

TRENDS CONCERNING FOUR MISCONCEPTIONS IN
STUDENTS' INTUITIVELY-BASED PROBABILISTIC
REASONING SOURCED IN THE HEURISTIC
OF REPRESENTATIVENESS

by

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ABSTRACT

Student difficulty in the study of probability arises in intuitively-based misconceptions derived from heuristics. One such heuristic, the one of note for this research study, is that of representativeness, in which an individual informally assesses the probability of an event based on the degree to which the event is similar to the sample from which it is selected or the degree the event is characterized by the notable features of the system from which it is derived. Four misconceptions were examined in this that study arise from this heuristic: the representativeness misconception, positive and negative recency effects, the distinction between compound and simple events, and the effect of sample size.

Furthering the research of Fischbein and Schnarch (1997), this research sought Spearman correlations between frequencies of responses to items testing for the above misconceptions and grade level of students (7th, 9th, or 11th). A significant positive Spearman correlation was found for positive and negative recency effects and a significant negative correlation was found for the effect of sample size.

Spearman correlations were also sought between correctness of student responses and perceived self-efficacy in those responses via a five-point Likert scale at each of the three grade levels. Significant positive correlations were found for positive and negative recency effects (all three grades), the distinction between compound events (7th), and the representativeness misconception (7th and 11th); significant negative correlations were found for the effect of sample size (11th) and the distinction between compound and simple events (9th and 11th).

DEDICATION

I would like to dedicate this dissertation to my wife, Amber, for all of her help in pushing me when I needed to be pushed, putting up with me and my time management issues, and, in spite of these things, loving me when I needed it the most. Without her, this dissertation would have never been realized.

LIST OF ABBREVIATIONS AND SYMBOLS

| | |
|----------|---------------------------|
| α | alpha, significance level |
| N | population size |
| n | sample population size |
| p | probability |
| r_s | Spearman correlation |

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CHAPTER I:

INTRODUCTION

Intuition is one of those terms with a variety of interpretations (Cheyne, 1997). Asked to explain his or her definition of the term *intuition*, chances are that definition will include words or phrases such as a *gut feeling*, *instinct*, *guess*, or *previous experience*. The definition may even include the concepts of *extra-sensory perception* or *ESP*, or the *sixth sense*. However viewed, humankind sees intuition as a necessary tool for survival (Eddy, 1982).

Within the research community, one definition advocated in research sees intuition as a predictive cognitive tool used to effectively find the most pragmatic strategy when undertaking a particular task (Fischbein, 1987). Another complementary definition holds that intuition predicts outcomes of certain tasks, both mathematical and otherwise in social cultural situations (Kahneman & Tversky, 1982a). Thus, researchers themselves can view intuition differently.

Specifically, within mathematics, the term intuition brings up complex queries regarding mathematics as an exact science. For instance, what role, if any, does intuition play in the science of mathematics? What role does intuition play with games of chance in weighing possible mathematical outcomes against each other? Does the individual follow his or her personal intuition in either of these circumstances? What other socio-historical influences affect an individual's mathematics decision-making abilities? Classroom mathematics teachers often use their pedagogical intuition to determine how students may react to a question; teachers' expertise can influence their interpretations of students' intuition. At the same time that

mathematics teachers are relying upon their intuition, students are relying upon their own intuitions during a mathematics lesson.

Intuition dependency alone can, at times, lead individuals to propose connections that are nonexistent (Greer, 2005), especially when faced with a dilemma that is either too difficult to reason through or lacks experiential familiarity. This usually occurs when feedback is theoretical, as is often the case in mathematics. Confusion in making sense about applying theoretical logical solutions may look differently when empirically applied to a problem (Borovcnik & Peard, 1996; Hastie & Dawes, 2001). When intuition is applied to an “exact” science, such as mathematics, the complex nature of intuition may become a strong reasoning tool used in conjunction with logic rather than against logic. This research study investigated the correlations, if any, between students’ misconceptions in probabilistic reasoning and their grade level in grades 7, 9, and 11 in a K-12 public school district.

The Definitions of Intuition

Intuition is not a term easily defined, and what definitions are used can sometimes be unclear (Fischbein, Tirosh, & Melamed, 1981). Hastie and Dawes (2001) provided an informal description of intuition, describing it as a type of automatic thinking in which other tasks and thought processes can be performed simultaneously. Fischbein and Gazit (1984) defined intuition as “a global, synthetic, non-explicitly justified evaluation or prediction” (p. 2). Fischbein (1987) defined intuition as a cognitive thinking process that appears self-evident and extrapolative, and different within each content area, as well as dealing with a source of knowledge in the context of an inductive method of reasoning. Extrapolation can be seen as a type of intuitive thinking, and quite often, mathematical statements need proof, but proof of a statement often requires the use of extrapolation to determine what pattern proves that statement.

Simmons and Nelson (2006) defined intuition as “the first answer that springs to mind when one is required to make a decision” (p. 409). Dreyfus and Eisenberg (1982) referred to intuition as “the mental representations of facts that appear self-evident” (p. 360). Winerman (2005) defined intuition as “the act or process of coming to direct knowledge or certainty without reasoning or inferring” (p. 50). Regardless of differing definitions, there are two key elements the definitions of intuition have in common: (a) immediate answers with little or no reasoning, and (b) a perception of obviousness resulting from the thought process employed through self-efficacy beliefs.

Fischbein and Grossman (1997) described intuition as “. . . generally the global, subjective effect of systematic, well established, highly integrated, sequential structures” (p. 31). They further drew differences between intuitive guesses and random guesses by stating that intuitive guesses are previously experienced information, while random guesses are more trial and error because of limited previous information. Thus, intuition is often described as a type of cognitive thinking (Fischbein, 1987; Noddings & Shore, 1984; Sutherland, 2005) which is “directly grasped without . . . any need for explicit justification or interpretation” (Fischbein, 1987, p. 3). The self-efficacy beliefs that arise from information drawn from previous experiences are attributed to the subjective nature of intuition. These self-efficacy beliefs are personal truths and, hence, may not be identical to another person’s self-efficacy beliefs (Bandura, 1986).

With respect to the compendium of definitions and descriptions, this study viewed students’ intuition as Kant (1781/1958) postulated, that “[a]pppearances are the sole objects which can be given to us immediately, and that in them which relates immediately to the object is called intuition” (p. 86). While Kant’s definition contains the two common key elements (immediacy

and obviousness), he differs from others with reference to the immediate perception of a single object rather than the perception as a general concept. Despite inconsistencies in definitions of the term intuition, mathematics is a quest for certitude that seeks determining conceptual knowledge strategies between different possible outcomes in solving posed problems in mathematics classrooms (Fischbein, 1987).

Heuristics and Probabilistic Misconceptions

Research has shown that most individuals, when asked to determine an unknown quantity or when faced with an uncertain event, rather than assessing probabilities of these unknowns, employ a small number of heuristics (Kahneman & Tversky, 1972, 1973; Tversky & Kahneman, 1974). Heuristics are simple and efficient learned rules that explain how an individual makes quick decisions, solves problems, or makes judgments with only partial information (Gilovich & Griffin, 2002; Kahneman & Tversky, 1972, 1973; Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993; Myers, 2007; Nickerson, 2004; Pratt, 2000; Shaughnessy, 1977; Tversky & Kahneman, 1974, 1982a). Heuristics reduce the complexity of determining probabilities of events to simpler operations involving the person's judgments on perceivable aspects of those events (Tversky & Kahneman, 1974), and therefore heuristics can be helpful to individuals employing them. However, heuristics may also lead to systematic errors in logic that will create misconceptions in thinking where probability is concerned (Cox & Mouw, 1992; Gilovich & Griffin; Kahneman & Tversky, 1982b; Kruglanski & Ajzen, 1983; Nickerson; Shaughnessy; Tversky & Kahneman, 1974). Although there are several heuristics that cause misconceptions in probabilistic reasoning when employing judgment about probabilistic events, the heuristic of particular importance to this study is that of representativeness.

Representativeness is one of the most often used heuristics in probability assessment (Kahneman & Tversky, 1972, 1973). An individual employing this heuristic evaluates the probability of an event based on the degree to which the event is similar to the sample space from which it is selected (Bar-Hillel, 1982; Gilovich & Savitsky, 2002; Kahneman & Frederick, 2002; Kahneman & Tversky, 1973; Kruglanski & Ajzen, 1983; Shaughnessy, 1977, 1981; Tversky & Kahneman, 1974) or the degree to which the event is characterized by the notable features of the system from which it is derived (Fischbein, 1987; Kahneman & Tversky, 1973; Shaughnessy, 1977). For example, in the research of Fischbein and Schnarch (1997), when asked which was more likely of a person given a description of that person, students tended to assume more about the person, based on the person's interests that were prominently featured in the question. Those two manifestations of the representativeness heuristic are at the root cause of probabilistic misconceptions.

Helping students overcome these probabilistic misconceptions is of great importance in the mathematics classroom, and steps have been taken to see that instruction in probability addresses these misconceptions. The state of Alabama aligns a portion of its course of study document, *Alabama Course of Study: Mathematics* (2003), to standards put in place by the National Council of Teachers of Mathematics (NCTM) (2000), and the document reflects that alignment by demonstrating ways and methods that teachers can teach to the different standards. Data Analysis and Probability, one of NCTM's content standards, is implemented at every level of K-12 education. Beginning with the middle school grade levels, this document includes implementation in the classroom of concepts that are the source of probabilistic misconceptions; these concepts include compound events, combinations, and independent and dependent events.

The misconceptions of concern in this study were the following: (a) the representativeness misconception (Bar-Hillel, 1982; Fischbein, 1987; Fischbein & Schnarch, 1997; Gilovich & Savitsky, 2002; Kahneman & Frederick, 2002; Kahneman & Tversky, 1973; Kruglanski & Ajzen, 1983; Shaughnessy, 1977, 1981; Tversky & Kahneman, 1974); (b) the negative recency effect, also known as “the gambler’s fallacy” (Cox & Mouw, 1992; Fischbein & Schnarch; Fischhoff, 1982; Gal & Baron, 1996; Shaughnessy, 1981), along with the positive recency effect (Fischbein, 1975; Fischbein, Nello, & Marino, 1991; Gilovich, Vallone, & Tversky, 2002); (c) the distinction between simple and compound events (Cox & Mouw; Fischbein & Schnarch; Lecoutre, 1992; Lecoutre & Durand, 1988; Quinn, 2004; Rubel, 2007; Tversky & Kahneman, 1982c); and (d) the effect of sample size in determining probabilities (Bar-Hillel; Fischbein & Schnarch; Tversky & Kahneman, 1974, 1982c). These probabilistic misconceptions are examined further in the review of the literature.

Statement of the Problem

Probability is a difficulty mathematics topic for students in the middle- and high-school levels. Misconceptions exist and persist in student thinking where probability is concerned. To better understand the misconceptions students have in the study of probability, examining how the misconceptions are related to grade level will keep researchers informed by providing them with some information on the topic. Examination of the misconceptions may also suggest ways to combat these misconceptions in the classroom. This research was informed by two key pieces of research, Fischbein and Schnarch (1997) and Rubel (2002), which also examined misconceptions by means of similar methods.

Purpose of the Study

The first purpose of this study was to determine if the appearances of certain probabilistic misconceptions were related to grade level for students in middle- and high-school grade levels. The second purpose of this study was to determine if student achievement in the study of probability with respect to these misconceptions was related to self-efficacy at three different grade levels (7th, 9th, 11th). The research's first purpose was informed by other research on the topic, namely Fischbein and Schnarch (1997) and Rubel (2002). The research's second purpose has not been studied with respect to probabilistic misconception, so this research will add to the knowledge base by addressing concerns not yet studied.

Research Questions

There were two questions for this research study. They included the following:

1. What is the relationship between students' 7th-, 9th-, or 11th-grade level and each of the four intuitively-based probabilistic misconceptions of the representativeness misconception, positive and negative recency effects, the distinction between compound and simple events, and the effect of sample size; and
2. What is the relationship between students' responses in the 7th-, 9th-, and 11th-grade levels, to questions involving probability tasks and their perceived self-efficacy in answering those questions?

Methodology and Data Collection

A survey, adapted from earlier work by Fischbein and Schnarch (1997), was administered to participants in 7th-, 9th-, and 11th-grade levels at three schools in a large public school district in central Alabama. This survey had four probabilistic reasoning questions designed to detect the presence of four intuitively-based probabilistic misconceptions derived from the

representativeness heuristic. The survey also included a five-point Likert scale item concerning participants' self-efficacy for each of the four questions.

Data from this study were compared in a limited manner to that collected by Fischbein and Schnarch (1997); the limited comparison is due to the descriptive nature of their statistics. Fischbein and Schnarch compared percentages of students providing the correct mathematical response to each question across grade levels and made informal assessments about correlations between grade level and responses to the mathematical items based on those percentages. The data collected was analyzed using SAS 9.2, and Spearman correlations were computed to determine the correlative nature of the grade level of the participants and those participants' responses to each item on the survey. A Spearman correlation was also used to compute the correlative nature of the participants' responses, within individual grade levels, to each item on the survey and their perceived self-efficacy in responding to those items.

Definitions of Terms

Compound event: An event consisting of at least two outcomes of an experiment (Jaynes, 2003).

Event: Any subset of the sample space of outcomes of an experiment (Jaynes, 2003).

Heuristic: A simple and efficient learned rule explaining how an individual makes quick decisions, solve problems, or makes judgments with only partial information (Kahneman & Tversky, 1972, 1973; Shaughnessy, 1977; Tversky & Kahneman, 1974, 1982a).

Intuition: The appearances or aspects of a single object immediately perceived and related to the object (Kant, 1781/1958).

Probabilistic misconceptions: Incorrect notions or conceptions regarding questions dealing with probability (Rubel, 2002)

Probability: The study of mathematics in which events are assigned a numerical value between 0 and 1, inclusive, that indicates how likely the event is, with numbers with smaller values corresponding to less likely events than events with numbers with larger values (Jaynes, 2003).

Probability of an event: A number between 0 and 1 assigned to an event, which is the ratio of the number of favorable outcomes to the total number of outcomes of an experiment (Konold, 1991).

Representativeness: A heuristic in which an individual assesses the probability of an event based on the degree to which the event is similar to the sample space from which it is selected or the degree the event is characterized by the notable features of the system from which it is derived (Bar-Hillel, 1982; Fischbein, 1987; Gilovich & Savitsky, 2002; Kahneman & Frederick, 2002; Kahneman & Tversky, 1973; Kruglanski & Ajzen, 1983; Shaughnessy, 1977, 1981; Tversky & Kahneman, 1974).

Sample space: The set of all possible outcomes of an experiment (Jaynes, 2003).

Self-efficacy: “people’s judgments of their capabilities to organize and execute courses of action required to attain designated types of performances” (Bandura, 1986, p. 391); “a judgment of one’s capability to accomplish a certain level of performance” (Bandura, 1986, p. 391).

Simple event: An event consisting solely of one outcome of an experiment (Jaynes, 2003).

Limitations of the Study

This research study did have limitations. First, the research was performed within a single metropolitan school district. Thus, the results of this study could only be generalized to the population of students in this school district or districts with similar demographics.

Depending on various factors, participants from other districts or geographical regions may react differently to the same survey items for this research. In tandem, more diverse student populations potentially may respond differently than the students in the current study.

A second limitation is that the sample for this research study was chosen as a convenience sample. Randomizing the sample may help to generalize the results of this study to other populations. Since a large sample size was ideal for this study, many more students would need to be surveyed in order to be able to draw a random sample that was as large as the samples used in this study.

A third limitation is that students volunteered to participate in the study, which may skew the data slightly from that of a more homogeneous pool sample of student participants. Having a more diverse group of students may also yield results that are more easily generalized to other populations, just as randomizing the sample may produce a similar effect.

A fourth limitation is that the study's investigation results only focused on the correlations between the variables. While many studies have investigated causal relationships between variables, the inclusion of combining correlations between variables could provide alternative consideration to understanding mathematical probability choices. Equally, for this type of investigation, the development of a relational survey instrument would be necessary.

A fifth limitation is that the researcher coded all responses to the open-ended survey for the pilot study. Multiple readers of the same items may have arrived at different conclusions for students' responses. Having multiple readers code the responses would have increased inter-rater reliability and strengthened the pilot study data.

Assumptions of the Study

1. The participants of this study interpreted items in the way intended by the researcher and the authors of the instrument to the best of their abilities.
2. The participants of this study answered items based on the way they understood the problems present in the items on the instrument.
3. No outside influence was given to participants in answering the items on the instrument.
4. Observations between the pilot study and the research study were independent. No students talked to each other about the surveys between the pilot study and the research study.
5. Students taking the multiple choice and open-ended surveys were tested simultaneously but were seated separately during the pilot study data collection.

Summary

Chapter I introduced probability, intuition, and intuition's influence on probabilistic misconceptions. The discussion included the purpose and research questions of the research study. Chapter II provides a review of the literature that supported the theoretical framework for this study. Chapter III provides a detailed description of the methodology that was implemented in data collection and analysis. Chapter IV provides the results of the study. Chapter V provides a discussion of the findings of the study, including an interpretation of the results, implications of the study, and possibilities for future study.

CHAPTER II:

REVIEW OF THE LITERATURE

The purpose of this research study was twofold: first, to investigate relationships existing between perceived mathematics probability misconceptions among students at the 7th-, 9th-, and 11th-grade levels; and second, to investigate relationships existing between students' perceived probabilistic misconceptions and their self-efficacy beliefs with intuitive mathematics probability. This chapter includes sections on (a) the difficulties of probability in K-12 education, (b) mathematical intuition and its characteristics, (c) intuition's role in probability, (d) intuition and rationality, (e) prior research for the proposed study, (f) representativeness and its four associated misconceptions, and (g) mathematical identity and self-efficacy.

The Difficulties of Probability in K-12 Education

Probability is one of the most difficult concepts for students to learn in the mathematics classroom (Greer, 2001); however, research into the misconceptions students have in this conceptually difficult area of mathematics is fairly recent, with the first studies conducted in the 1970s by Kahneman and Tversky and Fischbein (Garfield & Ahlgren, 1988; Konold, 1991). Prior to the 1960s, conventional wisdom and underlying assumptions in research were that an individual's everyday reasoning, when confronted with uncertainty in specific situations, was similar to that of statisticians (Peterson & Beach, 1967), meaning that people viewed statistics using perfect logic, as precisely what they were rather than loading unreasonable expectations onto them. Research indicates that this reasoning is often flawed due to the availability of only

limited information, experience, and the use of several heuristics that people use to make decisions (Fischbein, 1987; Kahneman & Tversky, 1973; Konold, 1989). The most widely used of these heuristics is representativeness (Kahneman & Tversky, 1972, 1973), but there are several others that are used. Some of those heuristics include availability, in which an individual assesses the probability of an event based on how easily it is to think of instances that are representative of or reflect the event (Fischbein; Schwarz & Vaughn, 2002; Shaughnessy, 1977; Tversky & Kahneman, 1974, 1982a); anchoring and adjustment, in which an individual estimates the probability of an event by adjusting some potentially unrelated piece of information (Tversky & Kahneman, 1974); the time-axis fallacy, in which an individual has difficulty assessing the effect on a conditioning event by the event it conditions (Fischbein, 1999; Shaughnessy, 1992); and the outcome approach, in which an individual tries to assess the probability of an event by successfully predicting the outcome of a single trial of the event (Konold, 1989). While these heuristics can provide estimates of probabilities when faced with uncertainties, limitations occur in tandem with the experiences of the person using them. Consequently, the results of cognitive estimations employing the heuristics can often be at odds with the actual scenario in which the heuristics are being applied (Konold, 1991).

Over the past twenty years, probability has received much more attention in middle- and high-school mathematics education (Batanero, Godino, & Roa, 2004; Ebersbach & Wilkening, 2007; Keeler & Steinhorst, 2001; Schlottmann, 2001), primarily due to efforts by NCTM (2000) to include the study of probability in their K-12 mathematics curriculum. Probability must be taught at the middle- and high-school levels so that students can be exposed to its intricacies while they are still developing probabilistic reasoning (Schlottmann, 2001); instruction has been found to have a positive effect on misconceptions in mathematical understanding (Ben-Zeev &

Star, 2001; Fischbein & Gazit, 1984). Having a working knowledge of probability is necessary in several areas of study. Probability plays a prominent role in the study of many disciplines, including business, biology, and statistics. People are inundated with information in the form of probabilities with respect to health insurance, automobile warranties, and advertisements for various products.

The appearance of probability and statistics in many facets of life, as well as the expectation that entering college freshmen have an understanding of probability concepts, has caused mathematics educators to advocate the need to include a probability focus in the K-12 curriculum. NCTM (2000) has suggested state and national mathematics standards that include the learning of probability thinking throughout grades K-12. Due to the difficulties students have in dealing with and understanding probability (Garfield & Ahlgren, 1988), these standards sought to implement the teaching of probability that will effect change in students' understanding of probability.

What makes probability a difficult topic both to teach and to learn? There are four major probability attributions that contribute to this difficulty: (a) the number of different ways to think about probability problems; (b) the theoretical nature of probability and the disconnection between theoretical and experimental results; (c) the abstract thinking required in the study of probability; and (d) the different formulations of the study of probability (Aspinwall & Shaw, 2000; Borovcnik & Bentz, 1991; Cosmides & Tooby, 1996; Garfield & Ahlgren, 1988; Konold, 1989; Schlottmann, 2001). The first attribution refers to the different ways an individual can approach solving problems involving probability. For classroom students, a degree of experiential knowledge using probability strategies already exists. However, critical thinking is required for transferring that experiential knowledge to new situations and subject areas (Dawes,

1988; Halpern, 1998, 1999); this critical intuitive cognitive process is often overlooked in regular classroom instruction (Ebersbach & Wilkening, 2007; Hawkins & Kapadia, 1984; Schlottmann).

Specifically relating to mathematics, the study of probability would require students to be clearly guided through instruction in probable cause thinking. Such an instructional method would require a mathematics teacher who is adept in probability conceptual thinking to guide struggling students toward strengthening their intuitive probability choices. For those mathematics classroom teachers who are ill-adept in theoretical probability, further research in providing appropriate professional development opportunities needs to be conducted (Batanero, Godino, & Roa, 2004; Castro, 1998; Garfield & Ahlgren, 1988; Keeler & Steinhorst, 2001).

The second probability attribution that makes probability more difficult to teach to students is the disconnection between theoretical and experimental resulting choices (Borovcnik & Bentz, 1991). For example, students may think that a coin would land on heads half the number of times the coin was tossed regardless of the number of tosses, or that if heads is tossed on the coin, the next toss necessarily would result in the coin landing as tails. This is an instance where a student's intuitive reasoning leads to inappropriate probability judgments. Through instruction, a teacher can demonstrate students' intuitive reasoning in probability may not always be correct by posing a question regarding how many times heads would occur with an odd number of tosses. Using this probability question, students may realize their intuitive reasoning choice is situational and not absolute. Such probability questioning highlights the disconnection between theoretical and experimental reasoning. Thus, conflicting experimental intuitive reasoning with theoretical probabilities usually occurs with a student's limited conceptual intuitive reasoning. The purview in strengthening the conflicting disconnections in mathematics instruction should be advocated within the curriculum.

A third probability attribution, the abstract thinking used in the study of probability, requires an individual's ability in higher order abstract thinking. While most branches of mathematics and most mathematics equations require an abstract thinking process, there are instances when this is not the case. Konold (1989, 1995) described a method of informal reasoning that he called the "outcome approach" in which students tried to predict the trial outcomes of an experiment. For example, a student who implemented this type of reasoning, when asked if a six was likely to be rolled on a single die, interpreted the question as a request to predict the outcome of rolling the die once; when a six was rolled, the student said that the six was very likely to be rolled, despite the fact that this was not the case. The students who thought about probability in this way reduced their responses to merely "yes" or "no" responses rather than determining probabilities attached to the outcomes. This method of informal reasoning was an attempt to make intuitive predictions in real world situations, but in the realm of theoretical probability, this method lacked validity (Konold, 1989). For instance, graphing a function gives an equation a certain amount of concreteness, but there are different ways to think about that mathematical function; there still does not remain one formula or method which can wholly describe and illustrate a probability result.

The fourth probability attribution, the different formulations of the study of probability, refers to the difficulty in there being not a single "calculus of probability" (Cosmides & Tooby, 1996, p. 2) (i.e., a mathematical basis for the study of probability). Konold (1989, 1991) posited a number of probability results reflecting this difficulty: probability exists in many forms and meanings. The classical interpretation, for instance, assigns an event a number equal to the ratio of the number of favorable outcomes to the number of total outcomes of the event. The frequentist or experimental interpretation assigns the same ratio as in the classical interpretation

except when conducted in an actual experiment rather than a theoretical interpretation. The subjectivist or personalist interpretation indicates the degree to which a person believes an event will occur (e.g., coin tossing). These variations in seeking probability choices can occur either in calculus or in discrete mathematics where backgrounds often differ.

Teaching probability can be a challenging task, and therefore a teacher's role is to assist students in overcoming their difficulties in comprehending probability and helping students recognize the perceived incongruities with theoretical and experimental reasoning of probability. When engaging in teaching probability, priority must be given to students' level of recognizing ratio and proportion needs examining, since these concepts are used in calculating and interpreting probabilities, and they form a cornerstone of mathematical reasoning skills (Cramer, Post, & Currier, 1993; Garfield & Ahlgren, 1988; Hines & McMahon, 2005; NCTM, 2000). To address misconceptions students are guided to recognize and confront common errors within their intuitive probabilistic reasoning. Critically reflecting on their conceptions, students were better able to develop appropriate alternative cognitive thinking skills when they confronted probability tasks (Borovcnik & Bentz, 1991; Scholz, 1991). To create situations that represent the students' current world views, the inclusion of concrete examples during probability instruction helped to differentiate between theoretical and experimental situations (Aspinwall & Shaw, 2000; Fischbein, 1975; Garfield & Ahlgren; Hawkins & Kapadia, 1984; Konold, 1994). Including such strategies during mathematics classroom instruction helped promote meaningful learning regarding probability skills that continued developing in later school grades, further ensuring students had appropriate grounding and greater confidence in their ability to engage in probability when they enrolled in college mathematics courses (Aspinwall & Shaw; Hawkins & Kapadia, 1984; Konold, 1994; Shaughnessy, 1977).

Probability is a conceptually difficult branch of mathematics. Classroom teachers who instructionally include the following four probability attributes can increase their students' probability-strategic knowledge: (a) the number of different ways to think about probability problems; (b) the theoretical nature of probability and the disconnection between theoretical and experimental results; (c) the abstract thinking required in the study of probability; and (d) the different formulations of the study of probability. Having knowledge of these sources of difficulty, teachers can attempt to effect change in students' understanding of probability through various instructional strategies.

Mathematical Intuition and its Characteristics

As mentioned in the previous section, the study of probability is difficult due to the level of abstraction and the disconnection between experimental and theoretical results of probability simulations. Some degree of knowledge about probabilistic situations must be exhibited by students (Konold, 1991), and this is often in the form of intuition. The study of intuition, in particular the intuitively-based misconceptions that arise in student thinking, is necessary since intuition is vital to an understanding of the intricacies of problems that involve probability (Frantz, 2005).

The definition of intuition is a conscious term individuals assume knowing but often find little consensual agreement with others in meaning. Intuition research can be studied as a psychological cognitive thinking concept, as well as studied within contextual settings, such as Kant's (1781/1958) axioms of intuition. Kant posited that mathematics would not be possible without certain intuition known *a priori* during development of those mathematics. Sutherland (2005) suggested that Kant's axioms of intuition are derived from personal experience in and

with the world and subject to an individual's personal biases and misconceptions in the way he or she experienced the world (Greer, 2001; Sosa, 2006).

Haslanger and Saul (2006) found that individuals' intuition perspectives are often naturally occurring socially constructed categories, such as race and gender. Those socially constructed categories changed over time as more experiences contributed in expanding those categories. Hence, these categorizations were not reliable in capturing the meaning of the terms associated with the study of probability (Haslanger & Saul, 2006). Intuition can be applied to everyday situations in order to make those situations easier, but individual's intuitions can misrepresent intuitive connections that do not exist in the situation (Greer, 2005), or by contradicting what is believed to be intuitively true with the existing information (Fischbein, 1987).

Intuition, as part of human beings' rational thinking process, is absolutely necessary (Greer, 2005), and it is used to support a variety of situations. For instance, intuitions can be used to predict the outcome of an experiment (Van Dooren, De Bock, Weyers, & Verschaffel, 2004), to make comparisons between objects (Chiu, 1996), or to choose between different alternative possibilities of a task (Simmons & Nelson, 2006).

Intuition, in mathematics, is primarily used as a bridge between mathematical concepts and the real world (Fischbein, 1987; Isaacson, 1994; Parsons, 1994; Van Dooren et al., 2004), as well as used to formulate the axioms upon which mathematics is built (Fischbein, 1987; Kant, 1781/1958). Fischbein's research drew a large distinction between any empirical evidence, namely real objects and occurrences, and mental objects such as constructs, ideals, and mathematics-like systems of logic (Fischbein, 1987; Van Dooren et al., 2004). Individuals intuit

attempts to make sense of objects or occurrences from the real world by comparing them to or using objects or methods from the world of mental objects (Fischbein, 1987).

That individuals make this connection between objects in the real world and objects in the world of mental objects was further confirmed by Van Dooren et al. (2004), who determined that students employing the intuitive rules of ‘More A-more B’ and ‘Same A-same B’ consistently made fewer errors than students who did not answer the same questions systematically. According to Fischbein (1987), intuitions arose as adaptations of the learner’s environment and were shaped by the experiences of that learner. Results indicated that intuition could easily mislead individuals who were looking for patterns that did not exist (Fischbein, 1987; Fischhoff, 1982; Myers, 2002)—for example, a pattern of natural numbers whose first five terms are consecutive powers of two fails to continue in that way beginning with the sixth term (see Parker, 2005). Therefore, each learner garnered his or her own intuitions, and those intuitions could—and often did—vary from learner to learner (Borovcnik & Peard, 1996; Einhorn, 1982; Sosa, 2006).

A question Fischbein (1987) proposed is if intuitions were adaptive, how did individuals fail in so many situations where weighing probability choices was a factor? Part of the answer seemed to lay in that being adaptive, intuitions did fail in part, but over the long term, intuitions adjusted to accurately depict the situation (Myers, 2002). Another part of the answer was that intuition did differ from person to person because of individual’s unique social experiences. For every person whose intuitions were appropriate for a particular task, there could have been one hundred individuals whose intuitions did not develop in the same way (Myers, 2002). Experience is an important factor in developing intuitions. Another reason for the failure of intuition was trying to adapt the faulty real situation to an ideal probabilistic situation, with the

obvious problem being that reality does not often follow an ideal path (Borovcnik & Bentz, 1991; Greer, 2001).

Although Fischbein (1987) identified that students came into the classroom with prior knowledge about probability, he was a proponent of teaching probabilistic reasoning to students through activities in the classroom in order to hone that knowledge derived from prior experiences (Fischbein, 1975; Fischbein & Gazit, 1984; Mariotti & Fischbein, 1997). The teaching of probability often leaves probability in the realm of abstract thinking; providing activities in the classroom gave students a hands-on approach to determining probabilities of events, which in turn augmented probabilistic intuition. Fischbein and Gazit (1984) proposed their view for the use of activities in the classroom, which summarized the attitudes of all such proponents:

Our point of view is that new intuitive attitudes can be developed only through the personal involvement of the learner in a practical [*sic*] activity. Intuitions (cognitive beliefs) cannot be modified by verbal explanations only. Therefore, a teaching programme which intends to develop an improved and efficient intuitive background for probability concepts and strategies, along with the corresponding formal knowledge, must provide the learner with frequent opportunities to experience actively, even emotionally, stochastic situations. In such situations, the learner will confront his plausible expectations with empirically obtained outcomes. (pp. 2-3)

The combination of students' intuitions, based on previous knowledge and experiences, and new, hands-on approaches to probability can guide students to hone their intuition skills by providing them with available situations to view future decisions regarding probability and prediction.

Fischbein (1987) cited eight intuition properties: (a) self-evidence, (b) intrinsic certainty, (c) perseverance, (d) coerciveness, (e) theory status, (f) extrapolativeness, (g) globality, and (h) implicitness. Of these, self-evidence and intrinsic certainty were the two characteristics that often led to confusion in student probabilistic reasoning (Fischbein). Despite their differences,

all of the definitions given in the first chapter of this paper featured the property of self-evidence. By self-evidence, Fischbein (1987, 1999) meant that when a statement was posited to someone, that person understood the statement to be true without any proof or justification and that the statement was self-explanatory. Many mathematical statements are self-evident despite the fact that the proofs for those statements are simple; examples of such statements include the following: (a) every whole number has a successor (Fischbein, 1987, 1999), (b) everything can be divided into two parts (Stavy & Tirosh, 2000), and (c) two lines parallel to a third line are parallel to each other (Fischbein, 1987).

Intrinsic certainty, on the other hand, was slightly different than self-evidence; “[the two properties] are highly correlated but they are not reducible one to the other” (Fischbein, 1987, p. 45). Intrinsic certainty was a feeling of certainty that students had with the mathematics of the statement posited to them. When a student felt that a mathematical statement was correct, that student viewed the statement as having intrinsic certainty. Confidence in one’s answer to a question or belief in the accuracy of a mathematical statement was almost synonymous with intrinsic certainty.

Additionally, intuitive cognitions that seemed self-evident often, for the student, did not require any sort of necessary justification while intuitive cognitions that had intrinsic certainty seemed to have a fairly easy explanation, although those cognitions may not have been self-evident (Fischbein, 1987). Students were certain of many of the theorems and equations that were used in everyday mathematics classrooms despite the fact that those theorems and equations may not have been self-evident (Fischbein, 1987; Fischbein, Tirosh, & Melamed, 1981). Students felt that statements were self-evident even though they may not have been sure that the answer they believed was correct was, in fact, the correct one. The problem that these

two cognitive intuitions created for students was that they led students to dismiss certain types of problems without delving deeper into the structure and the intricacies of those problems, and this often led to a misunderstanding of those problems.

The other properties that also characterized intuition, as noted from Fischbein (1987), helped to shape the perception of intuition, but they did so to a lesser degree than self-evidence and intrinsic certainty. Perseverance referred to the ability of intuitions to persist once established, and formal instruction generally did little to change intuitive perceptions. Coerciveness referred to the ability of intuitions to make individuals disregard or exclude alternate answers to mathematical statements. Theory status referred to the ability of individuals to take intuitive statements, whether correct or incorrect, and make them into universal statements rather than into specific statements for the experiment at hand; extrapolativeness was closely related to theory status in that it was the ability to transcend the information at hand by generalizing the data into a somewhat universal scenario. Globality referred to the ability of individuals to apply intuitive realizations to parallel situations in which they may or may not have applied. Finally, implicitness referred to the ability of individuals to unquestioningly believe their intuitions despite the fact that the statements may not have been intuitively obvious. Each of these properties characterized intuitions, and no one of the properties was directly reducible to any of the others.

Mathematical intuition is necessary in the study of mathematics, but this intuition must be given time in order to develop (Fischbein, 1987). The problem with developing intuition in students, though, is that intuition is based on experience, making the intuition of one student markedly different than that of another. Despite this, intuition is characterized by the properties

of self-evidence, intrinsic certainty, perseverance, coerciveness, theory status, extrapolativeness, globality, and implicitness.

Intuition's Role in Probability

Bunge (1962/1975) suggested the role of intuition in learning mathematics as “the intuitionist thesis of mathematical intuitionism”:

Since mathematics is not derived from either logic or experience, it must originate in a special intuition that presents us the basic concepts and inferences of mathematics as immediately clear and secure. “A mathematical construction ought to be so immediate to the mind and its results so clear that it needs no foundation whatsoever” (Heyting, 1956, p. 6). We should consequently choose as basic notions the most immediate ones, such as those of natural number and existence. (p. 39)

Bunge's mathematical intuition, much like that of Kant (1781/1958), forms a cornerstone in the construction of all mathematics, and it can be particularly useful to validate such advanced mathematical constructs as the axioms of set theory. The intuitionist thesis of mathematical intuitionism is applied to learning all mathematics, including probability. With this intuition, individuals have an a priori understanding of mathematical objects such as numerals and shapes in geometry (Parsons, 1994; Resnik, 2000; Sutherland, 2005).

Intuition seems to be a perfectly viable solution to most problems, but in the realm of mathematics, where proof is essential, intuition seems to play a much smaller role in what is demonstrated publicly. However, mathematicians use intuition in their private work to help them determine what methods of proof may be applicable to their work at hand. With respect to the importance of proof in the mathematics classroom, Jaynes (2003) said that “a responsible scientist...will not assert the truth of a general principle, and urge others to adopt it, merely on the strength of his own intuition” (p. 144). However, most proof begins with some intuition—each student must be able to make some educated guess about where a proof begins. Probability

may be most useful in mathematics, primarily because of its practical applications and broad use in multiple disciplines. Probability exchanges are in games of chance, as well as in sporting events, such as in baseball players' batting averages, fielding percentages, earned run average, et cetera. Modeling weather patterns, planning finances, and determining lengths for warranties on vehicles are other examples of daily probability dependency.

Probability is a branch of mathematics where students must rely upon intuition. Intuition plays a key role in students' thinking about probability that can have either a positive or negative effect. All students have some sort of intuitive notion about probability, but sometimes that notion caused a misunderstanding of the material (Shaughnessy, 1981). A thorough knowledge of probability guided a student's skillfulness on applying which intuitive and procedural problem solving strategies helped to address the dilemma, as well as to enhance retention of that choice for future problem solving (Fischbein, 1999).

Intuition and Rationality

Individuals tend to trust their intuition rather than other thought-out courses of action, even when information that contradicts their intuition is available (Denes-Raj & Epstein, 1994; Kirkpatrick & Epstein, 1992; Simmons & Nelson, 2006). The bias toward intuition is apparent, but where does this bias originate? Much of the research on this topic suggested that intuitive bias originates in the interaction between two systems of the mind (Cloninger, 2006; Epstein, 1994; Epstein, Lipson, Holstein, & Huh, 1992; Kahneman, 2003; Myers, 2007; Simmons & Nelson; Sloman, 1996, 2002; Stanovich & West, 2002).

The first system, referred as the intuitive system, is "a relatively effortless system that relies on prior knowledge, judgmental heuristics, immediate experience, and affect in order to rapidly and crudely assess the decision alternatives" (Simmons & Nelson, 2006, p. 409). This

system was one of implicit learning (Cloninger, 2006; Myers, 2007) that required little effort, was fast, and was automatic (Myers, 2007; Simmons & Nelson, 2006). The intuitive system, characterized by feelings and emotions (Myers, 2007), ultimately overrode any rational thought concerning the decision at hand (Schwarz, 2002). The intuitive system allowed individuals to make quick decisions based on a cursory evaluation of options and a quick judgment. Objects were similar in the intuitive system based on the perceived similarity they had to previous objects (Sloman, 2002).

In direct opposition to the first system, the second system is referred to as the rational system. Simmons and Nelson (2006) described this system as “a slower, effortful, resource-dependent, rule-based system that monitors and updates [the intuitive system’s] assessment in light of information that [the intuitive system] neglected to consider” (p. 410). This system was one of explicit learning (Cloninger, 2006; Myers, 2007). Two underlying principles of the rational system are that it was productive (i.e., the system could encode any number of propositions) and that it was systematic (i.e., the fact that the system could encode certain propositions implied an ability to encode other related ones) (Sloman, 2002). The rational system also corrected any decision, judgment, or course of action decided upon through use of the intuitive system solely (Kahneman & Frederick, 2002). There is a caveat with this system in that individuals can be led astray by their intuition, even in light of information contradicting their intuition (Denes-Raj & Epstein, 1994; Kirkpatrick & Epstein, 1992; Simmons & Nelson). Such occurrences were viewed as failure of the rational system successfully correcting the individual’s intuitive system (Simmons & Nelson, 2006).

The intuitive system often overrode the rational system. There were three suggested reasons for these occurrences: (a) individuals either were unmotivated to determine what

corrections should be made, (b) the intuitive system was an emotionally-charged system with an individual's vast life experiences which relied on whatever worked in the past, and (c) individuals were unable to process the information sufficiently to correct the intuitive system (Myers, 2007; Schwarz, 2002; Simmons & Nelson, 2006). Individuals often chose to rely upon their intuitive reasoning system (Gilbert, 2002; Tversky & Kahneman, 1974). Intuitive emotions, in these situations, were much more powerful tools than logic, resulting in a hasty decision to follow one's intuition (Armor & Taylor, 2002; Schwarz, 2002).

The intuitive system (emotional), more primitive than the rational system (logic and abstract thinking), was just as capable of influencing probability choices as the rational system (Kahneman & Frederick, 2002). Gradually, rational processes moved into the intuitive system as skill was developed in rational thinking, and the rational system corrected errors in thinking derived from the intuitive system (Kahneman & Frederick, 2002). For this reason, the intuitive system was of particular importance within this research.

There are two systems of the mind: the rational system, where logic and abstract thinking occurred, and the intuitive system, one characterized by emotion. The intuitive system often overrode decisions made by the rational system, and it is in the intuitive system where most probabilistic misconceptions arose (Cloninger, 2006; Epstein, 1994; Epstein, Lipson, Holstein, & Huh, 1992; Kahneman, 2003; Myers, 2007; Simmons & Nelson; Sloman, 1996, 2002; Stanovich & West, 2002).

Prior Research for the Study

Fischbein, Tirosh, and Hess (1979) tested the persistence of student misconceptions involving the concept of infinity. In that research, students in fifth through ninth grades responded to 10 questions, some divided into multiple parts, concerning different aspects of

infinity. The series of questions, in a survey format, were generally broken down into two, four, or five groups based on similarity of the questions. Data results were percentages within the respondents' grade levels, with general trends observed by the researchers.

The study of intuitively-based probabilistic misconceptions arose out of the study of these heuristics and the systematic errors in thinking about probability. It is through these heuristics that most probabilistic analyses are made, and it is in these heuristics where most probabilistic misconceptions originate. This current research study was an extension of an initial study by Fischbein and Schnarch (1997) on intuitively-based probabilistic misconceptions. In their study on probabilistic misconceptions, students in Israel who had not previously received any formal instruction in probability were asked to assess situations involving probability. Twenty students in each of 5th-, 7th-, 9th-, and 11th-grade levels, along with 18 college students who were prospective teachers specializing in mathematics, were investigated; the 11th-grade students were in the average ability level of the three levels of instruction in Israeli high schools, and the sample of students reflected a broad range of students with respect to socioeconomic level and cultural background. Fischbein and Schnarch's research mirrored the template of Fischbein, Tirosh, and Hess (1979) as it gave those students a questionnaire consisting of a series of seven questions, two of which had two parts, all related to common intuitively-based probabilistic misconceptions. Students wrote their answers to each problem.

The data gathered from that study were used to determine informal correlations for each misconception between grade level and the presence of the misconception based on the student's answer to the question testing for that particular misconception. Percentages of students who answered a question correctly and percentages of students who answered a question incorrectly were reported within each grade level, with general trends observed by the researchers. In

addition, results noted whether students chose the correct mathematical answer or the answer reflecting what was called the “main misconception”. No formal correlations were determined in this previous research as it was the first part of a planned, multi-stage research project.

The results from Fischbein and Schnarch (1997) for the misconceptions that were relevant to this research study were as follows: for the item that tested on the representativeness misconception, which asked students to judge what combination of six digits was more likely to win as a lottery combination, the trend was for the misconception to decrease with age. Of the students in the fifth-grade level, 70% thought the combination with the non-consecutive numbers had a more likely chance of winning, and for the remaining 7th-, 9th-, and 11th-grade levels, along with college undergraduates, those percentages were 55%, 35%, 35%, and 22%, respectively. For the item that tested on positive and negative recency effects, which asked students what was more likely to happen on the toss of a coin after three consecutive heads were tossed, the negative recency effect decreased with age while the positive recency effect remained relatively nonexistent. Of the students in the fifth-grade level, 35% thought that a toss of tails was more likely to occur; for the remaining 7th-, 9th-, and 11th-grade levels, along with college undergraduates, those percentages were 35%, 20%, 10%, and 0%, respectively. Only 5% of students in the seventh-grade level and 6% of undergraduate students thought that heads was more likely to be tossed, signifying the positive recency effect; no other students answered in this way.

For the item that tested on the distinction between compound and simple events, which asked students if the pair 5-6 or the pair 6-6 was more common when rolling two dice, the trend was that this misconception was relatively stable across all five age groups. For the five age groups, 70% of students at both the fifth- and seventh-grade levels, 75% of both the 9th- and

11th-grade levels, and 78% of undergraduates thought that both results had the same probability, demonstrating the equiprobability bias described by Lecoutre (1992). For the item that tested on the effect of sample size, which asked if tossing at least two heads out of three tosses of a coin had the same probability as tossing at least 200 heads out of 300 tosses of a coin, the trend was that the misconception increased with age across the 5th-, 7th-, 9th-, and 11th-grade levels, with a slight decrease in the percentage from the 11th-grade level students to the college students. For those five grade levels, 30% of students at the fifth-grade level, 45% at the seventh-grade level, 60% at the ninth-grade level, and 75% at the 11th-grade level thought these two events had the same probability; 44% of undergraduates thought these events to have equal probabilities as well.

The purpose of the study by Fischbein and Schnarch (1997) was to obtain, through the collection of empirical data, an overall feel for how probabilistic misconceptions evolved over time. The results contradicted that of Piaget and Inhelder (1951/1975) that these misconceptions about probability became stable during the formal operational period of development. This study was a preliminary one as it was the first stage of what was to become a mixed-methods study that involved an interview protocol in addition to the questionnaire data collected. However, with the death of Fischbein in 1998, further studies were not realized. The current research study extended the research of Fischbein and Schnarch (1997) in their investigation into the persistence of probabilistic misconceptions in students in 7th-, 9th-, and 11th-grade levels.

Representativeness and its Four Associated Misconceptions

When faced with an uncertain event, or when asked to determine an unknown quantity, what does an individual rely upon in order to assess the probability attached to the event or to achieve the unknown information sought? Calculating simple probabilities cognitively assesses

the particular situation at hand (Kahneman & Tversky, 1982c). However, research has shown that most individuals, rather than assessing the probabilities to these unknowns, employed a small number of heuristics, which are simple and efficient learned rules that explain how an individual makes quick decisions, solves problems, or makes judgments with only partial information (Gilovich & Griffin, 2002; Kahneman & Tversky, 1972, 1973; Konold et al., 1993; Myers, 2007; Nickerson, 2004; Pratt, 2000; Shaughnessy, 1977; Tversky & Kahneman, 1974, 1982a). Heuristics, in effect, reduced the complexity of determining probabilities of events to simpler operations involving the person's judgments on perceivable aspects of those events (Tversky & Kahneman, 1974).

Heuristics were helpful in this sense, but heuristics also led to systematic error in estimations that, depending on the situation, could have been severe (Cox & Mouw, 1992; Gilovich & Griffin, 2002; Kahneman & Tversky, 1982b; Kruglanski & Ajzen, 1983; Nickerson, 2004; Shaughnessy, 1977; Tversky & Kahneman, 1974); these biases were the measure of overconfidence or underconfidence, depending on the situation (Yates, Lee, Sieck, Choi, & Price, 2002). Of particular interest in this study was the heuristic of representativeness; this heuristic gives rise to several probabilistic misconceptions, including the representativeness misconception, positive and negative recency effects, the effect of sample size, and the distinction between compound and simple events.

Representativeness was one of the most often used heuristics when employing judgment about probabilistic events (Kahneman & Tversky, 1972, 1973). An individual who employed the representativeness heuristic evaluated the probability of an event based on the degree to which the event was similar to the sample from which it was selected (Bar-Hillel, 1982; Gilovich & Savitsky, 2002; Kahneman & Frederick, 2002; Kahneman & Tversky, 1973; Kruglanski &

Ajzen, 1983; Shaughnessy, 1977, 1981; Tversky & Kahneman, 1974) or the degree the event was characterized by the notable features of the system from which it was derived (Fischbein, 1987; Kahneman & Tversky, 1973; Shaughnessy, 1977). Individuals viewed those samples as highly representative of their population, regardless of their population size (Tversky & Kahneman, 1982b). While some events reflect their parent populations, other events do not. The representativeness heuristic often resulted in errors that were derived from the following factors posited by Tversky and Kahneman (1974) and Shaughnessy (1977): (a) insensitivity to prior probabilities and disregard for population proportions, (b) insensitivity to the effects of sample size on predictive accuracy, (c) unwarranted confidence in a prediction based on invalid input data, (d) misconceptions of chance, (e) the illusion of validity, and (f) misconceptions of regression.

The Representativeness Misconception

The first of four intuitively-based probabilistic misconceptions at the focus of this study was the representativeness misconception. An example of individuals who employed the representativeness misconception was that of determining which combination of lottery numbers was more likely to occur (Cox & Mouw, 1992; Fischbein, 1999; Fischbein & Schnarch, 1997). In testing for this misconception, subjects were asked which of two combinations was more likely to occur in a single trial: a specific combination where all six numbers were consecutive (e.g., 1, 2, 3, 4, 5, 6) or a specific combination where that was not the case (e.g., 1, 8, 10, 13, 27, 41). Despite the fact that two specific combinations of numbers were equally likely to occur, subjects generally expected the second combination to be more likely; this was due in part to the fact that the specific six-number combination was a member of the class of combinations that contained non-consecutive numbers, and this class is much larger than the one that contains six-

number combinations that are consecutive. Misstatements occur^{3e} when the specific six-number combination with non-consecutive numbers was construed as any six-number combination with that property.

Similar to this example of the representativeness misconception is that of ordered sequences of the sex of children or of results of the flip of a coin. Individuals tended to think that the sequence BGBGBG was more likely to occur in a family with six children than, for example, either of the two sequences BBBBGB or BBBGGG (Borovcnik & Bentz, 1991; Kahneman & Tversky, 1972; Konold, 1991, 1995; Konold et al., 1993; Shaughnessy, 1977, 1981, 1992), or the same when a fair coin was tossed six times and the sequences of heads and tails tossed with that coin were compared (Borovcnik & Bentz, 1991; Griffiths & Tenenbaum, 2001; Hawkins & Kapadia, 1984; Konold, 1995; Konold et al.; Nickerson, 2004; Rubel, 2006; Tenenbaum & Griffiths, 2001). Shaughnessy (1977) asked students to compare, both before and after teaching these concepts, two families who each had six children; the birth orders for these two families were BGGBGB and BBBBGB. Over 70% of students prior to being taught the concepts chose the first sequence as more prevalent because it was more representative of realistic scenarios, having three boys and three girls; 55% of students also chose this option after being taught. Shaughnessy also asked students to compare the sequence BGGBGB to BBBGGG in the same way, this time giving students the additional option that the sequences had about the same chance. Of these respondents, 40% still chose BGGBGB to be more prevalent, citing the perceived randomness of the sexes in that first sequence that the second sequence seemingly lacked; however, about 35% of students chose that the sequences had about the same chance based on the fact that each sequence had three boys and three girls.

The problem of sequences of coin tosses is also susceptible to representativeness. Kahneman and Tversky (1972) found that students generally chose the sequence of coin tosses HTHTTH as being more likely to occur than HTHHHH since three heads and three tails was more representative of the number of heads and tails in a sequence of six tosses; most students disregarded that the sequence was in question. Kahneman and Tversky also found that students perceived the sequence of coin tosses HTTHTH as more random, and therefore more likely to occur, than the sequence HHHTTT. For a similar question on the outcomes of coin tosses, Rubel (2002) found that only about 39% of students perceived no difference in likelihood in any of the sequences based on independence of the coin tosses; students also perceived the sequence HTHTHT as too ordered, and less likely, comparatively speaking, than the sequence HHTHTT.

Positive and Negative Recency Effects

Two other misconceptions derived from the representativeness heuristic that were examined in this study were the negative recency effect, also known as “the gambler’s fallacy” (Cox & Mouw, 1992; Fischbein & Schnarch, 1997; Fischhoff, 1982; Gal & Baron, 1996; Shaughnessy, 1981), and the positive recency effect (Fischbein, 1975; Fischbein, Nello, & Marino, 1991; Fischbein & Schnarch; Gilovich et al., 2002). An individual who believed the outcome of one independent trial of an event was more likely to occur due to the absence of that outcome in earlier trials demonstrated thinking due to the negative recency effect. An individual who coin tossed heads several times might think that tails was more likely to be tossed the next time because the number of both heads and tails should have been approximately equal; this same effect occurred with rolling dice in craps games or other games of chance, hence the “gambler’s fallacy” moniker. Konold et al. (1993) asked students from varying levels of education, to choose a sequence of coin tosses that was most likely, with 72% able to do so.

When given sequences from which to choose one that was most likely, the majority of students in even the remedial level were able to do so. Konold et al. (1993) also asked students to pick which sequence of several given was least likely; 43% of those in the remedial level who answered the “most likely” question correctly indicated a least likely outcome. Hirsch and O’Donnell (2001) found, in their adaptation of the problem of sequences of coin tosses by Konold et al., that 44% of college students correctly identified that any sequence of five tosses of a coin was equally as likely as any other sequence of five tosses, but their research did not include students who answered in that way picking a specific sequence of tosses as having been least likely.

Alternately, if an individual felt that a specific outcome of a single trial of several independent trials of an event was more likely to occur because it has occurred in several previous trials, that person would have demonstrated thinking due to the positive recency effect. Many games of skill, such as blackjack and basketball, are viewed by outsiders as having components subject to positive recency. Basketball players who make several shots in a row are said to be “on fire”, the implication being that they would not miss any of the next several shots (Gilovich et al., 2002). A blackjack player who “rides a hot streak” may continue to gamble, believing he or she has a good chance to continue winning. As this misconception rarely manifested itself in previous research (Tversky & Kahneman, 1974), it was less noteworthy and less studied than the negative recency effect. Both the negative recency effect and the positive recency effect arose out of individuals’ misconceptions of what it meant for an event to be left up to chance (Tversky & Kahneman, 1974).

The Distinction Between Compound and Simple Events

Another misconception that arose out of the representativeness heuristic was the distinction between simple and compound events (Cox & Mouw, 1992; Fischbein & Schnarch, 1997; Lecoutre, 1992; Lecoutre & Durand, 1988; Quinn, 2004; Rubel, 2007; Tversky & Kahneman, 1982c). An individual who applied this misconception would have had trouble differentiating between a compound event, such as a roll of a five and a six with two fair dice, and a simple event, such as a roll of two sixes with two fair dice (Lecoutre & Durand, 1988). A roll of one five and one six is a compound event because there are two instances of it out of the 36 possible outcomes of rolling the two dice: a five on one die and a two on the other, or a two on the first die and a five on the other. A roll of two sixes is a simple event because there is only the one instance of rolling two sixes out of the same 36 possible outcomes.

Many studies were concerned with distinctions, or lack thereof, students made between these two events. Fischbein and Gazit (1984) had students compute probabilities of different sums of two dice; most students used 12 as the size of their sample space. Fischbein, Nello, and Marino (1991) asked students to compare the probabilities of tossing two heads with one head and one tail; 41% of elementary students and 54% of middle school students indicated these events had equal probabilities. Lecoutre (1992) asked college students to compare the probabilities of rolling a five and a six with two dice to rolling two sixes; about 50% of those students identified these events as having equal probabilities.

The Effect of Sample Size

A final misconception that arose from the representativeness heuristic is the effect that sample size plays in determining probabilities (Bar-Hillel, 1982; Fischbein & Schnarch, 1997; Tversky & Kahneman, 1974, 1982c). The law of large numbers states that as the number of

trials of an experiment increases, the frequency of a particular event relative to the number of trials will tend toward the theoretical probability (Rubel, 2002). An individual generally assessed probabilities without regard to the magnitude of a sample (Tversky & Kahneman, 1982c), having incorrectly tried to apply results from the law of large numbers to small samples (Tversky & Kahneman, 1982b). Because of the law of large numbers, the larger a sample, the more likely the probability of an event was to be to the actual probability in its parent population.

Kahneman and Tversky (1972) found that only 20% of college undergraduates correctly identified that the smaller of two hospitals was more likely to yield more days in which 60% of its births yielded a male child when both hospitals yielded similar proportional frequencies of male and female births. Shaughnessy (1977) replicated this study, finding that 60% of students discerned no difference in the two hospitals. Bar-Hillel (1982) found similar results, but also found that when the frequency was increased to 70% or 80% male births, more students chose the smaller hospital. Fischbein and Schnarch (1997) found that most of the younger students who even attempted to answer the question answered that the larger hospital would have more such births, while about 80% of the older students answered that there was no difference between the two hospitals. Watson and Moritz (2000) adapted the same problem and found that 61% of students found no difference in the two hospitals. A second example that utilized differences in sample size was that of comparing the probabilities of having tossed at least two heads in three coins, which has a probability of 50%, to having tossed at least two hundred heads in three hundred coins, which has a probability much less than 1% (Fischbein, 1999; Fischbein & Schnarch; Tirosh & Stavy, 1999).

Heuristics were often used to make quick judgments in situations where calculating probabilities was too time-consuming or too cumbersome. The most often used heuristic in

probability assessment was representativeness, and it yielded four intuitively-based misconceptions: the representativeness heuristic, positive and negative recency effects, the distinction between compound and simple events, and the effect of sample size (Bar-Hillel, 1982; Cox & Mouw, 1992; Fischbein, 1975, 1987, 1999; Fischbein, Nello, & Marino, 1991; Fischbein & Schnarch, 1997; Fischhoff, 1982; Gal & Baron, 1996; Gilovich et al., 2002; Lecoutre, 1992; Lecoutre & Durand, 1988; Quinn, 2004; Rubel, 2007; Shaughnessy, 1981; Tversky & Kahneman, 1974, 1982c). However useful the representativeness heuristic could have been, it was also susceptible to these intuitively-based probabilistic misconceptions, which were the roadblock to thorough understanding in the study of probability.

Mathematical Identity and Self-Efficacy

Why have some students had success with calculating probabilities (and likewise, mathematics in general) while others did not? There are factors that influence this question, but two were of the utmost importance: mathematical identity and mathematical self-efficacy. Mathematical identity is the way in which students view themselves with respect to the mathematics classroom; Forster (2000) defined mathematical identity as “students’ confidence in learning mathematics, exhibited in relationship with others” (p. 225). Mathematical identity was represented in students by such statements as “I am not a math person.” (negative mathematical identity) or “I am good at math.” (positive mathematical identity), and it was even seen in students stereotyping themselves with respect to gender, race, culture, or some other social category (Byrnes, 2005; Royer & Garofoli, 2005; Shih, Pittinsky, & Trahan, 2006). Mathematical identity and learning interconnected in the way in which Davis (1999) stated, “Learning, then, is not a matter of ‘adding to’ what is already there. Our learning, rather, is entangled in our becoming” (p. 333).

Closely related to mathematical identity was mathematical self-efficacy. Bandura (1986) defined self-efficacy as “people’s judgments of their capabilities to organize and execute courses of action required to attain designated types of performances” (p. 391); he further called self-efficacy “a judgment of one’s capability to accomplish a certain level of performance” (p. 391) in some task. A student’s mathematical self-efficacy often had the effect of raising or lowering that student’s mathematical identity, depending on if that self-efficacy was positive or negative.

Self-efficacy lent itself to the study of mathematics because of students’ attitudes toward their abilities to solve mathematical problems. Collins (1982) showed that students with high self-efficacy abandoned faulty methods of working problems, worked more problems correctly, and reworked and solved more difficult problems more often than students who had low self-efficacy. Student achievement and understanding were, therefore, positively correlated to a student’s self-efficacy. Similarly, Pajares and Miller (1994) found that self-efficacy was more predictive of problem solving than mathematics self-concept, perceived usefulness of mathematics, prior experience, or gender. Lent, Lopez, and Bieschke (1993) also found this correlation was true, that self-efficacy was a good predictor of student grades in mathematics. Similarly, Hackett and Betz (1989) found that self-efficacy moderately correlated with student achievement.

How have students’ mathematical identity and mathematical self-efficacy influenced their achievement in mathematics? Collins (1982) found that students with high mathematical self-efficacy were more willing to work problems and to return to those problems they had worked incorrectly in order to find the mistake with their work than students with low mathematical self-efficacy. Students with high mathematical self-efficacy not only examined the specific problems at hand, but also they examined structures of the mathematics problems they

encountered (Bandura, 1997; Hackett & Betz, 1989; Lent, Lopez, & Bieschke, 1993; Pajares & Miller, 1994). Likewise, students with low mathematical self-efficacy usually worked mathematics problems without examining the underlying principles of the specific problems and the general structure of the problems. Concerning mathematical identity, students who had a positive mathematical identity were those who felt they could work mathematics problems; therefore, just like with mathematical self-efficacy, those students were more successful in their study of mathematics over the long term. Students with a negative mathematical identity were more likely to give up rather than examine the hindrances they encountered in working mathematics problems (Hackett & Betz, 1989; Lent, Lopez, & Bieschke; Pajares & Miller, 1994).

Should studies attempt to identify mathematical identity in the pursuit toward mathematical understanding? Psychologists believe that psychology and teaching pedagogy are dependent upon each other to inform actual classroom situations (Alexander, 2006). Fischbein (1975) stated that student conceptual development in probability came from classroom instruction and interaction between students in the social classroom setting. For effective communication of problems to occur in a classroom setting, it was necessary for instruction to consider the students' backgrounds, their attitudes toward mathematics, and their personal mathematical identities (Bandura, 1986, 1997; Pajares & Kranzler, 1995; Pajares & Miller, 1994). One of the difficulties in teaching is the lurking question about whether students comprehend the topics being discussed; when viewed as the teacher's effectiveness, this was teacher's self-efficacy, which formed some part of a basis for student cognitive development (Bandura, 1997). Teachers needed to be able to anticipate what students' difficulties in learning probability were in order to help them develop problem-solving strategies (Batanero, Godino, &

Roa, 2004); this in turn had the effect of boosting students' personal mathematical identity and mathematical self-efficacy. Students should be aware of their own personal mathematical identities because that awareness helped to structure one's thought process; this helped the student to understand the causal nature behind his personal mathematical identity and self-efficacy and his ability to perform mathematics successfully (Duval & Wicklund, 1982).

In order to achieve student success in mathematics, mathematical proficiency influenced how a student perceived the ability to successfully achieve the goal (Collins, 1982; Matsui, Matsui, & Ohnishi, 1990). Self-efficacy, therefore, was the important linking factor to this perception. Another factor was a student's perception of himself in the social setting of the mathematics classroom, which was his identity (Bandura, 1986). The cited factors measured a student's mathematical understanding and, therefore, need attention. Although there was an apparent strong correlation between mathematical identity and mathematical self-efficacy, these two attributes did not necessarily have to agree for any one particular person. Consider a student who had a positive mathematical identity—this student would almost certainly possessed high mathematical self-efficacy because of the nature of his attitude toward himself. However, a student with a persistent attitude when running into obstacles and might not see himself as a mathematics student—this student would have a negative mathematical identity but high self-efficacy, and while this would lead to some level of mathematical understanding, it probably would not be long-lasting.

Teacher and students' mathematics beliefs influenced the quality of probabilistic reasoning in instruction and in problem solving (Charalambous, Philippou, & Kyriakides, 2008). A student who believed mathematics to be a useful tool in solving problems across many curricula would be successful in the study of mathematics (Duval & Wicklund, 1982; Lent,

Lopez, & Bieschke, 1993). Teachers who believed mathematics to be a source of certainty were enthusiastic about teaching probability to students, and those teachers were usually more willing and able to delve deeper into the structure of a topic rather than providing an outlet conducive only to a surface understanding (Bettencourt, Gillett, Gall, & Hull, 1983). Conversely, students and/or teachers who had a negative attitude toward mathematics, or if the teachers were not comfortable teaching probability, the quality of probabilistic reasoning went down because students did not learn as much and to a deeper level of understanding (Bettencourt et al., 1983).

Probabilistic reasoning is cemented in student cognitive intuition. As was the case with much mathematics, students must have had some idea about probability concepts before they were introduced (Fischbein, 1987; Piaget & Inhelder, 1951/1975), and this required deep intuitive thought for those students. Self-efficacy and identity, in relation to mathematics, must be considered in any study of student intuition and probabilistic reasoning since these psychological concepts would also play a part in the development of student intuition and probabilistic reasoning. Individuals must become better informed when having made any choices in which probabilistic reasoning were employed (Shaughnessy, 1977, 1981; Scholz, 1991).

Mathematical identity and self-efficacy were also necessary in an examination of student success in mathematics. Several studies, including Collins (1982), Hackett and Betz (1989), Lent, Lopez, and Bieschke (1993), Pajares and Kranzler (1995), and Pajares and Miller (1994) found positive correlations between mathematical self-efficacy and student achievement in mathematics. If mathematical identity and self-efficacy could be increased in a positive manner, student achievement should increase as well, thus providing students with the tools they needed to be successful in furthering their abilities in mathematics.

Summary

This chapter contained sections that cover attributions of probability that make it a difficult topic in mathematics, mathematical intuition and its characteristics, and the incorporation of intuition into the study of probability. There were also sections on intuitive thought versus rational thought, heuristics used to make quick decisions regarding probability tasks, misconceptions that arose from these heuristics, and mathematical identity and self-efficacy.

Intuition is a multi-faceted concept in the psychology of the mind, and the use of heuristics in assessing probabilities is an evolution of the mind to make these assessments quickly. Research indicated that there are common misconceptions that arose from the use of heuristics in these situations, especially in the developmental stages of rational thinking that took place in 5-12 education. Research showed that the persistence of probabilistic misconceptions was correlated with grade level. Certain misconceptions may be correlated to students' self-efficacy in being able to handle complicated probability tasks and probabilistic topics.

CHAPTER III:

METHODOLOGY

Introduction

The purpose of this quantitative study was twofold. First, the study investigated relationships between perceived mathematics probability misconceptions among students at the 7th-, 9th-, and 11th-grade levels. Second, the study investigated relationships existing between students' perceived probabilistic misconceptions and their self-efficacy beliefs with intuitive mathematics probability. This study extended the research of Fischbein and Schnarch (1997) concerning misconceptions in probability while offering insight into the relationship between self-efficacy and achievement with respect to probability.

This research consisted of both a pilot study and the main research study. The participants for both studies were chosen from the 7th-, 9th-, and 11th-grade levels. These grade levels were chosen to mirror the research of Fischbein and Schnarch (1997), who used those grade levels as well as students at the fifth-grade level in their initial study. However, the fifth-grade level was omitted in this study, due to the fact that most fifth-grade mathematics classes have not covered those probability topics, based on the *Alabama Course of Study: Mathematics* (2003).

Research Questions

There were two questions for this research study. They included the following:

1. What is the relationship between students' 7th-, 9th-, or 11th-grade level and each of the four intuitively-based probabilistic misconceptions of the representativeness misconception,

positive and negative recency effects, the distinction between compound and simple events, and the effect of sample size; and

2. What is the relationship between students' responses in the 7th-, 9th-, and 11th-grade levels, to questions involving probability tasks and their perceived self-efficacy in answering those questions?

Participants

The setting for the study was a large public school district located in Alabama; the school district is in a suburban city of a large metropolitan area of the state. Other 2003 estimates for the city included that 57.5% of the population held a post-secondary degree and that the median household income was \$71,964. The 2003 demographic estimates for the city, the latest available such statistics, are reported in Table 1. The demographics for the school district for the 2009-2010 school year are reported in Table 2. The demographics for the 7th-, 9th-, and 11th-grade schools used in the research for the 2009-2010 school year are reported in Table 3.

Table 1

City Population Demographics, 2003 Estimates

| Demographic Category | Percentage of Total |
|---------------------------|---------------------|
| White | 87.0% |
| Black | 7.0% |
| Asian or Pacific Islander | 3.5% |
| Other | 2.5% |

Note. Estimated population 72,235.

Table 2

School District Population Demographics, 2009-2010 School Year

| Demographic Category | Percentage of Total |
|---------------------------|---------------------|
| White | 65.8% |
| Black | 21.7% |
| Asian or Pacific Islander | 5.7% |
| Other | 0.3% |

Note. Approximate population 12,900; 19.3% qualified for free or reduced-price lunches.

Table 3

Individual School Population Demographics, 2009-2010 School Year

| Demographic Category | <u>School Year</u> | | |
|---------------------------|--------------------|-----------------|------------------|
| | 7 th | 9 th | 11 th |
| White, Not Hispanic | 69.5% | 64.8% | 66.2% |
| Black, Not Hispanic | 20.0% | 23.9% | 22.5% |
| Hispanic | 6.2% | 6.1% | 4.0% |
| Asian or Pacific Islander | 4.3% | 4.7% | 7.2% |
| Other | 0.0% | 0.5% | 0.1% |

The percentages of students in each school chosen for the study most closely resembled the demographics for the school district when compared with similar schools within the district. This was the reason for choosing these particular schools. Also, these three schools feed each other, so some students in the 9th- or 11th-grade levels may have had the same teachers in previous grades.

The participants for the pilot study were 308 students total: 103 students in the seventh-grade level, 101 students in the ninth-grade level, and 104 students in the 11th-grade level. The participants for the main study were 546 students total: 174 students in the seventh-grade level, 188 students in the ninth-grade level, and 184 students in the 11th-grade level. In order to get a variety of students, a convenience sample of participants for each part of the study was selected by selecting teachers in the school district who teach students at multiple levels for each respective grade and requesting student participation from those teachers' classes.

Permission for the Study

Prior to the study, permission was requested from the Institutional Review Board (IRB) at the researcher's university. The researcher contacted the assistant superintendent of the school district to explain the purpose of the study and to obtain permission to conduct a survey at the selected school sites. Permission was then sought from principals at the selected school sites. Once this was achieved, teachers at those sites were contacted concerning their willingness to have their classes participate in the collection of data.

Once permission was obtained by the assistant superintendent, school principals, and teachers, the potential participants had the study described to them, and any questions or concerns they may have had regarding the study were addressed. Students were given consent forms (see Appendix A) to give to their parent(s) or guardian(s); these consent forms explained the purpose of the study, what was being asked of their child to participate in the study, and other information that was relevant to the study. The consent form assured parents and guardians that their child's participation was voluntary, students' grades were not affected by participation or lack thereof, and there were no incentives to participate in the study. Students' names were not used, the researcher was the sole person who collected and examined the data, and the results

were confidential. Three days were provided to students to receive consent from their parent or guardian. Assent from the students was then sought, which indicated their own willingness to participate in the study. Once consent and assent were received, there was an additional brief period for face-to-face questions concerning the study. Afterward, students responded to the survey. At that time, the only questions that participants were allowed to ask were clarifications of a general nature; no questions concerning procedural aspects of each mathematical item would be answered.

All materials and collected data, both on paper and electronically on a flash drive, were kept in a locked drawer of a file cabinet, the only key to which was kept by the researcher, as per the stipulations made in the IRB application. Keeping materials and data locked in a file cabinet preserved the anonymity of students who participated in the study while maintaining those materials if needed again.

Data Source and Collection

Pilot Study

The pilot study was conducted in a large public school district in Alabama. The school district is a suburban part of a large metropolitan area of the state. The purpose of the pilot study was to determine if the researcher could use the multiple-choice format of the survey in place of using the open-ended format of the survey (see Appendices B and C for surveys). Both the open-ended survey and the multiple-choice survey consisted of four items that tested for probabilistic misconceptions derived from the heuristic of representativeness; the items on both surveys parallel each other. The items were modified from the original survey given by Fischbein and Schnarch (1997).

The four items on both surveys tested for intuitively-based probabilistic misconceptions derived from the heuristic of representativeness. The four probabilistic misconceptions examined in this study were the representativeness misconception, positive and negative recency effects, the distinction between compound and simple events, and the effect of sample size. Each of the four items on the survey corresponded to one of those probabilistic misconceptions. On the multiple-choice survey, three answer choices were given: one was the correct answer and two were incorrect answers. The distractors were developed out of responses found in the Fischbein and Schnarch (1997) study. In the survey developed by Fischbein and Schnarch, items were open-ended, and for their data analysis, student responses were coded into groups. For each item, those groups included correct responses and incorrect responses, but among the incorrect responses, they referred to one particular incorrect response as the “main misconception”—the incorrect response that occurred with the largest frequency in their research. The other incorrect answers occurred with lesser frequency in their research.

The two surveys were developed over the course of one month by the researcher and committee members with respect to the original questions used by Fischbein and Schnarch (1997). The items were narrowed down to those involving probabilistic misconceptions derived from the representativeness heuristic. Wording for items was developed using sample problems from seventh-grade textbooks used in the school district, *Alabama Course of Study: Mathematics* (2003), NCTM’s (2000) *Principles and Standards for School Mathematics*, and Rubel (2002), who also studied these probabilistic misconceptions. All wording was developed using a seventh-grade level while still maintaining the essence of each item as asked on the survey by Fischbein and Schnarch.

Each of the mathematical items on the surveys had a corresponding five-point Likert scale item concerning students' self-efficacy in answering the item. Students were asked to judge their own certainty that each of their responses was mathematically correct, where 1 indicated very little certainty, 2 indicated little certainty, 3 indicated neither certainty nor uncertainty, 4 indicated certainty, and 5 indicated much certainty in the mathematical correctness of the response. Self-efficacy was not examined in the study by Fischbein and Schnarch (1997), but it was examined in this research study as it related to student response at each of the 7th-, 9th-, and 11th-grade levels. In addition to responses to both the mathematical item and the Likert scale item, two demographic items also were sought: grade level and gender.

Parental consent and student assent were gained via the procedures described in the Permission section of this chapter. After both consent and assent were achieved, students were gathered on a designated day in the library of the school, during their "academy" period for the seventh-grade students and after school for 9th- and 11th-grade students, for ease of distributing and administering the surveys. Students were given a brief, ten-minute period to ask questions to the researcher, after which time the surveys were administered to the students by the researcher. Students were given thirty minutes in which to take the survey, all of whom finished well before that time limit. Students were not allowed to ask questions concerning procedural aspects of each mathematical item during this thirty-minute period; only questions concerning clarifications of a general nature were allowed.

The population for the pilot study consisted of four classes, two from each of two teachers, at each of grades 7, 9, and 11 within the school district, for a total of twelve classes. One class from each teacher was a middle-to-high-achieving class and the other was a low-to-middle-achieving class. There were a total of 332 students in these twelve classes, and a total of

308 volunteered to participate in the study, yielding a 93% response rate. At the seventh-grade level, 50 students took the open-ended survey and 53 students took the multiple-choice survey. At the ninth-grade level, 51 students took the open-ended survey and 50 students took the multiple-choice survey. At the 11th-grade level, 52 students each took the open-ended and multiple-choice surveys. The frequency of each type of response for each item on each survey format is reported in Tables 4-7.

The researcher coded all responses on the open-ended survey. Codings for “main misconception” and alternate incorrect responses were derived from the codings made for responses on the study by Fischbein and Schnarch (1997). Such types of responses that distinguished “main misconception” answers from alternate incorrect answers were also reported in that previous study. Correct responses were also coded in the previous study. The researcher’s experience in teaching probability was also used to distinguish different types of responses.

Table 4

Frequencies of Responses, Pilot Study, Item 1

| Grade | Open Ended Survey | | | Mutiple Choice Survey | | |
|-------|-------------------|----|----|-----------------------|----|----|
| | C | MM | AI | C | MM | AI |
| 7 | 20 | 20 | 10 | 23 | 21 | 9 |
| 9 | 34 | 8 | 9 | 33 | 9 | 8 |
| 11 | 26 | 19 | 7 | 24 | 21 | 7 |

Note. C = correct; MM = main misconception; AI = alternate incorrect.

Table 5

Frequencies of Responses, Pilot Study, Item 2

| Grade | Open Ended Survey | | | Mutiple Choice Survey | | |
|-------|-------------------|----|----|-----------------------|----|----|
| | C | MM | AI | C | MM | AI |
| 7 | 14 | 33 | 3 | 14 | 35 | 4 |
| 9 | 29 | 22 | 0 | 29 | 20 | 1 |
| 11 | 24 | 27 | 1 | 25 | 26 | 1 |

Note. C = correct; MM = main misconception; AI = alternate incorrect.

Table 6

Frequencies of Responses, Pilot Study, Item 3

| Grade | Open Ended Survey | | | Mutiple Choice Survey | | |
|-------|-------------------|----|----|-----------------------|----|----|
| | C | MM | AI | C | MM | AI |
| 7 | 40 | 5 | 5 | 41 | 6 | 6 |
| 9 | 27 | 20 | 4 | 26 | 22 | 2 |
| 11 | 29 | 21 | 2 | 28 | 22 | 2 |

Note. C = correct; MM = main misconception; AI = alternate incorrect.

Table 7

Frequencies of Responses, Pilot Study, Item 4

| Grade | Open Ended Survey | | | Multiple Choice Survey | | |
|-------|-------------------|----|----|------------------------|----|----|
| | C | MM | AI | C | MM | AI |
| 7 | 7 | 41 | 2 | 7 | 43 | 3 |
| 9 | 22 | 24 | 5 | 24 | 24 | 2 |
| 11 | 25 | 25 | 2 | 26 | 24 | 2 |

Note. C = correct; MM = main misconception; AI = alternate incorrect.

For the coding of student responses on the open-ended survey, the researcher's experience in teaching probability was also required to determine in a few instances what the student's intended response was in order to clarify any ambiguity in responses. Since the last two items on the survey presented two options from which students were to select a response, potential ambiguity of responses was at its highest in the first two items. Examples of such ambiguities, whose descriptions follow, appear in Appendix D.

For the seventh-grade student, on the first item, she stated that heads would be expected on the fourth flip, but she then clarified that either heads or tails could occur with a 50/50 probability. Her use of the word "well" indicated that she had rethought this scenario and believed either heads or tails could occur on the fourth flip. For this reason, this response was coded as either heads or tails was equally likely to occur. For the same student's response on the second item, she stated that any numbers of heads and tails could occur and gave two possibilities: 49 heads and 51 tails or 7 heads and 93 tails. This response was coded as about equal number of heads and tails because although she interpreted the question to predict the

exact outcome of 100 flips of the coin, she recognized that each flip of the coin is independent, meaning about half the flips would result in heads.

The ninth-grade student's response to the second item also could be interpreted as ambiguous. In his response, he stated that there was no way to determine the breakdown of heads and tails because there was a fifty-fifty chance on each flip of the coin. This response mirrored the response of the seventh-grade student in recognizing the independence of the flips, despite his interpretation of the question as having sought an exact distribution of the number of heads and tails in 100 flips of the coin.

For the 11th-grade student, her response to the first item indicated heads was would occur on the fourth flip of the coin after three tails were flipped on the previous three flips of the coin. She did state that it was unlikely to land on tails again but also stated there was a 50/50 chance on one flip of the coin. However, unlike the seventh-grade student, she interpreted this 50/50 chance in a different way. To this student, the probability was very high of flipping a heads on the fourth flip, with the 50/50 chance lessening that probability slightly. Thus past performance of the coin flips was interpreted as having an impact on the current coin flip. To this student, the combination of three tails being flipped previously weighed more on the next flip than the 50/50 chance of heads on the fourth flip. For this reason, this answer was coded as heads, an instance of the negative recency effect. For this student's response on the second item, she stated that every ten flips of the coin would result in at least seven heads. Even though she did not indicate this explicitly, this student appeared to believe in maintaining the proportional outcomes of heads and tails, since at least seven heads every ten flips would have resulted in at least 70 heads. For this reason, this student's response was coded as about 70 heads, neglecting the sample size of the two scenarios.

Pilot Study Results

The frequencies of the types of responses between the two formats of the survey were comparable for each item and at each grade level. Most frequencies differed by only one or two; the largest difference in frequencies was three, which occurred twice between the two formats. One occurrence was between correct responses at the seventh-grade level (20 correct responses on the open-ended survey versus 23 correct responses on the multiple-choice survey); the other occurrence was between alternate incorrect responses at the ninth-grade level (five on the open-ended survey compared to two on the multiple-choice survey). Based on the similarities of the responses at all grade levels and for all items, the multiple-choice survey (see Appendix C) was used for the research study. For the research study, the multiple-choice survey was preferable in order that each student's response to each item was clear; an open-ended survey could have caused different readers of student responses to disagree on what was the student's actual intended answer.

Instrument

Based on the findings from the pilot study, the multiple-choice survey was used for the research study. The multiple-choice survey featured four questions, one each testing for the presence of four intuitively-based probabilistic misconceptions: the representativeness misconception, positive and negative recency effects, the distinction between compound and simple events, and the effect of sample size.

For each item on the survey, students were to select one of the answer choices given and were also offered an opportunity to explain the choice they selected. Additionally, the students were then to rate themselves on a five-point Likert scale assessing how certain they were that the answer they just gave was correct. Each item on the survey allowed students to offer an

explanation for the answer they selected, although very few students actually did offer such an explanation.

The first question on the survey tested for the presence of the misconception of positive and negative recency effects. The question concerned the likelihood of a specific outcome on one flip of a coin based on the results of three previous flips of the coin, all of which resulted in tails.

The second question on the survey tested for the presence of the misconception of effect of sample size. The question concerned which of two events, if either, was more likely to occur: a certain number of flips resulting in heads out of a small amount of flips or a proportional number of flips resulting in heads out of a much larger amount of flips.

The third question on the survey tested for the presence of the misconception of the distinction between compound and simple events. The question asked students to determine which of two events, if any, was more likely to occur when tossing a pair of fair dice: the simple event of two sixes or the compound event of one five and one six.

The fourth question on the survey tested for the presence of the representativeness misconception. The question asked of students to determine which of two sets of numbers, if any, drawn in a lottery was more likely to occur: a set in which the numbers are consecutive (8, 9, 10, 11) or a set in which the numbers are not consecutive (4, 28, 17, 19).

Research Study Participants

Based on the pilot study results, the multiple-choice survey was used for the research study. The research study took place in the same school district, and the population for the pilot study consisted of students at the same schools as those who participated in the pilot study but excluded students who participated in the pilot study. In the seventh-grade school, due to the

size of the school, the remaining seventh-grade students made up the population for that grade level. The ninth-grade population consisted of students in Algebra 1, Algebra 1-A, Geometry, and Geometry Math Team classes. The 11th-grade population consisted of students in Algebra 2, Algebra 3, Precalculus, Pre-AP Precalculus, and Precalculus Math Team classes. There were a total of 713 students in these classes, and a total of 546 volunteered to participate in the study, yielding a 77% response rate. Of this number, 174 students were in the seventh grade, 188 students were in the ninth grade, and 184 students were in the 11th grade.

Parental consent and student assent were gained via the procedures described in the Permission section of this chapter. Once consent and assent were achieved, students who wished to participate met on a designated day in the library of the respective schools. At the seventh-grade school, students met during their “academy” period, and at the 9th- and 11th-grade schools, students met after school for the seventh-grade students and after school for 9th- and 11th-grade students; this was for ease of distributing and administering the surveys. Students were given a brief, 10-minute period to ask questions to the researcher, after which time the surveys were administered to the students. Students had 30 minutes to take the survey, and most students finished well before that time limit. Students were not allowed to ask questions concerning procedural aspects of each mathematical item during this 30-minute period; only questions concerning clarifications of a general nature were allowed.

Data Analysis

Research Question One

The first goal of the research study was to determine if significant correlations existed between the grade level of the participants and both the frequencies of their responses to each mathematical item on the survey. These correlations, which were computed for each of the

probabilistic misconceptions, were to determine if those misconceptions were linked to the grade level of students taking the survey. Fischbein and Schnarch (1997) also were interested in this correlation between these variables for each misconception, but their informal correlations were determined by making assessments and examining trends based on percentages of responses reported.

Since both of the variables in this research study were viewed as categorical data, Spearman correlations were determined in order to achieve this goal. Spearman correlations were used rather than Pearson correlations because normality assumptions about the variables were not necessary; Pearson correlations require that the variables are normally distributed (Lomax, 2001). Also, Spearman correlations are used to detect monotonic trends in data, whereas Pearson correlations are used to detect linear trends specifically (Gravetter & Wallnau, 2009). With little previous research on this topic, no assumptions were made about the variables in order to reduce bias. For each item on the survey, a Spearman correlation was computed to determine if an association existed between the two variables of grade level and frequencies of responses to the item. A post-hoc analysis followed which omitted the alternate incorrect response given as an answer choice in addition to the correct and “main misconception” responses.

The null research hypotheses for the first research question were the following:

H₀₁: There is no significant correlation between grade level and frequency of responses with respect to positive and negative recency effects.

H₀₂: There is no significant correlation between grade level and frequency of responses with respect to the effect of sample size.

H₀3: There is no significant correlation between grade level and frequency of responses with respect to the distinction between compound and simple events.

H₀4: There is no significant correlation between grade level and frequency of responses with respect to the representativeness misconception.

Research Question Two

The second goal of the research study was to determine if a significant correlation existed within each of the three grade levels between the correctness of participants' responses to each mathematical item on the survey and their perceived self-efficacy in responding to those items. This portion of the research study was not studied by Fischbein and Schnarch (1997), and no other research was located linking self-efficacy to these probabilistic misconceptions. These data were again viewed as categorical, and a Spearman correlation was determined in order to see if such an association existed. The same assumptions for using the Spearman correlation for the first research question (no normality assumption required, monotonic trends sought rather than specifically linear trends, limited availability of previous research) also applied to the second research question. For each item on the survey and within each grade level, a Spearman correlation was computed in order to determine what relationship, if any, existed between the correctness of student responses to the mathematical item and their rating on the five-point Likert scale assessing certainty of those responses.

The null research hypotheses for the second research question were the following:

H₀1: There is no significant correlation between frequency of correct responses and perceived certainty with those responses with respect to positive and negative recency effects for students at the 7th-, 9th-, or 11th-grade levels.

H₀2: There is no significant correlation between frequency of correct responses and perceived certainty with those responses with respect to the effect of sample size for students at the 7th-, 9th-, or 11th-grade levels.

H₀3: There is no significant correlation between frequency of correct responses and perceived certainty with those responses with respect to the distinction between compound and simple events for students at the 7th-, 9th-, or 11th-grade levels.

H₀4: There is no significant correlation between frequency of correct responses and perceived certainty with those responses with respect to the representativeness misconception for students at the 7th-, 9th-, or 11th-grade levels.

All data were processed for both research questions using SAS 9.2 with the previously mentioned correlations sought. In addition to those correlations, the breakdown of descriptive statistics such as grade level and responses to items were computed.

Summary

This study investigated the correlations existing between responses given on probability items testing for probabilistic misconceptions and student grade level for students at the 7th-, 9th-, or 11th-grade level. This study also investigated the correlations between the correctness of those student responses and their perceived self-efficacy in answering those probability items. This study was an extension of that of Fischbein and Schnarch (1997); however, formal correlative statistics were determined from this study that were not examined in the previous study.

CHAPTER IV:

RESULTS

Probability is a difficult mathematical topic at the middle- and high-school levels. Student difficulty in the study of probability arises out of the use of multiple heuristics. The intended effect of these heuristics is to simplify the thought processes needed to work problems involving probability. However, the unintended consequence of using heuristics is that misconceptions occur in these thought processes that lead students to incorrect answers on those problems. This study was concerned with trends between grade level and four intuitively-based probabilistic misconceptions arising out of the heuristic of representativeness first introduced by Fischbein and Schnarch (1997).

A pilot study was administered to determine if any differences in student responses existed across two formats of an instrument adapted from Fischbein and Schnarch (1997). One instrument contained open-ended items, the second contained multiple-choice items. Once data were collected, comparisons were made across the two formats of the instrument. The frequencies of responses across all items and grade levels were similar, and by comparing the frequencies of responses across the two formats, the decision to use the multiple-choice format of the survey was validated.

The purpose of the research study was twofold. First, the study determined if significant correlations between grade level and frequencies of responses to the multiple-choice survey items existed for each item. Second, the study determined if significant correlations existed for each item and within each individual grade level (7, 9, or 11) between the correctness of student

responses and their perceived self-efficacy in answering those survey items by way of a five-point Likert scale selection for each item. Spearman correlations were determined to answer each of these research purposes.

Research Study

The statistical analysis for the research study consisted of three steps. The first step was to compute descriptive statistics and frequencies in order to provide a demographic profile of the research study participants. The second step was to compute a Spearman correlation for each item between grade level and student responses. The third step was to compute a Spearman correlation for each item and within each grade level between correctness of student responses and frequency of Likert scale responses that indicated their perceived self-efficacy in answering the item. Significance of all statistics in the research study was determined at the $\alpha = .05$ level.

Analysis of Research Question One

Item 1

The first item of the survey was concerned with the probabilistic misconception of positive and negative recency effects, as described in the Pilot Study section of the previous chapter. Students could respond in one of three ways: with the correct answer, with an incorrect answer indicating the “main misconception”, and with an alternate incorrect answer. The frequencies of these responses appear in Table 8.

Table 8

Frequencies of Responses, Research Study, Item 1

| Grade | C | MM | AI |
|-------|-----|----|----|
| 7 | 121 | 40 | 13 |
| 9 | 146 | 31 | 11 |
| 11 | 158 | 19 | 7 |

Note. C = correct; MM = main misconception; AI = alternate incorrect.

A Spearman correlation was computed comparing grade level to student response for this item. Statistical analysis showed there was a significant negative correlation between grade level and student response [$r_s = -.157, p < .001, n = 546$]. This statistic indicated that the frequency of incorrect responses decreased as grade level increased. Because the frequency of alternate incorrect answers was so low, a post-hoc analysis was performed excluding those responses; thus, only correct and “main misconception” responses were considered in this second analysis. Statistical analysis showed there was again a significant negative correlation between grade level and student response [$r_s = -.150, p < .001, n = 515$]. Because the correlations were similar, this further analysis showed that the alternate incorrect response did not influence the data to any substantial degree.

Item 2

The second item of the survey was concerned with the probabilistic misconception of the effect of sample size, as described in the Pilot Study section of the previous chapter. Students could respond in one of three ways: with the correct answer, with an incorrect answer indicating the “main misconception”, and with an alternate incorrect answer. The frequencies of these responses appear in Table 9.

Table 9

Frequencies of Responses, Research Study, Item 2

| Grade | C | MM | AI |
|-------|----|-----|----|
| 7 | 46 | 113 | 15 |
| 9 | 39 | 129 | 20 |
| 11 | 29 | 134 | 21 |

Note. C = correct; MM = main misconception; AI = alternate incorrect.

A Spearman correlation was computed comparing grade level to student response for this item. Statistical analysis showed there was a significant positive correlation between grade level and student response [$r_s = .101, p = .018, n = 546$]. This statistic indicated that the frequency of incorrect responses increased as grade level increased. Because the frequency of alternate incorrect answers was so low, a post-hoc analysis was performed excluding those responses; thus, only correct and “main misconception” responses were considered in this second analysis. Statistical analysis showed there was again a significant positive correlation between grade level and student response [$r_s = .107, p = .018, n = 490$]. Because the correlations were similar, this further analysis showed that the alternate incorrect response did not influence the data to any substantial degree.

Item 3

The third item of the survey was concerned with the probabilistic misconception of the distinction between compound and simple events, as described in the Pilot Study section of the previous chapter. Students could respond in one of three ways: with the correct answer, with an incorrect answer indicating the “main misconception”, and with an alternate incorrect answer. The frequencies of these responses appear in Table 10.

Table 10

Frequencies of Responses, Research Study, Item 3

| Grade | C | MM | AI |
|-------|----|-----|----|
| 7 | 42 | 123 | 9 |
| 9 | 46 | 135 | 7 |
| 11 | 46 | 136 | 2 |

Note. C = correct; MM = main misconception; AI = alternate incorrect.

A Spearman correlation was computed comparing grade level to student response for this item. Statistical analysis showed there was no significant correlation between grade level and student response [$r_s = -.035$, $p = .410$, $n = 546$]. This statistic indicated that no relationship existed between the frequency of responses and the grade level. Because the frequency of alternate incorrect answers was so low, a post-hoc analysis was performed excluding those responses; thus, only correct and “main misconception” responses were considered in this second analysis. Statistical analysis showed there was again no significant correlation between grade level and student response [$r_s = .002$, $p = .969$, $n = 528$]. Because the correlations were similar, this further analysis showed that the alternate incorrect response did not influence the data to any substantial degree.

Item 4

The fourth item of the survey was concerned with the representativeness misconception, as described in the Pilot Study section of the previous chapter. Students could respond in one of three ways: with the correct answer, with an incorrect answer indicating the “main misconception”, and with an alternate incorrect answer. The frequencies of these responses appear in Table 11.

Table 11

Frequencies of Responses, Research Study, Item 4

| Grade | C | MM | AI |
|-------|-----|----|----|
| 7 | 109 | 62 | 3 |
| 9 | 125 | 60 | 3 |
| 11 | 130 | 53 | 1 |

Note. C = correct; MM = main misconception; AI = alternate incorrect.

A Spearman correlation was computed comparing grade level to student response for this item. Statistical analysis showed there was no significant correlation between grade level and student response [$r_s = -.071$, $p = .097$, $n = 546$]. This statistic indicated that no relationship existed between the frequency of responses and the grade level. Because the frequency of alternate incorrect answers was so low, a post-hoc analysis was performed excluding those responses; thus, only correct and “main misconception” responses were considered in this second analysis. Statistical analysis showed there was again no significant correlation between grade level and student response [$r_s = -.063$, $p = .144$, $n = 539$]. Because the correlations were similar, this further analysis showed that the alternate incorrect response did not influence the data to any substantial degree.

Analysis of Research Question Two

For each item on the survey, a follow-up question was posed to students immediately afterward, asking them to assess how certain they were that the answer they chose for that item was correct. This was a five-point Likert scale item with the following levels: “1 – Not Very Certain,” “2 – Not Certain,” “3 – Unsure,” “4 – Certain,” and “5 – Very Certain.” Students were to indicate their certainty level with their answer by circling the appropriate number.

Item 1

The first item dealt with positive and negative recency effects. Responses were broken down by correctness of answer and by rating on the Likert scale. The results for the seventh-grade students appear in Table 12, those for the ninth-grade students appear in Table 13, and those for the 11th-grade students appear in Table 14.

Table 12

Likert Scale Responses, Item 1, 7th Grade

| Likert Scale Selection | | | | | |
|------------------------|---|----|----|----|----|
| Response | 1 | 2 | 3 | 4 | 5 |
| Correct | 0 | 8 | 20 | 51 | 42 |
| Incorrect | 4 | 12 | 15 | 17 | 5 |

Table 13

Likert Scale Responses, Item 1, 9th Grade

| Likert Scale Selection | | | | | |
|------------------------|---|---|----|----|----|
| Response | 1 | 2 | 3 | 4 | 5 |
| Correct | 1 | 4 | 11 | 38 | 92 |
| Incorrect | 0 | 5 | 22 | 11 | 4 |

Table 14

Likert Scale Responses, Item 1, 11th Grade

| Response | Likert Scale Selection | | | | |
|-----------|------------------------|---|----|----|-----|
| | 1 | 2 | 3 | 4 | 5 |
| Correct | 3 | 5 | 7 | 42 | 101 |
| Incorrect | 1 | 4 | 13 | 8 | 0 |

A Spearman correlation was computed comparing student response to Likert scale response for this item at each individual grade level. Statistical analysis showed there was a significant positive correlation between correctness of student response and Likert scale response at the seventh-grade level [$r_s = .385, p < .001, n = 174$]. This statistic indicated that the frequency of correct responses increased as the number selected on the Likert scale increased at the seventh-grade level, meaning that students who answered correctly were more likely to be confident in their answers. Secondly, there was also a significant positive correlation between correctness of student response and Likert scale response at the ninth-grade level [$r_s = .521, p < .001, n = 188$]. This statistic indicated that the frequency of correct responses increased as the number selected on the Likert scale increased at the ninth-grade level, again meaning students who correctly responded to the problem were more confident with their responses. Lastly, there was a significant positive correlation between correctness of student response and Likert scale response at the 11th-grade level [$r_s = .519, p < .001, n = 184$]. This statistic indicated that the frequency of correct responses increased as the number selected on the Likert scale increased at the 11th-grade level, again implying students who answered the problem correctly tended to be more confident with their answers.

Item 2

The second item dealt with the effect of sample size. Responses were broken down by correctness of answer and by rating on the Likert scale. The results for the seventh-grade students appear in Table 15, those for the ninth-grade students appear in Table 16, and those for the 11th-grade students appear in Table 17.

Table 15

Likert Scale Responses, Item 2, 7th Grade

| Likert Scale Selection | | | | | |
|------------------------|---|----|----|----|----|
| Response | 1 | 2 | 3 | 4 | 5 |
| Correct | 0 | 5 | 17 | 14 | 10 |
| Incorrect | 7 | 18 | 34 | 44 | 25 |

Table 16

Likert Scale Responses, Item 2, 9th Grade

| Likert Scale Selection | | | | | |
|------------------------|---|---|----|----|----|
| Response | 1 | 2 | 3 | 4 | 5 |
| Correct | 0 | 2 | 6 | 18 | 13 |
| Incorrect | 1 | 9 | 31 | 54 | 54 |

Table 17

Likert Scale Responses, Item 2, 11th Grade

| Response | Likert Scale Selection | | | | |
|-----------|------------------------|---|----|----|----|
| | 1 | 2 | 3 | 4 | 5 |
| Correct | 0 | 3 | 10 | 9 | 7 |
| Incorrect | 2 | 6 | 29 | 63 | 55 |

A Spearman correlation was computed comparing student response to Likert scale response for this item at each individual grade level. Statistical analysis showed there was no significant correlation between correctness of student response and Likert scale response at the seventh-grade level [$r_s = .037, p = .624, n = 174$]. This statistic indicated that no relationship existed between frequency of responses to the item and frequency of Likert scale responses at the seventh-grade level. Secondly, there was also no significant correlation between correctness of student response and Likert scale response at the ninth-grade level [$r_s = .018, p = .811, n = 188$]. This statistic indicated that no relationship existed between frequency of responses to the item and frequency of Likert scale responses at the ninth-grade level. Lastly, however, there was a significant negative correlation between correctness of student response and Likert scale response at the 11th-grade level [$r_s = -.148, p = .045, n = 184$]. This statistic indicated that the frequency of correct responses decreased as the number selected on the Likert scale increased at the 11th-grade level, meaning that students who answered the problem correctly were less likely to be confident with that answer than students who answered the problem incorrectly.

Item 3

The third item dealt with the distinction between compound and simple events.

Responses were broken down by correctness of answer and by rating on the Likert scale. The results for the seventh-grade students appear in Table 18, those for the ninth-grade students appear in Table 19, and those for the 11th-grade students appear in Table 20.

Table 18

Likert Scale Responses, Item 3, 7th Grade

| Likert Scale Selection | | | | | |
|------------------------|----|----|----|----|----|
| Response | 1 | 2 | 3 | 4 | 5 |
| Correct | 1 | 3 | 5 | 22 | 11 |
| Incorrect | 14 | 18 | 33 | 38 | 29 |

Table 19

Likert Scale Responses, Item 3, 9th Grade

| Likert Scale Selection | | | | | |
|------------------------|---|---|----|----|----|
| Response | 1 | 2 | 3 | 4 | 5 |
| Correct | 3 | 8 | 14 | 13 | 8 |
| Incorrect | 5 | 9 | 31 | 56 | 41 |

Table 20

Likert Scale Responses, Item 3, 11th Grade

| Response | Likert Scale Selection | | | | |
|-----------|------------------------|----|----|----|----|
| | 1 | 2 | 3 | 4 | 5 |
| Correct | 2 | 2 | 22 | 12 | 8 |
| Incorrect | 2 | 11 | 32 | 48 | 45 |

A Spearman correlation was computed comparing student response to Likert scale response for this item at each individual grade level. Statistical analysis showed there was a significant positive correlation between correctness of student response and Likert scale response at the seventh-grade level [$r_s = .190, p = .012, n = 174$]. This statistic indicated that the frequency of correct responses increased as the number selected on the Likert scale increased at the seventh-grade level, meaning that students who responded correctly to this item tended to be more confident with that correct response. Secondly, there was also a significant negative correlation between correctness of student response and Likert scale response at the ninth-grade level [$r_s = -.202, p = .006, n = 188$]. This statistic indicated that the frequency of correct responses decreased as the number selected on the Likert scale increased at the ninth-grade level; thus, students who answered this question correctly were less confident with their answer than students who answered incorrectly. Lastly, there was a significant negative correlation between correctness of student response and Likert scale response at the 11th-grade level [$r_s = -.189, p = .010, n = 184$]. This statistic indicated that the frequency of correct responses decreased as the number selected on the Likert scale increased at the 11th-grade level, again meaning students

who answered correctly tended to be less confident with their responses than students who answered incorrectly.

Item 4

The fourth item dealt with the representativeness misconception. Responses were broken down by correctness of answer and by rating on the Likert scale. The results for the seventh-grade students appear in Table 21, those for the ninth-grade students appear in Table 22, and those for the 11th-grade students appear in Table 23.

Table 21

Likert Scale Responses, Item 4, 7th Grade

| Likert Scale Selection | | | | | |
|------------------------|----|---|----|----|----|
| Response | 1 | 2 | 3 | 4 | 5 |
| Correct | 3 | 7 | 15 | 46 | 38 |
| Incorrect | 17 | 8 | 19 | 15 | 6 |

Table 22

Likert Scale Responses, Item 4, 9th Grade

| Likert Scale Selection | | | | | |
|------------------------|---|----|----|----|----|
| Response | 1 | 2 | 3 | 4 | 5 |
| Correct | 2 | 12 | 26 | 38 | 47 |
| Incorrect | 2 | 6 | 17 | 20 | 18 |

Table 23

Likert Scale Responses, Item 4, 11th Grade

| Response | Likert Scale Selection | | | | |
|-----------|------------------------|---|----|----|----|
| | 1 | 2 | 3 | 4 | 5 |
| Correct | 1 | 2 | 20 | 46 | 61 |
| Incorrect | 1 | 8 | 14 | 26 | 5 |

A Spearman correlation was computed comparing student response to Likert scale response for this item at each individual grade level. Statistical analysis showed there was a significant positive correlation between correctness of student response and Likert scale response at the seventh-grade level [$r_s = .459, p < .001, n = 174$]. This statistic indicated that the frequency of correct responses increased as the number selected on the Likert scale increased at the seventh-grade level, so students who responded correctly to this question were more likely to be confident in their answers. Secondly, there was no significant correlation between correctness of student response and Likert scale response at the ninth-grade level [$r_s = .091, p = .212, n = 188$]. This statistic indicated that no relationship existed between frequency of responses to the item and frequency of Likert scale responses at the ninth-grade level. Lastly, there was a significant correlation between correctness of student response and Likert scale response at the 11th-grade level [$r_s = .386, p < .001, n = 184$]. This statistic indicated that the frequency of correct responses increased as the number selected on the Likert scale increased at the 11th-grade level; thus students who answered correctly tended to be more confident with their response than students who answered incorrectly.

Summary

Presented in this chapter were all of the data relevant to both the pilot and the research study. The pilot study results indicated that no significant differences between any of the corresponding items on the open-ended and multiple-choice formats of the survey existed. The multiple-choice format of the survey was more desirable for the study since the coding of responses on the open-ended survey by more than one person could yield different sets of coded data due to interpretation of meaning of student responses. The next chapter features discussion of these data and implications of this study as well as possibilities for future study building on this research.

CHAPTER V:

DISCUSSION

Introduction

This chapter features an overview of the research study, a restatement of the research questions for the study, a brief review of the literature, information relevant to the instrument, and a brief description of the population used in this study. Afterward are an interpretation of the findings, conclusions based on the findings, implications of the study, and recommendations for future research.

Probability is one of the most versatile branches of mathematics in terms of its usage, and yet it is one of the most difficult branches of mathematics for pre-college students to understand and master. The intuitively-based misconceptions that were the focus of this study revealed a source for this difficulty. Understanding how these misconceptions relate to grade level will help educators so that they can emphasize these specifics during instruction. The findings of this study have implications for future empirical studies designed to assess the presence and persistence of intuitively-based probabilistic misconceptions. This study introduced these misconceptions in the context of self-efficacy, an area in which no previous research was found.

Overview of the Study

The purposes of the research study were to determine how each of four intuitively-based probabilistic misconceptions correlated to grade level and how, at each of the 7th-, 9th-, and 11th-grade levels self-efficacy correlated with achievement on items that tested for each of the misconceptions. Students from each of those grade levels participated by taking a survey that

contained one item for each misconception, along with a corresponding item for them to measure their confidence with their response on a five-point Likert scale. Spearman correlations were computed to determine if these variables were related.

Research Questions

There were two questions for this research study. They included the following:

1. What is the relationship between students' 7th-, 9th-, or 11th-grade level and each of the four intuitively-based probabilistic misconceptions of the representativeness misconception, positive and negative recency effects, the distinction between compound and simple events, and the effect of sample size; and
2. What is the relationship between students' responses in the 7th-, 9th-, and 11th-grade levels, to questions involving probability tasks and their perceived self-efficacy in answering those questions?

Review of the Literature

Two pieces of literature, Fischbein and Schnarch (1997) and Rubel (2002), examined these misconceptions as they related to grade level. Fischbein and Schnarch found that “main misconception” responses (a) decreased as grade level increased for positive and negative recency effects and, to a lesser degree, the representativeness misconception; (b) increased as grade level increased for the effect of sample size; and (c) remained constant as grade level increased for the distinction between compound and simple events. Rubel found that “main misconception” responses (a) decreased significantly as grade level increased for the effect of sample size and (b) had an insignificant association with grade level for positive and negative recency effects and the distinction between compound and simple events. The differences are probably due to the populations used in the studies. This study and the study by Fischbein and

Schnarch (1997) were done with relatively homogeneous populations, whereas the study by Rubel (2002) featured a more heterogeneous, and potentially more random, sample.

How student achievement and self-efficacy correlate with respect to probabilistic misconceptions was a focus of this study. Although mathematics achievement correlates positively with self-efficacy in mathematics in general, no research was located correlating self-efficacy with achievement in the context of these probabilistic misconceptions.

Instrument

Items used on the multiple-choice and open-ended instruments were developed over the course of one month by referencing both Fischbein and Schnarch (1997) and Rubel (2002), along with seventh-grade textbooks used in the school district in which the research took place and *Alabama Course of Study: Mathematics* (2003). A pilot study was conducted to determine if responses on both instruments were similar. The pilot study found that frequencies of responses were similar among similarly sized groups of students at each grade level. This led to the conclusion to use the multiple-choice instrument in order to minimize potential ambiguity in student responses.

Study Population

The research study took place in a large public school district in Alabama. The district is located in a suburban area of a large metropolitan area of the state. The population was made up of students in the 7th-, 9th-, and 11th-grade levels. In the 7th grade, 174 students took part in the study; those numbers in the 9th and 11th grades were 188 and 184, respectively. Students in ninth grade who participated in the study were taking Algebra 1, Algebra 1-A, Geometry, and Geometry Math Team classes; in the 11th grade, students were in Algebra 2, Algebra 3, Precalculus, Pre-AP Precalculus, and Precalculus Math Team classes.

Interpretations of Findings

The interpretations of the findings in Chapter IV based on both research questions are presented in this section and arranged according to item. For each item, to address the first research question, a Spearman correlation was found between the grade level of students and the numbers of responses of each type. Positive correlations in each of the statistical analyses indicated incorrect answers were more prevalent in higher grades than lower grades, while negative correlations indicated the opposite. For each item, to address the second research question, a Spearman correlation was found at each grade level between the correctness of student response to the item and the frequencies of student selections on the five-point Likert scale. Positive correlations in each of the statistical analyses indicated higher frequencies of selections on the Likert scale at the high end for students who answered correctly than for students who answered incorrectly; negative correlations indicated higher frequencies of selections on the Likert scale at the high end for students who answered incorrectly than for students who answered correctly.

Comparisons to the findings of Fischbein and Schnarch (1997) and Rubel (2002) were also made concerning the first research question. No research was located dealing with self-efficacy in the context of any of the probability tasks used on the survey, although many pieces of research indicated a positive correlation between self-efficacy in mathematics and mathematics achievement (Collins, 1982; Hackett & Betz, 1989; Lent, Lopez, & Bieschke, 1993; Pajares & Kranzler, 1995; Pajares & Miller, 1994). Therefore, no comparisons to other pieces of literature could be made for the results of the statistical analyses for the second research question.

Item 1 – Positive and Negative Recency Effects

The first item on the survey was concerned with the probabilistic misconception of positive and negative recency effects. For this item, statistical analysis showed a significant negative Spearman correlation between grade level of student and student response when all three types of responses were considered and when only the correct response and the “main misconception” were considered. These negative correlations indicated that the frequency of incorrect responses tended to be lower in the higher grades. Thus, students in the higher grades tended to correctly answer the item concerning positive and negative recency effects more so than students in the lower grades, being less swayed in their responses by either effect than students in the lower grades. Conversely, students in the lower levels tended to show responses that reflected the “main misconception”, which corresponded to the negative recency effect, more than students in the higher grades.

For the Likert scale item related to this probabilistic misconception, statistical analyses showed significant positive Spearman correlations between correctness of student response and frequencies of selections on the five-point Likert scale for each of grades 7, 9, and 11. These positive correlations indicated that the frequency of selections on the Likert scale on the high end, which indicated much certainty in the answer to the item, were higher for students who answered the item correctly than for students who answered the item incorrectly at each of the three grade levels. Thus, students who answered the item correctly tended to be more confident in that answer than students who answered incorrectly. Conversely, students who answered the item incorrectly were more likely to be unsure of their answer than students who answered the item correctly.

While neither of the previous studies by Fischbein and Schnarch (1997) or by Rubel (2002) examined formal correlations among the grade levels they studied for any of the survey items, both were concerned with trends across all grade levels in their respective studies. Fischbein and Schnarch made the same conclusion in their study, though informally, with regard to the “main misconception” responses, while Rubel found a statistically insignificant association in correct responses across the three grade levels.

Item 2 – The Effect of Sample Size

The second item on the survey was related to the probabilistic misconception of the effect of sample size. For this item, statistical analysis showed a significant positive Spearman correlation between grade level of student and student response when all three types of responses were considered and when only the correct response and the “main misconception” were considered. These positive correlations indicated that the frequency of incorrect responses tended to be lower in the lower grades; therefore, students in the higher grades answered the item concerning the effect of sample size incorrectly more often than students in the lower grades. Alternately, students in the lower levels tended to show responses that reflected the “main misconception” less than students in the higher grades.

For the Likert scale item concerned with this probabilistic misconception, statistical analysis showed a significant negative Spearman correlation between correctness of student response and frequencies of selections on the five-point Likert scale only for students in 11th grade. At the 11th-grade level, this negative correlation indicated that the numbers of students who indicated high certainty of the correctness of their response tended to respond incorrectly to the item. Therefore, 11th-grade students who responded to the probability item incorrectly had more confidence with their selection than students who responded correctly to the probability

item. Alternately, students who responded to the probability item correctly had less confidence with their selection than students who responded incorrectly to the probability item. Correlations for students in both grades 7 and 9 were not significant in the statistical analyses. Therefore, at both of those grade levels, there was no difference in the frequencies of high certainty responses on the Likert scale for students who answered the item correctly or students who answered the item incorrectly. Students who answered correctly and students who answered incorrectly in both of those grade levels had the same level of self-efficacy in their responses.

Fischbein and Schnarch (1997) found that frequency of “main misconception” responses increased with grade level, and Rubel (2002) found statistically significant associations in both correct responses (increase across grade levels) and “main misconception” responses (decrease across grade levels). The results of this study for the first research question with respect to this probabilistic misconception were more in line with that of Fischbein and Schnarch than that of Rubel.

Item 3 – The Distinction Between Compound and Simple Events

The third item on the survey was concerned with the probabilistic misconception of compound and simple events. For this item, statistical analysis showed no significant Spearman correlation between grade level of student and student response when all three types of responses were considered and when only the correct response and the “main misconception” were considered. Therefore, grade level was not related to student response with respect to the distinction between compound and simple events. Based on the frequencies of each of the three types of responses, roughly equal numbers of students answered this item both correctly and incorrectly.

For the Likert scale item related to this probabilistic misconception, statistical analysis showed a significant positive Spearman correlation between correctness of student response and frequencies of selections on the five-point Likert scale for students in 7th grade. At the 7th-grade level, this positive correlation indicated that the numbers of students who indicated much certainty of their response's correctness answered the item correctly. Therefore, students who answered the item correctly were more confident in that answer than students who answered the item incorrectly. Alternately, students who answered the item incorrectly were less confident with their response than students who answered the item correctly.

However, at both the 9th- and 11th-grade levels, statistical analyses showed significant negative Spearman correlations between correctness of student response and frequencies of selections on the five-point Likert scale. These negative correlations indicated that the frequencies of students who indicated high certainty with their response actually answer the item incorrectly more than students who answered the item correctly. Students who answered the item correctly were less confident with their answer than students who answered the item incorrectly. Likewise, students who answered the item incorrectly were much more confident with that incorrect answer than students who answered the item correctly.

Fischbein and Schnarch (1997) found that frequency of "main misconception" responses stayed consistent with grade level. Rubel (2002) did not have a comparable item in her research, but among items testing for similar concepts, no statistically significant associations were found between correct responses and grade level.

Item 4 – The Representativeness Misconception

The fourth item on the survey was related to the probabilistic misconception of representativeness. For this item, statistical analysis showed no significant Spearman correlation

between grade level of student and student response when all three types of responses were considered and when only the correct response and the “main misconception” were considered. Therefore, grade level was not related to student response with respect to the representativeness misconception. Based on the frequencies of each of the three types of responses, a general upward trend emerged for correct responses, but this trend was not statistically significant.

For the Likert scale item concerned with this probabilistic misconception, statistical analyses showed significant positive Spearman correlations between correctness of student response and frequencies of selections on the five-point Likert scale for students in both the 7th and 11th grades. At those two grade levels, the positive correlations indicated that the numbers of students who indicated high certainty of the correctness of their response tended to respond correctly to the item. Therefore, students who answered the item correctly were more confident in that answer than students who answered the item incorrectly. Alternately, students who answered the item incorrectly were less confident with that incorrect answer than students who answered the item correctly. The related correlation for students in grade 9 was not significant. Therefore, at the ninth-grade level, there was no difference in the frequencies of high certainty responses on the Likert scale for students who answered the item correctly or students who answered the item incorrectly.

Fischbein and Schnarch (1997) found that frequency of correct responses decreased across grades 7 and 9, while “main misconception” responses increased across these same grade levels. They also found that there was no difference in responses between grades 9 and 11 for either correct or “main misconception” responses. Rubel (2002) did not have a comparable item in her research.

Conclusions

Item 1 – Positive and Negative Recency Effects

Correlations for this probabilistic misconception indicated the following: (a) more students answered the item correctly in the higher grade levels than in the lower ones, and (b) students in all three grade levels who answered this item correctly tended to be more confident with their answer than those who answered the item incorrectly. This was probably due in part to the exposure of this specific type of problem to students in those grade levels, which is fairly common in the study of probability. Looking at the responses across grade levels, the ratio of correct answers to “main misconception” answers at the 7th-, 9th-, and 11th-grade levels were 3:1, 5:1, and 8:1, respectively. Clearly a good job is being done in the district where this research took place to address this particular probabilistic misconception. Additionally, of students who answered this item correctly, the percentages of students who ranked their confidence a 4 or 5 on the Likert scale in the 7th-, 9th-, and 11th-grade levels were 77%, 89%, and 91%, respectively. Therefore, not only did more students answer the item correctly as grade level increased, their confidence in their responses also increased. These percentages explain to some degree the positive correlations found for the Likert scale portions at each of the three grade levels.

At the 10th-grade level, independent events are generally taught through the use of playing cards as a manipulative rather than coins. Students may have had difficulty equating independent events with other manipulatives such as coins or dice, having not realized that coin flips were independent events. The emphasis placed on one type of manipulative may have made students unaware of the concept of independent events across multiple probabilistic settings.

Item 2 – The Effect of Sample Size

Correlations for this probabilistic misconception indicated the following: (a) more students were answering the item incorrectly in the higher grades than in the lower grades, and (b) at the 11th-grade level, students who answered this item correctly tended to be less confident with their answer than those who answered the item incorrectly. This particular type of problem is a difficult one, due to the fact that sample size was rarely regarded in middle- and high-school probability instruction. Further, no mention is made in *Alabama Course of Study: Mathematics* (2003) regarding sample size in probability instruction. Students typically must “learn” on their own about the effect sample size has on probabilities, and unfortunately, few actually do learn this effect, as evidenced by the results of this study. Responses across the three grade levels indicated the ratio of “main misconception” responses to correct responses increased from 3.5:1 to 4:1 to 5.5:1 across the 7th-, 9th-, and 11th-grade levels, respectively. Percentages of students who correctly answered the question ranked their confidence a 4 or 5 on the Likert scale in the 7th-, 9th-, and 11th-grade levels 52%, 80%, and 55% of the time, respectively.

Not only did the occurrence of incorrect answers increase across grade level, confidence in answers was higher with students at the 11th-grade level who incorrectly answered the question than that for students who correctly answered the question. Although 55% of students at the 11th-grade level who correctly answered this question ranked their confidence a 4 or 5, the percentage of students at this grade level who incorrectly answer this question and ranked their confidence a 4 or 5 was 76%, which also led to the negative correlation for this item with respect to the first research question. Another anomaly was the 80% confidence for ninth-grade students who answered the question correctly, which was out of line compared with the same percentages of students at the 7th- and 11th-grade levels. This concept may have been taught or

mentioned recently at that level, or a higher percentage of those students may have had a teacher who was comfortable with teaching probability. Additionally, students at the seventh- or ninth-grade levels may not have understood if they were working the problem correctly.

This probabilistic misconception is certainly one that needs more emphasis placed on it during instruction. Teachers would expect that frequencies of misconceptions would decrease after instruction; it was clear that in this school district, not enough emphasis was placed on this misconception in previous courses. With the increase in the frequency of “main misconception” responses with increasing grade level, it was evident that students either did not understand that sample size needed to be taken into consideration or they neglected that piece of information as extraneous.

Item 3 – The Distinction Between Compound and Simple Events

Correlations for this probabilistic misconception indicated the following: (a) there was no significant correlation between the numbers of correct or “main misconception” responses for students in the three grade levels, (b) at the seventh-grade level, students who answered this item correctly tended to be more confident with their answer than those who answered the item incorrectly, and (c) at the 9th- and 11th-grade levels, students who answered this item correctly tended to be less confident with their answer than those who answered the item incorrectly.

According to previous research, students have trouble with this misconception in that they see a difference in two identical dice being rolled and two dice that have some distinguishing featured, such as having differently colored surfaces, being rolled (Lecoutre, 1992; Lecoutre & Durand, 1988). If identical dice were used, students often did not understand that a 5 on one die and a 6 on the other was a different roll than a 6 on the first die and a 5 on the second. The frequencies of “main misconception” responses to correct responses stayed constant at a 3:1 ratio across all

three grade levels. Additionally, the percentages of students who ranked their correct response to the question as a confidence level of 4 or 5 on the Likert scale were 79%, 46%, and 43% in the 7th-, 9th-, and 11th-grade levels, respectively.

Confidence in correct responses was high for students at the seventh-grade level. The 79% rate of high confidence for correct responses at this grade level, compared to the 51% high confidence for incorrect responses at this grade level, explained some of the reason behind this positive correlation. The low percentages of students who ranked their correct response with high confidence at the 9th- and 11th-grade levels were 46% and 43%, respectively, compared with 68% and 67% as their incorrect response counterparts. Because the ratios of “main misconception” responses to correct responses stayed even across the three grade levels, combined with these percentages on the Likert scale portion of the item, it is easy to see that this concept was probably not expanded on through the grade levels. If the material was taught in the same way with the same manipulatives, student who incorrectly understood the material the first time may have had their misunderstanding bolstered by an unchanging delivery of the material or the non-use of other materials. Both of these scenarios could have hampered a better understanding of the distinction between compound and simple events. Regardless of the specific deficiency, increasing emphasis should be placed on this topic, which is probably touched on with little emphasis in the higher grade levels.

Item 4 – The Representativeness Misconception

Correlations for this probabilistic misconception indicated the following: (a) there was no significant correlation between the numbers of correct or “main misconception” responses for students in the three grade levels, and (b) at the 7th- and 11th-grade levels, students who answered this item correctly tended to be more confident with their answer than those who

answered the item incorrectly. Perceived randomness of specific outcomes was generally extrapolated to the entire class of outcomes with perceived randomness, rather than having observed the specific outcome as individual. The frequencies of correct responses were higher than “main misconception” responses for this item, but not by an overwhelming number. The ratios of correct responses to “main misconception” responses ranged from 1.5:1 to 2.5:1 for all three grade levels, which indicated little correlation with this misconception to grade level.

Students at both the 7th- and 11th-grade levels tended to rate their correct responses with high certainty. At the seventh-grade level, 71% of students who answered the question correctly ranked their confidence as a 4 or 5, compared to 32% of their counterparts who answered the question incorrectly. For the 11th-grade level, those respective percentages were 82% and 57%. At the ninth-grade level, though, no significant correlation was found; this can be seen with 68% of students ranking their correct responses with high confidence compared to 60% of students ranking their incorrect responses likewise. Students at the 11th-grade level would have undoubtedly encountered this type of question previously and were good at distinguishing one specific outcome from an entire class of random outcomes. Seventh-grade students may not have even taken into consideration that one outcome could have possibly represented, in their minds, several outcomes. Just as with the distinction between compound and simple events, increasing emphasis needs to be placed on these concepts in the higher grades.

Implications

Theoretical

The two studies that focused on the same misconceptions as this study and had similar results were Fischbein and Schnarch (1997) and Rubel (2002). The findings from this research study are similar to the findings in both previous studies. Theoretically, this study could initiate

a dialogue in the state of Alabama about the ways that probability is taught in the middle- and high-school grades, especially as regards the misconceptions of effect of sample size and distinction between compound and simple events, both of which had higher frequencies of incorrect responses in the higher grade levels. This study also highlights the need to emphasize sample size in the study of probability, as it is rarely taught or mentioned in probability curricula and students seemed to have the most difficulty with that particular misconception.

To better increase students' exposure to these misconceptions, specific references need to be made in future editions of *Alabama Course of Study: Mathematics* (2003) and local curriculum guides. These references should include the individual misconceptions along with the correlations found in this research, especially those of the effect of sample size and the distinction between compound and simple events, as these had the most alarming outcomes when it came to student retention of conceptual understanding. Emphasis should be placed on using multiple manipulatives to cover concepts for which they would aid in student understanding (such as using coins or dice for independent events).

Another implication of this study is that it may encourage the study of self-efficacy, an already much researched topic, specifically in the study of probability. After an exhaustive search, no research was found dealing with self-efficacy in the context of these specific probability tasks. With self-efficacy being a key predictor in achievement not only in mathematics but in other disciplines, it is necessary to foster positive self-efficacy as a portion of the probability curriculum. Students may benefit from increased positive self-efficacy in such a way that their abilities in the study of probability would also increase.

Practical

The practical implications from the results are compelling. NCTM (2000) has emphasized the importance for students to be able to read data and come to conclusions based on that data, which requires of those students a working knowledge of probability. Among the expectations, NCTM strongly advocates that by the time students graduate from high school, they should be able to (a) “understand how to compute the probability of a compound event”, (b) “use simulations to construct empirical probability distributions”, (c) “understand the concepts of conditional probability and independent events”, and (d) “use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations.” Lack of knowledge in any one of these expectations may lead to the misconceptions arising from the use of intuition in the field of probability, a focus of this study.

Considering each misconception individually, teachers can adapt or modify instructional materials such as procedures, examples, or activities to include discussions about those misconceptions, including students’ misunderstandings when choosing incorrect answers to questions where those mathematical misconceptions may arise. Teachers will be better informed for what instruction is appropriate regarding students’ misconception choices. The study’s investigation focus on classroom instruction looked into how misconceptions related to three grade levels (7th, 9th, 11th). The probabilistic misconceptions of effect of sample size and distinction between compound and simple events could have more emphasis placed on them for future study since they had the most troubling results. Responses concerned with positive and negative recency effects, for example, correlated with grade level in the way that was expected, where frequency of correct answers increased over the three grade levels.

Writers of curricula can use the results to adapt benchmarks in probability to reflect testing for misconceptions. Teachers at grade levels below grade 7 will also know what misconceptions tend to increase with grade level so that they can begin covering those topics early with students, possibly helping to minimize the effect those misconceptions will have in the study of probability. With a better understanding of probability and its facets, students may become more proficient in fields where probability is a necessary and vital tool and may become better at making sense of data that is presented in those fields.

A clearer understanding of probability should also help to bolster student self-efficacy in the area of mathematics instruction. By guiding students, teachers give those students thorough backgrounds for a high degree of probability comprehension; this effectively provides students with the purpose and uses of probability, which may lead to a higher degree of positive self-efficacy, since self-efficacy correlates positively with mathematics achievement (Collins, 1982; Duval & Wicklund, 1982; Hackett & Betz, 1989; Lent, Lopez, & Bieschke, 1993; Matsui, Matsui, & Ohnishi, 1990; Nicolaou & Philippou, 2007; Pajares & Kranzler, 1995).

Future Research

Extensions of Research Study

The lack of research in the area of intuitively-based probabilistic misconceptions suggests there is a great need for research in this area. This research study was only concerned with the four specific misconceptions of positive and negative recency effects, the effect of sample size, the distinction between compound and simple events, and the representativeness misconception and their correlations with grade level, as well as students' self-efficacy in the context of these misconceptions. Further research could extend this study by attempting to produce similar

outcomes in understanding the presence or absence of misconceptions at the 7th-, 9th-, and 11th-grade levels.

Further research could focus on multiple items reflecting the same misconception, which would bring some internal consistency reliability to the instrument. Because there was only one item related to each of the four misconceptions on the survey, internal consistency reliability was not tested for the instrument used in this study; the researcher chose not to test for this reliability since the items were viewed individually. A more in-depth survey using multiple items testing for the same misconceptions would further advance research in this area of study.

A third area of research that could be pursued is with regard to student self-efficacy. Self-efficacy in general mathematics has been studied, but the lack of research on self-efficacy as it regards to probability, specifically to the misconceptions caused by the heuristic of representativeness, suggests more investigation is warranted.

A fourth area of research which is probably necessary is that regarding the effect of sample size. This probabilistic misconception was strongest in the later grades in this study, suggesting a possible compounding effect of this misconception throughout probability instruction, either by explicit instruction or lack thereof. Regarding self-efficacy, students at the 11th-grade level were incredibly confident in their wrong answers for the item concerning this probabilistic misconception, again suggesting some deficiency in the coverage of this topic. An entire research study could probably focus only on this misconception.

Modifications of Research Study

The results indicated that several modifications could be made as a means of future research. The research of Fischbein and Schnarch (1997) and Rubel (2002) examined other intuitively-based probabilistic misconceptions that were not examined in this research; those

misconceptions include the conjunction fallacy, availability, and the effect of the time axis; Rubel specifically examined students' misconceptions with counterintuitive conditional probability problems. The same approach used in the present study's investigation could be applied to the other misconceptions studied by Fischbein and Schnarch and Rubel, examining not only the correlations of the misconceptions with grade level but also with self-efficacy. Such research would strengthen the study of self-efficacy to those other misconceptions, broadening the current self-efficacy research.

A second modification would be to examine students in grades other than 7, 9, or 11. In addition to students at the 7th-, 9th-, and 11th-grade levels, Fischbein and Schnarch (1997) also examined college students who were prospective teachers specializing in mathematics and fifth-grade students in their research; Rubel (2002) examined fifth-grade students in her research as well. Research modification within this area could include fifth-grade or college students or students from other grade levels not examined by any of the three studies to determine if those correlations extended beyond the 7th- through 11th-grade levels with respect to the misconceptions presented in this research study or any of the previously mentioned misconceptions that were a part of the other two pieces of research.

A third modification could include examinations of differences in responses at any of the grade levels. Chi-square statistics could be computed to determine what differences may exist for any of the misconceptions presented in this research study or any of the other misconceptions presented in the other two pieces of research. This data was found by Rubel (2002) for certain misconceptions, but no further research was located looking at these specific statistics.

A fourth modification could be an examination of the misconceptions based on gender. Not only could future research look at the differences that exist among grade levels, it could also

examine differences at grade level with respect to gender, taking into account three-way associations of grade level, gender, and responses to the survey items. Differences in self-efficacy with respect to gender could also be examined. Lent, Lopez, and Bieschke (1993) found that males had a higher self-efficacy with mathematics in general, but future study could look at that self-efficacy as it relates to probability and the misconceptions specifically.

Closing Summary

This study was an extension of the research by Fischbein and Schnarch (1997) on intuitively-based probabilistic misconceptions; however, this study looked only at those misconceptions sourced in the heuristic of representativeness: positive and negative recency effects, the effect of sample size, compound and simple events, and representativeness. Only correlations between student responses to items testing for the misconception and the grade level of those students were investigated. Investigation also sought to address correlations between the participants' responses and their self-efficacy as measured on a five-point Likert scale.

The major findings of this study were that the only correlations existing between correct student responses and grade level were a positive correlation for positive and negative recency effects and a negative correlation for the effect of sample size. Regarding the self-efficacy aspect of this study, positive correlations existed for self-efficacy and correct responses for positive and negative recency effects (at all three grade levels), the distinction between compound and simple events (at the seventh-grade level), and the representativeness misconception (at the 7th- and 11th-grade levels); negative correlations existed for the effect of sample size (at the 11th-grade level) and the distinction between compound and simple events (at the 9th- and 11th- grade levels).

Implications from the study have potential for classroom teachers who teach probability. Instructional materials can be adapted from students' probability misconceptions that could provide a positive efficacy during choice considerations. Teachers can discuss misconceptions with students to understand why certain intuitive answers to probability tasks may not be correct ones. Conceptual probability knowledge, an NCTM standard, is essential for students' mathematical success; this study proffered several essential practical tools in the teaching of probability to pre-college students.

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APPENDIX A

INFORMED CONSENT AND ASSENT FORMS

UNIVERSITY OF ALABAMA
Informed Consent Statement for Parental Permission

Dear Parent/Guardian:

It is being requested of your child to participate in a research study that deals with misconceptions about tasks involving probability. The study is called “The Appearance of Intuitively Based Probabilistic Misconceptions in Secondary Education Students.” The study is being conducted by Paul N. Kustos, a doctoral candidate in the Department of Curriculum & Instruction in the College of Education at The University of Alabama. Mr. Kustos is supervised by his dissertation Chair, Dr. Craig S. Shwery, Associate Professor in the Department of Curriculum & Instruction in the College of Education.

What is this study about?

The purpose of this study is to determine if there are correlations among students’ grade level, students’ perceptions of themselves in being able to answer problems involving probability correctly, and students’ ability in being able to answer questions involving probability correctly. Probability is the branch of mathematics dealing with the likelihood that an event is to occur.

Why is this study important? What good will the results do?

The results may help teachers who teach probability to students in those grades by helping them to see how students think about probability and how to help students overcome obstacles in their thinking about probability.

Why has my child been asked to take part in this study?

Your child has been asked to participate because he or she is in one of the above mentioned grades and is representative of the larger population of students in one of the above mentioned grades.

How many people besides my child will be in this study?

About 800 students will be in this study.

What will my child be asked to do in this study?

Should you decide to allow your child to participate in this study, you will then sign the attached permission form. Your child will then be explained the study and then asked if he or she would like to participate. There will be a second permission form that your child will sign stating that he or she would like to participate. If your child provides this assent to be in the study, he or she will be asked to complete a couple of demographic items (grade and gender) and to complete an instrument consisting of nine questions regarding probability and nine accompanying Likert-type

questions asking for his or her perception of the answer given to each probability question. The total anticipated participation time for your child will be approximately 20 minutes.

Will being in this study cost me or my child anything?

There will be no cost to you for allowing your child to be in this study. The only cost to your child will be his or her time in responding to the survey instrument and demographic information. There is no compensation to students for being in the study.

Can the researcher take me out of this study?

Your child's participation in this study is completely voluntary. Refusal to participate will involve no penalty. Your child's grade in school will not be affected by his or her refusal to participate, and there will be no behavioral or other penalty imposed by your child's teacher. At anytime during the survey, your child may skip a question or stop from completing the survey at any time.

What are the benefits of being in this study?

There are no direct benefits to you or your child from being in this study. However, teachers may benefit from the knowledge gained in this study which will help future students.

What are the risks (dangers or harm) to my child if he or she is in the study?

There are no foreseeable risks or discomforts involved in this study.

How will my child's confidentiality (privacy) be protected? What will happen to the information the study keeps on my child?

To assure anonymity of responses, provide your child's name only on the permission form at the end of this consent statement. Your child will not provide his or her name anywhere on the instrument, and no other information, other than grade level and gender, will be gathered. Confidentiality of responses will be maintained by the researcher serving as the sole individual with access to the data.

What are the alternatives to being in this study? Do I have other choices?

Your child's participation in this study is completely voluntary.

What are my child's rights as a participant?

Taking part in this study is voluntary—it is your free choice whether to allow your child to participate. Your child may choose not to take part in the study at all. If your child starts the study, he or she may stop at any time. Leaving the study will not result in any penalty or loss of any benefits your child would otherwise receive.

The University of Alabama Institutional Review Board (IRB) is the committee that protects the rights of individuals, such as your child, in research studies. The IRB may review study records from time to time to be sure that individuals in research studies are being treated fairly and that the study is being carried out as planned.

Who do I call if I have questions or problems?

If your child has questions about the study, he or she may ask them during the study. If you have questions about the study now or later on, please contact the investigator, Paul N. Kustos, either by e-mail at pkustos@hoover.k12.al.us, or by phone at 205-492-9866. You may also contact my dissertation Chair and advisor, Dr. Craig S. Shwery, at 205-348-1181. If you have any questions about your child's rights as a research participant, you may contact Ms. Tanta Myles, The University of Alabama Research Compliance Officer, at 205-348-5152.

UNIVERSITY OF ALABAMA
Parental Permission Form

Please keep the informed consent statement presented on the previous front and back page for your records. Please return this parental permission form if you choose to have your child participate in the study.

I hereby give consent for my child, _____, to participate in the study called "The Appearance of Intuitively Based Probabilistic Misconceptions in Secondary Education Students."

Parent/Guardian Signature: _____

Parent/Guardian Name (please print): _____

Date: _____

UNIVERSITY OF ALABAMA
Participant Assent Form

Dear Potential Participant:

I am from the University of Alabama, and I am doing a study to try to learn how people your age are at working probability questions. I am asking your help because I do not know very much about this topic and want to learn more about it.

If you agree to be in my study, I am going to give you a survey that has some probability questions on it. These questions will ask your opinion on several questions, and they will also ask how sure you are that your answers to those questions are correct. The survey should take no more than 20 minutes to complete.

You can ask questions that you might have about this study at any time. Also, if you decide at any time not to finish, you may stop whenever you want. Remember, these questions are only about what you think. This is not a test, and you will not be graded on it. Also, no one will be able to tell how you answered the questions on this survey.

If you sign this paper, it means that you have read this and that you want to be in the study. If you do not want to be in the study, do not sign the paper. Remember, being in the study is up to you, and no one will be upset if you do not sign this paper or even if you change your mind later.

Sincerely,

Paul N. Kustos

Signature of Participant: _____

Date: _____

APPENDIX B

PROBABILITY AND SELF-EFFICACY INSTRUMENT (FREE RESPONSE)

Demographics

Grade (circle one): 7 9 11

Gender (circle one): Female Male

Probability and Self-Efficacy Instrument

For each of the following probability questions please indicate the answer you believe to be correct by placing the letter of the answer in the blank provided. Please provide only one answer per question, and make sure to answer every question. In addition, for each question, indicate the degree to which you are certain that your answer is correct by circling the appropriate number, using the following scale:

| | | | | |
|------------------|-------------|--------|---------|--------------|
| Not Very Certain | Not Certain | Unsure | Certain | Very Certain |
| 1 | 2 | 3 | 4 | 5 |

1a) A quarter has two sides: heads or tails. On each flip of the quarter, either heads or tails comes up. If Casey flips a quarter three times, and all three flips are tails, what would happen on a fourth flip? Explain your answer.

1b) How certain are you that your answer to question 1a is correct?

| | | | | |
|------------------|-------------|--------|---------|--------------|
| Not Very Certain | Not Certain | Unsure | Certain | Very Certain |
| 1 | 2 | 3 | 4 | 5 |

2a) You flip a quarter 10 times and get 7 heads. What is the most likely thing to happen if you flip the quarter 100 times? Explain your answer.

2b) How certain are you that your answer to question 2a is correct?

| | | | | |
|------------------|-------------|--------|---------|--------------|
| Not Very Certain | Not Certain | Unsure | Certain | Very Certain |
| 1 | 2 | 3 | 4 | 5 |

3a) Two six-sided dice have faces numbered 1, 2, 3, 4, 5, and 6. If the two dice are rolled together, which is more likely to be rolled face up: (a) one 5 and one 6, or (b) two 6's? Explain your answer.

3b) How certain are you that your answer to question 3a is correct?

| | | | | |
|------------------|-------------|--------|---------|--------------|
| Not Very Certain | Not Certain | Unsure | Certain | Very Certain |
| 1 | 2 | 3 | 4 | 5 |

- 4a) Draw 4 numbers from a bag which contains the numbers 1 through 30. Which set of numbers would most likely be drawn: (a) 8, 9, 10, 11, or (b) 4, 28, 17, 19? Explain your answer.

- 4b) How certain are you that your answer to question 4a is correct?

| | | | | |
|------------------|-------------|--------|---------|--------------|
| Not Very Certain | Not Certain | Unsure | Certain | Very Certain |
| 1 | 2 | 3 | 4 | 5 |

* Adapted from Fischbein and Schnarch (1997) & Rubel (2002)

APPENDIX C

PROBABILITY AND SELF-EFFICACY INSTRUMENT (MULTIPLE CHOICE)

Demographics

Grade (circle one): 7 9 11

Gender (circle one): Female Male

Probability and Self-Efficacy Instrument

For each of the following probability questions, select the answer you think is correct by writing the letter of the answer in the blank. Please only put one answer per question, and make sure to answer every question. Also, for each question, circle the number for how sure you are about your answer being correct, using the following scale:

| | | | | |
|------------------|-------------|--------|---------|--------------|
| Not Very Certain | Not Certain | Unsure | Certain | Very Certain |
| 1 | 2 | 3 | 4 | 5 |

- 1a) A quarter has two sides: heads or tails. On each flip of the quarter, either heads or tails comes up. If Casey flips a quarter three times, and all three flips are tails, what would happen on a fourth flip?

_____heads _____tails _____same chance of heads or tails

Explain the choice you selected.

- 1b) How certain are you that your answer to question 1a is correct?

| | | | | |
|------------------|-------------|--------|---------|--------------|
| Not Very Certain | Not Certain | Unsure | Certain | Very Certain |
| 1 | 2 | 3 | 4 | 5 |

2a) Which is more likely to happen?

_____you flip a quarter 10 times and get 7 heads

_____you flip a quarter 100 times and get 70 heads

_____both have the same chance

Explain the choice you selected.

2b) How certain are you that your answer to question 2a is correct?

| | | | | |
|------------------|-------------|--------|---------|--------------|
| Not Very Certain | Not Certain | Unsure | Certain | Very Certain |
| 1 | 2 | 3 | 4 | 5 |

3a) Two six-sided dice have faces numbered 1, 2, 3, 4, 5, and 6. If the two dice are rolled together, which is more likely to be rolled face up?

_____one 5 and one 6 _____two 6's _____both have the same chance

Explain the choice you selected.

3b) How certain are you that your answer to question 3a is correct?

| | | | | |
|------------------|-------------|--------|---------|--------------|
| Not Very Certain | Not Certain | Unsure | Certain | Very Certain |
| 1 | 2 | 3 | 4 | 5 |

4a) Draw 4 numbers from a bag which contains the numbers 1 through 30. Which set of numbers would most likely be drawn?

_____ 8, 9, 10, 11

_____ 4, 28, 17, 19

_____ both sets of numbers are equally likely to be drawn

Explain the choice you selected.

4b) How certain are you that your answer to question 4a is correct?

Not Very Certain
1

Not Certain
2

Unsure
3

Certain
4

Very Certain
5

* Adapted from Fischbein and Schnarch (1997) and Rubel (2002)

APPENDIX D

SAMPLE STUDENT RESPONSES, FREE RESPONSE SURVEY

Demographics

Grade (circle one): 7 9 11

Gender (circle one): Female Male

Probability and Self-Efficacy Instrument

For each of the following probability questions please indicate the answer you believe to be correct by placing the letter of the answer in the blank provided. Please provide only one answer per question, and make sure to answer every question. In addition, for each question, indicate the degree to which you are certain that your answer is correct by circling the appropriate number, using the following scale:

| | | | | |
|------------------|-------------|--------|---------|--------------|
| Not Very Certain | Not Certain | Unsure | Certain | Very Certain |
| 1 | 2 | 3 | 4 | 5 |

- 1a) A quarter has two sides: heads or tails. On each flip of the quarter, either heads or tails comes up. If Casey flips a quarter three times, and all three flips are tails, what would happen on a fourth flip? Explain your answer.

Heads... Well it could be either one
be cause its a 50,50 probability

- 1b) How certain are you that your answer to question 1a is correct?

| | | | | |
|------------------|-------------|--------|---------|--------------|
| Not Very Certain | Not Certain | Unsure | Certain | Very Certain |
| 1 | 2 | 3 | 4 | 5 |

- 2a) You flip a quarter 10 times and get 7 heads. What is the most likely thing to happen if you flip the quarter 100 times? Explain your answer.

Any thing could happen.

like i said its a fifty-fifty chance.

It could be 49 heads & 51 tails, or

7 heads & 93 tails, its all possible

- 2b) How certain are you that your answer to question 2a is correct?

Not Very Certain Not Certain Unsure Certain Very Certain
1 2 3 4 5

- 3a) Two six-sided dice have faces numbered 1, 2, 3, 4, 5, and 6. If the two dice are rolled together, which is more likely to be rolled face up: (a) one 5 and one 6, or (b) two 6's? Explain your answer.

Either, its all probability.
but i think A.

- 3b) How certain are you that your answer to question 3a is correct?

Not Very Certain Not Certain Unsure Certain Very Certain
1 2 3 4 5

- 2a) You flip a quarter 10 times and get 7 heads. What is the most likely thing to happen if you flip the quarter 100 times? Explain your answer.

You have no clue because it is a fifty fifty chance every time.

- 2b) How certain are you that your answer to question 2a is correct?

Not Very Certain Not Certain Unsure Certain Very Certain
1 2 3 4 5

- 3a) Two six-sided dice have faces numbered 1, 2, 3, 4, 5, and 6. If the two dice are rolled together, which is more likely to be rolled face up: (a) one 5 and one 6, or (b) two 6's? Explain your answer.

It has the same chance of rolling on 5 & 6 or 6 & 6.

- 3b) How certain are you that your answer to question 3a is correct?

Not Very Certain Not Certain Unsure Certain Very Certain
1 2 3 4 5

Demographics

Grade (circle one): 7 9 11

Gender (circle one): Female Male

Probability and Self-Efficacy Instrument

For each of the following probability questions please indicate the answer you believe to be correct by placing the letter of the answer in the blank provided. Please provide only one answer per question, and make sure to answer every question. In addition, for each question, indicate the degree to which you are certain that your answer is correct by circling the appropriate number, using the following scale:

| | | | | |
|------------------|-------------|--------|---------|--------------|
| Not Very Certain | Not Certain | Unsure | Certain | Very Certain |
| 1 | 2 | 3 | 4 | 5 |

- 1a) A quarter has two sides: heads or tails. On each flip of the quarter, either heads or tails comes up. If Casey flips a quarter three times, and all three flips are tails, what would happen on a fourth flip? Explain your answer.

It would be heads because the chance of it landing on tails again would be very unlikely, but there is a 50/50 chance, making heads more likely

- 1b) How certain are you that your answer to question 1a is correct?

| | | | | |
|------------------|-------------|--------|----------|--------------|
| Not Very Certain | Not Certain | Unsure | Certain | Very Certain |
| 1 | 2 | 3 | <u>4</u> | 5 |

- 2a) You flip a quarter 10 times and get 7 heads. What is the most likely thing to happen if you flip the quarter 100 times? Explain your answer.

every 10 times that you
flip atleast 7 or more will land
on heads

- 2b) How certain are you that your answer to question 2a is correct?

| | | | | |
|------------------|-------------|--------|---------|--------------|
| Not Very Certain | Not Certain | Unsure | Certain | Very Certain |
| 1 | 2 | 3 | 4 | 5 |

- 3a) Two six-sided dice have faces numbered 1, 2, 3, 4, 5, and 6. If the two dice are rolled together, which is more likely to be rolled face up: (a) one 5 and one 6, or (b) two 6's? Explain your answer.

one 5 and one 6 because
it is less likely that both
dice will not land on the same
side.

- 3b) How certain are you that your answer to question 3a is correct?

| | | | | |
|------------------|-------------|--------|---------|--------------|
| Not Very Certain | Not Certain | Unsure | Certain | Very Certain |
| 1 | 2 | 3 | 4 | 5 |