wave function profiles.

Next let us consider mass terms on the IR and UV branes for two bulk hypermultiplets such as

$$\begin{aligned} \mathcal{L}_m = & \int d^2\theta \omega^3 \frac{m_1}{M_5} H_a(y = \pi) H_b(y = \pi) \\ & + \int d^2\theta \frac{m_2}{M_5} H_a(y = 0) H_b(y = 0) + \text{h.c.} \end{aligned} \quad (2.13)$$

Here two mass terms on the IR and UV branes have been generally introduced. If two bulk fields are localized around the IR brane ($C_{a,b} + Q_{a,b}\alpha > 0$), we obtain the mass term in effective four dimensional theory as

$$m_{4D} \sim m_1 + m_2\omega. \quad (2.14)$$

Although there is no suppression due to the wave function profiles in this case, the mass term on the IR brane is warped down. This is the characteristic feature of RS models mentioned above. More general cases are, again, analogous, and we find that suppression factors (in addition to the warp factor) appear in the effective mass according to the wave function overlap.

In the next section, we apply these results to explain the naturalness of type I and type II seesaw in the minimal SO(10) model. We will see that this goal can more or less be achieved except we still need to do one fine-tuning.

III. RELEVANT ASPECTS OF THE MINIMAL SUSY SO(10) MODEL

In order to apply the discussion of the previous section to the minimal SO(10) model, we provide a brief reminder of the salient aspects of these models. All the couplings and mass parameters in this model refer to four dimensions and we omit the superscript 4D for all of them for simplicity. As long as we allow only renormalizable couplings, the model has only two Yukawa coupling matrices: (i) h for the **10** Higgs and (ii) f for the **126** Higgs. SO(10) has the property that the Yukawa couplings involving the **10** and **126** Higgs representations are symmetric. Therefore if we assume that CP violation arises from other sectors of the theory (e.g., squark masses) and work in a basis where one of these two sets of Yukawa coupling matrices is diagonal, then there are only nine parameters describing the Yukawa couplings. Noting the fact that the **45** and $\bar{\mathbf{5}}$ SU(5)-submultiplets of $\bar{\mathbf{126}}$ have a pair of standard model doublets in addition to the **5** and $\bar{\mathbf{5}}$ multiplets of **10** that contributes to charged fermion masses, one can write the quark and lepton mass matrices as follows [4]:

$$\begin{aligned} M_u &= h\kappa_u + f v_u, & M_d &= h\kappa_d + f v_d, \\ M_\ell &= h\kappa_d - 3f v_d, & M_D &= h\kappa_u - 3f v_u, \end{aligned} \quad (3.1)$$

where $\kappa_{u,d}$ are the VEVs of the up and down standard model type Higgs fields in the **10** multiplet and $v_{u,d}$ are the corresponding VEVs for the same doublets in **126**. This gives 13 parameters describing the fermion masses and mixings (for both leptons and quarks). If we input six quark masses, three lepton masses, and three quark mixing angles and weak scale, these are a total of 13 parameters and all

parameters are now determined. Thus all parameters of the model that go into fermion masses are determined. The neutrino sector therefore has no free parameters except for two overall scales (v_L and v_R) as we see below:

$$\mathcal{M}_\nu = 2f v_L - M_D^T (2f v_R)^{-1} M_D. \quad (3.2)$$

If type I or type II seesaw dominates, except for an overall scale, all the rest of the parameters of the neutrino mass matrix are predicted. The problem addressed in this paper is to what extent one can understand the naturalness of parameters that make either type I or type II dominate. As noted earlier, a simple understanding of the large neutrino mixings [6,7] as well as an explanation of the value of $\sqrt{\frac{\Delta m_{23}^2}{\Delta m_{21}^2}}$ as being of order of the Cabibbo angle comes about in the case of type II dominance.

When one tries to understand Cabibbo-Kobayashi-Maskawa CP violation in these models, it is useful to extend it by the inclusion of a **120** Higgs field that couples to standard model fermions [23]. We omit the **120** field from our considerations since our main point is not affected by this.

To see what fine-tunings are needed to make type II seesaw dominate, let us write down the superpotential for the 4D SUSY SO(10) model that we are discussing. Denoting the **126** fields by Σ , and **210** ones by Φ , we have

$$W = M_\Sigma^{4D} \Sigma \bar{\Sigma} + M_\Phi^{4D} \Phi^2 + \lambda_1^{4D} \Sigma \bar{\Sigma} \Phi + \lambda_2^{4D} \Phi^3, \quad (3.3)$$

where we have used the superscript 4D to denote that this is a 4D theory. It is helpful to write down the SU(5) \times U(1) X submultiplets of the various SO(10) multiplets used here:

$$\begin{aligned} \mathbf{210} &= \mathbf{1}_0 \oplus \mathbf{5}_{-8} \oplus \bar{\mathbf{5}}_8 \oplus \mathbf{10}_4 \oplus \bar{\mathbf{10}}_{-4} \oplus \mathbf{24}_0 \oplus \mathbf{75}_0 \oplus \mathbf{40}_{-4} \\ &\quad \oplus \bar{\mathbf{40}}_4, \\ \mathbf{126} &= \mathbf{1}_{-10} \oplus \bar{\mathbf{5}}_{-2} \oplus \mathbf{10}_{-6} \oplus \bar{\mathbf{15}}_{+6} \oplus \mathbf{45}_2 \oplus \bar{\mathbf{50}}_{-2}, \\ \mathbf{10} &= \mathbf{5}_2 \oplus \bar{\mathbf{5}}_{-2}. \end{aligned} \quad (3.4)$$

And the decomposition of matter field **16** is

$$\mathbf{16} = \mathbf{1}_{-5} \oplus \bar{\mathbf{5}}_3 \oplus \mathbf{10}_{-1}. \quad (3.5)$$

The supergraph responsible for type II seesaw term is given in Fig. 1. An inspection of this graph reveals that the following conditions must be satisfied for the type II seesaw to be important for neutrino mass discussion:

- (i) $M_{15} \sim f 10^{-2} M_{\text{GUT}}$;
- (ii) coupling $\bar{\mathbf{15}} \cdot \mathbf{5} \cdot \mathbf{5} \subset \mathbf{210} \cdot \mathbf{126} \cdot \mathbf{10}$ or $\bar{\mathbf{15}} \cdot \mathbf{5} \cdot \mathbf{5} \subset \mathbf{210} \cdot \mathbf{126} \cdot \bar{\mathbf{126}}$ must not be suppressed and be of order one.

We will show in the next section how we can have an understanding of these two conditions within a miniwarped model using the technique outlined in Sec. II.

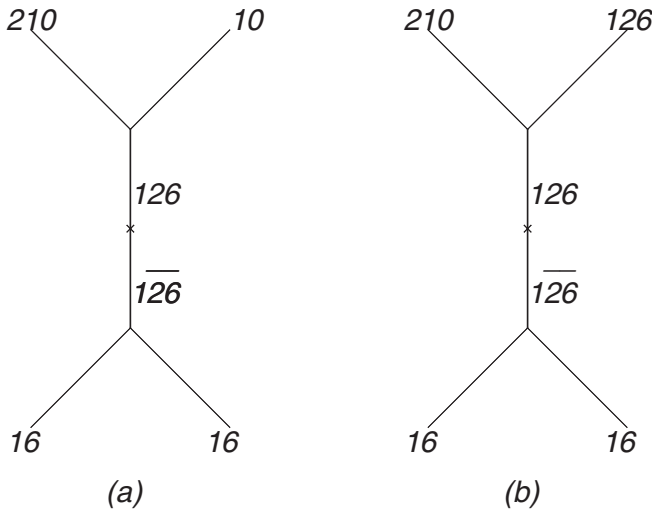


FIG. 1. Supergraph for type II seesaw.

IV. MINIMAL SO(10) THEORY IN FIVE DIMENSIONS

We take the $N = 1$ SUSY SO(10) model in five dimensions and put all the fields (matter as well as Higgs) in the bulk with different bulk mass terms for different fields. Note that all fields are paired with their complex conjugate fields so that the bulk mass terms are allowed by gauge invariance and supersymmetry. Note that these mass terms play the role of a parameter describing the wave function profile of the field and are not the mass terms of 4D theory.

We put the interaction terms on both IR and UV branes. Both **126** and **10** mass terms are on the IR brane, and the mass term of **210** is on the UV brane. Because of the nonrenormalization theorem of supersymmetry, such a choice is technically quite natural. The relevant part of the Lagrangian can be written as $\mathcal{L} = \int d^2\theta W_{\text{IR}} + \int d^2\theta W_{\text{UV}} + \text{h.c.}$, where

$$\begin{aligned}
 W_{\text{IR}} &= \omega^3 \left[\frac{M_\Sigma}{M_5} \Sigma \bar{\Sigma} + \frac{M_H}{M_5} H^2 + \frac{\lambda_1}{M_5^{3/2}} \Phi^3 + \frac{\eta_1}{M_5^{3/2}} \Phi \Sigma \bar{\Sigma} \right. \\
 &\quad \left. + \frac{1}{M_5^{3/2}} \Phi H (\alpha_1 \Sigma + \bar{\alpha}_1 \bar{\Sigma}) \right]_{y=\pi}, \\
 W_{\text{UV}} &= \left[\frac{M_\Phi}{M_5} \Phi^2 + \frac{\lambda_2}{M_5^{3/2}} \Phi^3 + \frac{\eta_2}{M_5^{3/2}} \Phi \Sigma \bar{\Sigma} \right. \\
 &\quad \left. + \frac{1}{M_5^{3/2}} \Phi H (\alpha_2 \Sigma + \bar{\alpha}_2 \bar{\Sigma}) \right]_{y=0}.
 \end{aligned} \tag{4.1}$$

Suppose that the couplings on the UV and IR branes are of the same order.

Now we assume that the adjoint chiral multiplet of $U(1)_X$ has nonzero VEV as in Eq. (2.4) [24] and gives additional contributions to the bulk mass parameters for the bulk fields. In the following, we denote each chiral field of SU(5)-submultiplets in H_i as $H_{im} = (\Phi_m, H_m, \Sigma_m, \bar{\Sigma}_m)$,

where m specifies the dimension of the submultiplets. The zero mode solution of H_{im} is described as

$$H_{im}(x, y) = \kappa_{im} \sqrt{k} e^{kr_c |y|} e^{(C_i + \alpha Q_{im}) kr_c |y|} h_{im}(x), \tag{4.2}$$

where $\kappa_{im} \equiv \sqrt{\frac{2(C_i + \alpha Q_{im})}{e^{2(C_i + \alpha Q_{im}) kr_c \pi} - 1}}$. On the IR brane $H_{im}(x, \pi) = \kappa_{im} \sqrt{k} \omega^{-1} \omega^{-(C_i + \alpha Q_{im})} h_{im}(x)$ while $H_{im}(x, 0) = \kappa_{im} \sqrt{k} h_{im}(x)$ on the UV brane.

We take M_Σ and M_H to be $\sim M_P$ and M_Φ to be $\sim M_{\text{GUT}}$. Because of the warp factor ω , the 4D effective masses of the IR brane are warped down to $\omega M_P \simeq M_{\text{GUT}}$. Next note that

$$e^{(C_i + \alpha Q_{im}) kr_c \pi} \gg 1, \quad \kappa_{im} \simeq \sqrt{2(C_i + \alpha Q_{im})} \omega^{C_i + \alpha Q_{im}} \tag{4.3}$$

$$e^{(C_i + \alpha Q_{im}) kr_c \pi} \ll 1, \quad \kappa_{im} \simeq \sqrt{-2(C_i + \alpha Q_{im})} \tag{4.4}$$

$$e^{(C_i + \alpha Q_{im}) kr_c \pi} = 1, \quad \kappa_{im} \simeq \sqrt{-\frac{1}{\ln \omega}}. \tag{4.5}$$

The extent of the suppression of couplings and masses in effective four dimensional theory is determined by parameters C_i and α . In this paper, we choose the parameters as listed in the tables: C_{16} in Table I, C_{10} in Table II, C_{126} in Table III, $C_{\bar{126}}$ in Table IV, and C_{210} in Table V.

A. Masses of submultiplets of 126

As noted in Sec. III, one main problem for the minimal 4D SO(10) is that the SU(5)-submultiplets **15**, **50**, and **45** have the same mass M_Σ (up to the Clebsch-Gordan coefficients) [18]. When we lower the **15** Higgs mass so as to obtain type II dominance, other Higgs fields accordingly become light. As a result, gauge couplings blow up before they unite at the GUT scale. As we show now, the situation is very different in the miniwarped model.

Under the SU(5) decomposition, the mass term of the **126** pair on the IR brane can be written as

 TABLE I. $C_{16} = 1/2$ and $\alpha = -1/4$.

16 components	$C_{16} + \alpha Q_i$
1 ₋₅	7/4
10 ₋₁	3/4
5 ₃	-1/4

 TABLE II. $C_{10} = 1/2$ and $\alpha = -1/4$.

10 components	$C_{10} + \alpha Q_i$
5 ₂	0
5 ₋₂	1

TABLE III. $C_{126} = 0$ and $\alpha = -1/4$.

126 components	$C_{126} + \alpha Q_i$
$\mathbf{1}_{-10}$	5/2
$\mathbf{\bar{5}}_{-2}$	1/2
$\mathbf{10}_{-6}$	3/2
$\mathbf{\bar{15}}_6$	-3/2
$\mathbf{45}_2$	-1/2
$\mathbf{\bar{50}}_{-2}$	1/2

TABLE IV. $C_{\mathbf{126}} = 1$ and $\alpha = -1/4$.

$\mathbf{126}$ components	$C_{\mathbf{126}} + \alpha Q_i$
$\mathbf{1}_{10}$	-3/2
$\mathbf{5}_2$	1/2
$\mathbf{\bar{10}}_6$	-1/2
$\mathbf{15}_{-6}$	5/2
$\mathbf{45}_{-2}$	3/2
$\mathbf{50}_2$	1/2

$$\begin{aligned}
& \int d^2\theta \omega^3 \left[\frac{M_\Sigma}{M_5} \Sigma \bar{\Sigma} \right]_{y=\pi} \\
& \sim \int d^2\theta m_\Sigma [\epsilon_{\sigma 0} \epsilon_{\bar{\sigma} 0} \sigma_0 \bar{\sigma}_0 + \epsilon_{\sigma 15} \epsilon_{\bar{\sigma} 15} \sigma_{15} \bar{\sigma}_{15} \\
& \quad + \epsilon_{\sigma 10} \epsilon_{\bar{\sigma} 10} \sigma_{10} \bar{\sigma}_{10} + \epsilon_{\sigma 50} \epsilon_{\bar{\sigma} 50} \sigma_{50} \bar{\sigma}_{50} \\
& \quad + \epsilon_{\sigma 45} \epsilon_{\bar{\sigma} 45} \sigma_{45} \bar{\sigma}_{45} + \epsilon_{\sigma 5} \epsilon_{\bar{\sigma} 5} \sigma_5 \bar{\sigma}_5], \quad (4.6)
\end{aligned}$$

where $m_\Sigma = \omega M_\Sigma \sim M_{\text{GUT}}$, and $\epsilon_{im} \equiv \kappa_{im} \omega^{-(C_i + \alpha Q_{im})}$. From Tables III and IV, we have $\epsilon_{\sigma 15} \sim \omega^{3/2}$ and $\epsilon_{\bar{\sigma} 15} \sim 1$; therefore the mass of $\mathbf{15}$ is suppressed by the factor $\omega^{3/2}$ and $M_{\mathbf{15}} \sim \omega^{3/2} M_{\text{GUT}} \sim 10^{13}$ GeV. On the other hand, we read $\epsilon_{\sigma 50} = \epsilon_{\bar{\sigma} 50} \sim 1$, so the mass of $\mathbf{50}$ is $\sim M_{\text{GUT}}$. For $\mathbf{45}$, $\epsilon_{\sigma 45} \sim \omega^{1/2}$ and $\epsilon_{\bar{\sigma} 45} \sim 1$, and its mass is $\sim \omega^{1/2} M_{\text{GUT}} \sim$

$$\begin{aligned}
& \int d^2\theta \omega^3 \left[\frac{M_\Sigma}{M_5} \Sigma \bar{\Sigma} + \frac{\lambda_1}{M_5^{3/2}} \Phi^3 + \frac{\eta_1}{M_5^{3/2}} \Phi \Sigma \bar{\Sigma} \right]_{y=\pi} + \left[\frac{M_\Phi}{M_5} \Phi^2 + \frac{\lambda_2}{M_5^{3/2}} \Phi^3 + \frac{\eta_2}{M_5^{3/2}} \Phi \Sigma \bar{\Sigma} \right]_{y=0} \\
& \supset m_\Sigma \epsilon_{\sigma 0} \epsilon_{\bar{\sigma} 0} \sigma_0 \bar{\sigma}_0 + M_\Phi \kappa_{\phi 0}^2 \phi_0^2 + (\lambda_1 \epsilon_{\phi 0}^3 + \lambda_2 \kappa_{\phi 0}^3) \phi_0^3 + (\eta_1 \epsilon_{\phi 0} \epsilon_{\sigma 0} \epsilon_{\bar{\sigma} 0} + \eta_2 \kappa_{\phi 0} \kappa_{\sigma 0} \kappa_{\bar{\sigma} 0}) \sigma_0 \bar{\sigma}_0 \phi_0 \\
& \sim m_\Sigma \omega^{3/2} \sigma_0 \bar{\sigma}_0 + M_\Phi \phi_0^2 + (\lambda_1 \omega^6 + \lambda_2) \phi_0^3 + (\eta_1 \omega^{5/2} + \eta_2 \omega^{7/2}) \sigma_0 \bar{\sigma}_0 \phi_0. \quad (4.7)
\end{aligned}$$

F -flatness conditions for $\bar{\sigma}_0$ and ϕ_0 lead to

$$\begin{aligned}
& \sigma_0 [m_\Sigma \omega^{3/2} + (\eta_1 \omega^{5/2} + \eta_2 \omega^{7/2}) \phi_0] = 0, \\
& 2M_\Phi \phi_0 + 3(\lambda_1 \omega^6 + \lambda_2) \phi_0^2 + (\eta_1 \omega^{5/2} + \eta_2 \omega^{7/2}) \sigma_0 \bar{\sigma}_0 = 0, \quad (4.8)
\end{aligned}$$

and the solutions are

$$\begin{aligned}
& \langle \phi_0 \rangle \simeq -\frac{m_\Sigma}{\eta_1 \omega}, \\
& \langle \sigma_0 \bar{\sigma}_0 \rangle \simeq -\frac{2M_\Phi \langle \phi_0 \rangle}{\eta_1 \omega^{5/2}} \left(1 + \frac{3\lambda_2 \langle \phi_0 \rangle}{2M_\Phi} \right). \quad (4.9)
\end{aligned}$$

TABLE V. $C_{\mathbf{210}} = -2$ and $\alpha = -1/4$.

210 components	$C_{\mathbf{210}} + \alpha Q_i$
$\mathbf{1}_0$	-2
$\mathbf{5}_{-8}$	0
$\mathbf{\bar{5}}_8$	-4
$\mathbf{10}_4$	-3
$\mathbf{\bar{10}}_{-4}$	-1
$\mathbf{24}_0$	-2
$\mathbf{40}_4$	-3
$\mathbf{40}_{-4}$	-1
$\mathbf{75}_0$	-2

10^{15} GeV. In our miniwarped SO(10) model, there is no mass degeneracy between these submultiplets.

This mass splitting also leaves gauge coupling unification of MSSM unchanged, since the submultiplets are all full SU(5) multiplets. It is easy to check that the unified gauge coupling value at the GUT scale, i.e., $\alpha_{\text{GUT}} \sim 0.2$, is in the perturbative regime even though the $\mathbf{15} \oplus \mathbf{\bar{15}}$ multiplets with mass around 10^{13} GeV and the $\mathbf{45} \oplus \mathbf{\bar{45}}$ multiplets with mass around 10^{15} GeV are involved in the gauge coupling running.

B. Symmetry breaking

Here we examine the realization of the SO(10) symmetry breaking. Let us first see the SO(10) gauge symmetry breaking down to SU(5). There are three SU(5) singlets: one in $\mathbf{210}$ and one in each of the $\mathbf{126}$ pair with nonzero $B - L$ charge. Since supersymmetry must remain unbroken all the way down to the weak scale, F -flatness conditions determine vacuum expectation values. The relevant part in the superpotential in Eq. (4.1) is given by

SO(10) gauge symmetry is broken down to $\text{SU}(5) \times \text{U}(1)_X$ by $\langle \phi_0 \rangle$ at the scale $m_\Sigma / (\eta_1 \omega)$. More correctly, when we carefully consider the Clebsch-Gordan coefficients and normalization of submultiplets of SO(10) under SU(5), we have an extra factor 10 accompanying the coupling η_1 [18]. Thus, if we take, for example, $\eta_1 \sim 4\pi$, this symmetry breaking occurs around the GUT scale, $\langle \phi_0 \rangle \sim m_\Sigma / (10\eta_1 \omega) \sim M_{\text{GUT}}$. On the other hand, in order to arrange the $B - L$ breaking scale to be around the GUT scale, one needs to fine-tune the coupling λ_2 to be $\lambda_2 \sim 1 - \omega^{3/2}$.

Next we consider the SU(5) symmetry breaking by $\mathbf{24}$ VEV. The relevant superpotential is given by

$$\begin{aligned}
& \int d^2\theta \omega^3 \left[\frac{\lambda_1}{M_5^{3/2}} \Phi^3 \right]_{y=\pi} + \left[\frac{M_\Phi}{M_5} \Phi^2 + \frac{\lambda_2}{M_5^{3/2}} \Phi^3 \right]_{y=0} \\
& \supset M_\Phi \kappa_{\phi 24}^2 \phi_{24}^2 + (\lambda_1 \epsilon_{\phi 0} \epsilon_{\phi 24}^2 + \lambda_2 \kappa_{\phi 0} \kappa_{\phi 24}^2) \phi_0 \phi_{24}^2 + (\lambda_1 \epsilon_{\phi 24}^3 + \lambda_2 \kappa_{\phi 24}^3) \phi_{24}^3 \\
& \sim M_\Phi \phi_{24}^2 + (\lambda_1 \omega^6 + \lambda_2) \phi_0 \phi_{24}^2 + (\lambda_1 \omega^6 + \lambda_2) \phi_{24}^3.
\end{aligned} \tag{4.10}$$

Through the F -flatness condition for ϕ_{24} , we obtain

$$\langle \phi_{24} \rangle \sim -\frac{M_\Phi + \lambda_2 \langle \phi_0 \rangle}{\lambda_2} \sim M_{\text{GUT}}. \tag{4.11}$$

Once ϕ_0 gets the VEV, a new contribution appears to the mass of **15** through the superpotential,

$$\begin{aligned}
& \int d^2\theta \omega^3 \left[\frac{\eta_1}{M_5^{3/2}} \Phi \Sigma \bar{\Sigma} \right]_{y=\pi} + \left[\frac{\eta_2}{M_5^{3/2}} \Phi \Sigma \bar{\Sigma} \right]_{y=0} \supset [\eta_1 \epsilon_{\phi 0} \epsilon_{\sigma 15} \epsilon_{\bar{\sigma} 15} + \eta_2 \kappa_{\phi 0} \kappa_{\sigma 15} \kappa_{\bar{\sigma} 15}] \langle \phi_0 \rangle \sigma_{15} \bar{\sigma}_{15} \\
& \sim [\eta_1 \omega^{7/2} + \eta_2 \omega^{5/2}] \langle \phi_0 \rangle \sigma_{15} \bar{\sigma}_{15}.
\end{aligned} \tag{4.12}$$

Substituting the above $\langle \phi_0 \rangle$ into this formula, we find the additional contribution of order $\omega^{3/2} M_{\text{GUT}}$, that is the same order as the one from the tree level mass term in Eq. (4.6).

V. NEUTRINO MASS AND TYPE II DOMINANCE

In this section we show how type II dominance emerges in our model. Yukawa couplings on both the IR and UV branes are given by

$$\begin{aligned}
& \int d^2\theta \omega^3 \left[\frac{f_{1ab}}{M_5^{3/2}} \Psi_a \Psi_b \bar{\Sigma} + \frac{h_{1ab}}{M_5^{3/2}} \Psi_a \Psi_b H \right]_{y=\pi} \\
& + \left[\frac{f_{2ab}}{M_5^{3/2}} \Psi_a \Psi_b \bar{\Sigma} + \frac{h_{2ab}}{M_5^{3/2}} \Psi_a \Psi_b H \right]_{y=0},
\end{aligned} \tag{5.1}$$

where Ψ_a is the **16** matter field of the a th generation ($a = 1, 2, 3$).

We first consider the Yukawa coupling for $\bar{\mathbf{5}} \cdot \bar{\mathbf{5}} \cdot \mathbf{15}$, which is extracted as

$$\begin{aligned}
& [f_{1ab} \epsilon_{\psi \bar{5}}^2 \epsilon_{\bar{\sigma} 15} + f_{2ab} \kappa_{\psi \bar{5}}^2 \kappa_{\bar{\sigma} 15}] \psi_{\bar{5}} \psi_{\bar{5}} \bar{\sigma}_{15} \\
& \sim [f_{1ab} \omega^{1/2} + f_{2ab} \omega^{5/2}] \psi_{\bar{5}} \psi_{\bar{5}} \bar{\sigma}_{15}.
\end{aligned} \tag{5.2}$$

Now the effective Yukawa coupling in 4D is found to be $\sim f_{1ab} \omega^{1/2}$.

In Fig. 1, there are two vertices between Higgs fields involved in type II seesaw formulas, $\mathbf{210} \cdot \mathbf{126} \cdot \mathbf{10}$ or $\mathbf{210} \cdot \mathbf{126} \cdot \mathbf{126}$. From the superpotential in Eq. (4.1) the vertex in Fig. 1(a) can be read off as

$$[\alpha_1 \epsilon_{\phi 5} \epsilon_{h 5} \epsilon_{\sigma \bar{15}} + \alpha_2 \kappa_{\phi 5} \kappa_{h 5} \kappa_{\sigma \bar{15}}] \phi_5 h_5 \sigma_{\bar{15}}. \tag{5.3}$$

From the tables, $\epsilon_{\phi 5} \sim \epsilon_{h 5} \sim 1$, $\epsilon_{\sigma \bar{15}} \sim \omega^{3/2}$, and $\kappa_{\phi 5} \sim \kappa_{h 5} \sim \kappa_{\sigma \bar{15}} \sim 1$, so that we have the coupling $\sim \alpha_2 \phi_5 h_5 \sigma_{\bar{15}}$ unsuppressed. On the other hand, for the vertex in Fig. 1(b), we have

$$[\eta_1 \epsilon_{\phi 5} \epsilon_{\bar{\sigma} 5} \epsilon_{\sigma \bar{15}} + \eta_2 \kappa_{\phi 5} \kappa_{\bar{\sigma} 5} \kappa_{\sigma \bar{15}}] \phi_5 \bar{\sigma}_5 \sigma_{\bar{15}}. \tag{5.4}$$

This contribution is negligible compared to the previous one, since $\epsilon_{\bar{\sigma} 5} \sim 1$ and $\kappa_{\bar{\sigma} 5} \sim \omega^{1/2}$.

We are now ready to estimate the relative magnitudes of the two different seesaw contributions to neutrino mass in our model. For this purpose, we note that in terms of the original SO(10) Yukawa couplings $f_1 \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{126}$, we can rewrite the seesaw formula as

$$\mathcal{M}_\nu = 2f_1 v_L - M_D^T (2f_1 v_R)^{-1} M_D. \tag{5.5}$$

The magnitude of the neutrino mass from the type II seesaw contribution is estimated as

$$M_\nu^{II} \simeq \frac{2(f_1)_{33} \omega^{1/2} v_{10} v_{210} \alpha_2}{M_{\text{GUT}} \omega^{3/2}}, \tag{5.6}$$

where $v_{10,210}$ is the VEV of up-type Higgs doublets in **10** and **210**. If we take $(f_1)_{33} \sim 1$, $\alpha_2 \sim 0.5$, and assume $v_{10} \simeq v_{210} \sim 100$ GeV, we arrive at the reasonable value for the atmospheric neutrino oscillation data, $M_\nu^{II} \simeq 0.05$ eV. Note, however, that $b - \tau$ unification as well as charge fermion fitting implies that $(f_1)_{33} \sim 0.037$ [8]. In this case also one can get the type II term to be 0.046 eV if $\alpha_2 \sim 4\pi$ and is perturbative.

Next let us examine the type I seesaw contribution. The right-handed neutrino mass can be read as

$$[f_{1ab} \epsilon_{\psi 1}^2 \epsilon_{\bar{\sigma} 0} + f_{2ab} \kappa_{\psi 1}^2 \kappa_{\bar{\sigma} 0}] \langle \bar{\sigma}_1 \rangle \sim \omega^{3/2} f_{1ab} M_{\text{GUT}}. \tag{5.7}$$

Thus, the type I seesaw contribution is found to be

$$M_\nu^I = M_D^T M_R^{-1} M_D \simeq \frac{m_t^2 \omega^{1/2}}{2(f_1)_{33} M_{\text{GUT}} \omega^{3/2}}, \tag{5.8}$$

where m_t is the top quark mass, and we have used the natural relation $M_D \sim m_t$ in GUT models. Using $m_t \sim 100$ GeV at the GUT scale, the type I seesaw gives the contribution to the ‘‘heaviest’’ light neutrino mass as $m_3 \simeq 0.025$ eV for $(f_1)_{33} \sim 1$, which is already smaller than the

type II seesaw contribution. Again for the case of $(f_1)_{33} \sim 0.037$ obtained from charged fermion fitting in Ref. [8], even though the naive order of magnitude estimate for m_ν from type I seesaw may appear to be large, full matrix effects from M_D and M_R indeed give the desired neutrino masses. For example, if we use the explicit forms for the coupling matrices given in Ref. [8], with $(f_1)_{33} \approx 0.037$ using Eq. (5.7), we get the right order for m_3 even though naive estimates would have suggested $m_\nu \approx 0.68$ eV.

VI. CONCLUSION

In conclusion, we have shown that unlike the four dimensional minimal SUSY SO(10) models where it is not possible to achieve type II dominance of the seesaw formula, embedding into a miniwarped 5D space-time cures this problem and leads to an effective 4D theory where either type II or mixed seesaw can dominate the neutrino mass. Thus the simple understanding of the large neutrino mixings as well as the right solar mass difference square obtained in minimal SUSY SO(10) models is based on sound theoretical footing and no new Higgs fields need be

added. We have also analyzed the symmetry breaking of SO(10) down to the standard model in this framework, and we found that to maintain the SU(5) and SO(10) scales at 10^{16} GeV in this model, we need to fine-tune only one parameter by a factor of 10^{-3} . Note that in the minimal 4D SO(10) model, we could not even do any fine-tuning to get the desired feature of type II dominance. We have also checked that the SU(5) multiplets below the GUT scale not only do not affect unification as expected, but they also keep the GUT couplings $\alpha_{\text{GUT}} \sim 0.2$, meaning that one can use perturbation theory up to the GUT scale without any problem.

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