A METHOD FOR ATTITUDE CONTROL OF A SATELLITE IN CIRCULAR ORBIT BY ACTIVELY VARYING ITS INERTIA

by

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A DISSERTATION

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NOMENCLATURE

A	Cross sectional area of a liquid line.
A_{jk}	Elements of the matrix transforming the components of a vector in the orbital reference frame to components in the body reference frame.
A	$\dfrac{I_{yy} - I_{zz}}{I_{xx}}$.
a_{jk}	Elements of the control equation (3-18) relating I_{jk} and A_{jz} .
B_{jk}	Elements of the matrix transforming the components of a vector in the inertial reference frame to components in the orbital reference frame.
B	$\dfrac{I_{yy} - I_{xx}}{I_{zz}}$.
b_{jk}	Elements of the control equation (3-18) relating I_{jk} and ω_j .
C	$\dfrac{I_{xx} - I_{zz}}{I_{yy}}$.
$c\theta$	Cosine θ.
$\bar{c}$	Position vector from the origin of the body axis system to the center of mass.
cm	Center of mass.
$\bar{\bar{E}}$	Unit dyadic.
$\hat{e}$	Unit vector.

$\overline{H}$	Angular momentum about the mass center.
I	Inertia.
I_{jk}	Elements of the inertia matrix with respect to body axis.
J_{jk}	Elements of the inertia matrix with respect to orbital axes.
k	Elements of the matrix relating vehicle products of inertia and variation of liquid mass in the tanks. See equations (3-10) and (3-13).
$d\overline{l}$	Increment of length along a liquid line (Fig. 2-4).
l_i	Initial mass of liquid in tank i and thus $-l_i$ is the lower limit on the variation of liquid mass in tank i .
$\dot{l}_i$	Limit on mass flow rate out of tank i .
M	Total vehicle mass.
m_t	Mass of liquid in the tanks.
m_i	Mass of liquid in tank i .
Δm_i	Variation of liquid mass in tank i .
n	Number of tanks (lines) in the vehicle.
$\overline{p}$	A vector defined by the location of the fuel lines $\overline{p} = \int \overline{\rho} \times d\overline{l}$.
r	Distance from the center of the earth to the center of mass of the satellite vehicle.
$s\theta$	Sine θ.
s	Laplace transform variable.
$\overline{T}$	External torque about the center of mass. For equation (2-27) and subsequent $\overline{T}$ indicates all external torques acting on the vehicle except gravity gradient torques.

NOMENCLATURE (Continued)

$\overline{T}_G$	Gravity gradient torques.
t	Time.
u_i	The upper limit on variation of liquid mass in tank i ; hence, the capacity of the tank less the initial mass.
$\dot{u}_i$	Limit on mass flow rate into tank i .
x, y, z	A right-hand orthogonal axis system with an associated subscript which indicates the axis to be: I (inertial), O (orbital), or B (body-fixed).

Greek Letters

γ	Density of the fluid.
θ	First Euler rotation (about y axis).
μ	Gravitational constant per unit mass.
$\bar{\rho}$	Position of a mass element relative to the origin of the vehicle axes.
$\bar{\rho}_i$	Position of the center of mass of the fluid in tank i .
ϕ	Third Euler rotation (about x axis).
ψ	Second Euler rotation (about z axis).
$\overline{\Omega}$	Orbital angular velocity.
$\bar{\omega}$	Vehicle angular velocity.

Subscripts

B	Body.
c	Center of mass.
C	Commanded value.

NOMENCLATURE (Continued)

Subscripts (Con't.)

D	Desired value.
G	Gravity.
i	Summation index over number of tanks.
I	Inertial
j , k	Summation index over xyz .
L	Limiting value.
O	Orbital.
R	Reference value.
t	Tank.
V	Vehicle.
xyz	xyz axes.
0	Initial condition.

Superscripts

—	Denotes a vector.
=	Denotes a dyadic.
^	Denotes a unit vector.
-1	Denotes matrix inverse.
·	Denotes first derivative with respect to time. For vectors and dyadics, the notation denotes differentiation relative to the body reference frame in the absence of a qualifying notation.
· ·	Denotes second derivative with respect to time.

Symbols and Operators

[ ]	Encloses 3×3 matrices unless the equation is explicitly identified as a scalar equation.
{ }	Encloses a 3×1 matrix unless the equation is explicitly identified as a scalar equation.
$(\dot{\ })_I$	Denotes differentiation of a vector or dyadic in the reference frame indicated by the subscript.
[~]	Denotes a 3×3 skew-symmetric matrix formed from a 3×1 column matrix.

$$[\tilde{a}] = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \quad \text{where} \quad \{a\} = \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix}$$

Δ	Difference operator.
δ	Variational operator.
d	Differential operator.
$\sum$	Summation operator.
| |	Denotes the absolute value of the enclosed quantity.

CHAPTER I

INTRODUCTION

Background

Methods for controlling the attitude of satellite vehicles are based on one of the two fundamental concepts, application of torque to the vehicle or an exchange of angular momentum among vehicle components. The most common method of applying external torque to a satellite is by means of systems which interact with the earth's magnetic or gravitational fields. The most common momentum exchange devices are mass expulsion systems and spinning wheels.

Certain inherent limitations are associated with each of the two basic concepts. One common characteristic of systems that employ momentum exchange alone is a limited operating time. Mass expulsion systems become depleted and are unsuitable on some missions because of contamination to the surrounding environment and to exposed surfaces of scientific equipment. The momentum that can be stored and extracted from a spinning wheel is also limited. Although systems with momentum wheels are effective in controlling internal disturbances and external disturbances which are periodic in nature, any secular components of torque result in eventual saturation of the system unless some auxiliary method of desaturation is provided.

Under proper conditions, systems employing external torques may have a long operating lifetime, but these systems have in common a very low torque available for control. However, since the external disturbance torques encountered by the satellite are also low, barring such conditions as docking with another vehicle, it is frequently possible to devise a system that can generate sufficient torque for control from one of the earth's potential fields. This is particularly true when the desired attitude of the vehicle is fixed with respect to the earth.

Prior to the development of man-made satellites, the fact that a satellite could attain a stable orientation with respect to the earth was well known and clearly understood from studies of the attitude motion of the moon. These classical studies, which dated from the 17th century, were concerned primarily with expressing analytically the angular motions of the moon within the earth's gravitational field and determining the associated astronomical constants from observing these motions. Pringle and DeBra give brief historical summaries of these studies in their doctoral dissertations [1, 2].

With the development of artificial satellites, both the physical characteristics of the satellites and their angular motion created a basis for new emphasis and broader objectives to the study of this problem. In the late 1950's, Roberson published a number of significant papers outlining the problems of attitude control and analyzing the motion of a satellite in the gravitational field, including the effects of motions caused by moving parts [3, 4, 5]. DeBra and Delp parameterized the stability and natural frequency of a rigid body in circular orbit in a symmetrical force field [6]. The practical considerations of vehicle flexibility and multiple connected bodies were analyzed by a number of authors, who examined the advantages of different approaches and formulations of the problem. Some authors, Fletcher et al. [7] and Hooker [8] for example, preferred to formulate the problem directly from the equations of Newton and Euler, while others such as Pringle [1] and Nelson and Meirovitch [9] preferred the Lagrangian formulation. An excellent treatise on various formulations of vehicle equations for flexible space vehicles from the viewpoint of the control engineer was recently prepared by Likins [10].

The major engineering problems associated with designing a satellite to assume a fixed attitude with respect to an earth-oriented axis system utilizing gravitational forces may be categorized as follows:

1. Devising methods to damp any librations resulting from injection errors or subsequent disturbances.

2. Minimizing the perturbations resulting from the asymmetries associated with noncircular orbits, gravitational anomalies, and magnetic fields.

3. Devising methods for capturing the satellite from large angular motions (tumbling) to a constrained attitude about certain equilibrium positions.

DeBra, Pringle, and Bainum have each presented comprehensive analyses of the stability and response of passive satellites, treating the aspects of the problems that were mentioned previously [1, 2, 11]. Bainum also traces the development of energy dissipative connections as a means of damping, giving both important contributors to the analytical development and significant satellite configurations. The major attraction in constructing a satellite inherently stable in attitude is the associated simplicity and reliability; however, for many applications, such satellites are also characterized by poor accuracy and response. Thermal warping of the long extensible booms, for example, has created stability as well as accuracy problems [12, 13, 14].

Recently attempts have been made to supplement the inherent stability of such configurations by actively varying the vehicle's inertia characteristics. Many of these studies were associated with the deployment of the extensible booms and were directed toward the capture of a tumbling satellite by a properly controlled or programmed deployment of the booms [11, 15, 16, 17]. A few authors have extended this concept to the consideration of actively controlling the vehicle's inertia throughout its operating lifetime as a part of the system of attitude control. In 1968, Gatlin proposed the use of a gimbaled boom to supplement control gyros as a means of attitude control and to prevent gyro saturation [18]. Hooker, in a paper published the following year, considers three configurations: a gimbaled boom with three gyro wheels, a gimbaled boom with one gyro wheel, and four gimbaled booms with no wheels [19]. A recent paper expands the rigid rod model used by Gatlin and Hooker to include the effects of rod flexibility [20]. Doane addresses the problem of optimally varying the elements of the inertia matrix as a means of control, but does not define a physical configuration [21].

The concept of passive gravity gradient stabilization has been applied extensively over the past several years to small earth satellites. By the end of 1968, 39 known spacecraft had been launched which utilized this principle [22]. The addition of gravity gradient booms has also been considered as a method for stabilizing large manned space stations [23, 24]. As research continues, it is becoming apparent that the most promising direction for developing advanced control systems appears to be the utilization of one of the earth's potential fields to torque the vehicle by means of an active system on board the spacecraft. This dissertation proposes the approach of actively varying the vehicle's inertia to control its attitude and develops a method for implementing this concept for a particular configuration. The method of implementation is consistent with the configuration and mission requirements, because the inertia variation is accomplished by transferring liquid propellants among tanks which exist primarily to satisfy mission requirements rather than to provide a means for control. Active inertia management utilizes inertia variations to control the magnitude and direction of the gravitational torque on the vehicle and to adjust the vehicle components of angular momentum. In addition, the capability exists for minimizing or utilizing aerodynamic torques by shifting the mass center of the vehicle. The proposed method of control employs simple onboard sensors and requires no elaborate computation devices.

Preliminary Concepts

Before analyzing in detail the problem of control by active inertia management, let us explore briefly some underlying concepts and physical relationships that inspire such an approach. Consider a satellite in circular orbit about a symmetrical earth and assume one axis of the satellite is required to point toward the earth's center. Assume the satellite is placed approximately in its proper initial conditions by the launch vehicle, giving a condition such as that shown in Figure 1-1, where $\bar{H}$ is the angular momentum of the vehicle, $\bar{\omega}$ is the total angular velocity of the vehicle, and $\bar{\Omega}$ is the orbital angular velocity of the vehicle. The equivalent matrix relationship between $\bar{H}$ and $\bar{\omega}$ is given by

$$\{H\} = [I]\{\omega\} \tag{1-1}$$

where $[I]$ is the inertia matrix of the vehicle. For the vehicle to be in a fixed position with respect to the earth, it is required that $\bar{\omega} = \bar{\Omega}$. For the purpose of illustration,assume that external moments on the vehicle are zero; then $\bar{H}$, in addition to $\bar{\Omega}$, is fixed in space. This suggests that some inertia exists such that

$$\{H\} = [I]\{\Omega\} \quad . \tag{1-2}$$

Note that the elements or certain critical elements of $[I]$ will be invariable with respect to an axis system fixed in space, but generally not invariable with respect to axes which rotate with the vehicle. If, however, $\bar{H}$ and $\bar{\Omega}$ are colinear, then a condition may be found where $[I]$ is invariable with respect

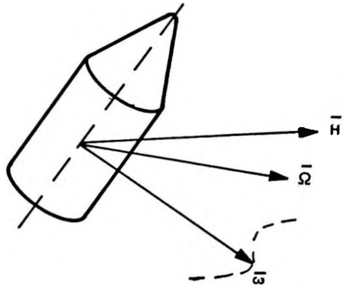


Figure 1-1. Condition of Satellite at Orbit Injection.

to a set of axes rotating with the vehicle, and equation (1-2) is also satisfied. This suggests that if $\bar{H}$ and $\bar{\Omega}$ are not initially colinear, then $\bar{H}$ should be aligned with $\bar{\Omega}$ so that the vehicle could remain in an earth-pointing attitude without continually varying the inertia with respect to the vehicle.

The gravitational field of the earth exerts a torque on the vehicle about the x axis approximately proportional to the product of inertia I_{yz} and a torque about the y axis approximately proportional to I_{xz} ; therefore, by proper selection of these two products of inertia, a torque may be exerted about any axis lying in the horizontal plane and during the course of one orbit about any direction in space (Fig. 1-2). Thus a means exists to alter the magnitude and direction of the momentum vector by varying the vehicle inertia and to bring it to a desired value. The inertia can also be varied to counteract angular accelerations resulting from external disturbances, such as aerodynamic torques, or internal disturbances from inertia variations not a part of the controlled inertia.

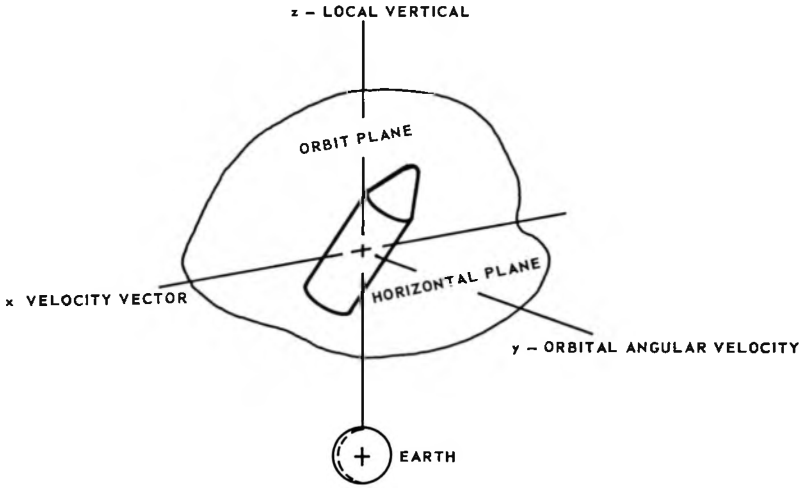


Figure 1-2. Orbital References.

It is not obvious that we can simultaneously satisfy equation (1-2) and select elements of the inertia matrix which cause $\overline{H}$ to approach some desired value. Also, the simple relationships presented thus far are not sufficient to assure that the concept is valid or realizable. The object of the succeeding chapters is to develop the concept in an orderly fashion and to demonstrate its practical application.

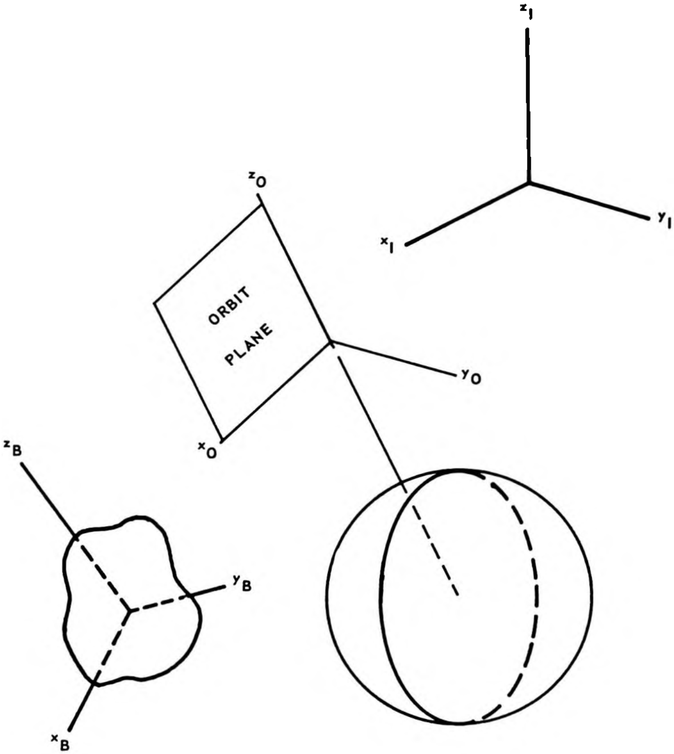
CHAPTER II

VEHICLE EQUATIONS

Kinematics

This chapter defines and develops the axis systems, kinematical relationships, vehicle equations of motion, and expressions for the gravity gradient torques acting on the vehicle. From a generalized formulation a specialized set of vehicle equations are developed which define angular motion for a configuration in which fluids are pumped among several tanks within the vehicle. Finally a set of linearized perturbation equations are developed for linear analysis of the inertia management of a space station. Various formulations for what may be called variable inertia satellites have been developed from early papers by Roberson to more recent papers by Hooker and Margulies and an excellent report by Likins [5, 25, 10]. However, it is desirable to develop the equations from elementary principles to present the consolidated set of equations to be used in this analysis in an orderly fashion.

Three right-hand orthogonal axis systems shown in Figure 2-1 will be used in the following analyses. The inertial axes $x_I y_I z_I$ are fixed in space. The orbital axes $x_O y_O z_O$ are initially parallel to the inertial axes and for a circular orbit rotate about the y axis at a constant angular



x_I y_I z_I – INERTIAL AXES

x_O y_O z_O – ORBITAL AXES

x_B y_B z_B – BODY AXES

Figure 2-1. Axis Systems.

velocity $\bar{\Omega}$. The x_O and z_O axes lie in the orbit plane with the x_O axis in the direction of the orbital velocity and the z_O axis in the direction of an outward normal to the planet surface. The body axes $x_B y_B z_B$ are initially coincident with the orbital axes, move with the body, and rotate at an angular velocity $\bar{\omega}$ with respect to the inertial axes. Note that for a practical problem the body consists of a set of incremental masses which remain fixed relative to the body axes plus a set of incremental masses which move relative to the body axes.

In developing the equations of motion, it is necessary to represent the angular position of the three axis systems with respect to one another. This can be done in a number of ways but the choice is usually dictated by the problem to be solved. The two best known representations are Euler angles and direction cosines. Aside from their familiarity, these appear best suited for the following analyses, because the angular positions to be considered are small enough to avoid the ambiguities and singularities sometimes associated with these coordinates. A single rotation about the y axis is sufficient to relate the orientation of the orbital axes with the inertial axes. A yzx Euler angle set, with corresponding angles θ, ψ, ϕ, is chosen to represent the position of the body axes with respect to the orbital axes. These angular relationships are shown in Figure 2-2.

The transformations and functional relationships between the Euler angles, the direction cosines, and the angular velocities of the three

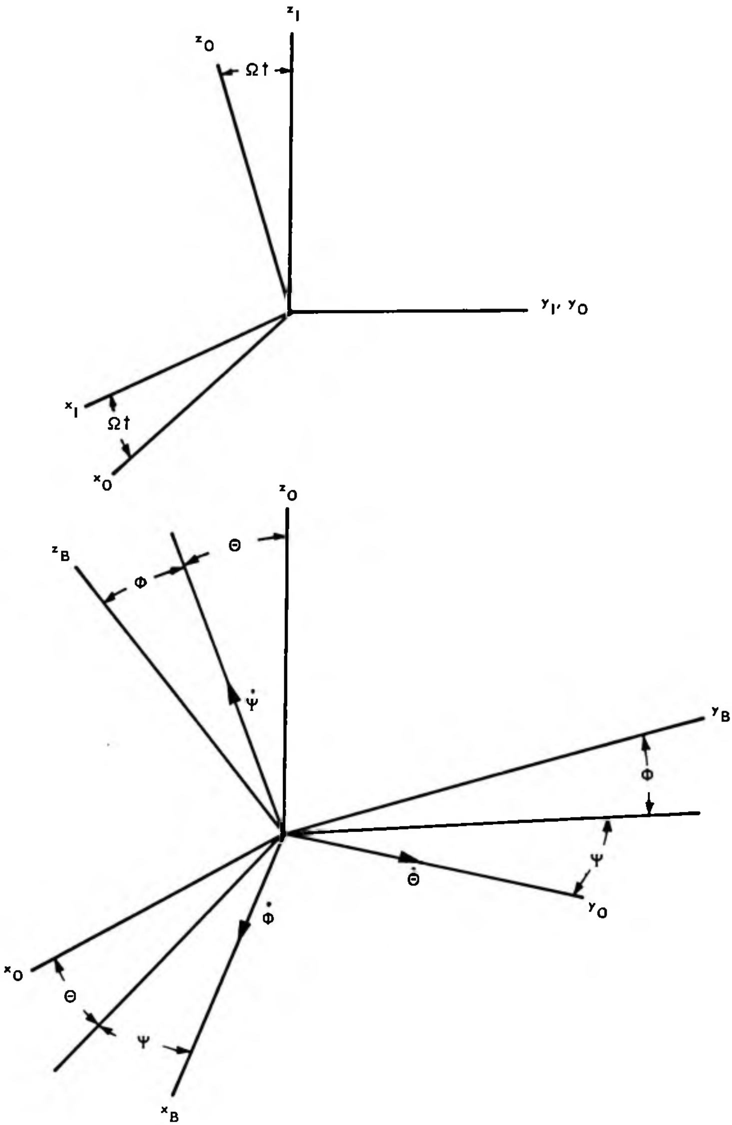


Figure 2-2. Angular Relationships.

coordinate systems are presented. Since the methods for developing these relationships are well known, only the formulas are listed [26].

$$\left\{ e_{Bj} \right\} = \left[A_{jk} \right] \left\{ e_{Ok} \right\} \tag{2-1}$$

$$\left[A_{jk} \right] = \begin{bmatrix} c\psi c\theta & s\psi & -c\psi s\theta \\ -c\phi s\psi c\theta + s\phi s\theta & c\phi c\psi & c\phi s\psi s\theta + s\phi c\theta \\ s\phi s\psi c\theta + c\phi s\theta & -s\phi c\psi & -s\phi s\psi s\theta + c\phi c\theta \end{bmatrix} \tag{2-2}$$

where $j = x, y, z$; $k = x, y, z$; $c\psi \equiv \cos\psi$, $s\psi \equiv \sin\psi$, etc.

$$\left\{ e_{Oj} \right\} = \left[B_{jk} \right] \left\{ e_{Ik} \right\} \tag{2-3}$$

$$\left[B_{jk} \right] = \begin{bmatrix} c\Omega t & 0 & -s\Omega t \\ 0 & 1 & 0 \\ s\Omega t & 0 & c\Omega t \end{bmatrix} \tag{2-4}$$

$$\left\{ \omega_j \right\} = \begin{bmatrix} 1 & s\psi & 0 \\ 0 & c\psi c\phi & s\phi \\ 0 & -s\phi c\psi & c\phi \end{bmatrix} \begin{Bmatrix} \dot{\phi} \\ \dot{\theta} + \Omega \\ \dot{\psi} \end{Bmatrix} \tag{2-5}$$

$$\begin{Bmatrix} \dot{\phi} \\ \dot{\theta} + \Omega \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} 1 & -c\phi s\psi/c\psi & s\phi s\psi/c\psi \\ 0 & c\phi/c\psi & -s\phi/c\psi \\ 0 & s\phi & c\phi \end{bmatrix} \left\{ \omega_j \right\} \tag{2-6}$$

The vehicle equation to be developed in the next section expresses angular information as a function of both $\bar{\omega}$ and the position of the unit outward normal to the planet surface, $\hat{e}_{Oz}$, relative to the body axis system. A functional relationship between $\hat{e}_{Oz}$ and $\bar{\omega}$ will then be required to solve the equation. Because $\hat{e}_{Oz}$ rotates relative to the inertial axes at an angular velocity $\bar{\Omega}$, and the body axes rotate relative to the inertial axes at an angular rate $\bar{\omega}$, then $\hat{e}_{Oz}$ rotates relative to the body axes at the angular velocity $(\bar{\Omega} - \bar{\omega})$ and

$$\left(\dot{\hat{e}}_{Oz}\right)_B = -(\bar{\omega} - \bar{\Omega}) \times \hat{e}_{Oz} = \hat{e}_{Oz} \times (\bar{\omega} - \bar{\Omega}) \tag{2-7}$$

or

$$\left(\dot{\hat{e}}_{Oz}\right)_B = \hat{e}_{Oz} \times \bar{\omega} + \Omega\, \hat{e}_{Ox} \quad . \tag{2-8}$$

The notation $\left(\dot{\hat{e}}_{Oz}\right)_B$ denotes time differentiation of the vector $\hat{e}_{Oz}$ relative to the body axis system. From equation (2-1), since $\hat{e}_{Oz}$ is a unit vector, its components in the body axes are simply the last column of the transformation matrix $\{A_{jz}\}$, $j = x, y, z$.

Derivation of Equations

Consider a body composed of a system of particles free to move in relation to a set of vehicle axes, x_B, y_B, z_B, moving with the body and rotating at some angular velocity $\bar{\omega}$ with respect to the inertial axes, $x_I y_I z_I$. In application the vehicle axes will usually be considered fixed in some portion of the body which is considered rigid, but the assumption is not required at this time. As shown in Figure (2-3), $\bar{\rho}$ is defined as the position of some mass element dm relative to the origin of the vehicle axes, $\bar{c}$ as the position of the center of mass relative to the origin of the vehicle axes, and $\bar{\rho}_c$ as the position of the mass element relative to the mass center. The angular momentum of the system of particles about its mass center is then defined as

$$\bar{H} = \int \bar{\rho}_c \times \left(\dot{\bar{\rho}}_c\right)_I \quad dm \tag{2-9}$$

or

$$H = \int (\bar{\rho} - \bar{c}) \times [\,(\dot{\bar{\rho}})_I - (\dot{\bar{c}})_I]\, dm \tag{2-10}$$

where $(\dot{\bar{\rho}})_I$ indicates the derivative of $\bar{\rho}$ relative to the inertial axis system.

To obtain a workable set of equations, we will express derivatives relative to the vehicle axes by employing the fundamental relationship

$$(\dot{\bar{\rho}})_I = (\dot{\bar{\rho}})_B + \bar{\omega} \times \bar{\rho} \quad , \tag{2-11}$$

and at this point the time derivative of a vector with respect to the body axis system will be denoted simply by a dot over the symbol; that is, $(\dot{\bar{\rho}})_B = \dot{\bar{\rho}}$. Equations (2-10) and (2-11) yield

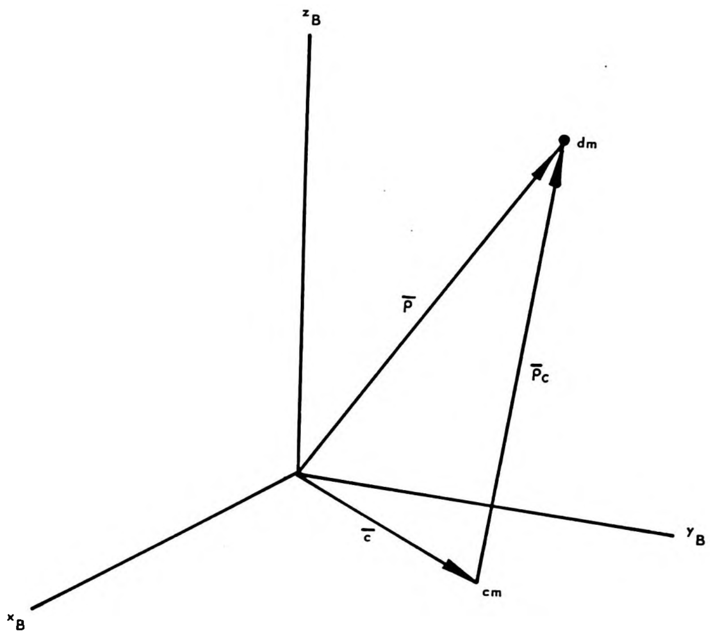


Figure 2-3. Some Vector Definitions.

$$\bar{H} = \int \bar{\rho} \times \dot{\bar{\rho}} \, dm + \int \bar{\rho} \times (\bar{\omega} \times \bar{\rho}) \, dm - \int \bar{\rho} \times \dot{\bar{c}} \, dm$$

$$- \int \bar{\rho} \times (\bar{\omega} \times \bar{c}) \, dm - \int \bar{c} \times \dot{\bar{\rho}} \, dm - \int \bar{c} \times (\bar{\omega} \times \bar{\rho}) \, dm$$

$$+ \int \bar{c} \times \dot{\bar{c}} \, dm + \int \bar{c} \times (\bar{\omega} \times \bar{c}) \, dm \quad . \qquad (2\text{-}12)$$

Some of the above terms can be simplified or eliminated by applying the relationships which define the mass center. These are

$$\int \bar{\rho}_c \, dm = \int (\bar{\rho} - \bar{c}) \, dm = 0 \quad , \qquad (2\text{-}13)$$

and

$$\int \bar{\rho} \, dm = \int \bar{c} \, dm = M\bar{c} \qquad (2\text{-}14)$$

where M is the total vehicle mass. Similarly

$$\int (\dot{\bar{\rho}} - \dot{\bar{c}}) \, dm = 0 \quad . \qquad (2\text{-}15)$$

Equations (2-12), (2-13), (2-14), and (2-15) yield

$$\bar{H} = \int \bar{\rho} \times \dot{\bar{\rho}} \, dm + \int \bar{\rho} \times (\bar{\omega} \times \bar{\rho}) \, dm$$

$$- M\bar{c} \times \dot{\bar{c}} - M\bar{c} \times (\bar{\omega} \times \bar{c}) \quad . \qquad (2\text{-}16)$$

At this point some simplification to the equations can be made by introducing the inertia dyadic

$$\bar{\bar{I}} = \int (\bar{\rho} \cdot \bar{\rho}\, \bar{\bar{E}} - \bar{\rho}\, \bar{\rho})\, dm \qquad , \tag{2-17}$$

where $\bar{\bar{E}}$ is the unit dyadic. Recognizing that

$$\begin{aligned} \bar{\bar{I}} \cdot \bar{\omega} &= \int (\bar{\rho} \cdot \bar{\rho}\, \bar{\bar{E}} - \bar{\rho}\, \bar{\rho})\, dm \cdot \bar{\omega} \\ &= \int (\bar{\rho} \cdot \bar{\rho}\, \bar{\omega} - \bar{\rho}\, \bar{\rho} \cdot \bar{\omega})\, dm \end{aligned} \tag{2-18}$$

and employing the vector identity

$$\bar{\rho} \times (\bar{\omega} \times \bar{\rho}) = \bar{\rho} \cdot \bar{\rho}\, \bar{\omega} - \bar{\rho} \cdot \bar{\omega}\, \bar{\rho} \qquad , \tag{2-19}$$

we can establish the following relationship:

$$\int \bar{\rho} \times (\bar{\omega} \times \bar{\rho})\, dm = \bar{\bar{I}} \cdot \bar{\omega} \qquad . \tag{2-20}$$

It should be noted that $\bar{\bar{I}}$ is defined for the origin of the vehicle coordinate system and not the center of mass. The final form of the equation for angular momentum is

$$\bar{H} = \int \bar{\rho} \times \dot{\bar{\rho}}\, dm + \bar{\bar{I}} \cdot \bar{\omega} - M\, \bar{c} \times (\dot{\bar{c}} + \bar{\omega} \times \bar{c}) \qquad . \tag{2-21}$$

To consider external torques on the vehicle, the familiar relationship

$$\bar{T} = (\dot{\bar{H}})_I \tag{2-22}$$

is introduced where $\bar{T}$ is the sum about the mass center of the external torques applied to the body and $(\dot{\bar{H}})_I$ is the rate of change of the angular momentum with respect to the inertial reference.

Equation (2-21), to be introduced into equation (2-22), must first be differentiated. The relationship

$$(\dot{\bar{\bar{I}}})_I = \dot{\bar{\bar{I}}} + \bar{\omega} \times \bar{\bar{I}} - \bar{\bar{I}} \times \bar{\omega} \tag{2-23}$$

is used with equation (2-11) to yield

$$(\dot{\bar{H}})_I = \bar{\bar{I}} \cdot \dot{\bar{\omega}} + \dot{\bar{\bar{I}}} \cdot \bar{\omega} + (\bar{\omega} \times \bar{\bar{I}}) \cdot \bar{\omega} + \int \bar{\rho} \times \ddot{\bar{\rho}} \, dm + \int \bar{\omega} \times (\bar{\rho} \times \dot{\bar{\rho}}) \, dm$$

$$- M \bar{c} \times \left[\ddot{\bar{c}} + 2(\bar{\omega} \times \dot{\bar{c}}) + \bar{\omega} \times (\bar{\omega} \times \bar{c}) + (\dot{\bar{\omega}} \times \bar{c}) \right] \quad . \tag{2-24}$$

In the development to follow, the torques on the body resulting from gravitational forces will be incorporated into the equations and the remaining torques will not be defined explicitly. This approach is taken because the gravitational torques can be described generally from the inertia characteristics of the body and its motion within the gravitational field,but the remaining torques must be associated specifically with the satellite external shape, magnetic properties, and the like. In addition, the proposed method for controlling the vehicle by varying the inertia utilizes the gravitational torques. The remaining torques which are not functionally related to changes in inertia are treated as disturbances.

The derivation of the gravity gradient torques acting on a body is well documented in the literature and will not be derived here [7, 27]. For a satellite in orbit about a spherical planet,the gravity gradient torque is

$$\bar{T}_G = \frac{3\mu}{r^3}\, \hat{e}_{Oz} \times \bar{\bar{I}}_c \cdot \hat{e}_{Oz} \quad , \tag{2-25}$$

and for a circular orbit it is

$$\bar{T}_G = 3\Omega^2\, \hat{e}_{Oz} \times \bar{\bar{I}}_c \cdot \hat{e}_{Oz} \tag{2-26}$$

where $\bar{T}_G$ is the gravity gradient torque about the center of mass, $\bar{\bar{I}}_c$ is the inertia dyadic about the center of mass, μ is the gravitational constant per unit mass, Ω is the orbital angular rate, $\hat{e}_{Oz}$ is a unit vector in the direction of the local vertical, and r is the distance from the center of the planet to the center of the satellite.

Equations (2-22), (2-24), and (2-26) or (2-25) may now be combined into one vector equation:

$$\bar{\bar{I}} \cdot \dot{\bar{\omega}} + \dot{\bar{\bar{I}}} \cdot \bar{\omega} + (\bar{\omega} \times \bar{\bar{I}}) \cdot \bar{\omega} + \int \bar{\rho} \times \ddot{\bar{\rho}}\, dm + \int \bar{\omega} \times (\bar{\rho} \times \dot{\bar{\rho}})\, dm$$

$$- M\bar{c} \times \left[\ddot{\bar{c}} + 2(\bar{\omega} \times \dot{\bar{c}}) + \bar{\omega} \times (\bar{\omega} \times \bar{c}) + (\dot{\bar{\omega}} \times \bar{c})\right]$$

$$= 3\Omega^2\, \hat{e}_{Oz} \times \bar{\bar{I}}_c \cdot \hat{e}_{Oz} + \bar{T} \tag{2-27}$$

where $\bar{T}$ is redefined to be all external torques acting on the satellite exclusive of the gravity gradient torques.

Specialized Vehicle Equations

Equation (2-27) is now specialized for a vehicle in which the inertia is varied by pumping fluid among several tanks. Tanks, pump, and lines are in some prescribed but arbitrary location with respect to the vehicle axes. The fluid in the tanks is assumed to be constrained in translation but unconstrained in rotation, such that it acts as a finite mass concentrated at one point in the vehicle. Thus the mass of fluid in each tank may be increasing or decreasing, but has no motion relative to the body axis system. Lines from the several tanks lead to a common pump and switching manifold so each line in the vehicle is associated with a particular tank (Fig. 2-4).

The solution of equation (2-27) can be greatly simplified by reformulating the two integrals to be a function only of the geometry of the lines. In this form the integrals need evaluating only one time for each vehicle configuration. Consider an element of fluid dm flowing in a line. This element may be written as

$$dm = \gamma A \, dl \tag{2-28}$$

where dl is an increment of length along the line, A is the cross sectional area of the line, and γ is the density of the fluid. From the continuity equation of fluid mechanics, the mass rate of flow in the line is given by

$$\dot{m} = \gamma \, \dot{\bar{\rho}} \cdot \bar{A} \tag{2-29}$$

or simply

$$\dot{m} = \gamma \dot{\rho} A \tag{2-30}$$

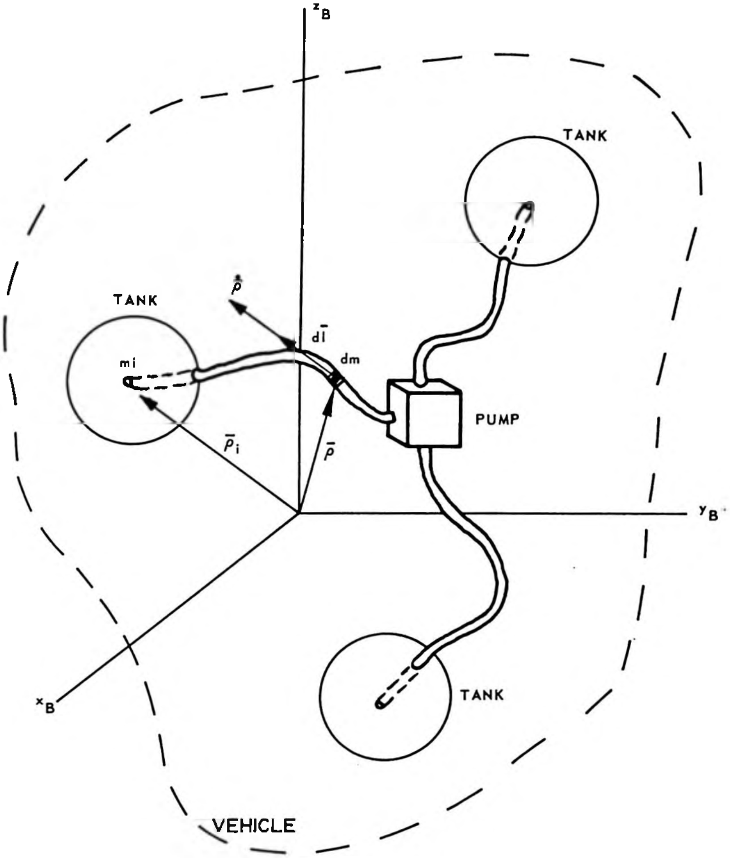


Figure 2-4. Schematic Representation of Vehicle With Liquid Transfer System.

where the flow is considered perpendicular to the cross section of the line. Using the sign convention that flow into a tank is positive, and observing that the flow velocity $\dot{\bar{\rho}}$ and an element of length along the line $d\bar{l}$ are parallel at every point in the line, we may write the expression $\dot{\bar{\rho}}\,dm$ as

$$\dot{\bar{\rho}}\,dm = \dot{\bar{\rho}}\gamma A\,dl = \dot{\rho}\gamma A\,d\bar{l} = \dot{m}\,d\bar{l} \qquad . \qquad (2\text{-}31)$$

Similarly

$$\ddot{\bar{\rho}}\,dm = \ddot{m}\,d\bar{l} \qquad , \qquad (2\text{-}32)$$

and the two integrals of equation (2-27) may be written as

$$\int \bar{\rho} \times \ddot{\bar{\rho}}\,dm = \ddot{m} \int \bar{\rho} \times d\bar{l} = \sum_{i=1}^{n} \ddot{m}_i \bar{p}_i \qquad (2\text{-}33)$$

and

$$\int \bar{\omega} \times (\bar{\rho} \times \dot{\bar{\rho}})\,dm = \bar{\omega} \times \dot{m} \int \bar{\rho} \times d\bar{l} = \bar{\omega} \times \sum_{i=1}^{n} \dot{m}_i \bar{p}_i \qquad . \qquad (2\text{-}34)$$

The vector $\bar{p}_i$ is a constant with respect to the body axes and can be evaluated for each line between the pump and the several tanks, n is the number of tanks (or lines), and $\dot{m}_i$ and $\ddot{m}_i$ are the mass flow rate and acceleration of the fluid in line i . Since the lines remain filled with liquid at all times, the flowing of liquid in the lines affects only the two integral terms just evaluated.

The position of the center of mass $\bar{c}$ may be determined directly from equation (2-14). A more detailed expression for $\bar{c}$ can be developed by recognizing that

$$m_t \bar{c}_t = \sum_{i=1}^{n} m_i \bar{\rho}_i \tag{2-35}$$

where m_t is the total mass of fluid in the tanks, $\bar{c}_t$ is the position of the center of mass of fluid in the tanks, and $\bar{\rho}_i$ is the center of mass of the liquid in tank i. The center of mass may now be expressed as

$$\bar{c} = \frac{M - m_t}{M} \bar{c}_V + \frac{1}{M} \sum_{i=1}^{n} m_i \bar{\rho}_i \tag{2-36}$$

where M is total vehicle mass and $\bar{c}_V$ is the position of the center of mass of the total vehicle less the mass of the fluid in the tanks. Notice that $\bar{c}_V$ includes the mass of the fluid in the lines. Because m_t is constant, $\bar{\rho}_i$ is constant with respect to the body axes, and

$$\sum_{i=1}^{n} \dot{m}_i = 0 \quad . \tag{2-37}$$

Then

$$\dot{\bar{c}} = \frac{1}{M} \sum_{i=1}^{n} \dot{m}_i \bar{\rho}_i \quad , \tag{2-38}$$

and

$$\ddot{\bar{c}} = \frac{1}{M} \sum_{i=1}^{n} \ddot{m}_i \bar{\rho}_i \quad . \tag{2-39}$$

The term $\dot{\bar{\bar{I}}}$ is only a function of the fluid in the tanks. If the inertia of the fluid in the tanks is given by

$$\bar{\bar{I}}_t = \sum_{i=1}^{n} m_i (\rho_i^2 \bar{\bar{E}} - \bar{\rho}_i \bar{\rho}_i) \quad , \tag{2-40}$$

then

$$\dot{\bar{\bar{I}}} = \dot{\bar{\bar{I}}}_t = \sum_{i=1}^{n} \dot{m}_i (\rho_i^2 \bar{\bar{E}} - \bar{\rho}_i \bar{\rho}_i) \quad . \tag{2-41}$$

It is frequently more convenient to work with equations in matrix form. The basic vehicle equation and the supporting relationships can be written by inspection in matrix form as

$$[I]\{\dot{\omega}\} + [\dot{I}]\{\omega\} + [\tilde{\omega}][I]\{\omega\} + \sum \ddot{m}_i \{p_i\} + [\tilde{\omega}] \sum \dot{m}_i \{p_i\}$$

$$- M [\tilde{c}]\{\ddot{c}\} - 2M [\tilde{c}][\tilde{\omega}]\{\dot{c}\} - M [\tilde{c}][\tilde{\dot{\omega}}]\{c\} - M [\tilde{c}][\tilde{\omega}][\tilde{\omega}]\{c\}$$

$$= 3\Omega^2 [\tilde{e}_{Oz}][I_c]\{e_{Oz}\} + \{T\} \tag{2-42}$$

$$[I] = [I_V] - \sum m_i [\tilde{\rho}_i][\tilde{\rho}_i] \tag{2-43}$$

$$[\dot{I}] = \sum \dot{m}_i [\tilde{\rho}_i][\tilde{\rho}_i] \tag{2-44}$$

$$\{c\} = \frac{M - m_t}{M}\{c_V\} + \frac{1}{M} \sum m_i \{\rho_i\} \tag{2-45}$$

$$\{\dot{c}\} = \frac{1}{M} \sum \dot{m}_i \{\rho_i\} \tag{2-46}$$

$$\{\ddot{c}\} = \frac{1}{M} \sum \ddot{m}_i \{\rho_i\} \tag{2-47}$$

$$[I_c] = [I] + M[\tilde{c}][\tilde{c}] \tag{2-48}$$

where $[I_V]$ represents the inertia of the total vehicle relative to the body axes less the inertia of the liquid contained in the tanks. Matrix notation is defined in the nomenclature.

Perturbation Equations

In this section a set of linearized perturbation equations are developed for the vehicle to be analyzed in the next chapters. This simplification is introduced to make the control aspects of the problem more visible, while retaining the essential characteristics of the vehicle. The following assumptions and constraints are applicable to these equations. A more complete description of the vehicle will be presented in the following chapters.

1. Liquids are transferred in such a manner that the vehicle center of mass remains in a fixed position within the vehicle (the body axis origin is selected to be the center of mass).

2. The liquid lines interconnecting the tanks are arranged so that the flow of liquids in the lines produces a negligible effect on vehicle motion. For example, if the pump and switching manifold were located at the center of mass and radial lines connected tanks and pump, then $\bar{\rho} \times \ddot{\bar{\rho}} = \bar{\rho} \times \dot{\bar{\rho}} = 0$.

3. The related control equation is written in terms of the vehicle inertia; thus, it is not necessary to introduce mass explicitly.

From these constraints and assumptions, equation (2-42) may be rewritten, introducing $\{A_{jz}\}$ for $\{e_{Oz}\}$, as follows:

$$[I]\{\dot{\omega}\} + [\dot{I}]\{\omega\} + [\tilde{\omega}][I]\{\omega\} = 3\Omega^2 [\tilde{A}_{jz}][I]\{A_{jz}\} + \{T\} \quad . \quad (2\text{-}49)$$

The necessary kinematical relationship (equation 2-7) is rewritten in matrix form:

$$\{\dot{A}_{jz}\} = [\tilde{A}_{jz}] \{\omega - \Omega\} \quad . \quad (2\text{-}50)$$

Employing elementary perturbation techniques and retaining only the linear terms, we may write equations (2-49) and (2-50) as

$$\begin{aligned}
&[I_0]\{\delta\dot{\omega}\} + [\delta I]\{\dot{\omega}_0\} + [\dot{I}_0]\{\delta\omega\} + [\delta\dot{I}]\{\omega_0\} \\
&+ [\delta\tilde{\omega}][I_0]\{\omega_0\} + [\tilde{\omega}_0][\delta I]\{\omega_0\} + [\tilde{\omega}_0][I_0]\{\delta\omega\} \\
&= 3\Omega^2 [\delta\tilde{A}_{jz}] [I_0] \{A_{jz0}\} + 3\Omega^2 [\tilde{A}_{jz0}] [\delta I] \{A_{jz0}\} \\
&\quad + 3\Omega^2 [\tilde{A}_{jz0}] [I_0] \{\delta A_{jz}\} + \{\delta T\} \quad , \quad (2\text{-}51)
\end{aligned}$$

and

$$\{\delta\dot{A}_{jz}\} = [\delta\tilde{A}_{jz}] \{\omega_0 - \Omega\} + [\tilde{A}_{jz0}] \{\delta\omega\} \quad (2\text{-}52)$$

where the subscript 0 denotes the initial condition or unperturbed value of the variable. The control equation to be developed in a subsequent chapter will be of the form $I = f(A_{jz}, \omega)$, giving three matrix equations in the three variables I, ω, A_{jz}.

Alternately A_{jz} and ω may be expressed in terms of the Euler angles utilizing equations (2-2) and (2-5). This approach has some advantage for specialized cases. For example, if the equations are solved using approximate methods, a normality constraint must be satisfied for A_{jz}, but no such constraint is required for the Euler angles. When small perturbations are considered, the equations can be linearized to two matrix equations (vehicle and control) in two unknowns, $[I]\{\theta\}$, where $\{\theta\}$ represents a column matrix of the Euler angles. A specialized set of linear equations are presented below.

Consider the condition where vehicle and orbital axes are aligned and rotating at the same angular velocity. The vehicle is assumed to be initially rigid so that the following set of initial conditions are satisfied:

$$\begin{aligned} \{\dot{\theta}\},\ \{\theta\},\ \{\dot{\omega}\} &= \{0\} \\ [\dot{I}] &= [0] \\ \{\omega\} &= \{\Omega\} \quad . \end{aligned} \tag{2-53}$$

Additionally if

$$\begin{aligned} & I_{yy} > I_{xx} > I_{zz} \\ & I_{jk} = 0 \quad , \quad j \neq k \quad , \end{aligned} \tag{2-54}$$

then the vehicle is in a stable attitude [6].

For this case equations (2-51) and (2-52) become

$$\begin{aligned} & [I_0]\{\delta\dot{\omega}\} + [\tilde{\Omega}][I_0]\{\delta\omega\} + [\tilde{\Omega}][\delta I][\Omega] + [\delta\tilde{\omega}][I_0]\{\Omega\} + [\delta\dot{I}]\{\Omega\} \\ & = 3\Omega^2 [\tilde{A}_{jz0}]\ [I_0]\ \{\delta A_{jz}\} + 3\Omega^2 [\tilde{A}_{jz0}]\ [\delta I]\ \{A_{jz0}\} \\ & \quad + 3\Omega^2 [\delta\tilde{A}_{jz}]\ [I_0]\ \{A_{jz0}\} + \{\delta T\} \quad , \end{aligned} \tag{2-55}$$

and

$$\{\delta\dot{A}_{jz}\} = [\tilde{A}_{jz0}]\ \{\delta\,\omega\} \qquad . \tag{2-56}$$

To express the vehicle equation in terms of Euler angles, the relationships of equations (2-2) and (2-5) are introduced. For this case under consideration, these become

$$[A_{jk0}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad , \quad [\delta A_{jk}] = \begin{bmatrix} 0 & \delta\psi & -\delta\theta \\ -\delta\psi & 0 & \delta\phi \\ \delta\theta & -\delta\phi & 0 \end{bmatrix} \tag{2-57}$$

and

$$\{\delta\omega\} = \begin{Bmatrix} \delta\dot{\phi} + \Omega\ \delta\psi \\ \delta\dot{\theta} \\ \delta\dot{\psi} - \Omega\ \delta\phi \end{Bmatrix} \qquad . \tag{2-58}$$

These relationships may now be introduced into equation (2-55) to yield the vehicle perturbation equation, which is most conveniently written in the form of the following three scalar equations:

$$\begin{aligned} &I_{xx0}\ \delta\ddot{\phi} + 4\Omega^2\,(I_{yy0} - I_{zz0})\ \delta\phi + I_{xy0}\ \delta\ddot{\theta} + 2\Omega\ I_{zy0}\ \delta\dot{\theta} \\ &- 3\Omega^2\,I_{xy0}\ \delta\theta + I_{xz0}\ \delta\ddot{\psi} + \Omega\ (I_{xx0} - I_{yy0} + I_{zz0})\ \delta\dot{\psi} \\ &+ \Omega^2\,I_{xz0}\ \delta\psi + \Omega\ \delta\dot{I}_{xy} + 4\Omega^2\ \delta I_{yz} - \delta T_x = 0 \qquad , \end{aligned} \tag{2-59}$$

$$I_{xy0}\,\delta\ddot{\phi} - 2\Omega\, I_{yz0}\,\delta\dot{\phi} - 4\Omega^2 I_{xy0}\,\delta\phi + I_{yy0}\,\delta\ddot{\theta} + 3\Omega^2 (I_{xx0} - I_{zz0})\,\delta\theta$$

$$+ I_{yz0}\,\delta\ddot{\psi} + 2\Omega\, I_{xy0}\,\delta\dot{\psi} - \Omega^2 I_{yz0}\,\delta\psi + \Omega\,\delta\dot{I}_{yy} - 3\Omega^2\,\delta I_{xz} - \delta T_y = 0 \quad , \tag{2-60}$$

and

$$I_{xz0}\,\delta\ddot{\phi} + \Omega\,(-I_{xx0} + I_{yy0} - I_{zz0})\,\delta\dot{\phi} + 4\Omega^2 I_{xz0}\,\delta\phi$$

$$+ I_{yz0}\,\delta\ddot{\theta} - 2\Omega\, I_{xy0}\,\delta\dot{\theta} + 3\Omega^2 I_{yz0}\,\delta\theta + I_{zz0}\,\delta\ddot{\psi}$$

$$+ \Omega^2\,[I_{yy0} - I_{xx0}]\,\delta\psi - \Omega^2\,\delta I_{xy} + \Omega\,\delta\dot{I}_{yz} - \delta T_z = 0 \quad . \tag{2-61}$$

CHAPTER III

CONFIGURATION AND CONTROL DEVELOPMENT

Requirements and Constraints

The application of the concept of active inertia management as a means of attitude control depends primarily upon the successful development of a suitable configuration. Such a configuration must have a simple means for varying the vehicle's inertia, but one that gives a sufficient variety of inertia characteristics. The vehicle configuration requirements for control should not impose severe limitations upon other mission requirements, particularly weight and reliability.

In Chapter I reference was made to recent studies of small earth satellites where active control was achieved through the use of gimbaled booms. This research considers instead the transfer of fluids, and although this means for inertia variation can be applied to small satellites, its most obvious application is to large earth orbiting space stations. Such stations are particularly suited to this method because relatively large amounts of liquid propellants are frequently carried while in orbit for reasons other than attitude control.

The class of vehicles to be considered is the large manned space stations in circular earth orbit. The mission of these stations is that of scientific observation of the earth, other celestial bodies, the environment of space, and in many instances the condition of weightlessness. Attitude requirements for such a mission are not so diverse as those for the small satellites. It is reasonable to assume that the attitude control system of the booster that places the space station in orbit is also capable of placing the station very near the proper attitude for orbital operation. This is in contrast to small earth satellites that are frequently ejected from the booster in such a manner that large initial attitude and angular rate errors are encountered. Although maneuverability requirements often exist for large space stations, the usual operational requirement is to maintain the station in some fixed orientation while many pieces of specialized scientific equipment, each containing its own attitude control system, are pointed in specified directions or scan specified areas. This is again in contrast to the small satellite with a single or small number of experiments where the attitude of the experiments is controlled in many cases by the satellite control system. In the work to follow, the reference attitude of the space station will be assumed fixed with respect to the earth. This is to some extent arbitrary because a reference fixed with respect to inertial space is desirable in some experiments and an earth fixed reference is desirable in others; but certainly the problem of attitude control by active inertia variation is more tractable in the orbital reference frame. The important fact is that it is easy to anticipate conditions where the amount of propellant or other liquid available to vary the inertia is very small; yet if the vehicle is in a passively stable though undamped condition, small inertia variations may still be adequate for control. It is therefore assumed that the vehicle is in a passively stable attitude within the earth's gravitational field; that is, the principal axis of least inertia points toward the center of the earth while the axis of greatest inertia is parallel to the orbital angular velocity.

It is natural to compare the concept of active inertia variation with the concepts for passively damped gravity gradient satellites discussed in Chapter I, since the same gravitational and inertial forces are involved. The intent of this research is to develop a method for applying these forces more effectively while retaining as much of the reliability and simplicity of the passive configurations as possible. There are two reasons for this criteria of simplicity. In the absence of detailed analysis, simplicity is a valid substitute for such criteria as reliability and cost. A simple system for inertia variation and control also aids in understanding the principles involved by retaining only the essential features of the concept. The constraints introduced in Chapter II in the development of the perturbation equations, for example, are retained.

The control system development should employ simple onboard sensors and a minimum of computation, avoid such calculations as coordinate transformations, and keep the order of the characteristic equation as small as possible. The vehicle structure is assumed to be rigid. Although vehicle flexibility must be considered in design, the space station is a relatively rigid compact structure and the frequencies of rigid and flexible motion are well separated.

In summary, the requirements and constraints for vehicle and control system development are listed below.

1. The attitude of the station will be constrained in an earth pointing mode.

2. The initial attitude errors of the space station at injection will be small.

3. The reference attitude of the rigid space station will be stable within the gravity field. The function of the control system will be to provide damping, increase response, and decrease attitude errors caused by disturbances acting on the vehicle.

4. The control system will be a simple onboard system without elaborate sensing or computing devices.

5. The configuration for varying vehicle inertia will be compatible with the configuration as determined by mission requirements.

6. The vehicle will be assumed to be a rigid body with the exception of the mass which moves in the manner prescribed by the control system.

Configuration Development

No precise method exists for selecting a vehicle configuration, but an approach or rationale can be developed by examining the perturbation equations (2-59), (2-60), and (2-61). For the equilibrium condition, the initial values of the products of inertia are equal to zero, and the following observations can be made:

1. Variations of only four of the six unique elements of the inertia matrix are effective in producing vehicle motion; δI_{yy} and δI_{xz} produce motion about the y axis, while δI_{xy} and δI_{yz} produce motion about the x and z axes.

2. Equation (2-60) is uncoupled from equations (2-59) and (2-61); that is, motion about the y axis is uncoupled from motion about the x and z axes.

3. The equilibrium or reference position can be maintained in the presence of external torques provided the products of inertia are varied appropriately. This condition is not limited to small perturbations but holds for the nonlinear equations as well. Thus, only variations of the three products of inertia are essential for control. An arrangement of propellant tanks that would allow each product of inertia to be varied independently would permit a simple control logic to be applied and probably minimize the amount of fluid motion required.

Tank Arrangement and Propellant Transfer Logic

A unique tank arrangement and propellant transfer logic are proposed which not only permit each product of inertia to be varied independently but also accomplish this without varying the vehicle moments of inertia or the position of the center of mass. The arrangement assumes the use of two separate fluids, a condition which will commonly be encountered with propellants on board satellite vehicles. Eight propellant tanks, four containing fuel and four containing oxidizer, are placed at the corners of a rectangular prism as shown in Figure 3-1. The faces of the prism are parallel (or perpendicular) to the body axes, and the tanks containing each propellant are located in opposite corners of each face of the prism. Consider first the inertia of the fuel system calculated with respect to the center of the prism. Transferring equal amounts of fuel from tanks 3F and 4F to tanks 1F and 2F decreases I_{xy} but produces no change in any other elements of the inertia matrix. Similarly I_{yz} and I_{xz} may be varied independently by filling and draining the appropriate tanks. By transferring an equal mass of oxidizer in a similar manner, the products of inertia can be varied individually with no associated shift in the center of mass. It is not necessary for the center of the prism to coincide with the vehicle center of mass. From the equation

$$[I_c] = [I] + M\,[\tilde{c}]\,[\tilde{c}], \qquad (3\text{-}1)$$

it can be seen that when the mass center is not shifted, a variation of inertia is the same with respect to any point. Also, the arrangement may utilize two different sized prisms, one for fuel and one for oxidizer, provided they are similar in shape and the ratio of fuel to oxidizer mass transfer is inversely proportional to the ratio of the size of the fuel and oxidizer prisms. Another feature of this configuration is the capability to shift the mass center, if so desired, without changing the inertia with respect to the origin of the axis system. This is accomplished by transferring equal amounts of propellant from each tank on one face of the prism to each tank on the opposite face.

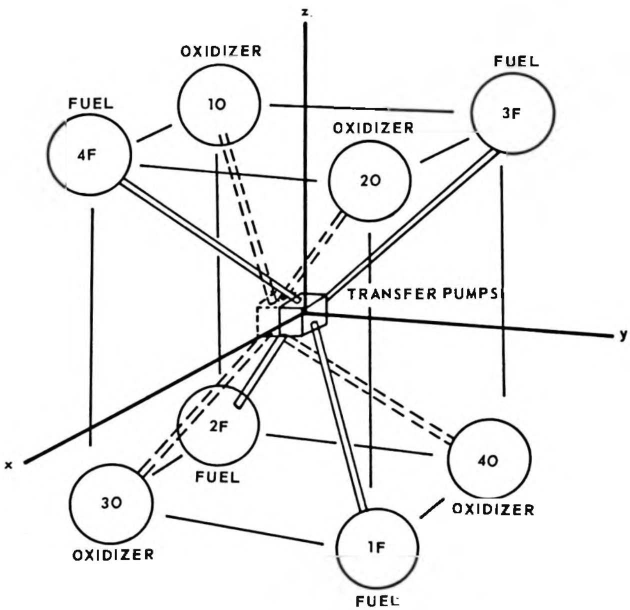


Figure 3-1. Propellant Tank Arrangement.

Although this capability is not considered in the following analysis, it may have application in minimizing external disturbances, such as aerodynamic torques, or maintaining a zero gravity environment.

The effects of propellants flowing in the lines can be calculated from equations (2-33) and (2-34). Note that the integral, $\int \bar{\rho} \times d\bar{l}$, is a function of the geometry of the propellant lines only and thus is evaluated only once for any configuration. Furthermore, if the lines are placed along radials from the origin, or in some other manner such that the integral is made equal to zero, then these effects can be eliminated. This does not imply that such an arrangement is desirable, since it is possible that this effect may be used to improve vehicle response through proper location of the propellant lines.

Other Tank Arrangements

The tank arrangement and propellant transfer logic just described will be used in the following analyses, but two other vehicle configurations should be mentioned. If a single propellant or other liquid is available, a four tank system identical to one of the pair just described could be used. The center of mass would shift within the vehicle, but this effect is formulated in Chapter II; thus, unless mission constraints prohibited the resulting vehicle motion, no obvious problems are seen to exist as a result.

An arrangement involving two partitioned tanks is shown in Figure 3-2. A similar scheme for transferring propellants can be devised for this configuration assuming fuel in one and oxidizer in the other. This

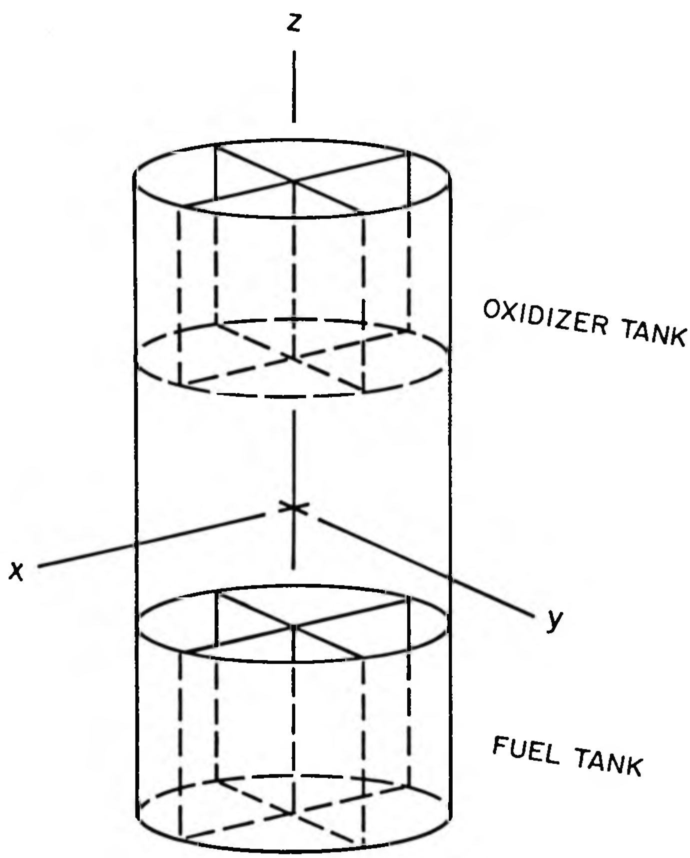


Figure 3-2. Partitioned Tank Arrangements.

arrangement has most of the features of the first configuration. However, propellant flow effects should be considered in this instance, and the ability to shift the mass center is limited to the xy plane.

Equations of the Vehicle

The vehicle equations developed in Chapter II were specialized in that section to apply to a configuration such as that just described. Now, relationships must be developed between the inertia and the propellant mass in the tanks, particularly those resulting from the physical constraints of the configuration. The extent to which the vehicle inertia can be varied is limited by the mass of the propellant, the size of the tanks, the location of the tanks, or some combination of these three factors. As discussed previously, the fuel and oxidizer systems are separate systems controlled in the same manner so that the relationship between variations in propellant location and the products of inertia is given as

$$\frac{I_{xy}}{2} = - \sum_{i=1}^{4} \Delta m_i \, x_i \, y_i \quad ,$$

$$\frac{I_{xz}}{2} = - \sum_{i=1}^{4} \Delta m_i \, x_i \, z_i \quad ,$$

and

$$\frac{I_{yz}}{2} = - \sum_{i=1}^{4} \Delta m_i \, y_i \, z_i \tag{3-2}$$

where Δm_i represents the deviation of the fuel (oxidizer) mass in tank i from its nominal value and $x_i \, y_i \, z_i$ are the coordinates of the center of mass of the fuel in tank i. Since the total mass of fuel in the system is constant,

$$\sum_{i=1}^{4} \Delta m_i = 0 \quad . \tag{3-3}$$

The size of the tank and the initial mass of fluid contained in the tank determine the limits of Δm_i,

$$-l_i \leqq \Delta m_i \leqq u_i \tag{3-4}$$

where l_i is the initial mass of fluid in tank i and u_i is the capacity of tank i less the initial mass.

The simplest relationship occurs when the center of the prism is at the origin of the body axis system, the tanks initially contain equal masses of fluid, and the tanks hold twice the initial mass. Letting

$$|x_i| = x, \quad |y_i| = y, \quad |z_i| = z \quad , \tag{3-5}$$

we rewrite equations (3-2) and (3-3) as

$$\begin{Bmatrix} \dfrac{I_{xy}}{2xy} \\ \dfrac{I_{xz}}{2xz} \\ \dfrac{I_{yz}}{2yz} \\ 0 \end{Bmatrix} = \begin{bmatrix} -1 & -1 & +1 & +1 \\ +1 & -1 & +1 & -1 \\ +1 & -1 & -1 & +1 \\ +1 & +1 & +1 & +1 \end{bmatrix} \begin{Bmatrix} \Delta m_1 \\ \Delta m_2 \\ \Delta m_3 \\ \Delta m_4 \end{Bmatrix} \quad . \tag{3-6}$$

Inverting the 4×4 matrix and dropping the last column along with the zero element in the I matrix yields

$$\begin{Bmatrix} \Delta m_1 \\ \Delta m_2 \\ \Delta m_3 \\ \Delta m_4 \end{Bmatrix} = \begin{bmatrix} -1 & +1 & +1 \\ -1 & -1 & -1 \\ +1 & +1 & -1 \\ +1 & -1 & +1 \end{bmatrix} \begin{Bmatrix} \frac{I_{xy}}{8xy} \\ \frac{I_{xz}}{8xz} \\ \frac{I_{yz}}{8yz} \end{Bmatrix} \quad . \tag{3-7}$$

Equation (3-4) becomes

$$|\Delta m_i| \leqq |\Delta m_L| \quad , \tag{3-8}$$

where Δm_L is the initial mass in each tank. Equations (3-7) and (3-8) may be combined to yield

$$\left|\frac{I_{xy}}{xy}\right| + \left|\frac{I_{xz}}{xz}\right| + \left|\frac{I_{yz}}{yz}\right| \leqq 8\,\Delta m_L \quad . \tag{3-9}$$

Geometrically equation (3-9) can be represented as the values of $\frac{I_{xy}}{xy}$, $\frac{I_{xz}}{xz}$, and $\frac{I_{yz}}{yz}$ bounded by an octahedron composed of equilateral triangles as shown in Figure 3-3.

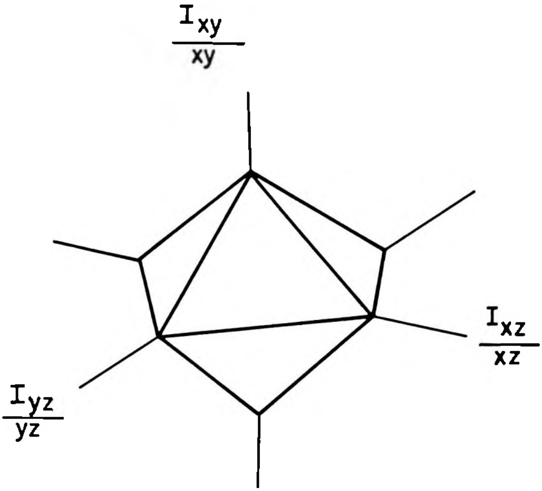


Figure 3-3. Bounds of Vehicle Products of Inertia.

The general expression for the inertia limits can be written by combining equations (3-2) and (3-3) into the form

$$\left\{ \frac{I}{0} \right\} = [k] \quad \{\Delta m\} \tag{3-10}$$

or

$$\{\Delta m\} = [k]^{-1} \left\{ \frac{I}{0} \right\} \quad , \tag{3-11}$$

and

$$-l_i \le \Delta m_i \le u_i \tag{3-12}$$

where I is the column of the products of inertia I_{xy}, I_{xz}, I_{yz}; the elements of k are defined from equations (3-2) and (3-3) as $2x_iy_i$ for the first row, $2x_iz_i$ for the second, $2y_iz_i$ for the third, and unity for the fourth. The values of Δm are the mass deviations in the tank as before. A similar set of expressions for $\dot{I}_{jk}$ can be written as

$$\left\{\frac{\dot{I}}{0}\right\} = [k] \quad \{\dot{m}\} \quad , \tag{3-13}$$

$$\{\dot{m}\} = [k]^{-1} \left\{\frac{\dot{I}}{0}\right\} \quad , \tag{3-14}$$

and

$$\dot{l}_i \leqq \dot{m}_i \leqq \dot{u}_i \tag{3-15}$$

with the additional constraint

$$\dot{m}_i = 0 \quad \text{when} \quad \Delta m_i = l_i \text{ or } u_i \quad . \tag{3-16}$$

The limits $\dot{l}_i$ and $\dot{u}_i$ are determined from the characteristics of the propellant transfer mechanism and obviously are not related to l_i or u_i.

Control System Development

A frequent control system development procedure, and the procedure followed here, is to linearize the equations about an equilibrium condition, to develop candidate systems using linear techniques, to evaluate the effects of system nonlinearities, and finally to assure satisfactory operation of the non-linear system by a computerized solution of the complete nonlinear equations.

The basic objective of the control system development, as stated previously, is to provide the simplest onboard system capable of satisfactorily controlling the vehicle. A major concern is that the variation in inertia available by propellant transfer is sufficient to provide control for a realistic vehicle and mission.

Linear Analysis

The linear analysis to develop a candidate control system consists of selecting those vehicle parameters to be sensed, defining a control law for the system, and analyzing the linear stability.

Sensor Definition and Selection

Four types of sensing devices are considered for attitude control. These are devices which sense:

1. The local vertical, such as horizon sensors.
2. A direction in space, such as startrackers.
3. Absolute angular velocity, such as rate gyros.
4. Attitude relative to an internally established reference, such as inertial platforms.

The horizon sensor and the rate gyro give information in the simplest and most direct form for controlling the vehicle relative to the orbital reference frame. Attitude information from such devices as startrackers or inertial platforms must be transformed into the orbital reference frame, and the computations to perform this transformation must be periodically updated because of variations in the orbit. If an inertial platform is used, additional means must be provided to update the platform drift; therefore, consideration is limited to information which can be generated from devices such as horizon sensors or rate gyros, provided a suitable control system can be developed using this information.

Selection of the Control Law

Before developing a control law in detail, the manner in which a horizon sensor and rate gyro are used to align the vehicle to the orbital reference frame will be described. A horizon sensor measures the attitude of the vehicle with respect to the horizon plane. This information may be presented in a variety of ways; as the angles between x_B and y_B axes and the horizon plane, as the angles between the x_B and y_B axes and the local vertical, and as the direction cosines of the local vertical. The horizon sensor obviously cannot measure rotation about the local vertical. A rate gyro measures the absolute rotation of the vehicle about some axis and, if the vehicle is in a fixed attitude with respect to the orbital reference frame, the rate gyro measures the component of the orbital rate about that axis. From equation (2-5),

$$\omega_x = \Omega \sin \psi \quad . \tag{3-17}$$

This implies that the orbital axes and the vehicle axes may be aligned by requiring the two direction cosines A_{xz} and A_{yz} and the angular velocity

ω_x to be zero (the condition where x_B and x_O are misaligned by 180 degrees presents no problem in practice). Note that this scheme does not involve integration of the angular rate, so any errors present in the system are not accumulative. The scheme is based upon the same concept as that whereby a gyro compass determines true north from the local vertical and the earth's rotation.

It was established that variation of the three products of inertia was a simple and effective means of influencing vehicle motion for small perturbations about the equilibrium position. The horizon sensor and rate gyro have been selected as the simplest means for sensing vehicle motion about an orbital reference axis. A candidate control law that functionally relates these is

$$\begin{Bmatrix} I_{xy} \\ I_{xz} \\ I_{yz} \end{Bmatrix} = \begin{bmatrix} a_{jk} \end{bmatrix} \begin{Bmatrix} A_{xz} - A_{xzR} \\ A_{yz} - A_{yzR} \\ 0 \end{Bmatrix} + \begin{bmatrix} b_{jk} \end{bmatrix} \begin{Bmatrix} \omega_x - \omega_{xR} \\ \omega_y - \omega_{yR} \\ \omega_z - \omega_{zR} \end{Bmatrix} \tag{3-18}$$

where A_{jzR} and ω_{jR} are reference values. The elements a_{jk} and b_{jk} are not generally constant but can be considered constant when the inequalities of equations (3-4) and (3-15) are satisfied and when the dynamic behavior of the mechanism which effects the inertia variation is sufficiently well separated in frequency from the dynamic behavior of the vehicle. The consideration is examined in more detail in the following chapter.

The linearized form of equation (3-18) is

$$\begin{Bmatrix} \delta I_{xy} \\ \delta I_{xz} \\ \delta I_{yz} \end{Bmatrix} = \begin{bmatrix} a_{jk} \end{bmatrix} \begin{Bmatrix} \delta A_{xz} \\ \delta A_{yz} \\ 0 \end{Bmatrix} + \begin{bmatrix} b_{jk} \end{bmatrix} \begin{Bmatrix} \delta\omega_x \\ \delta\omega_y \\ \delta\omega_z \end{Bmatrix} \tag{3-19}$$

or

$$\begin{Bmatrix} \delta Ixy \\ \delta Ixz \\ \delta Iyz \end{Bmatrix} = \begin{bmatrix} a_{jk} \end{bmatrix} \begin{Bmatrix} -\delta\theta \\ \delta\phi \\ 0 \end{Bmatrix} + \begin{bmatrix} b_{jk} \end{bmatrix} \begin{Bmatrix} \delta\dot{\phi} + \Omega\delta\psi \\ \delta\dot{\theta} \\ \delta\dot{\psi} - \Omega\delta\phi \end{Bmatrix} \tag{3-20}$$

from equations (2-57) and (2-58). Considering the objective of keeping the vehicle motion about the y axis uncoupled from motion about the x and y axes, the equation reduces to

$$\begin{Bmatrix} \delta I_{xy} \\ \delta I_{xz} \\ \delta I_{yz} \end{Bmatrix} = \begin{bmatrix} 0 & a_{12} & 0 \\ a_{21} & 0 & 0 \\ 0 & a_{32} & 0 \end{bmatrix} \begin{Bmatrix} -\delta\theta \\ \delta\phi \\ 0 \end{Bmatrix} + \begin{bmatrix} b_{11} & 0 & b_{13} \\ 0 & b_{22} & 0 \\ b_{31} & 0 & b_{33} \end{bmatrix} \begin{Bmatrix} \delta\dot{\phi} + \Omega\delta\psi \\ \delta\dot{\theta} \\ \delta\dot{\psi} - \Omega\delta\phi \end{Bmatrix} . \tag{3-21}$$

Further simplification of equation (3-21) and analysis of the linear stability of the system are accomplished by the application of the Laplace transform and the conventional techniques of linear analysis. These details are given in the next chapter where the characteristics of a specific vehicle are calculated. However the simplified control equation for this specific vehicle will be used in the following sections to examine nonlinear considerations and to define some of the operating limits of the system. The complete set of transformed equations for vehicle and control and the characteristic equation for the system are given as follows.

$$\begin{bmatrix} I_{xx0}(s^2+4\Omega^2 A) & I_{zz0}(1-B)\Omega s & \Omega s & 4\Omega^2 & 0 & 0 \\ -I_{xx0}(1-A)\Omega s & I_{zz0}(s^2+\Omega^2 B) & -\Omega^2 & \Omega s & 0 & 0 \\ b_{11}s+(a_{12}-b_{13}\Omega) & b_{13}s+b_{11}\Omega & -1 & 0 & 0 & 0 \\ b_{31}s+(a_{32}-b_{33}\Omega) & b_{33}s+b_{11}\Omega & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{yy0}(s^2+3\Omega^2 C) & -3\Omega^2 \\ & & & & b_{22}s-a_{21} & -1 \end{bmatrix} \begin{Bmatrix} \delta\phi \\ \delta\psi \\ \delta Ixy \\ \delta Iyz \\ \delta\theta \\ \delta Ixz \end{Bmatrix} = \begin{Bmatrix} \delta T_x \\ \delta T_z \\ 0 \\ 0 \\ \delta T_y \\ 0 \end{Bmatrix} \tag{3-22}$$

and

$$\begin{aligned} &\Big\{ I_{xx0} I_{zz0}\left[s^4+\Omega^2\left(3A+AB+1\right)s^2+4\Omega^4 AB\right] + a_{12} I_{zz0}\left[s^2+\Omega^2\right] \\ &+ a_{32} I_{zz0}\Omega^2\left[\left(3+B\right)s^2+4\Omega^2 B\right] + b_{11}\Big[I_{zz0}\Omega s^4+\left(-I_{xx0}A\Omega^3+I_{zz0}\Omega^3\right)s^2 \\ &\qquad + \left(-I_{xx0}A\,4\Omega^5\right)\Big] \\ &- b_{13}s\left[\left(I_{xx0}A+I_{zz0}\Omega^2\right)s^2+\left(4\Omega^2 I_{xx0}A+I_{zz0}\Omega^4\right)\right] \\ &+ b_{31}s\left[\left(I_{xx0}\Omega^2+I_{zz0}\Omega^2\left(3+B\right)\right)s^2+\left(I_{xx0}4\Omega^2+I_{zz0}4\Omega^2 B\right)\right] \\ &+ b_{33}\left[I_{xx0}\Omega s^4+\left(I_{xx0}4\Omega^3-\Omega^3 I_{zz0}\left(3+B\right)\right)s^2+\left(-I_{zz0}\Omega^5 4B\right)\right] \\ &+ \Big[\left(b_{11}b_{33}-b_{13}b_{31}\right)s^2+\left(b_{33}a_{12}-b_{13}a_{32}\right)s+\left(b_{11}b_{33}-b_{13}b_{31}\right)\Omega^2 \\ &\quad + \left(b_{31}a_{12}-b_{11}a_{32}\right)\Omega\Big]\,\Omega^2\left(s^2+4\Omega^2\right)\Big\} \quad \Big\{I_{yy0}s^2-3\Omega^2 b_{22}s \\ &\qquad + \left(3\Omega^2 a_{21}+3\Omega^2 C\,I_{yy0}\right)\Big\} = 0 \quad . \end{aligned} \tag{3-23}$$

where

$$A = \frac{I_{yy} - I_{zz}}{I_{xx}}$$

$$B = \frac{I_{yy} - I_{xx}}{I_{zz}} \qquad (3\text{-}24)$$

$$C = \frac{I_{xx} - I_{zz}}{I_{yy}}$$

Equation (3-21) is simplified in the following chapter to

$$\begin{Bmatrix} \delta I_{xy} \\ \delta I_{xz} \\ \delta I_{yz} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ a_{21} & 0 & 0 \\ 0 & a_{32} & 0 \end{bmatrix} \begin{Bmatrix} -\delta\theta \\ \delta\phi \\ 0 \end{Bmatrix} + \begin{bmatrix} b_{11} & 0 & b_{13} \\ 0 & b_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \delta\dot{\phi} + \Omega\delta\psi \\ \delta\dot{\theta} \\ \delta\dot{\psi} - \Omega\delta\phi \end{Bmatrix} . \qquad (3\text{-}25)$$

Nonlinear Considerations

Three types of nonlinearities will be considered:

1. The nonlinearities associated with the general forms of the vehicle and control equations and the kinematical relationships; for example, equations (2-49), (3-18), and (2-50).

2. Saturation of the magnitude of the components of I introduced by tank size and available propellant mass.

3. Saturation of the rate of change of the components of I introduced by a propellant mass flow rate limit.

When only the nonlinearities associated with the general forms of the equations are considered, the elements of the control equation a_{jk} and b_{jk} may still be considered constant, and the candidate system developed from the linearized equation is a good approximation. When saturation of I or $\dot{I}$ is encountered, the control equation is in effect defining a condition which is physically impossible for the vehicle to attain, and the form of the control equation just developed is no longer valid. The new functional relationships are highly dependent upon the detailed manner in which the propellant transfer mechanism is implemented. The regions in which the latter two nonlinearities occur will be determined, but a control system will not be defined for this condition. The following chapter shows that this condition is not of major significance for large earth orbiting spacecraft of the type considered in this study.

The limitation on inertia variation caused by tank size and available propellant mass can be used to determine, for equilibrium conditions, an equivalent limit on such system parameters as vehicle attitude or external torque. These limits may then be used to estimate the region of operation for a configuration within which only nonlinearities of the first type need be considered.

Allowable Magnitude of Disturbance Torques

Consider the vehicle to be rotating at orbital velocity with the body axes and orbital axes aligned and with some constant external torque applied to the vehicle. In such case, the vehicle equation (2-49) reduces to

$$\begin{Bmatrix} -4\Omega^2 I_{yz} \\ 3\Omega^2 I_{xz} \\ \Omega^2 I_{xy} \end{Bmatrix} + \begin{Bmatrix} T_x \\ T_y \\ T_z \end{Bmatrix} = 0 \quad . \tag{3-26}$$

Now, assuming the control system is capable of generating the required products of inertia (with negligible attitude error), then the physical limits on the products of inertia establish an upper bound on the external torques which the system can counteract.

Bounds on Vehicle Attitude

Two types of attitude bounds are considered. The first is the bound on stable reference attitudes which can be attained within the inertia limits. The only reference attitude discussed thus far has been when the body and orbital axes are aligned, and this attitude is a stable equilibrium position when

$$\begin{aligned} &I_{yy} > I_{xx} > I_{zz} \quad , \\ &I_{jk} = 0 \quad , \quad j \neq k \quad , \end{aligned} \tag{3-27}$$

and the vehicle is rotating at orbital velocity. Equation (3-18) presumes that some new reference position may be desired. In the new reference position, if the vehicle is rotating at orbital velocity and the inertia elements calculated with respect to the orbital axes satisfy equation (3-27), then this new attitude is obviously a stable equilibrium position also. Thus, the extent to which the principal axes can be shifted with respect to the body axes by varying the inertia determines the bounds of the stable reference attitudes.

The second attitude bound to be established is that on attitude errors. Having determined from equation (3-9) some bounded region in which the products of inertia may be varied, we find that a suitable transformation involving the control equation (3-25) determines an equivalent region of vehicle attitude errors in which the products of inertia will be less than the limiting value. If the vehicle is rotating at orbital angular velocity and the attitude errors are small, this region is defined by

$$\left| \frac{\Omega b_{11}}{xy} \psi - \frac{\Omega b_{13}}{xy} \phi \right| + \left| \frac{a_{32}}{yz} \phi \right| + \left| \frac{-a_{21}}{xz} \theta \right| \leq 8 m_L \qquad . \qquad (3\text{-}28)$$

The bounds on the Euler angles are an octahedron as before, but the triangles are not equilateral and two of the corners do not lie on the axes. Figures 3-4 and 3-5 show this octahedron and define the geometry in additional detail.

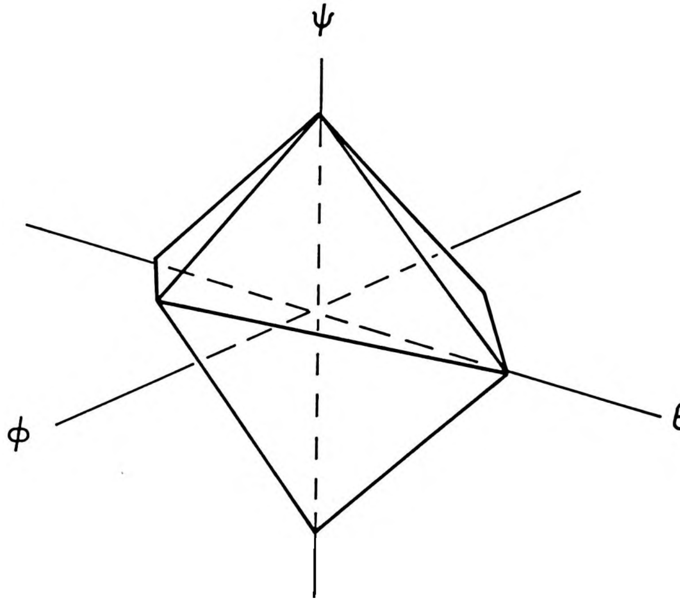
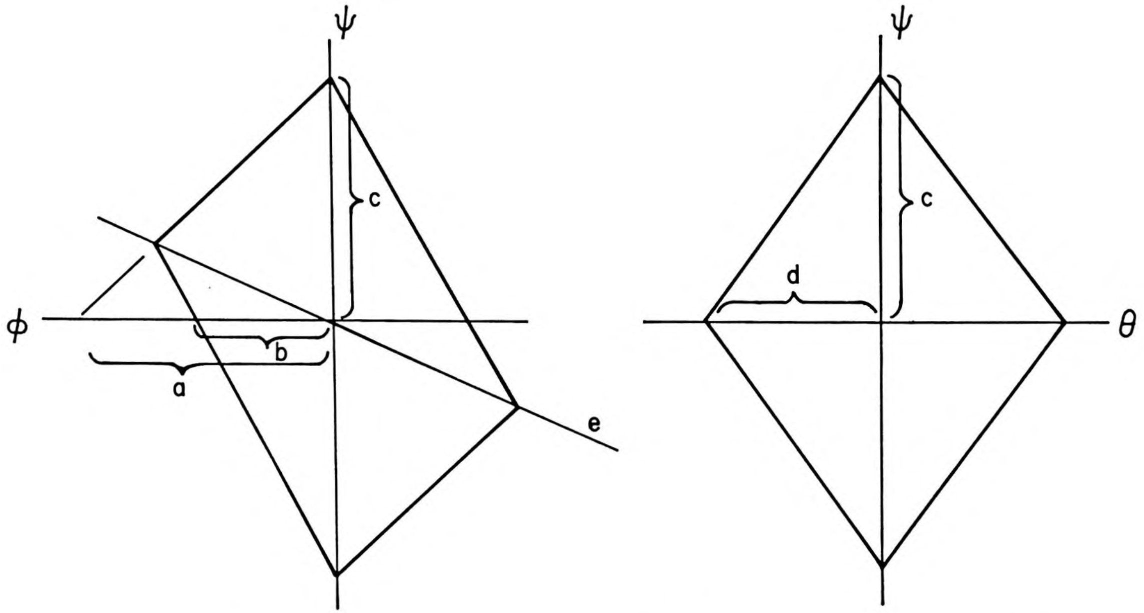


Figure 3-4. Bounds on Vehicle Attitude Error.

Bounds on Mass Flow Rate

The maximum value of the mass flow rates calculated from equation (3-14) depends upon the manner in which fluid transfer is implemented. This value could be a separate constant for each tank as in equation (3-15). If the transfer mechanism were a pump of limited capacity, the sum of all positive (and negative) flow rates would be the appropriate quantity to limit. Numerical examples in the next chapter show that limitations in flow rates are not expected to be a major problem.



$$a \sim \frac{8m_L}{a_{32}/yz - \Omega b_{13}/xy}$$

$$b \sim \frac{8m_L}{a_{32}/yz + \Omega b_{13}/xy}$$

$$c \sim \frac{8m_L}{\Omega b_{11}/xy}$$

$$d \sim \frac{8m_L}{a_{21}/xz}$$

$$e \sim b_{11}\phi = b_{13}\psi$$

Figure 3-5. Bounds on Vehicle Attitude Error — $\psi\phi$ Plane, $\psi\theta$ Plane.

CHAPTER IV

SPECIFIC VEHICLE CONFIGURATION

A method for controlling a satellite by transferring liquid among tanks on the vehicle was developed in Chapters II and III. As recognized during this development, a major limitation to its application is the amount of inertia variation that can be attained. Before this method can be considered to be an effective means for control, it must be shown, for a practical vehicle and mission, that the inertia can be varied sufficiently to overcome the disturbances the vehicle is expected to encounter. In addition the stability and response characteristics of the candidate control system must be demonstrated throughout its operating range using the more complete nonlinear equations.

Consider a configuration such as that shown in Figure 4-1. The mission of this vehicle is to store large quantities of propellants in earth orbit in support of other space operations. A mechanism for transferring propellants among tanks exists in such a vehicle, large quantities of liquid are on board, and the vehicle is required to remain in orbit for extended periods. Such a vehicle is ideally suited to the method of control just developed; however, the practical value of the concept will be proven more convincingly if a vehicle and mission are selected which are not so obviously suited to the method.

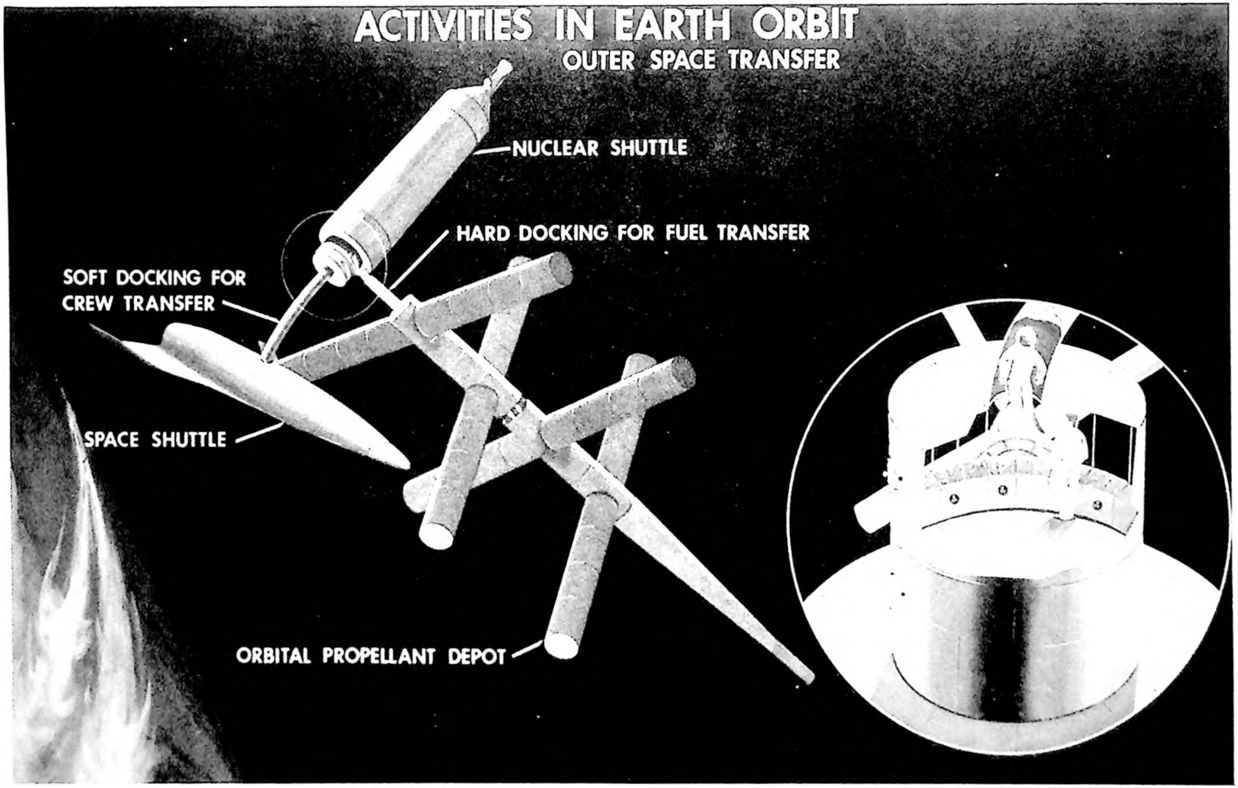


Figure 4-1. Orbital Propellant Depot.

The vehicle selected for analysis is an earth orbiting space station capable of being boosted into orbit by the Saturn V. On the aft end of the station is a final ascent module, commonly called a half stage or "kicker" stage, for achieving orbit and for orbit adjustment during the mission. At the forward end is a descent module for either the planned or emergency return of the crew to earth. The vehicle is placed in orbit at roughly its proper attitude and angular velocity under the control of the ascent module. To control the vehicle while in orbit, propellants are transferred among the descent module tanks which are full at injection and the ascent module tanks which are near depletion. At the end of the mission, all propellants are transferred to the descent module which separates from the station and returns to earth. The vehicle with its dimensions and physical characteristics is shown in Figures 4-2 and 4-3. The vehicle is in a circular orbit of approximately a 2-hour period ($\Omega \approx 0.001$ rad/sec) and the reference attitude is that described in Chapters II and III.

Linear Stability

The linear stability of this configuration is analyzed by mapping the roots of the characteristic equation (3-23) as a function of the coefficients, a_{jk} b_{jk} , of the control equation. The objective of this analysis, as previously stated, is to develop a simple candidate system with adequate linear stability, while anticipating the problems associated with the system nonlinearities.

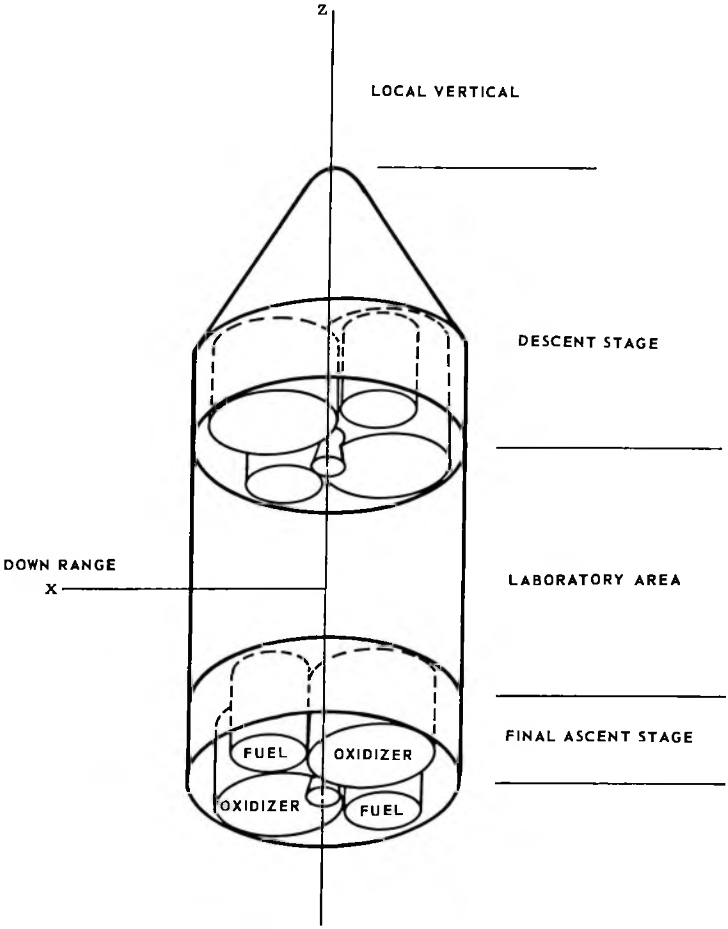
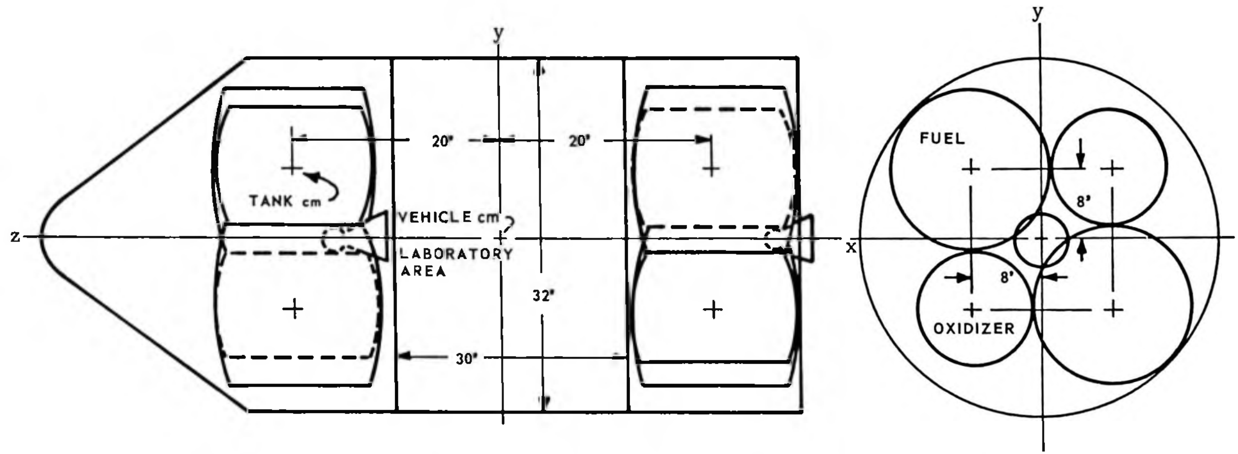


Figure 4-2. Earth Orbiting Space Station.



TOTAL WT. = 130,000 LB
PROPELLANT WT = 30,000 LB
(AVAILABLE TO VARY INERTIA)

$I_{xx} = 1.0 \times 10^6$ SLUG FT.^{2}
$I_{yy} = 1.2 \times 10^6$ SLUG FT.^{2}
$I_{zz} = 0.5 \times 10^6$ SLUG FT.^{2}

MAXIMUM PRODUCT OF INERTIA – SLUG FT.^{2}

BASED ON CONFIGURATION	ASSUMED FOR THE EXAMPLE
$I_{xy} = 0.6 \times 10^5$	0.5×10^5
$I_{xz} = 1.5 \times 10^5$	1.0×10^5
$I_{yz} = 1.5 \times 10^5$	1.0×10^5

Figure 4-3. Space Station — Major Dimensions and Numerical Data.

The two roots characterizing motion about the y axis are a function of only the two coefficients b_{22} and a_{21}. These roots are plotted in Figure 4-4. The four roots characterizing motion about the x and z axes cannot be presented so directly; however, it is recognized from equation (3-22) that the coefficients a_{32} and b_{11} are essential to reduce steady-state attitude errors resulting from external torques. The four roots are therefore mapped as a function of these two coefficients in combination with the remaining coefficients of the control equation. If only one additional coefficient is introduced, b_{13} is most effective in damping the vehicle, using the criterion that optimization in the sense of damping rate or transient response consists of selecting system parameters which result in minimizing the negative real parts of the roots. This criterion is used by Zajac and Hartbaum et al. in the study of configuration selection and optimization for passive gravity gradient satellites [28, 29]. The four roots characterizing motion about the x and z axes are plotted in Figure 4-5.

Additional equilibrium conditions of importance are (1) the reference attitude of equation (3-18) other than zero and (2) a constant external torque acting on the vehicle. The linear stability for small perturbations about these two conditions may be considered in the same manner as before. The initial conditions are

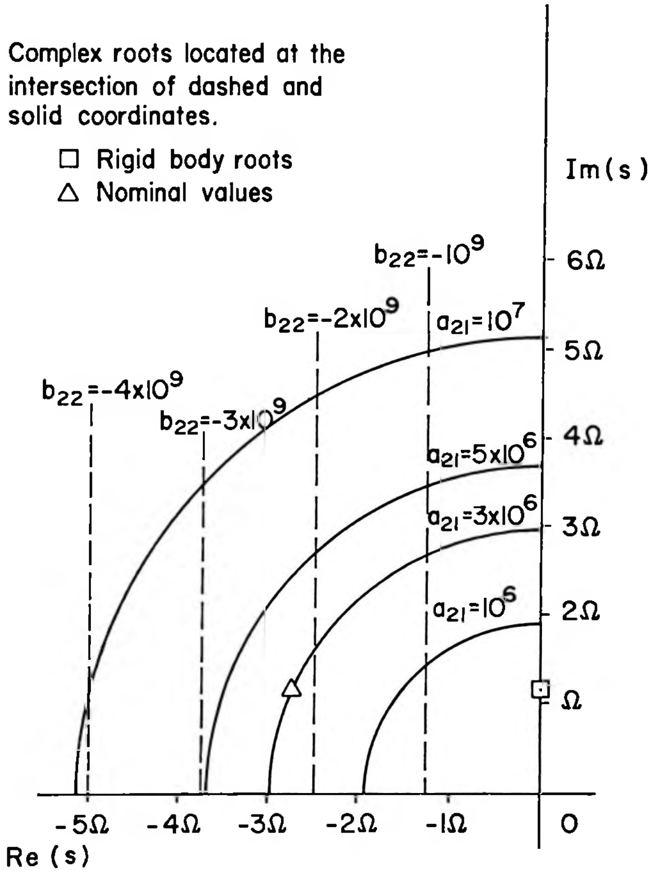


Figure 4-4. Locus of Roots of Equation (3-25) Characterizing Motion About the y Axis.

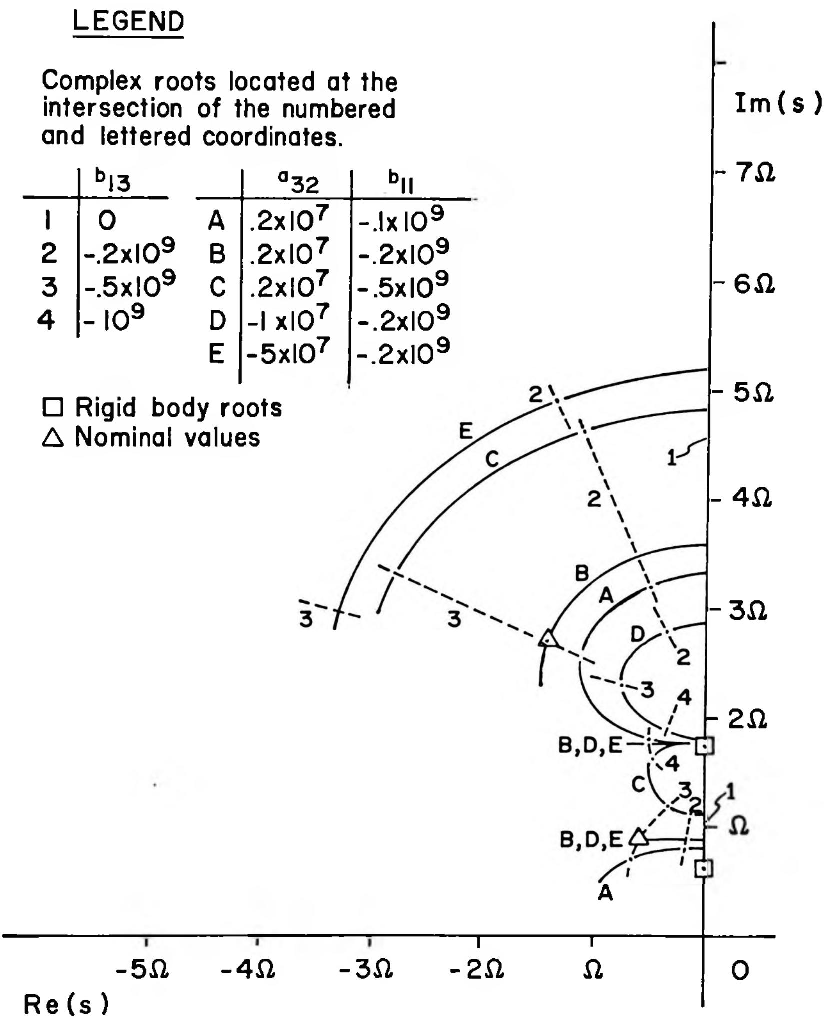


Figure 4-5. Locus of Roots of Equation (3-25) Characterizing Motion About the x and z Axes.

$$\{\dot{\theta}\} = \{\dot{\omega}\} = 0$$

$$[\dot{I}] = 0 \tag{4-1}$$

$$\{\Omega\} = \{\omega\} \quad ,$$

but

$$\{\theta\} \neq 0$$

$$\{I_{jk}\} \neq 0 \tag{4-2}$$

$$\{J_{jk}\} = 0 \quad , \; j \neq k$$

where J_{jk} are the products of inertia with respect to the orbital axes.

Perturbation equations (2-55) and (2-56) are still valid under these conditions, and the three resulting scalar equations in terms of Euler angles, although considerably more complex than equations (2-59), (2-60), and (2-61), are developed in a straightforward manner from equations (2-2) and (2-5). System stability under the condition where a constant external torque is applied to the vehicle is shown in Figure 4-6. System stability is still satisfactory within the limits established for inertia variation.

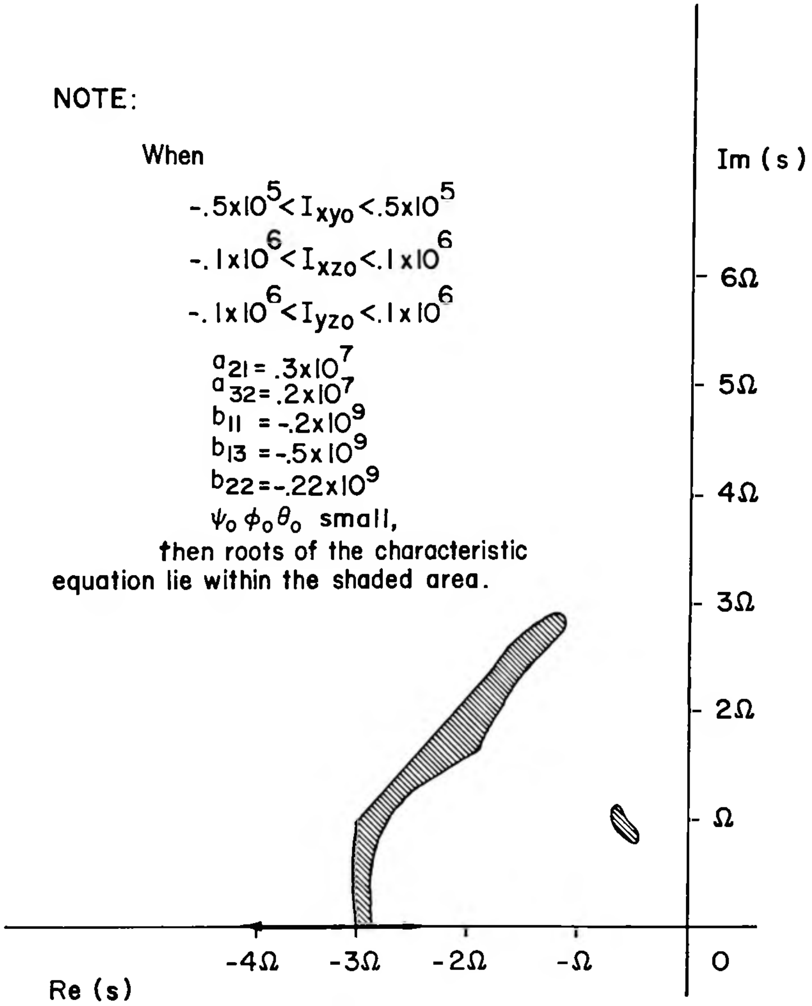


Figure 4-6. Locus of Roots for Variations in the Initial Value of the Products of Inertia.

Operating Limits of the System

In Chapter III relationships were developed which defined limits on vehicle attitude and external torque as a function of the limits imposed on the inertia variation. It will now be shown that the major internal and external disturbances which the vehicle is expected to encounter can be approximated by either an external torque or a vehicle attitude error; for the configuration selected, these disturbances are within the bounds established by the inertia limits. The bounds are given by equations (3-26) and (3-28) of Chapter III as

$$\begin{aligned} T_x &= 0.4 \text{ ft lb} \\ T_y &= 0.3 \text{ ft lb} \\ T_z &= 0.05 \text{ ft lb}, \end{aligned} \tag{4-3}$$

and

$$\begin{aligned} \theta &< 1.14 \text{ deg} \\ \psi &< 17 \text{ deg} \\ \phi &< 4.3 \text{ deg} \quad . \end{aligned} \tag{4-4}$$

For the limits placed on the products of inertia in Figure 4-3, the bounds within which the principal axes can be shifted with respect to the body axes are approximately

$$\psi < 16 \text{ deg}$$
$$\theta < 11 \text{ deg} \tag{4-5}$$
$$\phi < 10 \text{ deg} \quad .$$

Previously it was assumed that the mass flow rates commanded by the control system were close enough to the true flow rates to be considered identical when the tanks were not completely full or empty. This assumption can be examined in more detail now that a specific vehicle configuration has been defined, and the calculation in Appendix B shows the assumption is still valid. In the calculation the system response is shown to have a time constant of approximately 6 sec, which is negligible when compared to periods of greater than 1500 sec for the system oscillations. A pressure differential of two-tenths lb/in.^{2} between tanks produces mass flow rates of 15 lb/sec, which is greater than any commanded flow rate that does not cause the tanks to fill or empty completely (excluding transient responses of a few seconds duration resulting from step inputs). It is concluded that flow rate saturation is not a major problem and this effect is not considered further.

Magnitudes of Major Disturbances

The major disturbances encountered by the vehicle are now evaluated and compared to the bounds which have been established for the configuration.

Internal Disturbances

The most significant internal disturbances are crew motion, repositioning of crew or equipment, and intermittent operation of rotating machinery. In Appendix C the magnitudes of these disturbances are approximated by simple calculations. The disturbances are shown to be well within the system capability with the exception of the starting or stopping of rotating machinery with an angular momentum which is a large percentage of the spacecraft momentum. The torque created by leakage and the intentional venting of gases can be excessive if compensating techniques such as nonpropulsive venting are not used.

External Disturbances

The external disturbance of major concern results from aerodynamic torques on the vehicle; however, the proposed control system appears capable of overcoming these torques for most configurations and flight conditions. In those instances where the control is marginal or inadequate, the technique of shifting the center of mass may be employed to increase the capability of the system. Magnitudes of the aerodynamic torque and the technique of shifting the mass center are discussed in more detail in Appendix C.

The torques resulting from the interaction of the earth's magnetic field and the spacecraft are highly configuration dependent and difficult to calculate, particularly for large space stations. However, preliminary studies of large space station configurations indicate that they are much less than the aerodynamic torques and well within the capabilities of the proposed control system [24, 30]. In addition, techniques exist for further minimizing these torques if necessary (for example, rerouting electrical circuitry or reorienting electrical equipment).

Other sources of external torques include those which might be generated from the earth's electric field, the sun's radiation pressure, cosmic radiation, or electromagnetic radiation from the satellite. Although considerable uncertainty exists as to the magnitude of many of these sources, they are apparently small and well within the capabilities of the proposed system [30].

The docking of shuttle vehicles to the space station may easily produce disturbances which are beyond the capability of the proposed control system. Two conditions should be considered: the dynamics associated with the impact of the shuttle craft on the space station at docking and the inertial characteristics of the docked configuration. For most circumstances it appears that the proposed variable inertia system of the example will not provide sufficient control torque, response time, or inertia variation. The size of presently proposed space shuttles is the same order of magnitude as the station [31]. Some type of auxiliary system should be assumed for control in these conditions.

In summary, it appears that, except for unusual circumstances, the proposed control system will be adequate to overcome the expected external and internal disturbances which the station may encounter and the auxiliary systems or special provisions can be reasonably incorporated to control these conditions.

System Response

The response of the candidate system developed from linearized techniques is demonstrated in the more general formulation, retaining the nonlinearities of the vehicle equation and kinematical relationships and introducing the physical limits of the vehicle and the control equation. The method of solution of these equations is described in Appendix A. Initial attitude errors, angular velocities, external moments, and attitude commands are used to typify the injection errors and disturbances which might be expected in the operation of the system. Emphasis is primarily on the operating region in which the limits defined by propellant or tank capacity are not exceeded.

The mass flow rates presented in the following figures represent the total mass which passes through the fuel or oxidizer pump for the configuration shown in Figure 3-1.

Initial Attitude Errors

Initial attitude errors are expected at injection and Appendix C shows that an initial attitude error is a good approximation of the errors resulting from crew motions, such as the wall pushoff. Figures 4-7, 4-8, and 4-9 show the response of the system to initial attitude errors in ψ, ϕ, and θ of 0.02 radians. Note that motion characterized by ϕ and θ has essentially subsided within one orbit and the motion characterized by ψ is less than 10 percent of its initial value. Except when the initial error is in ψ alone, the

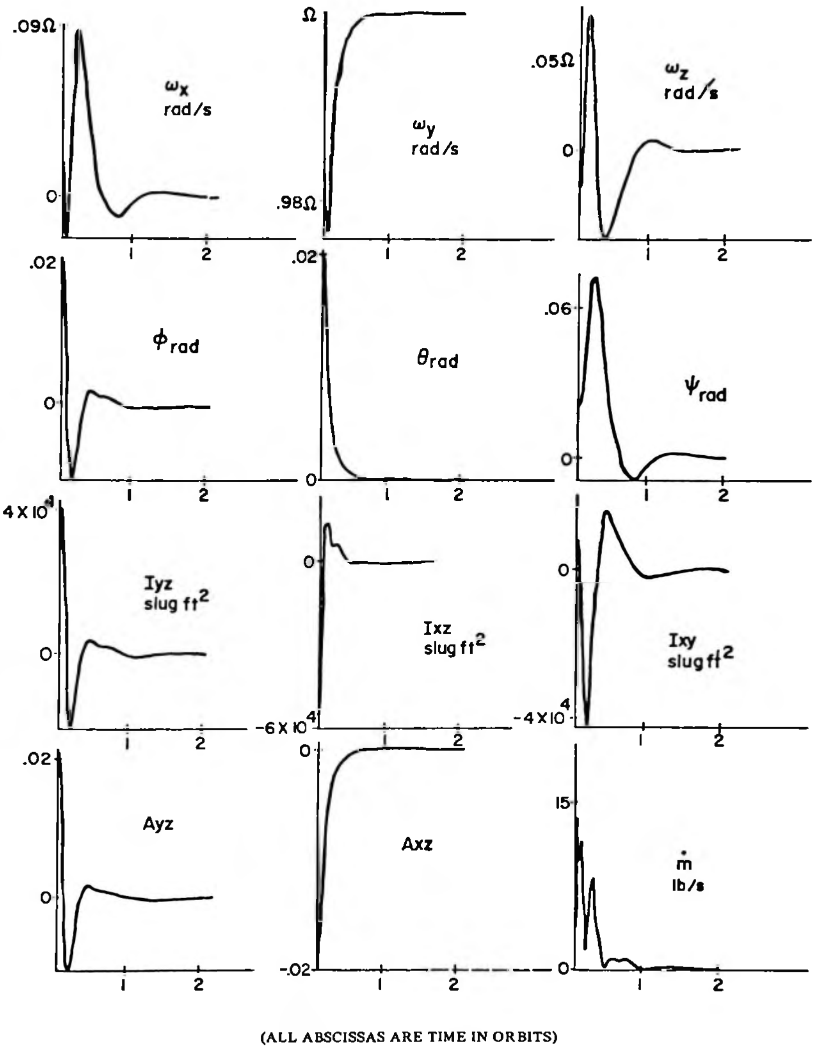
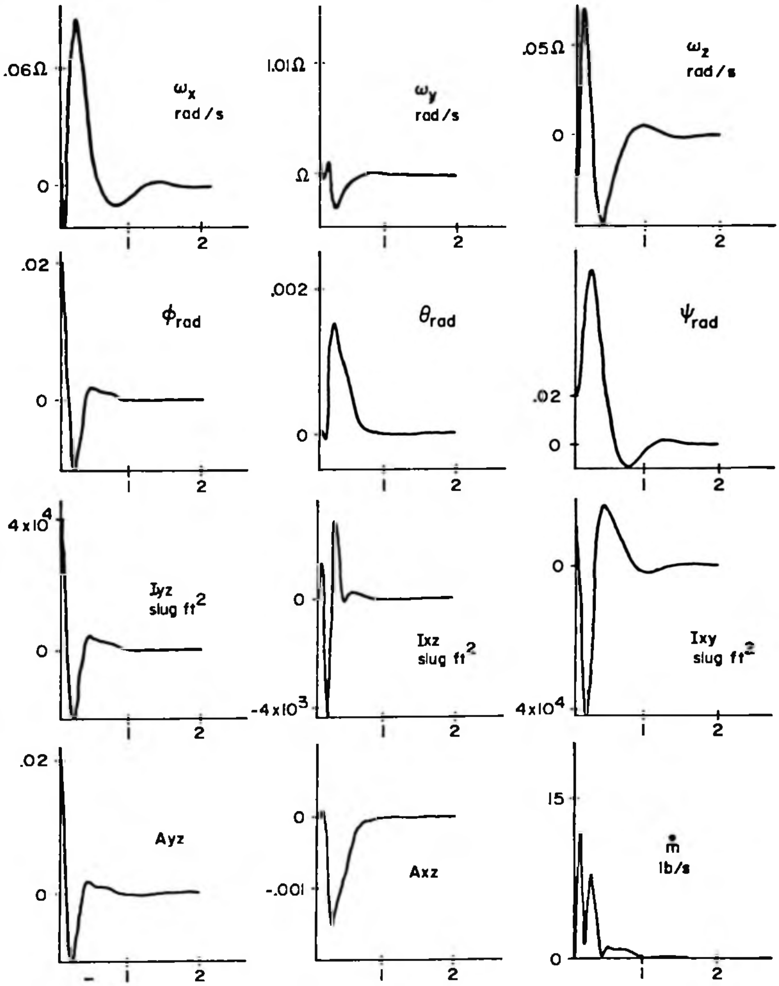


Figure 4-7. System Response to an Initial Attitude Error $\psi = \phi = \theta = 0.02$ rad.



(ALL ABSCISSAS ARE TIME IN ORBITS)

Figure 4-8. System Response to an Initial Attitude Error
$\psi = \phi = 0.02$, $\theta = 0$ rad.

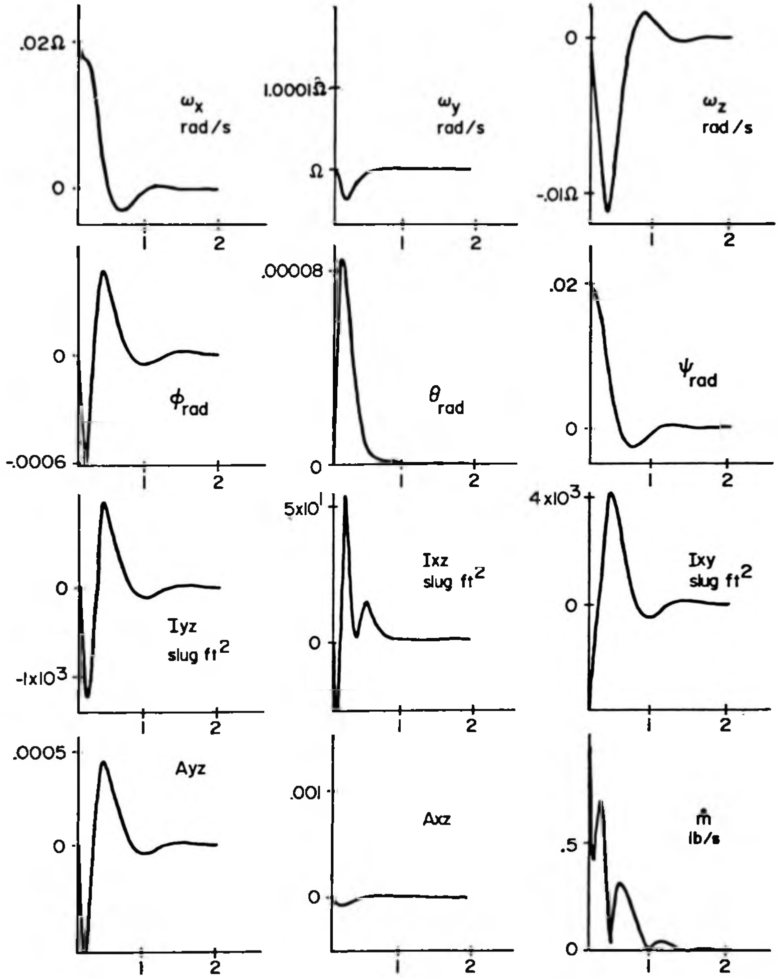


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Figure 4-9. System Response to an Initial Attitude Error $\psi = 0.02$, $\theta = \phi = 0$ rad.

ψ error increases initially and then subsequently subsides. As expected from the linear analysis, the motion characterized by ψ has the slowest response and is the least damped.

Initial Angular Velocities

Initial angular velocities will be expected at injection and will approximate the type error which occurs if a large piece of rotating machinery is suddenly stopped. Figures 4-10, 4-11, and 4-12 show the response of the system to various initial angular velocities. Again the system has essentially returned to its equilibrium position by the end of one orbit with the possible exception of motion about the z axis.

Initial External Moments

In most instances the external moments acting on the space station will be periodic in nature. Even the aerodynamic torques will be periodic because of variations in atmospheric density between the day and night portions of the orbit. Nevertheless a constant moment was selected to characterize the disturbing moment since this offers a more direct comparison with the other disturbances under consideration. The system response for various initial moments is given in Figures 4-13, 4-14, and 4-15. The major problem associated with such constant or slowly varying moments are the associated attitude errors which result.

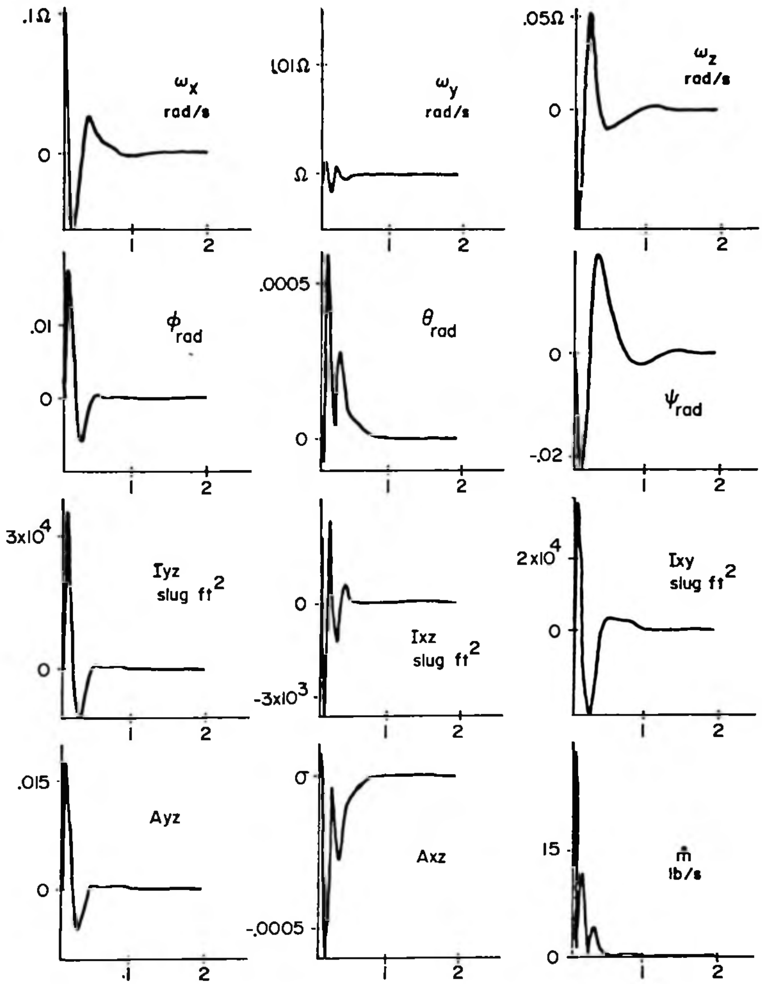

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Figure 4-10. System Response to an Initial Angular Velocity $\omega_x = 0.1\Omega$, $\omega_y = \Omega$, $\omega_z = 0$ rad/s.

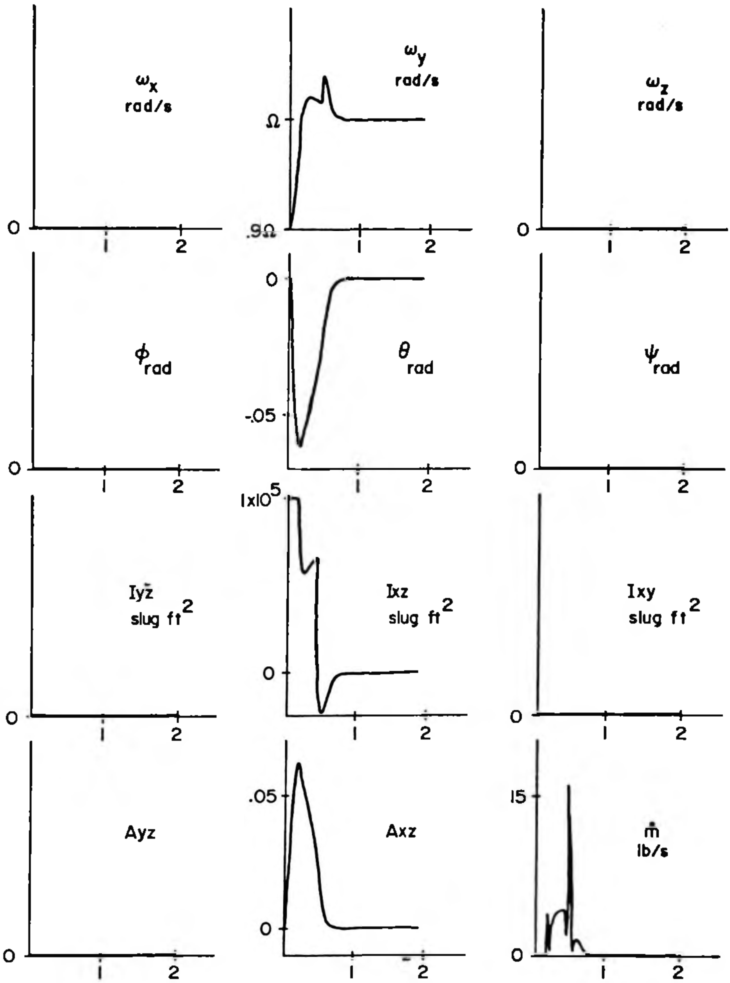

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Figure 4-11. System Response to an Initial Angular Velocity $\omega_y = 0.9\Omega$, $\omega_x = \omega_z = 0$ rad/s.

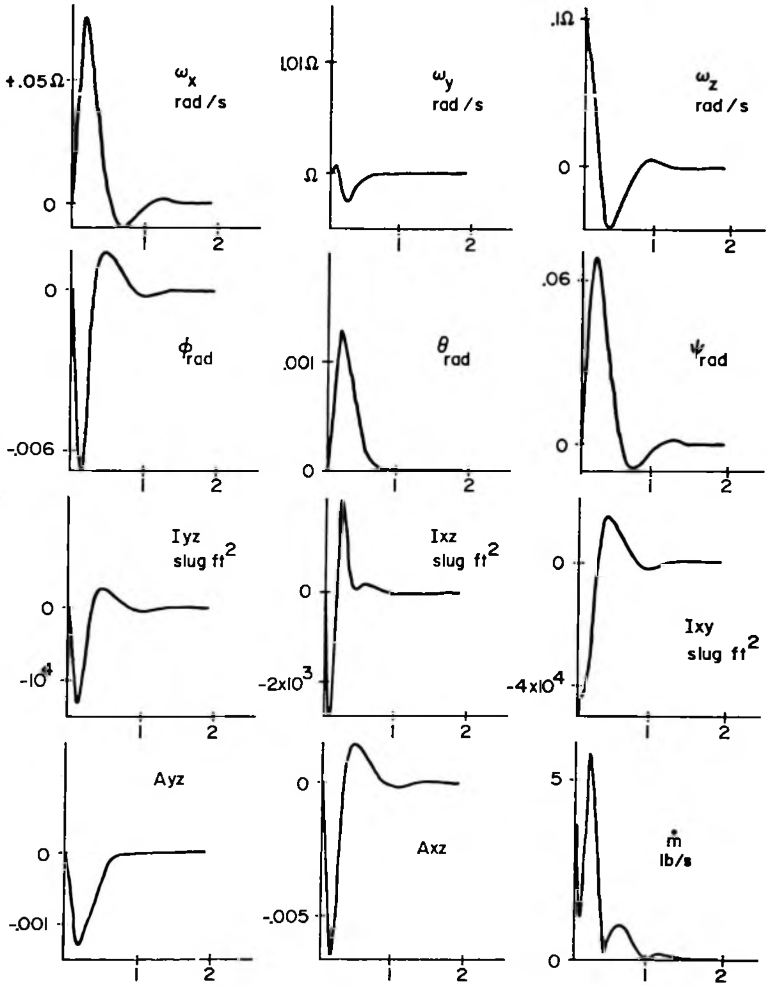

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Figure 4-12. System Response to an Initial Angular Velocity $\omega_z = 0.1\Omega$, $\omega_y = \Omega$, $\omega_x = 0$ rad/s.

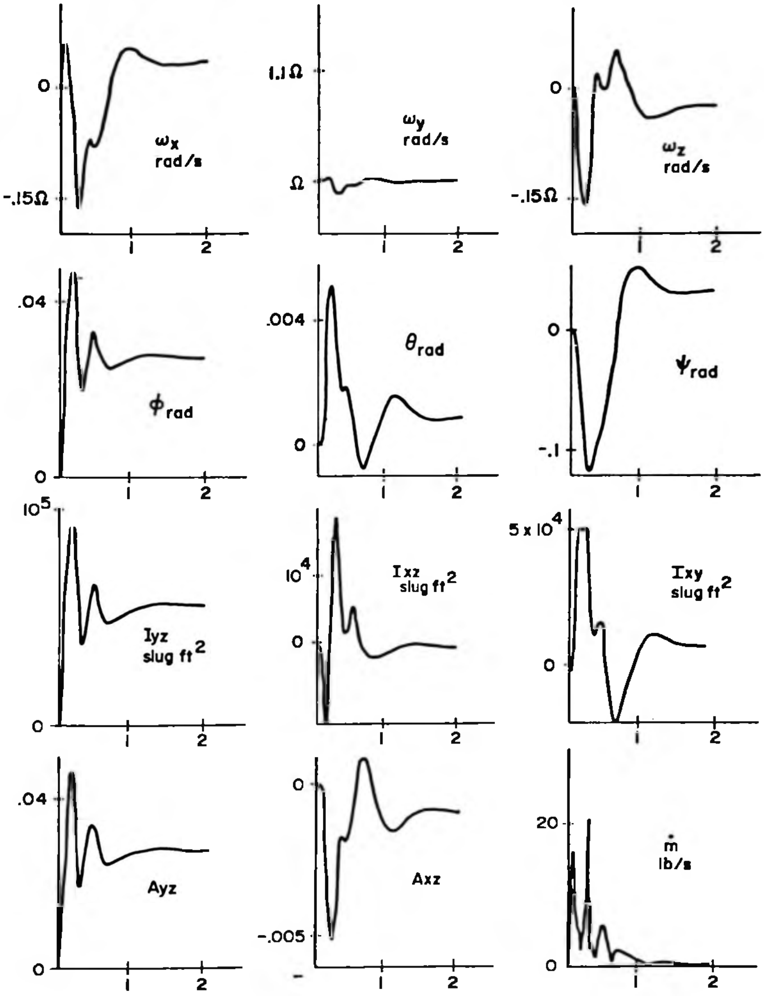

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Figure 4-13. System Response to an Initial External Moment $M_x = 0.3$, $M_y = M_z = 0$ ft-lb.

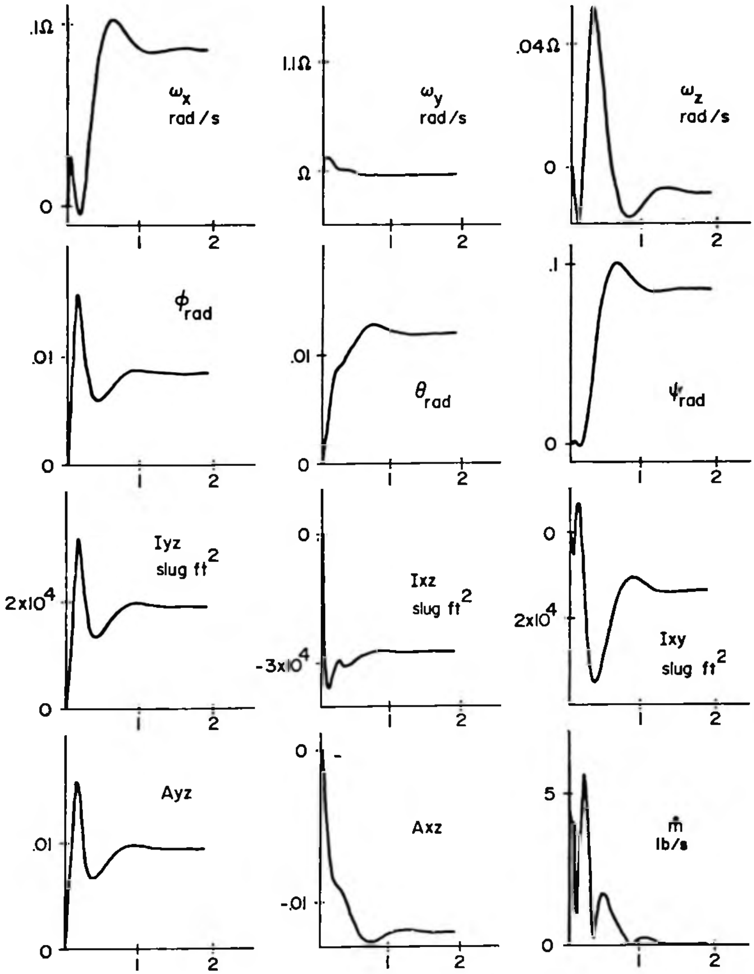

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Figure 4-14. System Response to an Initial External Moment $M_z = 0.03$, $M_x = M_y = 0.1$ ft-lb.

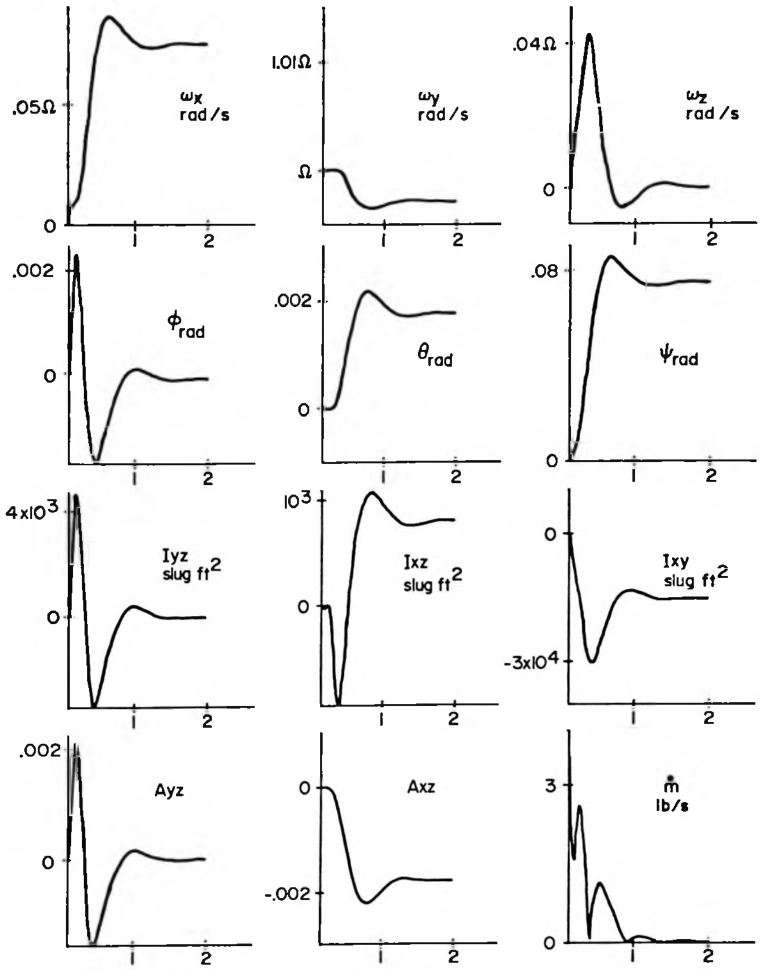

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Figure 4-15. System Response to an Initial External Moment $M_z = 0.03$, $M_x = M_y = 0$ ft-lb.

Should these errors be excessive for the mission requirements under study, methods exist for reducing them; for example, shifting the mass center or introducing a more elaborate control equation.

Previously, conditions were recognized where disturbance magnitudes may temporarily exceed the system capability. Requirements may also exist for a more rapid system response than has been demonstrated. In such circumstances the variable inertia system might be combined with a momentum storage device. This device would provide a high torque to counter large disturbances or allow more rapid maneuvering, while the variable inertia system provides a means for desaturating the momentum storage device without maneuvering the vehicle from its reference position.

Attitude Commands

When some vehicle attitude other than $\psi = \phi = \theta = 0$ is desired, it is necessary to calculate the reference values for ω_R and A_R as expressed in equation (3-18). Even when the external torques other than gravity torques are zero or may be neglected, $\psi_R\ \phi_R\ \theta_R$ associated with ω_R and A_R through equations (2-2) and (2-5) will not equal the desired attitude, $\psi_D\ \phi_D\ \theta_D$. The manner for calculating the reference values is as follows.

1. Observe that when the vehicle is in its equilibrium position the principal axes will be aligned with the orbital axes; that is, the vehicle products of inertia with respect to the orbital axes equal zero.

2. Define the desired attitude $\psi_D \; \phi_D \; \theta_D$.

3. Calculate $[A_D]$ from equation (2-2).

4. Calculate the products of inertia about the body axes from

$$J_{\ell m} = \sum_k \sum_j a_{\ell j} \, a_{mk} \, I_{jk} = 0 \tag{4-6}$$

where $a_{\ell j}$ and a_{mk} are the direction cosines for the ℓ and m orbital axes, $J_{\ell m}$ are the products of inertia with respect to the orbital axes and

$$[a_D] = [A_D]^{-1} \quad . \tag{4-7}$$

5. Substitute the calculated values of I_{jk} and the relationships of equations (2-2) and (2-5) into equation (3-18) and calculate $\psi_R \; \phi_R \; \theta_R$ from the implicit relationships thus formed.

6. Use the reference Euler angles $\psi_R \; \phi_R \; \theta_R$ to calculate $\{\omega_R\}$ and $[A_R]$ from equations (2-2) and (2-5).

Using small angle assumptions and the control equation (3-21) results in the following equations:

$$\psi_R = \frac{-\left[I_{xx} - I_{yy} + b_{11}\Omega \right] \psi_D - b_{13}\Omega \left[1 - \dfrac{a_{32} + I_{yy} - I_{zz}}{a_{32}} \right] \phi_D}{b_{11}\Omega} \quad , \tag{4-8}$$

$$\theta_R = \frac{\left[-\left(I_{zz} - I_{xx} \right) + a_{21} \right]}{a_{17}} \; \theta_D \quad , \tag{4-9}$$

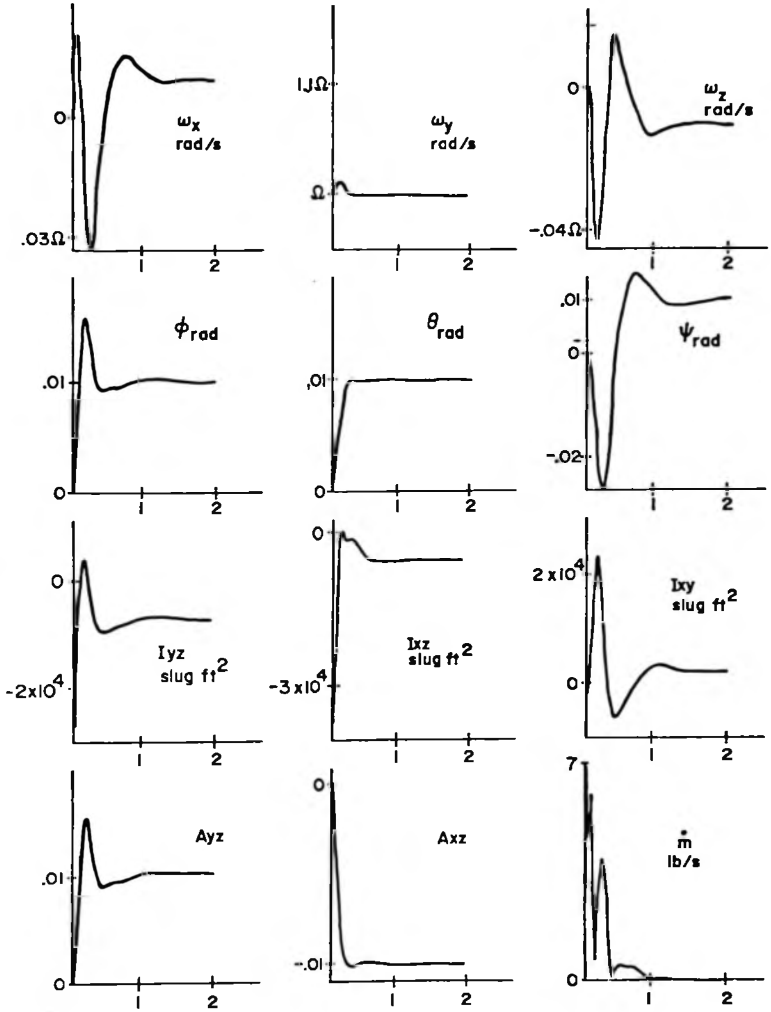

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Figure 4-16. System Response to an Attitude Command $\psi_D = \phi_D = \theta_D = 0.01$ rad.

CHAPTER V

SUMMARY

The problem considered in this research is that of developing an improved method for attitude control of satellite vehicles. Every attitude control system is based on two principles: the application of external torques to the vehicle and an exchange of angular momentum among vehicle components; the problem is one of applying these principles uniquely or more effectively than in the past. At present the most promising direction for research in new methods for control of satellite vehicles is the development of active onboard systems which react with one of the earth's potential fields.

The concept of controlling a satellite within a gravitational field by actively varying its inertia is applied to a vehicle which is in part rigid and in part fluid. Specifically, the fluid is contained in tanks and lines within the vehicle and is transferred among these tanks to control vehicle attitude.

Results and Conclusions

Important results are the formulation of equations of motion for vehicles with liquids flowing among tanks of arbitrary size and location, the development of vehicle configurations which permit the inertia to be varied effectively and yet are consistent with typical mission requirements, and the development of a configuration and control system for a representative mission to the extent necessary to demonstrate that the concept can be employed successfully for the control of large manned earth satellites.

In Chapter II equations are developed for a vehicle consisting of a system of particles which move in a generally prescribed fashion. These equations are then specialized to apply to a rigid vehicle containing a constant amount of fluid which is transferred among tanks on the vehicle. Various simplifications and specialized forms of the equations are derived to facilitate the vehicle configuration and control system development. The relationships between liquid mass in the tanks and inertia, which result from the liquid transfer logic and the physical constraints of the configuration, are formulated in Chapter III. Concepts and requirements for control of large earth orbiting satellites, typically a manned scientific space station, are presented; from these, a specific vehicle configuration, propellant transfer logic, and control system are developed. This development shows that the concept is realizable in a manner consistent with the practical considerations of configuration design and that a system can be devised which is simple enough to be competitive with passive systems for control in terms of lifetime and reliability.

Although not studied in detail, modifications and alternatives to the configuration are suggested which could result in further simplification of the system. In Chapter IV a particular vehicle is selected for analysis to show that, with notable exceptions, the inertia variation reasonably developed from propellants on board the vehicle is sufficient to overcome the major internal and external disturbances encountered during the mission.

The most important contribution of this research is the introduction of the method of fluid transfer as a means for satellite control and the development of configurations and fluid transfer logic that successfully apply this method to a large class of satellite vehicles.

Areas for Further Research

Three general areas are suggested for further research: the development of other vehicle configurations, the development of other methods for control, and the study of other initial or reference conditions.

A passively stable attitude was considered, where $I_{yy} > I_{xx} > I_{zz}$. This study can be expanded to consider other relationships among the moments of inertia; that is, different orders in the inequality, conditions where two or more of the moments of inertia are equal, and particularly those relationships which permit a larger region of maneuverability. Large initial angular displacements or rates can be investigated. Such conditions imply a more detailed development of the control system and are more representative of small satellites than large space stations.

Flow in the lines is not considered in the configuration development given in Chapter III and Chapter IV although the necessary formulation to describe these effects is developed in Chapter II. A detailed analysis would include these effects, but more important is the fact that arrangements of the liquid lines may be developed which use these effects as a momentum exchange device to induce more rapid vehicle response.

If the center of mass is allowed to move within the vehicle, then simpler configurations with fewer tanks can be developed. The vehicle equations of motion for configurations with variable center of mass are developed in Chapter II. Properly optimized, such configurations should be more desirable from weight and similar considerations and possibly may provide more effective control, particularly when aerodynamic forces can be utilized as a result. For scientific missions where the vehicle acceleration must be maintained as near zero as possible, controlling the vehicle center of mass by fluid transfer could be an effective means of reducing linear accelerations caused by internal disturbances.

Although the example and much of the development of the equations were directed toward configurations which transfer fluid as a means of inertia variation, this is not to suggest that other types of configurations may not be equally important, particularly for small satellites. As discussed in Chapter I, the concept of active inertia management is being considered for configurations having extensible booms [18, 19, 20, 21]. For such configurations, much wider variations in the elements of the inertia matrix are possible, and larger changes in equilibrium attitudes appear to be a reasonable goal under these conditions.

Hybrid systems consisting of control moment gyros and a fluid transfer mechanism are potentially a highly versatile approach; the fluid transfer mechanism provides for desaturation of the gyros and control of linear accelerations while the control moment gyros provide rapid response and maneuverability.

APPENDIX A

NUMERICAL SOLUTION OF THE NONLINEAR EQUATIONS

This appendix describes the approach and procedure for the numerical solution of the nonlinear equations for the vehicle, (2-49), the control system, (3-18), and the associated kinematics, (2-2), (2-6), and (2-50). Equations for the condition where the vehicle tanks are completely filled or emptied are approximated by placing limits on the individual products of inertia and setting the corresponding inertia derivatives equal to zero when these limits are reached. The mass flow rate of propellants is calculated from equation (3-14) and the relationship

$$\dot{m} = \sum_{i=1}^{4} \text{positive } \dot{m}_i \quad , \qquad \text{(A-1)}$$

where $\dot{m}$ is the total mass flow rate in each propellant (fuel or oxidizer) system. A limit is not imposed upon the propellant flow rate $\dot{m}$.

Integration of the equations may be reduced to the integration of two variables in the form

$$\dot{\bar{\omega}} = f(\bar{\omega}, \bar{A}) \tag{A-2}$$

and

$$\dot{\bar{A}} = f(\bar{\omega}, \bar{A}) \quad , \tag{A-3}$$

or

$$\dot{\bar{\omega}} = f(\bar{\omega}, \bar{\theta}) \tag{A-4}$$

and

$$\dot{\bar{\theta}} = f(\bar{\omega}, \bar{\theta}) \quad , \tag{A-5}$$

where $\bar{\theta}$ represents the three Euler angles ψ, ϕ, θ, and $\bar{A}$ represents the third column, A_{jz} , of equation (2-2). If $\bar{A}$ is chosen as one of the two variables to be integrated, the normality constraint

$$A_{xz}^2 + A_{yz}^2 + A_{zz}^2 = 1 \tag{A-6}$$

must be observed, since the method of integration is approximate. If, however, $\bar{\theta}$ is chosen and $\bar{A}$ is calculated from equation (2-2), then the normality of $\bar{A}$ is preserved. To facilitate the solution, the control equation (3-18) is rewritten in a form to yield a 3×3 inertia matrix rather than a column matrix of the elements. This form is

$$
\begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}
=
\begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} & g_{15} & g_{16} & g_{17} & g_{18} & g_{19} \\ g_{21} & g_{22} & g_{23} & g_{24} & g_{25} & g_{26} & g_{27} & g_{28} & g_{29} \\ g_{31} & g_{32} & g_{33} & g_{34} & g_{35} & g_{36} & g_{37} & g_{38} & g_{39} \end{bmatrix}
\begin{bmatrix} A_{xz} - A_{xzR} & 0 & 0 \\ A_{yz} - A_{yzR} & 0 & 0 \\ A_{zz} - A_{zzR} & 0 & 0 \\ 0 & A_{xz} - A_{xzR} & 0 \\ 0 & A_{yz} - A_{yzR} & 0 \\ 0 & A_{zz} - A_{zzR} & 0 \\ 0 & 0 & A_{xz} - A_{xzR} \\ 0 & 0 & A_{yz} - A_{yzR} \\ 0 & 0 & A_{zz} - A_{zzR} \end{bmatrix}
$$

$$
+ \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} & h_{15} & h_{16} & h_{17} & h_{18} & h_{19} \\ h_{21} & h_{22} & h_{23} & h_{24} & h_{25} & h_{26} & h_{27} & h_{28} & h_{29} \\ h_{31} & h_{32} & h_{33} & h_{34} & h_{35} & h_{36} & h_{37} & h_{38} & h_{39} \end{bmatrix}
\begin{bmatrix} \omega_x - \omega_{xR} & 0 & 0 \\ \omega_y - \omega_{yR} & 0 & 0 \\ \omega_z - \omega_{zR} & 0 & 0 \\ 0 & \omega_x - \omega_{xR} & 0 \\ 0 & \omega_y - \omega_{yR} & 0 \\ 0 & \omega_z - \omega_{zR} & 0 \\ 0 & 0 & \omega_x - \omega_{xR} \\ 0 & 0 & \omega_y - \omega_{yR} \\ 0 & 0 & \omega_z - \omega_{zR} \end{bmatrix}
$$

$$
+ \begin{bmatrix} I_{xxV} & I_{xyV} & I_{xzV} \\ I_{yxV} & I_{yyV} & I_{yzV} \\ I_{zxV} & I_{zyV} & I_{zzV} \end{bmatrix} \qquad \text{(A-7)}
$$

where the elements of the g and h matrix correspond to elements of the a and b matrix of equation (3-18); for example, $g_{11} = g_{12} = g_{13} = 0$, $g_{14} = g_{21} = 0$, $g_{15} = g_{22} = a_{12}$. This equation is written in a more compact notation as

$$[I] = [g]\left[\begin{array}{c|c|c} A - A_R & 0 & 0 \\ \hline 0 & A - A_R & 0 \\ \hline 0 & 0 & A - A_R \end{array}\right]$$

$$+ \quad [h]\left[\begin{array}{c|c|c} \omega - \omega_R & 0 & 0 \\ \hline 0 & \omega - \omega_R & 0 \\ \hline 0 & 0 & \omega - \omega_R \end{array}\right] + \left[I_V\right] \quad . \qquad (A\text{-}8)$$

Or, since g and h are constants,

$$[\dot{I}] = [g]\left[\begin{array}{c|c|c} \dot{A} & 0 & 0 \\ \hline 0 & \dot{A} & 0 \\ \hline 0 & 0 & \dot{A} \end{array}\right] + [h]\left[\begin{array}{c|c|c} \dot{\omega} & 0 & 0 \\ \hline 0 & \dot{\omega} & 0 \\ \hline 0 & 0 & \dot{\omega} \end{array}\right] \quad . \qquad (A\text{-}9)$$

To develop equation (A-2), equation (A-9) is substituted into the vehicle equation

$$[I]\{\dot{\omega}\} + [\tilde{\omega}][I]\{\omega\} + [\dot{I}]\{\omega\} = 3\Omega^2[\tilde{A}][I]\{A\} + \{T\} \qquad (A\text{-}10)$$

to yield

$$[I]\{\dot{\omega}\} + [\tilde{\omega}][I]\{\omega\} + [g]\left[\begin{array}{c|c|c} \dot{A} & 0 & 0 \\ \hline 0 & \dot{A} & 0 \\ \hline 0 & 0 & \dot{A} \end{array}\right]\{\omega\}$$

$$+ [h]\left[\begin{array}{c|c|c} \dot{\omega} & 0 & 0 \\ \hline 0 & \dot{\omega} & 0 \\ \hline 0 & 0 & \dot{\omega} \end{array}\right]\{\omega\} = 3\Omega^2 [\tilde{A}][I]\{A\} + \{T\} \quad . \qquad \text{(A-11)}$$

The solution for $\{\dot{\omega}\}$ is obtained by employing the identify

$$\left[\begin{array}{c|c|c} \dot{\omega} & 0 & 0 \\ \hline 0 & \dot{\omega} & 0 \\ \hline 0 & 0 & \dot{\omega} \end{array}\right]\{\omega\} = \left[\begin{array}{c} \lceil \omega_x \rfloor \\ \hline \lceil \omega_y \rfloor \\ \hline \lceil \omega_z \rfloor \end{array}\right]\{\dot{\omega}\} \qquad \text{(A-12)}$$

where

$$\left[\begin{array}{c} \lceil \omega_x \rfloor \\ \hline \lceil \omega_y \rfloor \\ \hline \lceil \omega_z \rfloor \end{array}\right] = \begin{bmatrix} \omega_x & 0 & 0 \\ 0 & \omega_x & 0 \\ 0 & 0 & \omega_x \\ \omega_y & 0 & 0 \\ 0 & \omega_y & 0 \\ 0 & 0 & \omega_y \\ \omega_z & 0 & 0 \\ 0 & \omega_z & 0 \\ 0 & 0 & \omega_z \end{bmatrix} \qquad \text{(A-13)}$$

Substituting this relationship into equation (A-11) yields

$$[I]\{\dot{\omega}\} + [h]\begin{bmatrix} \searrow\omega_x\searrow \\ \hline \searrow\omega_y\searrow \\ \hline \searrow\omega_z\searrow \end{bmatrix}\{\dot{\omega}\} = 3\Omega^2[\tilde{A}][I]\{A\} + \{T\} - [\tilde{\omega}][I]\{\omega\}$$

$$- [g]\left[\begin{array}{c|c|c} \dot{A} & 0 & 0 \\ \hline 0 & \dot{A} & 0 \\ \hline 0 & 0 & \dot{A} \end{array}\right]\{\omega\} = \{C\} \tag{A-14}$$

or

$$\{\dot{\omega}\} = \left[[I] + [h]\begin{bmatrix} \searrow\omega_x\searrow \\ \hline \searrow\omega_y\searrow \\ \hline \searrow\omega_z\searrow \end{bmatrix}\right]^{-1}\{C\} \quad . \tag{A-15}$$

When $\dot{\bar{A}}$ is expressed in terms of $\bar{A}$ and $\bar{\omega}$, employing the relationship

$$\{\dot{A}\} = [\tilde{A}]\{\omega - \Omega\} \tag{A-16}$$

$$\{\dot{A}\} = \begin{bmatrix} 0 & -c\psi c\theta & s\psi s\theta \\ \begin{matrix} -s\phi s\psi s\theta \\ +c\phi c\theta \end{matrix} & \begin{matrix} c\phi s\psi c\theta \\ -s\phi s\theta \end{matrix} & c\phi c\psi s\theta \\ \begin{matrix} -c\phi s\psi s\theta \\ -s\phi c\theta \end{matrix} & \begin{matrix} -s\phi s\psi c\theta \\ -c\phi s\theta \end{matrix} & -s\phi c\psi s\theta \end{bmatrix}\begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} \quad , \tag{A-17}$$

and [I] is expressed in terms of $\bar{A}$ and $\bar{\omega}$ from equation (A-8), then $\dot{\bar{\omega}}$ can be expressed explicitly as a function of $\bar{A}$ and $\bar{\omega}$. By introducing equation (2-2), $\bar{A}$ can be expressed in terms of $\bar{\theta}$; and $\dot{\bar{\omega}}$ can be written as an explicit function of $\bar{\omega}$ and $\bar{\theta}$.

In the previous development it was not necessary to integrate $\dot{I}$ to calculate I. However, this condition is based on the assumption that the inertia commanded by the control equation (A-8) is close enough to the actual inertia to be considered equal. When the physical limit defined by the tank size or available propellant is reached, this condition no longer holds. A detailed description of the fluid transfer mechanism and the manner in which the control is implemented is required to formulate exactly the relationships that occur when the inertia commanded by the control equation exceeds the physical capability of the vehicle to provide this inertia. As a study of this condition is not the purpose of this research, an approximation is used which is adequate to represent this condition for transients of small duration. In this approximation, limits are defined for the products of inertia I_{jk} and the two control signals,

$$[\alpha] = [g] \left[\begin{array}{c|c|c} A-A_R & 0 & 0 \\ \hline 0 & A-A_R & 0 \\ \hline 0 & 0 & A-A_R \end{array}\right] , \qquad \text{(A-18)}$$

and

$$[\beta] = [h] \left[\begin{array}{c|c|c} \omega-\omega_R & 0 & 0 \\ \hline 0 & \omega-\omega_R & 0 \\ \hline 0 & 0 & \omega-\omega_R \end{array}\right] \quad , \tag{A-19}$$

where

$$[I] = [\alpha] + [\beta] + [I_V] \quad . \tag{A-20}$$

When

$$I_{jk} \geq I_{jkL} \quad ,$$

$$\alpha_{jk} \geq \alpha_{jkL}$$

or

$$\beta_{jk} \geq \beta_{jkL}$$

in equation (A-7), then correspondingly

$$I_{jk} = I_{jkL} \quad , \qquad \dot{I}_{jk} = 0 \tag{A-21}$$

$$\alpha_{jk} = \alpha_{jkL} \quad , \qquad \dot{\alpha}_{jk} = 0$$

$$\beta_{jk} = \beta_{jkL} \quad , \qquad \dot{\beta}_{jk} = 0 \quad .$$

The equations are integrated using a four-pass Runge-Kutta method [32]. No instabilities associated with the integration scheme were noted; and when compared with exact solutions for rigid body motion, $[g] = [h] = [0]$, errors were negligible for several orbits. A flow diagram for the calculations is presented in Figures A-1a through A-1e.

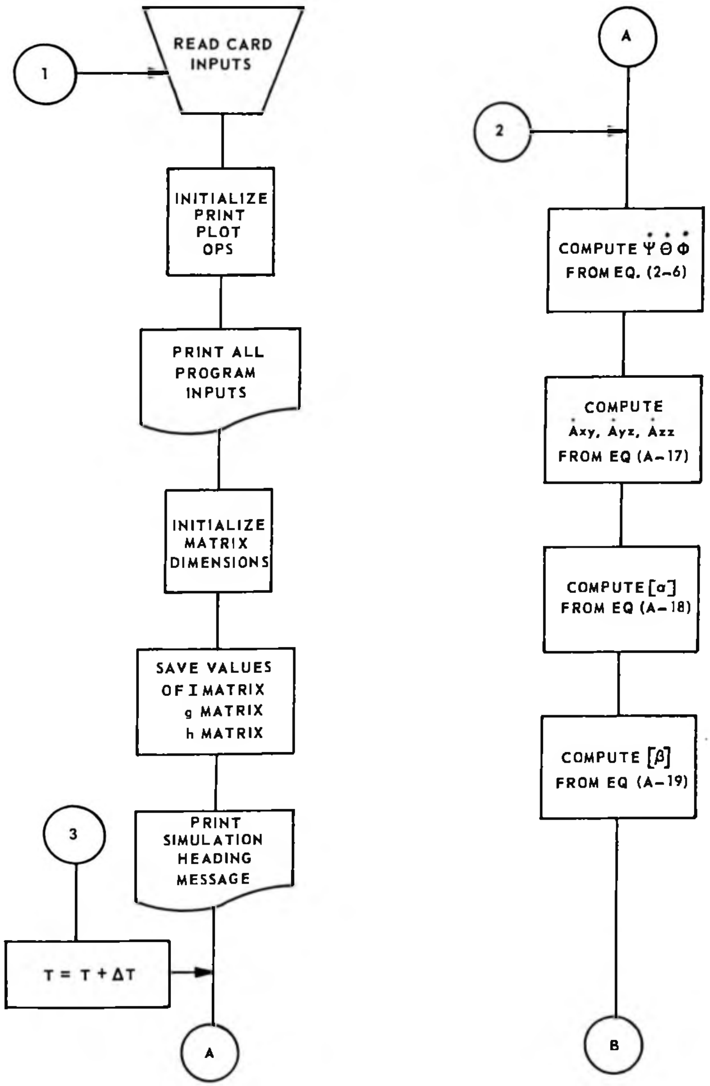


Figure A-1a. Computer Flow Diagram.

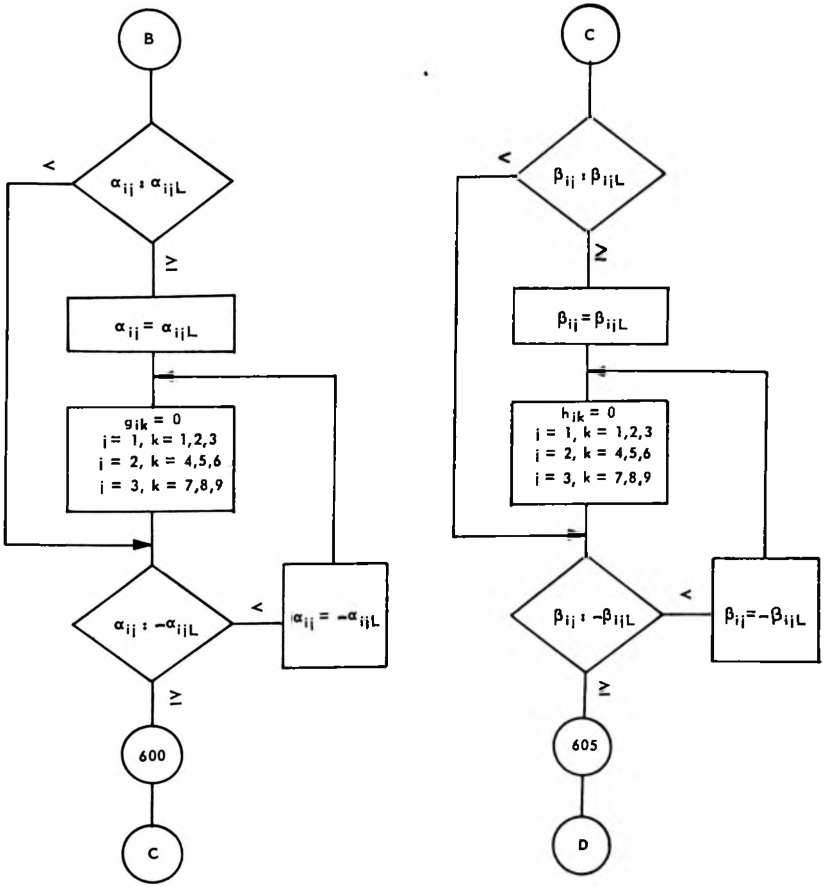


Figure A-1b. Computer Flow Diagram.

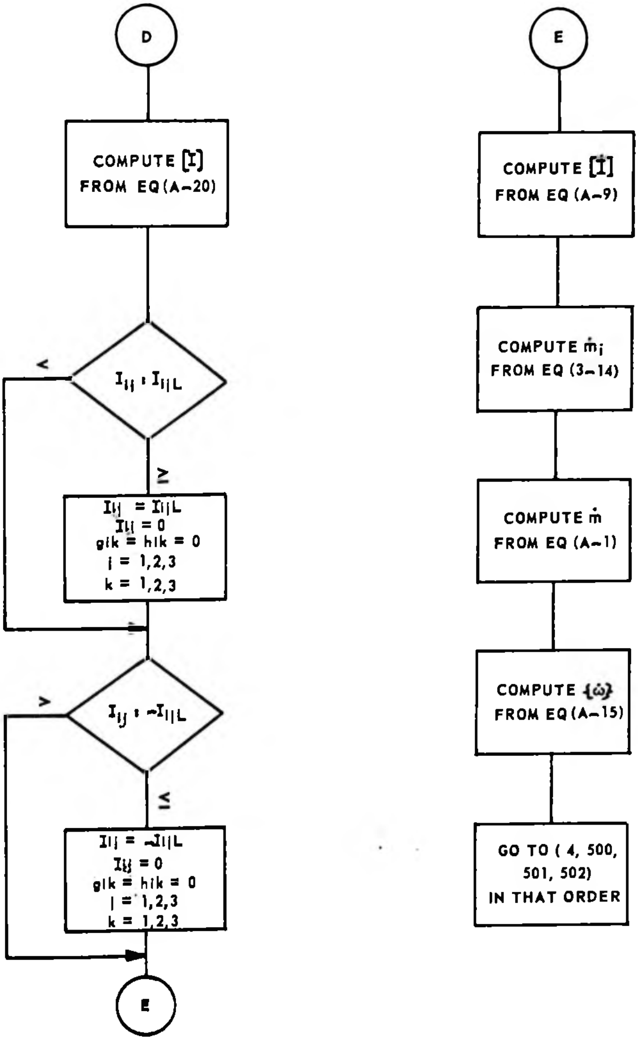
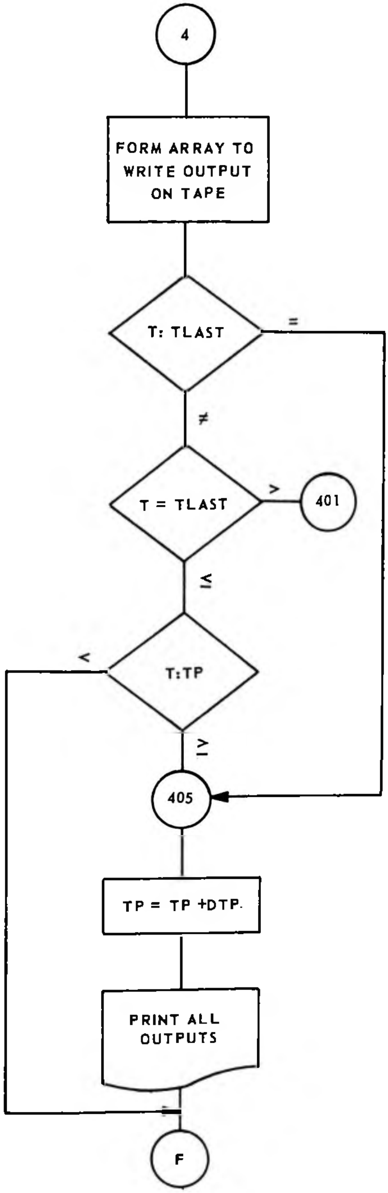


Figure A-1c. Computer Flow Diagram.



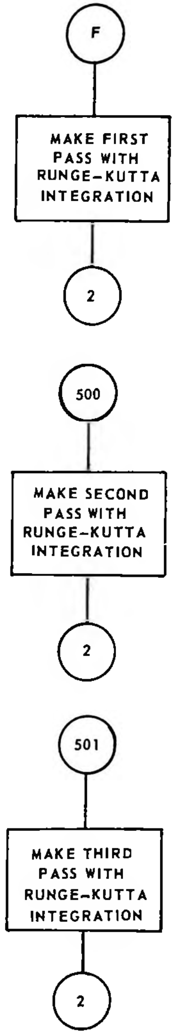


Figure A-1d. Computer Flow Diagram.

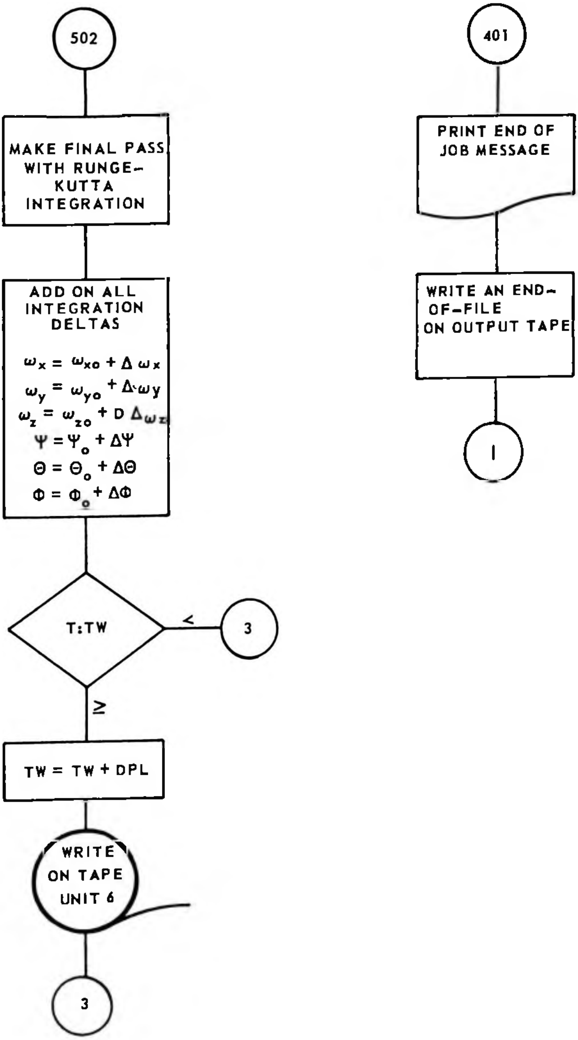


Figure A-1e. Computer Flow Diagram.

APPENDIX B

LIQUID FLOW RATES

The assumption was made earlier that the quantities of mass in the tanks and the mass flow rates between tanks commanded by the control system were close enough to the true mass and flow rates to be considered identical when the tanks were not completely filled or emptied. The following calculation will show this to be a reasonable assumption.

Assume a liquid transfer system similar to that shown in Figure 3-1 except that a pressure differential among the tanks will be used as the method of transferring liquid rather than a pump. Next assume that the liquid is being transferred among the tanks in a sinusoidal fashion at a frequency less than four times orbital frequency. This is in agreement with the linear stability analysis of the system discussed previously. Select cylindrical pipes 4 in. in diameter to transfer the liquid and assume the amplitude of the oscillation is just large enough to fill and empty completely each tank once each cycle.

The following is a tabulation of the data used in the problem:

Total liquid mass: 30,000 lb.

Liquid mass per tank: 3750 lb.

Length of pipe from tank to origin: 25 ft.

Density of LOX: 71 lb/ft^3.

Kerosene: 40 to 50 lb/ft^3.

Water: 62.4 lb/ft^3.

The example fluid will have the approximate characteristics of water

ρ = density = 2 slugs/ft^3,

μ = dynamic viscosity = 3×10^{-5} lb sec/ft^2,

and

ν = kinematic viscosity = 1.5×10^{-5} ft^2/sec.

The problem may be considered to consist of four sets of tanks similar to the set shown in Figure 3-1. During one oscillation the mass in each tank varies ±3750 lb and the peak flow rate becomes

$$\pm 4\Omega\,(3750) = \pm 15 \text{ lb/sec.}$$

where

$$4\Omega = 0.004 \text{ radians/sec.}$$

The pressure differential required to produce this flow rate may be calculated from the basic equations of fluid dynamics [33]. From the equation for continuity,

$$\dot{m} = \rho VA = \frac{15}{32.2} = 2V\frac{\pi}{4}\left(\frac{1}{3}\right)^2 \quad , \quad V = 2.67 \text{ ft/sec}$$

where A = area of pipe. The Reynolds number for the flow is

$$R = \frac{VD}{\nu} = \frac{2.67}{1.5\times 10^{-5}}\left(\frac{1}{3}\right) = 60,000 \quad ,$$

indicating turbulent flow. From empirical formulas for turbulent pipe flow, the pressure differential is:

$$P_1 - P_2 = \rho f \frac{L}{D} \frac{V_p^{\,2}}{2} + C_L \rho \frac{V_p^{\,2}}{2} \qquad ,$$

where

V_p = steady flow in pipe,

f = friction factor of pipe = 0.018,

C_L = loss coefficient of tank entrance and exit = 1.25,

and finally

$$P_1 - P_2 = \frac{2(2.67)^2}{2} \left[(0.018)(50)(3) + 1.25\right]$$

$$= 7.15\ [3.95] = 28.3\ \text{lb/ft}^2 = 0.2\ \text{lb/in.}^2 \ .$$

This small number indicates a system can be reasonably designed which will not flow saturate under conditions where it does not amplitude saturate.

Next consider the transient response of the flow to a step change in pressure. The equation of motion for a step application of pressure is given by:

$$A(P_1 - P_2) - A\frac{\rho}{2}\left(f\frac{L}{D} + C_L\right)V^2 = A\rho L \frac{dV}{dt} \qquad ,$$

which may be combined with the equation for steady flow above to yield

$$t = \frac{\rho L V_p}{2(P_1 - P_2)} \ln \frac{V_p + V}{V_p - V} \qquad .$$

Observe that the velocity reaches 99 percent of its final value in less than 17 sec, and the response may be approximated by

$$V = V_p \left(1 - e^{-\tau t}\right) \qquad ,$$

where $\tau \approx 6$ sec . Compared with a period of oscillation of greater than 1570 sec, the transient response of the flow may be neglected.

APPENDIX C

APPROXIMATE MAGNITUDES OF SOME MAJOR DISTURBANCES

Crew Motions

Analyses of crew activities in a typical space station indicate that the most important crew disturbance is the "wall pushoff," in which a crewman pushes off one wall of the spacecraft and travels freely through the spacecraft to the opposite wall where he arrests his motion. Experiments have shown that a pushoff of about 21-lb sec magnitude and 1-sec duration can be considered typical [34]. Considering the period of oscillation of the vehicle is much greater than the duration of the pulse or the time between pulses, the maneuver may be approximated by two impulses spaced some Δt apart. Furthermore, assuming the impulse response of the vehicle is approximately a damped sinusoid, we observe that during the interval between impulses the vehicle response may be approximated by a straight line (Fig. C-1). Under these circumstances, elementary relationships may be used to approximate the disturbance to the space station from a wall pushoff. Assume the moment applied by the crewman acts about a principal axis; thus,

$$M = I\dot{\theta} \,,$$

$$M = F\, y/2 \,,$$

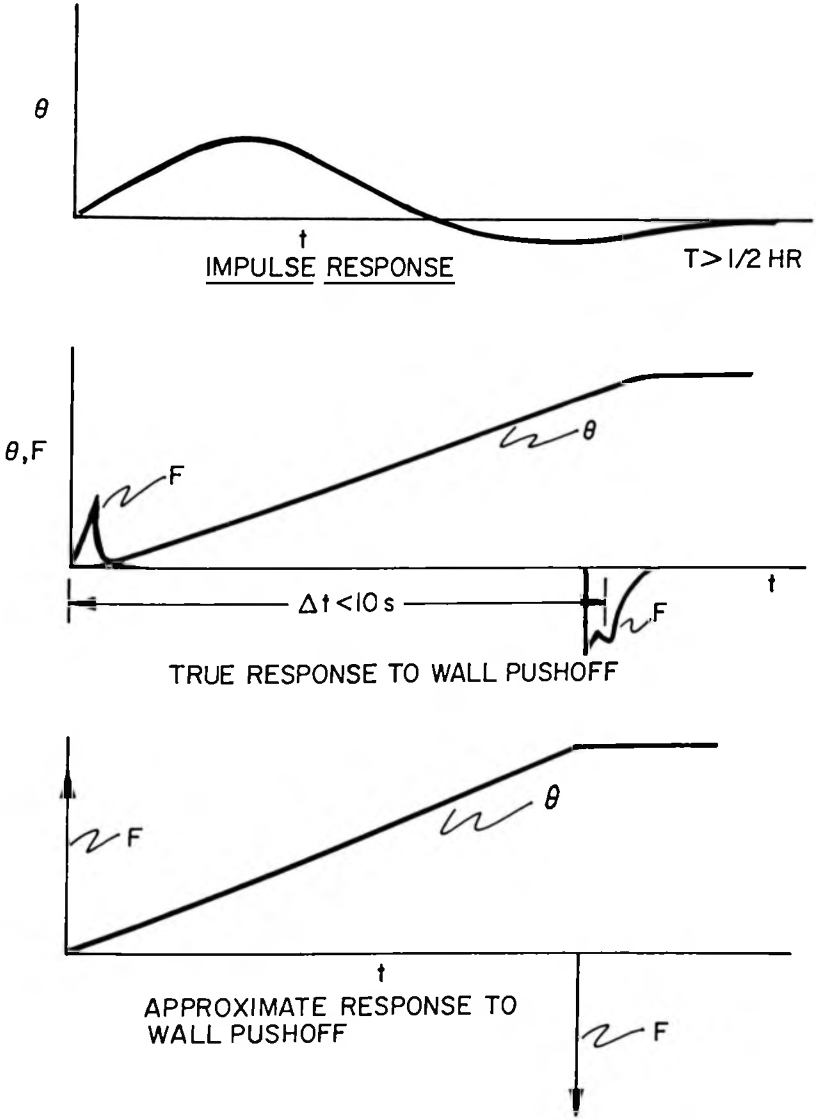


Figure C-1. Space Station Response.

$$F = m\nu ,$$

$$\Delta t = x/\nu ,$$

$$\Delta\theta = M\ \Delta t/I ,$$

$$\Delta\theta = \frac{m\ xy}{2I} ,$$

where

M = angular impulse ,

I = moment of inertia of the spacecraft ,

θ = angular rotation ,

F = linear impulse ,

m = mass of crewman ,

ν = velocity of crewman ,

Δt = time between impulses,

$\Delta\theta$ = angular rotation of spacecraft caused by wall pushoff ,

x, y = dimensions of the crew area.

The following conclusions can be made using the above relationships:

1. The angular displacement caused by the pushoff is independent of the magnitude of the pulse.

2. The control system is ineffective in reducing angular rates during the disturbance.

3. For the dimensions of Figure 4-3, $\Delta\theta < 0.15°$.

Repositioning of Crew or Equipment

For the concept of active inertia management as a method of vehicle control to be attractive, the random inertia variations caused by the position of crew and equipment must be an acceptably small percentage of the inertia variation available for control. In a static condition this variation may be evaluated from the basic equations for the system by assigning appropriate values to the vehicle inertia and calculating its effects on attitude error. A crew of 10 in the most adverse location within the laboratory area will produce a change in products of inertia which is less than 1.5 percent of the maximum change produced by propellant transfer. The maximum center of mass shift which can be created by a crew of 10 is approximately 2 in.

Intermittent Operation of Rotating Machinery

The space station will probably contain power generating equipment or experiments involving rotating machinery. If the angular momentum of this machinery is significantly large, it must be introduced into the vehicle equations. With reference to equation (2-21) for the case of an axially symmetric rotating body, the integral $\int \bar{\rho} \times \dot{\bar{\rho}}\, dm$ becomes $\int \bar{\rho} \times (\bar{s} \times \bar{\rho})\, dm$ where $\bar{s}$ is the angular velocity of the rotating body relative to the axis system. In a manner similar to equations (2-17) through (2-20), this integral reduces to

$$\int \bar{\rho} \times (\bar{s} \times \bar{\rho})\, dm = \bar{\bar{I}}_R \cdot \bar{s} = \bar{R} \quad ,$$

where $\bar{\bar{I}}_R$ is the inertia dyadic of the rotating machinery, and $\bar{R}$ is the angular momentum of the rotating machinery relative to the axis system. The two integrals of equation (2-24) can be expressed as

$$\int \bar{\rho} \times \ddot{\bar{\rho}}\, dm + \int \bar{\omega} \times (\bar{\rho} \times \dot{\bar{\rho}})\, dm = \dot{\bar{R}} + \bar{\omega} \times \bar{R} \quad .$$

If the introduction of $\bar{R}$ is carried through the perturbation equations, the following terms are added to the left side of equations (2-59), (2-60), and (2-61):

$$\text{Equation (2-59)} + \delta\dot{R}_x + R_z \delta\dot{\theta} - R_y \delta\dot{\psi} + R_y \Omega \delta\phi - \Omega \delta R_z \quad ,$$

$$\text{Equation (2-60)} + \delta\dot{R}_y + R_x \delta\dot{\psi} - R_x \Omega \delta\phi \quad R_z \delta\dot{\phi} - R_z \Omega \delta\psi \quad ,$$

and

$$\text{Equation (2-61)} + \delta\dot{R}_z + R_y \delta\dot{\phi} + R_y \Omega \delta\psi - R_x \delta\dot{\theta} + \Omega \delta R_x \quad .$$

Comparing terms leads to the assumption that if

$$R_y << \Omega I_{yy} \quad , \quad R_x << \Omega I_{xy\,max} \quad , \text{ and } \quad R_z << \Omega I_{yz}\ max \quad ,$$

then the effects of the rotating machinery may be ignored. Even when the effects are not ignorable, the system may still be controllable. In such case the sudden stopping or starting of machinery can be formulated in the following manner. The vehicle equations of motion before and after stopping (for example) are formulated in the manner outlined above. Since the momentum of the vehicle and the center of mass [equation (2-21)] is unchanged for a sudden (step) change in $\int \bar{\rho} \times \dot{\bar{\rho}}\, dm$ or $\bar{\bar{R}}$, a step change in $\bar{\bar{I}} \cdot \bar{\omega}$ may be associated with a step change in $\bar{R}$. The manner of treating $\bar{\bar{I}}$ for a step change in $\bar{R}$ will depend, of course, upon what assumptions are made concerning the relationship between actual and commanded inertia.

Aerodynamic Torques

Of all the vehicle disturbances under consideration, the aerodynamic torques have the greatest potential for exceeding the capability of the control system. however the following calculations will show that for orbits above 250 n. mi. and a reasonable center of mass location, the aerodynamic torques can be assumed to be controllable.

Considerable uncertainty still exists as to the atmospheric properties and aerodynamic coefficients for the conditions under consideration [30], but engineering estimates can be obtained without introducing the detail which would be required for a thorough analysis of the problem. For example, variations in atmospheric density with location in orbit (day or night side) and rotation of the earth's atmosphere are ignored. The normal force and the moment coefficients about the x and y axes for the configuration shown in Figure 4-2 are calculated using References 35 and 36. These coefficients are

Altitude (n. mi.)	α (deg)	C_N	C_M	C_A
100	100	5.38	-1.054	0.904
	90	5.44	-1.193	-0.027
	80	5.39	-1.066	-0.948
200	100	5.48	-1.074	0.923
	90	5.54	-1.202	-0.028
	80	5.49	-1.086	-0.966
300	100	5.52	-1.078	0.924
	90	5.59	-1.201	-0.031
	80	5.53	-1.092	-0.972
400	100	5.54	-1.080	0.925
	90	5.61	-1.201	-0.032
	80	5.55	-1.096	-0.976

The angle between the vehicle z axis and the relative wind is α ; the normal force and moment coefficients are defined as

$$C_N = \frac{2N}{\rho S V^2} ,$$

$$C_M = \frac{2M}{\rho S V^2 D} ,$$

$$C_A = \frac{2A}{\rho S V^2} ,$$

where

A = axial force,

N = normal force,

ρ = atmospheric density,

S = cross sectional area of the vehicle,

D = diameter of the vehicle,

V = velocity of the vehicle,

M = moment or torque,

and moments are calculated about the geometric center of the cylindrical portion of the vehicle. The aerodynamic torque on the vehicle may also be represented as

$$\overline{M} = \overline{F} \times \bar{d}$$

where $\bar{d}$ is the distance from the center of mass to the aerodynamic center, (the point about which the moment caused by aerodynamic forces equals zero) and $\overline{F}$ is the total force.

The aerodynamic torque is observed to be a weak function of angle of attack for small variations about 90 deg and a maximum at 90 deg. Therefore considering that the aerodynamic torque at 90 deg is representative of the torque in the attitude range of interest and that atmospheric density and relative velocity of vehicle and atmosphere are functions of altitude alone, then the aerodynamic torque may be expressed as a function of altitude and $\bar{d}$. The aerodynamic torques for the configuration in Figure 4-2 are calculated, using values for the atmospheric density from Reference 37 and the previous relationships, and presented in Figure C-2 along with the maximum torque available from the control system developed in Chapter IV. As an estimate of $\bar{d}$, note that if the mass of the satellite were uniformly distributed within its volume, $\bar{d}$ would lie in a positive direction along the z axis and be less than 3 ft in length.

Therefore, it is reasonable to conclude from Figure C-2 that aerodynamic torques will be easily controllable at altitudes above 250-n. mi. and, by carefully locating the satellite center of mass, at altitudes as low as 200 n. mi.

If large external appendages such as solar panels are a part of the configuration, the aerodynamic torques could be considerably higher and a stronger function of angle of attack. Notice however that the scheme for transferring propellants permits shifting the center of mass relative to the origin, without changing the moments or products of inertia about the origin. For example (Fig. 3-1), if equal masses of oxidizer are transferred from 1O and 4O to 3O and 2O and fuel from 3F and 2F to 1F and 4F, then the vehicle center of mass is shifted in the x direction without changing the moments or products of inertia about the vehicle origin. Thus the distance from the center of mass and the aerodynamic center can be eliminated or minimized. This transfer could be in the manner of a reballasting such as performed aboard a ship or airplane or it could be a part of the control system, utilizing the aerodynamic torques as a means of control if desired.

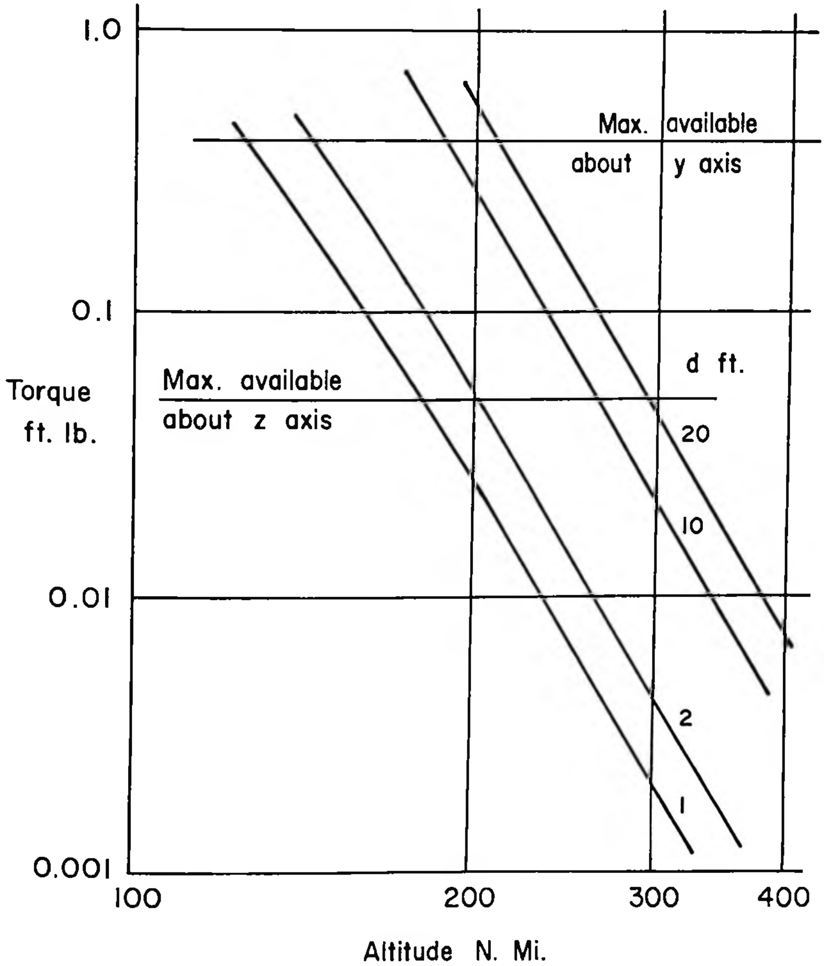


Figure C-2. Aerodynamic Torque on the Space Station in Figure 4-3.

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VITA

William Denleigh Clarke was born in Memphis, Tennessee, on June 7, 1925. He is the son of Charles K. Clarke and Belva (Massey) Clarke. On April 17, 1950, he was married to Jane Maddox. They have three sons, Matthew, Mark, and Luke, and one daughter, Margaret.

He graduated from Central High School in Memphis, Tennessee, in 1943. He attended Tulane University in New Orleans, Louisiana, for one year and Georgia Institute of Technology in Atlanta, Georgia, for three years, receiving a Bachelor of Aeronautical Engineering in 1946. He also attended the University of Washington in Seattle, Washington, for one year, receiving a Bachelor of Science in Industrial Engineering in 1949. He attended evening classes at Southern Methodist University in Dallas, Texas, from 1955 to 1959, graduating with a Master of Science in Aeronautical Engineering, and attended additional evening classes at the University of Alabama in Huntsville from 1960 to 1963. From September 1967 to September 1968, he attended the University of Alabama at Tuscaloosa as a full-time student.

In 1943, he enlisted in the U. S. Navy and was assigned to the V-12 college training program. Upon graduation, he was commissioned Ensign USNR and served as an aircraft maintenance officer at the U. S. Naval Air Station, Pensacola, Florida, until 1946. From 1946 until 1948, he was employed by

Boeing Aircraft Company in Wichita, Kansas, and Seattle, Washington, as an aerodynamicist. From 1949 to 1955, he was employed by the Coca-Cola Company in Atlanta, Georgia, and Dallas, Texas, as an industrial engineer. In 1955, he returned to the aircraft industry and was employed by Chance Vought Aircraft, Dallas, Texas, as an aerodynamicist. While at Chance Vought, he was a group supervisor responsible for the design and dynamic analysis of the automatic pilot, stability augmentation system, and manual control system for the F8U supersonic fighter aircraft. In 1960, he was employed by the Advanced Design Laboratory of the Army Ballistics Missile Agency in Huntsville, Alabama, as Chief of the Guidance Section, responsible for the development and preliminary design of guidance and control systems for battlefield missiles. In 1961, he transferred to the Astrionics Laboratory of the Marshall Space Flight Center, Huntsville, Alabama, where he is now employed. He has been responsible for the development and analysis of attitude control systems for the Saturn IB vehicle. Presently assigned to a research group, he is responsible for the development of new concepts, theories, and systems for the control of spacecraft.