THE EFFECT OF SUPPLEMENTARY GEOMETRY UNITS ON MATHEMATICAL ATTITUDES AND ACHIEVEMENT OF EIGHTH-GRADE STUDENTS

by

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CHAPTER I

OVERVIEW AND STATEMENT OF THE PROBLEM

Attitudes toward mathematics play an important though ambiguous role in mathematics education. Mathematics seems especially vulnerable to development of negative attitudes since the usual mathematics program consists of seven to nine years of somewhat uninteresting arithmetic drills. Attitudes toward arithmetic may be formed as early as the third grade (Fedon, 1958; Stright, 1960) though these attitudes are usually more positive than negative in elementary school (Stright, 1960). However, attitudes toward learning mathematics become less favorable as children progress through school (Anttonen, 1968), with the late elementary and the junior high school years usually regarded as being critical in the development of mathematical attitudes (Callahan, 1971; Taylor, 1970).

The new mathematics curriculum was partially designed to eliminate the overemphasis on arithmetic and to develop better attitudes toward mathematics. However, student boredom persists as a problem for both the old and the new mathematics. The result is a pervasive distaste for mathematics among the population (Lazarus, 1974, 1975; Poffenburger & Norton, 1959). In fact, disliking mathematics and admitting mathematical incompetence are very much in vogue. While few people admit to being poor in other subjects, a professed ignorance or dislike of mathematics is quite acceptable. The teacher of mathematics must attempt to eliminate this culturally entrenched feeling for mathematics by developing positive attitudes among students.

Introduction to the Problem

Despite vast curricular and methodological changes during the past 25 years, negative mathematical attitudes have remained a major concern. The importance of developing positive attitudes toward mathematics has been emphasized by various mathematics educators. Johnson and Rising (1972) argued that positive attitudes should be instilled to encourage further study of mathematics in the future. The same authors stated that attitudes were a major factor in retention of mathematics. Butler and Wren (1965) wrote that creating and maintaining interest, thus developing positive attitudes, was one of the important duties of mathematics teachers.

Various mathematics educators have espoused the need for more research in the affective domain. Recently, the Conference Board of the Mathematical Sciences formed the National Advisory Committee on Mathematical Education (NACOME) to study mathematics education over the past 20 years. After an extensive two-year study, NACOME (1975) listed the following among their recommendations for improving mathematics education:

> a) that the affective as well as cognitive domains in mathematics should be the subject of constant and programatic attention,
> b) that basic research into the affective domain specifically vis-a-vis mathematics should be pressed. (p. 142)

Kline (1973), a foremost critic of the new mathematics curriculum, made an even more sweeping challenge by stating that "when we reach the stage where fifty per cent of the high school graduates can honestly say that they like mathematics and appreciate its significance, then we shall have attained a large measure of success in the teaching of mathematics" (p. 170).

Several other educators have made similar recommendations. Because of the anxiety some children have when working with mathematics, numerous writers (Corcoran & Gibb, 1961; Proctor, 1965; Tulock, 1957) thought that teachers should strive to change mathematical attitudes to a more positive inclination. Anttonen (1968) suggested that actual attempts to change student attitudes in a more positive direction to bring about better performance should be investigated. Smithson (1974) commented that we need not concern ourselves with curricular changes as much as with finding reforms to ultimately develop an appreciation of mathematics. Silverman (1974) recommended that we develop units in mathematics that can effect changes in attitude.

Statement of the Problem

While the development of positive attitudes has been accepted as an educational goal, the selection of materials, content, and methods to achieve this goal have been less successful. Mathematical materials that may improve student attitudes or stimulate interest have been available in a wide variety of sources. However, there is a paucity of research to determine what specific materials actually do develop a more positive attitude toward mathematics. Therefore, specific materials that develop better attitudes toward a particular mathematical discipline have not been recommended.

Purpose of the Study

This study was devoted to the sole mathematical topic of geometry at the eighth-grade level. The purpose was to develop and to test supplementary geometry materials for eighth-grade general mathematics students to improve mathematical attitudes and achievement. In particular, the study attempted to determine if supplementary materials that stressed the relationship of geometry to art and nature developed better attitudes toward mathematics. In addition, the effect of the units on achievement and the relationship between attitude change and achievement change were investigated. This study also fulfilled the need for a readily available source of ideas for geometry which could be easily integrated into existing programs.

Methodology

The supplementary units for this study were designed for eighth-grade general mathematics students. Ten units were made for incorporation into existing programs on a regular basis during the three to five weeks of geometry instruction. The presentation of two units for every three days of instruction was suggested. This followed Johnson and Rising's (1972) idea that "an effective way of attaining the affective goals of instruction is an enrichment program that is part of the regular day-to-day instruction" (p. 260).

All units were teacher demonstrations, as opposed to student manipulations. These demonstrations were designed for teachers using traditional methods of instruction (lecture/discussion/homework).

The general goal of the units was to develop an appreciation of geometry by stressing the relationship of geometry to other disciplines. Therefore, a minimum of calculations was included to allow students to see the non-numerical aspects of geometry.

One major topic for the supplementary units was the relationship between geometry and nature. Kline (1973, 1976) argued that reform in mathematics should be in the direction of relating mathematics to the sciences and to nature. He stated that, historically, mathematics developed with the study of nature. Therefore, five of the units stressed the mathematics inherent in natural phenomena. Two units demonstrated the angles and mathematical principles involved in soap film configurations. Two units about snowflakes and honeycombs showed the efficiency of the hexagonal pattern in nature. A fifth unit, networks, was a slightly more abstract treatment of the branching patterns in nature.

The second major topic for the units was the relationship between geometry and art. Justification for including units on art came from Johnson and Rising's (1972) suggestions for building positive attitudes. They stated that "to develop appreciation of the elegance, power, and structure of mathematics . . . illustrate the harmony, symmetry, and beauty of mathematical patterns" (p. 264). Therefore, five units were constructed to show geometric patterns in art. One unit, geometric illusions, showed the role of geometric figures in altering perception. Units on line designs, string art, and tessellations illustrated the aesthetic appeal of certain geometrical configurations. A final unit on shapes in the environment showed how certain figures may elicit different emotions.

Johnson and Rising also suggested that any lesson should be organized as follows: objectives, motivation for the lesson, techniques and activities, materials, assignment, and evaluation. The ten units tested in the study followed this pattern closely by including seven sections: title, objectives, teacher information, classroom procedure, materials needed, optional student activities, and sources.

These units were not original works but were a composition of extant ideas. Providing objectives for direct incorporation into the classroom and testing the collection of ideas to determine attitude change have not been previously attempted.

Since some of the mathematical principles involved were quite complex, the units served only as an introduction to the given topic. While not definitive works, the units hopefully served to stimulate pupils to investigate the topic further.

To test the supplementary units, eighth-grade general mathematics classes were selected. The eighth grade was chosen since some researchers have found that the junior high school years are critical in developing attitudes toward mathematics (Callahan, 1971; Dutton, 1968). Eighth-grade classes in pre-algebra and Algebra I were not included since these classes generally had only the academically advanced pupils and since the content for pre-algebra, Algebra I, and general mathematics differ significantly.

Geometry content within the general mathematics curriculum was selected since Solheim (1971) found that students had significantly better attitudes toward mathematics in general than toward geometry. While the sample in the Solheim study was high school students, the results clearly implied the need for improvement in the geometry curriculum.

Five teachers of general eighth-grade mathematics in the four junior high schools of the Tuscaloosa County School System volunteered to use the supplementary units. These five teachers taught a total of 18 classes comprised of approximately 450 students. The teachers attended a three-hour instructional meeting in early February, 1977, in which all ten supplementary units were demonstrated by the researcher. Directions for administering achievement tests, attitude tests, and supplementary units were given to ensure uniformity among teachers.

The attitude instrument used in this study was the Attitude Toward Mathematics scale (Suydam & Trueblood, 1969). The scale was revised by the School District of Philadelphia for the purpose of assessing attitudes toward mathematics. It has been used at levels ranging from grade three to college. Reliability coefficients have varied between .87 and .96 for the levels tested.

The achievement test chosen was the <u>Mathematics Basic Concepts</u> test of the Sequential Tests of Educational Progress, Series II. Developed by the Educational Testing Service in 1969, the 50-item test had an administration time of 40 minutes. The two forms of the test were divided into three cognitive levels: recall facts and/or perform mathematical manipulation; demonstrate comprehension of mathematical concepts; and exercise ingenuity or higher mental processes. Five content areas were measured with reliability coefficients for the test ranging from .79 to .82.

In most schools geometry within the general mathematics classes began in April, 1977. Attitude and achievement tests were administered by each of the teachers prior to the geometry instructional period. To help control teacher effect, each teacher was assigned experimental and control groups. All ten units were used with the experimental groups during the instructional period of approximately three weeks. Notes of the content, the order of implementation of units, student reactions, and the general classroom environment were taken by the teachers in an attempt to detect intervening variables that might have affected the outcome of the experiment. Posttests on achievement and attitudes were administered after the instructional period. Also, teachers subjectively evaluated the effect of the supplementary units on the experimental groups.

The test results were used to analyze five experimental hypotheses. Analysis of variance and correlational techniques were the statistical procedures employed.

Limitations of the Study

Limitations of this study are listed below:

Results of this study could be generalized only to comparable junior high school mathematics classes in which traditional modes of instruction predominate.

Since the study included rural and suburban schools, generalization of results to urban schools without further testing should be attempted with caution.

All teachers volunteered for this study, hence, their willingness to use the supplementary materials may have biased the results in favor of the experimental groups.

General eighth-grade classes were selected while omitting pre-algebra and Algebra I classes.

A purely random sample was not taken since administrative control of scheduling was not feasible.

The geometry content in the eighth-grade mathematics classes was predetermined by the participating teachers. Also, the amount of time devoted to the study of geometry could not be dictated by the researcher.

Hence, the length of the instructional period was not under strict control.

Finally, as in much of educational research, a mere change from the ordinary classroom routine may have had an effect on the dependent variable.

Definitions

The major term in need of definition is <u>attitude</u>. An attitude in the context of this study is "an enduring emotional set or predisposition to react in a characteristic way toward a given person, object, idea, or situation" (Johnson, D.A., 1957, p. 114). A <u>positive attitude</u> toward mathematics is a favorable predisposition toward mathematics while a <u>negative attitude</u> toward mathematics is an unfavorable predisposition toward mathematics. Attitudes toward mathematics in this study are indicated by the score obtained on the attitude test. A positive or favorable attitude is denoted by an average score of 3.50 through 5.00 on a 1 to 5 scale. A neutral attitude toward mathematics is indicated by a score of 2.50 through 3.49. A negative or unfavorable attitude is indicated by an average score of 1.00 through 2.49.

For purposes of this study, positive attitudes toward mathematics, liking mathematics, and appreciation of mathematics are synonymous. Negative attitudes toward mathematics, disliking mathematics, and no appreciation of mathematics are also synonymous.

Supplementary units are the ten lessons relating geometry to nature and art that were developed for the study. All units are included in the appendix.

Organization of the Study

This study contains the following chapters. Chapter I, Overview and Statement of the Problem, describes the general nature of the study and the methodology. Chapter II, Review of Related Literature, summarizes important literature relating mathematical attitudes and achievement. Attempts to develop better attitudes toward mathematics are also reported in the chapter. Chapter III, Research Design, includes hypotheses tested, sample for the study, descriptions of the instruments, and statistical techniques employed in the study. Chapter IV, Analysis of Data, contains the statistical treatment of the data obtained from the study. Chapter V, Summary, Conclusions, and Recommendations, includes a general summary of the study, conclusions derived from the analysis of the data, and recommendations for improvement and further study.

CHAPTER II

REVIEW OF RELATED LITERATURE

The primary concern of this study was to determine if mathematical attitudes improved through the use of supplementary geometry units. A secondary consideration was the effect of these supplementary materials on achievement in mathematics. Unfortunately, little research has been completed relative to the mathematics materials that actually do develop better attitudes toward mathematics. The scant research available usually regarded mathematical attitudes as secondary to achievement. Nevertheless, a review of pertinent research in the area of mathematical attitudes should help clarify the present study.

The review of literature reported the views of authorities in mathematics education and specific research findings in the area of mathematical attitudes. One section of the chapter summarized research of the relationship between mathematical attitudes and mathematical achievement. A second section reviewed attempts to improve mathematical attitudes. Since the amount of research to improve attitudes was very limited, some studies conducted at the elementary, secondary, and collegiate levels relating to mathematics attitudes were included.

Relationship Between Attitudes and Achievement

Development of favorable attitudes has long been a major concern of mathematics educators. Attitudes are thought to be highly involved in retention of mathematics, further study of mathematics, and vocational choices (Johnson, R. A., 1957; Johnson & Rising, 1972). Attitudes are not as amenable to research as other areas, such as achievement, because of the self-reporting types of instruments used (Bassham, 1964). Thus, the reliability and validity of attitude tests are questionable (Aiken, 1970; Corcoran & Gibb, 1961). Despite these problems, which lead to difficulties in the interpretation of results, various correlational studies of mathematical attitudes and achievement have been conducted. While some of these studies report no relationship between attitudes and achievement (Demars, 1972; Wardrop, 1972), generally there is a small, but statistically significant, positive correlation between mathematical attitudes and achievement. In fact, despite a variety of attitude and achievement instruments, the correlation coefficient tends to be surprisingly consistent, usually between .2 and .4 (Neale, 1969).

Specific research findings included Alpert, et al. (1963), who reported a significant correlation between performance in mathematics and both anxiety and attitudes toward mathematics. Alpert, et al., surveyed 270 students in the seventh grade. Burbank (1970) also conducted a study of 411 seventh-grade students to determine the relationship between attitudes and achievement. He found there was a significant correlation between mathematical attitudes and achievement in mathematical reasoning, achievement in mathematical concepts, achievement in mathematical computations, and overall mathematical achievement.

Stephens (1960) compared accelerated, regular, and remedial seventh- and eighth-grade mathematics classes to determine differences in mathematical attitudes. The mean attitude score was significantly higher for the accelerated group than both the regular and remedial classes. He concluded that attitude scores could be used in conjunction with achievement scores for placement into classes. In a similar study of 22 eighth-grade high achievers and 22 low achievers in mathematics, Degnan (1967) found that the high achievers had more positive attitudes toward mathematics even though the high achievers were generally more anxious than the low achievers.

In a more comprehensive study of 755 students in junior and senior high schools to determine mathematical attitudes (Ellingson, 1956), significant positive correlations were found between attitude scores and each of the following: mathematics standardized test scores, teachers' grades, and overall grade point average. Students in college preparatory classes had somewhat more positive attitudes than those students in terminal or general mathematics classes.

Significant correlations between mathematical attitudes and achievement have been reported not only at the junior high school level but at all levels of mathematics. Low positive correlations between mathematical attitudes and achievement have been found at the elementary (Anttonen, 1962), high school (Solheim, 1971; Spickerman, 1970), and college undergraduate levels (Aiken & Dreger, 1961; Dreger & Aiken, 1957; Edwards, 1972; Fenneman, 1974; Whipkey, 1970; Wilson, J. M., 1973).

Despite this pervasive correlation between attitudes and achievement in mathematics, one must not assume any cause-effect relationship (Knaupp, 1973; Neale, 1969). Positive attitudes may cause achievement increases. However, an equally tenable alternative is that increased achievement causes development of better attitudes. Since correlational studies do not reveal cause or effect, Neale maintained that the role of mathematical attitudes in mathematics achievement is modest. In fact, Neale argued that obedience, patience, and compliance may be more important than attitudes in determining mathematical achievement.

Development and Modification of Attitudes

Due to the important role of attitudes in mathematics education, various attempts have been made to develop better attitudes or to modify negative attitudes toward mathematics. Studies of curricular changes, methodological changes, and instructional aids or materials have been conducted. Thus, these three important areas of research were reported in this section.

Of course the more widespread and publicized attempt to change student attitudes over the past 25 years was the creation and implementation of the new mathematics curriculum. Although many factors contributed to the need for developing the new curriculum, it was hoped that attitudes toward mathematics would improve as a result of the new mathematics programs. Overemphasis on drill and rote learning were considered to be major factors in developing negative attitudes toward mathematics (Bernstein, 1964; Wilson, G. M., 1961). Consequently, the new mathematics programs generally emphasized mathematical understanding and logical reasoning ability. Bernstein believed that such an organization of content would improve attitudes toward mathematics. Unfortunately, studies have usually shown that new or modern mathematics programs have not improved attitudes among students.

The most influential modern mathematics program was the School Mathematics Study Group (SMSG) curriculum developed at Stanford University. A comprehensive program from elementary through junior high school, the project included new content, instructional methods, and materials. One of the first studies to determine the effect of SMSG materials on student attitudes was by Alpert, et al. (1963). They compared SMSG and non-SMSG seventh-grade classes involving 270 students. Alpert, et al., found that the SMSG curriculum did not improve student attitudes over the academic year, either absolutely or in comparison with the non-SMSG curriculum. In fact, while attitudes of non-SMSG students remained constant from fall to spring, attitudes of the SMSG pupils actually decreased over the same period. They also found that a highly theoretical orientation on the part of the teacher led to positive attitudes in SMSG classes but not in non-SMSG classes. Therefore, placing a theoretical teacher in modern mathematics classes was suggested.

Osborn (1965) tested pupils who had studied SMSG materials for zero to three years in seventh, eighth, and ninth grades. Although achievement scores favored the SMSG students, there was a decrease in positive attitudes toward mathematics as the number of years of SMSG study increased.

Other SMSG curriculum studies included Phelps (1964), who found no significant differences between eighth-grade SMSG student attitudes and the mathematical attitudes of students in the traditional program. Hungerman (1967) obtained similar results at the sixth-grade level. In a more comprehensive study involving over 3500 pupils in the fourth, sixth, and eighth grades, Woodall (1967) found that the use of SMSG materials did not produce a more positive attitude toward mathematics than the conventional mathematics materials.

Another modern mathematics program with diverse appeal was created by the University of Illinois Committee on School Mathematics (UICSM). Comley (1967) compared mathematical attitudes of 338 college students who had the UICSM program in public school with those who had traditional mathematics. The UICSM group had significantly more positive attitudes toward mathematics than the non-UICSM group. In addition, the UICSM students took significantly more college mathematics than non-UICSM students.

Ryan (1967) compared three modern mathematics programs in secondary mathematics (Ball State, SMSG, and UICSM) involving 126 pairs of ninth-grade classes in a five-state area. The general result was that the modern mathematics programs had little effect on attitudes and interests of students over the academic year when compared to traditional programs. There was a tendency for the Ball State program to be associated with less favorable attitudes in comparison with the traditional program. The UICSM program was related to slightly more positive attitudes than was the conventional program. However, some UICSM studies have not found such positive results. Demars (1972) investigated the effect of a UICSM program on low achieving seventh-grade students. In addition to finding no significant correlation between attitudes and achievement, the UICSM program did not statistically improve attitudes or achievement when compared to conventional programs even though the means favored the UICSM group.

One factor to consider in the studies of the modern versus the traditional curriculum is that students in experimental programs may have been chosen because of their positive attitudes toward mathematics or because of their higher achievement levels. This selection possibly would have biased the results.

As Osborn (1965) suggested, the increased level of abstraction of modern mathematics may have been the reason for such poor attitudinal changes in comparison with conventional programs. More recent reports indicate that perhaps the new mathematics programs were not really implemented to any great extent nationally and that lack of proper teacher training adversely affected the modern mathematics programs (Hill, 1976; NACOME, 1975). For whatever reasons, it seems clear that the anticipated positive attitude changes have not developed as a result of studying modern mathematics.

The second major area of attempts to change student attitudes toward mathematics was instructional changes. Some of these instructional changes were suggested as a part of the modern mathematics programs while others were subsequently developed. Therefore, the effect of these instructional procedures on mathematical attitudes of students was investigated.

As noted previously, rote learning and excessive drill led to negative attitudes toward mathematics. What happened to student attitudes when more meaningful methods of instruction were used? Lyda and Morse (1963) tested the effect of a more meaningful method of teaching arithmetic on attitudes toward mathematics in elementary school. Their method stressed the concept of numbers, the understanding of the numeration system and place value, the use of fundamental operations, the rationale of computational forms, and relationships which make arithmetic a system of thinking. The experiment resulted in positive attitude changes and significant achievement gains in reasoning and computation for students receiving meaningful instruction.

Discovery methods of teaching mathematics were introduced to allow students to draw their own conclusions and to discover new concepts with the aid of the teacher. Keese (1972) used the discovery method to teach a unit of mathematics to 31 eighth-grade students. The control group studied the same unit for an equivalent time period of 12 days. The attitudes of students taught by discovery methods were significantly better after the instructional period than the attitudes of students taught by traditional or expository methods. Howitz (1966) compared discovery and expository methods in ninth-grade general mathematics classes. Discovery methods were used in six classes for the entire school year. Howitz found no significant differences in attitude changes between the experimental and control groups.

In the intermediate grades, similar mixed results have been found. Studer (1972) investigated the relationship of discovery methods in mathematics to creative thinking and to mathematical attitudes of fourth-grade and sixth-grade students. Among the conclusions, Studer noted that inner city school pupils had more positive attitudes toward mathematics than non-inner city pupils regardless of the teaching method and that inner city classes using expository methods had the most positive attitudes of all the groups. The discovery method clearly did not have any effect on student attitudes toward mathematics. However, Robertson (1971), in a study of 374 fourth-grade pupils, found that the discovery group had a significantly better attitude change than the expository group after seven months of instruction.

While the previous methods have not been investigated to any great extent, individualized instruction has been the subject of many studies. Results of research on the effect of individualized instruction on attitudes were ambiguous because of improper or inconsistent definitions. A good general description of individualized instruction and traditional instruction was given by Crosby, et al. (1960):

> Individualized instruction is defined as that in which each pupil participates in setting his own goals, works at his own rate (either alone or as a member of a small group) . . . and participates in evaluating his own progress. Traditional instruction is defined as all methods in which pupils are taught as a class. It includes homogeneous or heterogeneous grouping and does not preclude the use of audio-visual aids, committee work, or any other techniques traditionally used by teachers to help students learn. (p. 4)

At the junior high school level, attitudinal changes as a result of individualized instruction were generally neutral or negative. Crosby, et al. (1960), evaluated changes in attitudes and achievement of ninth-grade junior high school algebra students when taught by individual or conventional methods. They reported that attitudes for both the experimental and control groups dropped, with no significant differences found between the groups. Beul (1974) combined team teaching and individualized instruction in eight classes of seventh-grade students for one semester. Beul found that attitudes were higher at the beginning for the experimental group receiving team teaching and individualized instruction but that the attitudes decreased significantly over the testing period. There were no attitude changes in the seven groups using expository methods.

Corbin (1974) compared individualized instruction and traditional methods in seventh- and eighth-grade classes involving 204 pupils. There were no significant differences in attitudes toward mathematics between the experimental and control groups for either the seventh or eighth grade. Horvath (1976) conducted a more comprehensive study including seventh-, eighth-, and ninth-grade students over 30 months of instruction. In a comparison of groups taught by individual methods and traditional methods, there were no significant differences in either mathematical attitudes or achievement.

Malcom (1973) tested the effectiveness of an individualized mathematics program on seventh-, eighth-, and ninth-grade pupils. The individualized approach was more effective in developing positive attitudes than the traditional program for the seventh and ninth grades. However, no attitudinal differences were found for the eighth grade. Nix (1970) analyzed differences between three eighth-grade classes receiving individualized instruction and three classes receiving group instruction. According to Nix, "students developed a more favorable attitude toward school and mathematics . . . when taught general mathematics under the individualized method than under the group-oriented method" (p. 3367A).

Similar conflicting results have been found in the intermediate grades. Broussard (1971) and Scharf (1971) found that an individualized program had a more positive effect on attitudes than a traditional approach. Thomas (1972) found that the attitudes of sixth-grade students increased significantly under an individualized program while the attitudes of fifth-grade pupils remained constant in a comparable individualized program. Williams (1974) found no attitude changes among fourth-, fifth-, and sixth-grade students who participated in an individualized program.

In a comprehensive review of the subject of individualized instruction, Miller (1976) concluded that "individualized instruction has a limited effect on attitudes toward mathematics" (p. 350). He also noted that positive attitudes declined as the number of years of individualized instruction increased. Schoen (1976) similarly concluded that individualized instruction had a neutral effect on attitudes for students above the fourth-grade level.

The final instructional method investigated was the laboratory approach. With this approach, students are required to become involved in learning by using manipulative devices or completing activities in class. R. E. Johnson (1970) constructed activity-oriented lessons on number theory, rational numbers, and geometry for seventh-grade students. Johnson compared attitudes of students using the text only, activities only, and a combination of activities and text. No statistical differences were detected between the attitudes of the three groups but it was suggested that laboratory lessons in geometry might be appropriate for low and middle ability students. At the same level, Simpson (1974) compared the effects of a laboratory program with a traditional teacher-centered approach. Students that were identified as slow learners participated in the laboratory or the traditional program for 101 school days. A significant difference was found in attitude according to sex after the instructional period. That is, males in the experimental and control groups had more positive attitudes than females. However, no differences in attitudes were found according to the two treatments, laboratory and conventional instruction.

At the ninth-grade level, Finnell (1973) tested laboratory units written by teachers for general mathematics. The study involved three groups using the laboratory lessons and three control groups taught by conventional means. Significant differences were found in the affective measure in one of the three experimental groups while differences were not significant in the other two cases. In a study of sixth-, seventh-, and eighth-grade students (Smith, 1973), the experimental group of 82 pupils attended 40-minute laboratory sessions two times per week instead of attending class. After the instructional period, no significant differences existed between experimental and control group attitude scores. From these results, Smith concluded that attitudes played a limited role in mathematics achievement of middle school students.

Beal (1973) tested laboratory materials developed by teachers at the National Science Foundation Conference for Secondary Teachers of Low Achievers in Mathematics. The units were designed to aid students in attaining 26 mathematical competencies and to help develop better attitudes. Thirteen teachers in three different states taught twelfth-, ninth-, eighth-, and seventh-grade students with these materials. Each of the teachers instructed both experimental and control classes. The experiment resulted in a significant increase in attitudes in four of 15 experimental classes and three of 15 control classes. Attitudes were significantly better in two of the 15 experimental groups than the corresponding control group taught by the same teacher. Thus, Beal concluded that the results did not sufficiently support the claim that attitudes increased as a result of laboratory activities.

Investigations of laboratory procedures at the intermediate grade level lend further support to the claim that laboratory lessons have a neutral effect on student attitudes toward mathematics (Kujawa, 1976; Nowak, 1972; Ropes, 1973; Wilkinson, J. D., 1971).

The third major area of attempts to develop better mathematical attitudes was supplementary materials. Supplementary or instructional aids and materials included teacher demonstrations, games, enrichment, interdisciplinary approaches, and units emphasizing relevancy or applications.

Proctor (1965) recommended that mathematics be made more enjoyable to help build confidence within students. The use of laboratory materials, manipulative materials, and games was suggested. Thus, Burgess (1970) hypothesized that a game strategy would motivate low achieving secondary students and result in improved attitudes without reducing achievement. Games for the 248 subjects in the experimental group were developed for teaching concepts and skills with rational numbers. Pencil and paper activity sheets related to the text were used by the 240 pupils in the control group. Each class received half a period a day of normal instruction and half a period a day of games or activity sheets for eight weeks. Posttest attitude scores yielded significant differences in attitudes favoring the experimental group. Therefore, Burgess concluded that the game strategy improved the attitudes of low achieving mathematics students. In the achievement area, Burgess concluded that games did not reduce the learning of addition and subtraction concepts but that they may reduce learning in concepts dealing with division and multiplication.

Bryson (1972) investigated the effects of teacher designed materials and activities on the achievement and attitudes of ninth-grade pupils. Contract units which included educational games and field trips were designed for use in experimental classes. The control group used no games but participated in the field trips. Two teachers instructed one control and one experimental group each for a period of 18 weeks. A decrease in attitudes was found for the experimental group though not statistically significant. A significant increase in attitudes was found for the control groups. The achievement for both the control and experimental groups increased significantly. At the college level, Addleman (1972) also concluded that games and discovery experiences had a neutral effect on attitudes.

In a study of eighth- and ninth-grade pupils conducted by Ray (1961), teachers of experimental groups devoted four days to regular classroom activities and one day to enrichment activities. The enrichment activities included reading days, oral reports, committee work, films, projects, field trips, and guest speakers. After two years, students with enrichment activities developed greater achievement than control students in working with the four operations with decimal fractions, similar and right triangles, and algebra. While no overall attitude change was calculated, students with enrichment activities including oral and written reports felt they had a better chance to express their ideas in mathematics. In the achievement area, students with films, oral reports, committee days, and reading days developed higher achievement than regular students in work concerning similar and right triangles. Students with projects, guest speakers, field trips, written reports, and reading days developed higher achievement than regular students in the four operations with decimal fractions and in work concerning similar and right triangles. G. G. Wilkinson (1971) studied the effectiveness of mathematical objects, filmstrips, and films on attitudes of eighth-grade pupils. After the experimental period of 46 days, Wilkinson found that students taught with supplementary materials did not have a significant gain in attitudes. However, he concluded that the use of the supplementary materials did produce a significant gain in achievement.

Small group discovery lessons were the supplementary materials tested by Jordy (1976). The experimental and control groups for this study were both participating in a modern mathematics program designed for the top 15 to 20 percent of secondary school students. The ten small group discovery lessons written for use by eighth-grade students pertained to sets and groups. The four units written for the ninth-grade experimental group involved matrices and linear algebra. In the two experimental groups, the small group discovery lessons were used in conjunction with the text. The two control groups used the textbooks only. For the eighth grade, the experimental group expressed significantly higher attitudes than the control group after the units were used. However, there was no significant effect on attitudes of ninth-grade pupils in the experimental group. Jordy suggested that the difference between the results for eighth- and ninth-grade pupils may have been due to the small number of units constructed for the ninth grade. On the achievement test, the eighth-grade experimental group scored lower than the control group, though not significantly. For the ninth grade, achievement scores for the experimental group were significantly better than scores for the control group. Positive results in the area of attitudes have been found at the intermediate grade level by Silverman (1974) in a study of the effect of a measurement unit on attitudes.

The effects of interdisciplinary study, mathematical applications, and relevancy were also investigated. Higgins (1969) related mathematics and science to determine the effectiveness of the interdisciplinary units on attitudes of eighth-grade students. Pretest scores were used to separate students into eight attitudinal groups. After the experimental period, mean scores of the groups on ability, algebra achievement, and attitudes were compared. No significant differences were found for any of these scales. Higgins concluded that the formation of cohesive attitude groups was not a major factor for consideration in the design of interdisciplinary mathematics units.

To determine the effectiveness of an applications approach, Holtan (1963) chose 136 ninth-grade boys in general mathematics. A series of applications of mathematical knowledge was related to four areas of student interest: automobiles, farming, social utility, and intellectual curiosity. Students completed all four units, which dealt with mathematical inequalities. Holtan found that interest in the mathematics applications did make a significant differences in achievement in learning mathematics. For example, those that expressed interest in automobiles did better in that unit than those students with no interest in automobiles.

Wajeeh (1976) implemented a program of relevant and meaningful mathematics at the ninth-grade general mathematics level. Detroit inner city pupils participated in the program for 15 weeks. Wajeeh found that, after the instructional period, students in the experimental group had significantly higher attitude scores than the control group. Hence, she concluded that if general mathematics is made relevant and meaningful, attitudes increase. In achievement, students in the experimental group had significantly higher scores than students in the control group. At the college level, McNerney (1970) found that relevancy of materials had no effect on attitudes.

Summary

The purpose of this chapter was to review the literature related to the areas of mathematical attitudes and achievement. One section reported the relationship between attitudes and achievement in mathematics. At all levels, a significant positive correlation has been found between attitudes and achievement. However, no cause-effect relationship has been determined.

The last section of the chapter reported three major areas in which attempts have been made to improve attitudes. It was found that curricular changes generally had neutral or negative effects on attitudes when compared with the traditional curriculum. Attempts to change attitudes by using different methods of instruction were found to be similarly neutral. Studies of the discovery, individualized, and laboratory methods did not support the claim that teaching strategies change student attitudes to any great extent. One notable exception was that attitudes increased when a meaningful method of instruction was used. Conflicting results were found when using various supplementary materials to develop better attitudes. No clear advantage was found for using enrichment exercises, films, or games in mathematics classes. However, when supplementary units were related to student interests or when applications were used, attitudes toward mathematics increased. In addition, significant achievement increases were generally found for the experimental groups using various types of supplementary units.

CHAPTER III

RESEARCH DESIGN

The purpose of this study was to determine the effect of supplementary geometry units on student attitudes toward mathematics. A secondary consideration was the effect of these units on student achievement. This chapter describes the research design used and the procedures followed in testing the supplementary units.

Characteristics of the Participants

The units were tested in all four junior high schools in the Tuscaloosa County School System. While two other schools in the system included both the junior high school and the senior high school, the four schools in this study housed only seventh-, eighth-, and ninth-grade students. These four schools were selected for two reasons. First, traditional methods of instruction were used by all eighth-grade mathematics teachers in the Tuscaloosa County School System. Since the geometry units were designed especially for this type of teacher-directed instruction, it was necessary to select a school system in which traditional methods were prevalent.

The second reason for choosing schools from only the Tuscaloosa County School System was to ensure that the mathematical content for the eighth-grade classes was equivalent. Since all teachers in the system used the 1971 edition of Key Ideas in Mathematics 2 published by Harcourt Brace Jovanovich, the geometry content was equivalent for all the students. In addition, regular in-service meetings of the Tuscaloosa County schools allowed participating teachers to plan and coordinate the mathematical content for the eighth grade.

In these four junior high schools, five of the seven general eighth-grade mathematics teachers volunteered to use the units in their classroom. The five teachers ranged from 25 to 37 years of age. Four of the instructors had taught four to seven years while the fifth teacher had 14 years of experience. The highest degree for three of the teachers was the Bachelor of Science in Education while the other two had received a Master of Arts degree in Secondary Education.

The five junior high school mathematics teachers taught a total of 18 heterogeneously grouped classes. The average number of students per class was 25. Since administrative control of scheduling was not feasible, a purely random sample of students was not possible. However, classes for each teacher were randomly selected to serve as experimental groups. Thus, each teacher was assigned both experimental and control classes in an attempt to control for teacher effect.

Conducting the Investigation

In September, 1976, the researcher interviewed seven general eighth-grade mathematics teachers in the Tuscaloosa County School System. Five of the teachers expressed an interest in using supplementary geometry units to motivate pupils to study mathematics. Due to this positive response, the researcher developed 10 supplementary geometry units during the following three months. The units were submitted to the teachers in December for correction and revision.

In late February, 1977, an instructional meeting was held with the five teachers who agreed to use the units. After the researcher demonstrated each of the ten units, specific instructions concerning implementation into an eighth-grade classroom were given. Finally, directions for administering the attitude and achievement tests were given to ensure uniformity among teachers.

The geometry section of the eighth-grade mathematics classes began in March or April in the four schools. After the classes for each teacher were randomly assigned to an experimental or control group, pretests in attitudes and achievement were administered by each of the teachers. Since the five teachers used the same textbook, the geometry content during the instructional period was equivalent for all students. Major topics included basic geometric definitions, angles, perimeter, area, circles, and polygons. The number of days of geometry instruction varied from 15 to 25 for the experimental group and from 14 to 23 for the control group. Each teacher devoted approximately 12 to 18 minutes of class time to each of the 10 supplementary units. After the geometry section was completed, the teacher administered attitude and achievement posttests.

Instrumentation

Two instruments were used to gather data for this study. The attitude instrument used was Attitude Toward Mathematics developed by Suydam and Trueblood (1969). These two authors constructed the attitude instrument by combining original items with questions from other attitude scales. These questions were submitted to seven judges who sorted the items on an 11-point scale ranging from "not important" to "vital." A preliminary version of the scale was then constructed and given to prospective teachers. After an item analysis was completed, the test was further revised. The final result, the Attitude Toward Mathematics scale, consisted of 26 items with Likert-type response scales which required students to express an opinion or feeling about mathematics. Thirteen statements were worded positively while 13 were worded negatively. The possible responses for each of the 26 items ranged from "strongly disagree" to "strongly agree." No time limit was suggested for the final form of the test. Data from subsequent testing of 3000 students indicated that the internal reliability coefficient was approximately .95 (Suydam, 1974).

The achievement test chosen was the Mathematics Basic Concepts test of the Sequential Tests of Educational Progress, Series II. Developed by the Educational Testing Service in 1969, the 50-item test had an administration time of 40 minutes. The two forms of the test were divided into three cognitive levels: recall facts and/or perform mathematical manipulation; demonstrate comprehension of mathematical concepts; and exercise ingenuity or higher mental processes. Five content areas were measured: number and operation; geometry, measurement, logic, and proof; algebraic relations, sets, equations, and inequialities; application; and probability and statistics. Form 3A of the test contained 17 items in the geometry section while Form 3B contained 16 geometry items. For grade eight, the Educational Testing Service determined that the reliability coefficients for the test were .82 when Form 3A was given first and .79 when Form 3B was given first.

Test Administration and Scoring

The attitude instrument was administered by each teacher before the achievement test was given. Specific instructions were given to the students regarding the completion of the answer sheet. Students responded to each of the 26 statements by marking the appropriate box on the OPSCAN answer sheet. No time limit was imposed for completing the test.

In the actual marking of the answer sheet for the attitude instrument, the student selected response A if he strongly disagreed with the given statement. A numerical value of 1 was assigned to such a response. The student selected response B if he disagreed with the statement. A numerical value of 2 was assigned to this response. Similarly, the student marked C if he had neutral feelings or no opinion about the statement, D if he agreed with the statement, and E if he strongly agreed with the item. Numerical values of 3, 4, and 5, respectively, were assigned to each of these responses.

Before obtaining a total attitude score, the scores of the negatively stated items had to be reversed. For example, if the student marked response A (strongly disagree) for the statement "I am afraid of mathematics," his reversed score for that item was 5. Strongly disagreeing with such negative statements indicated favorable attitudes toward mathematics, hence the need for score reversals. After reversing the scores of the negatively stated items, a total attitude score was derived by adding the scores of all 26 items.

To differentiate between negative, neutral, and positive attitudes toward mathematics, the researcher devised the following scheme. A total score of 26 through 64, corresponding to an average of 1.00 through 2.49 for each item, indicated negative attitudes toward mathematics. Scores from 65 through 90, corresponding to an average of 2.50 through 3.49 for each item, indicated neutral attitudes toward mathematics. Scores from 91 through 130, corresponding to an average of 3.50 to 5.00 for each item, indicated positive attitudes toward mathematics.

All answer sheets for the attitude test were submitted to the University of Alabama Testing Service. The Testing Service obtained a punched computer card for each answer sheet by running the sheet through an OPSCAN machine. The researcher then submitted the punched cards to a statistical program that reversed the scores of the appropriate items and printed the total attitude score for each subject.

The achievement test also was administered by the classroom teacher. Since a standardized test was used, specific written instructions were provided in the test manual. Teachers adhered strictly to the 40-minute time limit. Students marked answers on an OPSCAN answer sheet. The completed answer sheets were then submitted to the University of Alabama Testing Service. The Testing Service punched cards for each of the answer sheets and graded the test. The researcher then checked the scores by submitting the punched cards to a statistical program that graded the tests and printed the scores for each subject.

Statistical Treatment of Data

After scoring the pretest and posttest for both attitudes and achievement, the following data were keypunched on computer cards for each of the subjects in the experiment:

1. Identification number
2. Attitude pretest score
3. Attitude posttest score
4. Attitude change score (the posttest attitude score minus the pretest attitude score)
5. Achievement pretest score
6. Achievement posttest score
7. Achievement change score (the posttest achievement score minus the prestest achievement score).

Only those students who completed the entire battery of four tests were included in the final analysis.

Hence, the experimental group contained 133 students and the control group contained 146 students, for a total of 279 subjects.

The results were used to test the following five experimental hypotheses:

H_1: There will be a significant difference in attitude improvement among students in the experimental and control groups.

H_2: When the experimental group is divided into negative, neutral, and positive attitude subgroups by pretest scores, there will be a significant improvement of attitudes within each subgroup.

H_3: When subjects are placed into low, average, and high achievement subgroups by pretest scores, there will be a significant difference in improvement of attitudes within each experimental and control subgroup.

H_4: There will be a significant correlation between attitude change scores and achievement change scores for all subjects.

H_5: There will be a significant difference in improvement of achievement among students in the experimental and control groups.

In order to test the first hypothesis, an analysis of variance was made using a Lindquist Type I design. The independent variables were treatment group (experimental and control) and repeated measures of attitude (pretest and posttest). The dependent variable was the attitude score.

For H_2, a Lindquist Type I design again was employed to complete the analysis of variance. Independent variables were attitude level (negative, neutral, and positive attitudes) and repeated measures of attitude. The dependent variable was the attitude score. For this hypothesis, only students in the experimental group were considered.

To test H_3, an analysis of variance was completed utilizing a Lindquist Type III design. The three independent variables were treatment group, achievement level (low, average, and high pretest achievement scores), and repeated measures of attitude. The dependent variable was the attitude score.

A Pearson product moment correlation coefficient was calculated for the fourth hypothesis. The gain or loss in both achievement and attitudes for all subjects in the study was computed. A correlation coefficient between attitude change and achievement change was then determined.

The last hypothesis was tested by an analysis of variance using a Lindquist Type I design. Independent variables were treatment group and repeated measures of achievement. The dependent variable was the achievement score.

Summary

In this chapter, characteristics of the participating schools, teachers, and students were reviewed. Next, the procedures followed in testing the supplementary geometry units in the classrooms were described. Characteristics of the instruments used to measure attitudes and achievement were given. Also, a thorough account of how the tests were administered was included. Subsequently, the five experimental hypotheses tested in the study were stated. Finally, the statistical procedures used to test the five hypotheses were outlined.

CHAPTER IV

ANALYSIS OF DATA

The purpose of this study was to determine the effectiveness of geometry units in terms of attitude and achievement change. Five hypotheses were formulated prior to the instructional period. This chapter reviews the method of data collection and analyzes the results in terms of the null hypotheses.

Collection of Data

During the spring of 1977, the subjects completed attitude and achievement pretests. Classroom teachers administered the pretests prior to the instructional period. Fifteen to 25 days of geometry instruction, within the general eighth-grade mathematics classes, followed the pretests. Posttests in attitude and achievement were given by the teachers when the geometry section was completed. Students marked responses for the attitude and achievement instruments on separate answer sheets. The University of Alabama Testing Service punched computer cards for each of the answer sheets and graded the achievement tests. The researcher checked the achievement scores by submitting the punched cards to a statistical program. Attitude scores were obtained by submitting the punched cards to a similar statistical program. Thus, there were four separate scores for each subject: attitude pretest, achievement pretest, attitude posttest, and achievement posttest. The researcher then punched a computer card with the following information for each individual: an identification number, four test scores, an attitude change score, and an achievement change score. These data were used in the subsequent analysis.

Analysis of Variance for Null Hypothesis 1

Ho_1: There will be no significant difference in attitude improvement among students in the experimental and control groups.

This hypothesis was tested by using a 2 x 2 Lindquist Type I statistical design (see the ANOV77 program, Barker & Barker, 1977). The treatment group either experimental or control, served as an independent variable. The other independent variable was repeated measures of attitude. The dependent variable was the attitude score.

Before beginning the analysis of variance, the experimental and control classes had to be homogeneously grouped. To check for homogeneity, Hartley's F-max test of group variance was applied. The variance of pretest attitude scores for the experimental group was calculated to be 284.07. The variance of pretest attitude scores for the control group was 329.97. Thus, the computed $\underline{F}$-max value was 1.16, which was not significant, $\underline{F}$-max$(2,60) = 1.67$, $\underline{p} = .05$. Therefore, the experimental and control groups were homogeneously grouped according to pretest attitude scores.

An assumption that needed validation was that experimental and control groups were essentially equal in pretest attitudes. From an inspection of pretest means (TABLE 1), it appeared that no statistical differences existed between pretest attitude scores for the two groups. Verification of this was incorporated in the actual statistical analysis.

TABLE 1

Attitude Means for Treatment Groups

Group	$\underline{n}$	Pretest Mean	Posttest Mean
Experimental	133	83.14	82.01
Control	146	81.23	80.78

Note. Minimum score possible = 26.

Maximum score possible = 130.

Analysis of the results (see TABLE 2) began with the interaction component. The obtained *F* value of .28 was substantially less than the required table value. This showed that neither group changed in attitudes more than the other. Neither of the main effects was significant. The nonsignificant F ratios for the treatment group main effect and the interaction component verified the aforementioned assumption of equal pretest scores for experimental and control groups. The lack of statistical differences was more understandable after an inspection of TABLE 1. Neither group made substantial gains in attitude. Actually, attitudes decreased slightly for both groups.

From this analysis, it was clear that the null hypothesis of no significant attitude improvement for the experimental group could not be rejected. As expected, there was no significant attitude change for the control group.

Analysis of Variance for
Null Hypothesis 2

Ho_2: When the experimental group is divided into negative, neutral, and positive attitude subgroups by pretest scores, there will be no significant improvement of attitudes within each subgroup.

TABLE 2

Analysis of Variance of Attitude
Scores for the Treatment Groups

Source	SS	df	MS	F
Between experimental and control	342.25	1	342.25	.65
Error between experimental and control	146,230.66	277	527.91	
Total between experimental and control	146,572.91	278		
Between pretest and posttest	82.84	1	82.84	1.42
Interaction	16.16	1	16.16	.28
Error within subjects	16,106.50	277	58.15	
Total within subjects	16,205.50	279		

Note. $F(1, 200) = 3.89$, $p = .05$.

This hypothesis was tested by using a 3 x 2 Lindquist Type I statistical design. Only students in the experimental group were included in this hypothesis. The independent variables were pretest attitude level (negative, neutral, and positive pretest attitudes) and repeated measures of attitude. The dependent variable was the attitude score.

To divide the experimental group into three pretest attitude levels, subjects who scored from 26 through 64 on the attitude pretest were placed into the negative-attitude group; subjects who scored from 65 through 90 on the attitude pretest were placed into the neutral-attitude group; and subjects who scored from 91 through 130 on the attitude pretest were placed into the positive-attitude group.

To check for homogeneity of groupings, Hartley's F-max test of group variance was applied. The variance for each of the three attitude subgroups was: negative-attitude, 25.75; neutral-attitude, 44.42; and positive-attitude, 46.81. The computed F-max value was not significant, $F(3,66) = 1.82$, $p > .05$. Therefore, the three experimental attitude subgroups were homogeneously grouped according to pretest attitudes.

Analysis of results (see TABLE 3) began with the interaction component. Since the obtained F ratio for interaction was significant, the individual cell means

TABLE 3

Analysis of Variance of Attitude Scores
for Experimental Students Grouped
by Pretest Attitude Scores

Source	SS	df	MS	F
Between levels of attitude	51,113.17	2	25,556.59	239.64*
Error between levels of attitude	13,863.98	130	106.65	
Total between levels of attitude	64,977.16	132		
Between pretest and posttest	84.58	1	84.58	1.42
Interaction	1,034.05	2	517.02	8.67*
Error within subjects	7,749.38	130	59.61	
Total within subjects	8,868.00	133		

*p < .01.

varied according to the pretest attitude level. Therefore, inspection of individual cell means was justified. By looking at the cell means (TABLE 4), it appeared that statistical differences between pretest scores did exist. However, this was expected since the groups were formulated by such an attitude difference. Hence, the only important *t* tests to perform were between pretest and posttest for each of the attitude levels. By referring to TABLE 4, the results of the *t* test showed that the only statistical differences were between pretest and posttest attitude scores for the positive-attitude group. That is, for those subjects in the experimental group with positive pretest attitudes, the attitude score decreased significantly.

The *F* ratio between levels of attitude was also significant. However, this was of no consequence since the three subgroups were formulated originally by such attitude differences. Finally, the *F* ratio for the other main effect of repeated measures was not significant.

From this analysis, the null hypothesis of no improvement in attitudes for each subgroup could not be rejected. In fact, a significant decrease in attitude scores occurred for those in the positive-attitude group.

TABLE 4

t Values Between Pretest and Posttest Means for each Attitude Level of the Experimental Group

Pretest attitude level	*n*	Pretest mean	Posttest mean	Error for *t* test	*df*	*t*
Negative	18	56.89	57.50	2.57	17	.24
Neutral	67	76.78	78.91	1.33	66	1.60
Positive	48	101.85	95.52	1.58	47	4.01*

*$p < .01$.

Analysis of Variance for
Null Hypothesis 3

Ho_3: When subjects are placed into low, average, and high achievement subgroups by pretest scores, there will be no significant difference in improvement of attitudes within each experimental and control subgroup.

This hypothesis was tested by using a 2 x 3 x 2 Lindquist Type III statistical design (see the ANOV10 program, Barker & Barker, 1977). The three independent variables were treatment group (experimental and control), achievement level (low, average, and high pretest achievement scores), and repeated measures of attitude. The dependent variable was the attitude score.

One of the constraints placed on the Lindquist Type III design is that the number of subjects per cell must be equal. To accommodate this restraint, the normal curve was used to initially separate the subjects into three numerically equal subgroups. By referring to the normal curve, the range of scores .44 of a standard deviation above and below the mean contains 34% of all scores. In this case, the mean for all 279 subjects on the achievement pretest was 19.93 with a standard deviation of 6.86. Therefore, the range of scores .44 of a standard deviation above and below the mean was

16.91 to 22.95. Since fractional scores were not possible for individuals on the achievement test, the range of scores was 17 through 23 for those subjects in the average achievement subgroup. Students in the high achievement subgroup, which included 33% of all subjects, had pretest achievement scores of 24 or above. Students in the low achievement subgroup, which included 33% of all subjects, had pretest achievement scores of 16 or below.

After the range of scores for each of the three achievement subgroups was determined, each subgroup was divided into experimental and control subjects. The least number of subjects in any of the six resulting cells was 36. Subjects for the other cells were randomly selected so that the number of subjects in each cell was 36. Therefore, in the subsequent analysis, 216 subjects were included.

To test for homogeneity of groupings, Hartley's $\underline{F}$-max test of group variance was applied to the pretest attitude scores. The variances for the six groups ranged from 205.51 to 394.85. The computed $\underline{F}$-max value of 1.92 was not significant, $\underline{F}\text{-max}(6, 30) = 2.91$, $p = .05$. Thus, the six achievement level subgroups were homogeneously grouped according to pretest attitude scores.

The means and the results of the analysis are presented in tabular form (TABLE 5 and TABLE 6) due to the complexity of the Lindquist Type III design. None of the interaction components were significant. Neither

TABLE 5

Pretest and Posttest Attitude
Means for Subjects Grouped by
Pretest Achievement Scores

Achievement level	Treatment group	
	Experimental	Control
Low		
Pretest	76.47	79.28
Posttest	76.69	77.81
Average		
Pretest	83.56	80.58
Posttest	83.81	81.83
High		
Pretest	91.28	89.06
Posttest	88.36	85.92

Note. *n* = 36 for each cell.

the treatment group main effect nor the repeated measures effect was significant. The only significant *F* ratio was for the achievement level main effect. However, this was of no consequence in this experiment since it applied to

TABLE 6

Analysis of Variance of Attitude
Scores for Subjects Grouped by
Pretest Achievement Scores

Source[a]	SS	df	MS	F
A	97.28	1	97.28	.19
B	8,897.81	2	4,448.91	8.77*
A x B	456.81	2	228.41	.45
Error (A)	106,510.06	210	507.19	
C	101.13	1	101.13	.27
A x C	2.47	1	2.47	.01
B x C	263.22	2	131.61	.36
A x B x C	32.78	2	16.39	.05
Error (B)	77,061.91	210	366.96	

[a] A = Treatment group. B = Achievement level. C = Repeated measures of attitude.

*$p < .01$.

the combined means of pretest and posttest attitude scores for both treatment groups at each of the three levels of achievement.

From this analysis, the null hypothesis of no attitude improvement for each achievement level of the experimental group could not be rejected.

Correlational Study for
Null Hypothesis 4

Ho_4: There will be no significant correlation between attitude change scores and achievement change scores for all subjects.

For this hypothesis, a Pearson $\underline{r}$ (see CORR01 program, Barker & Barker, 1977) was calculated between the attitude change scores and the achievement change scores for all subjects. The calculated $\underline{r}$ value of .06 was not significant, $\underline{F}(1,277) = 1.01$, $\underline{p} > .05$. Therefore, the null hypothesis of no correlation between attitude change scores and achievement change scores could not be rejected.

Analysis of Variance for
Null Hypothesis 5

Ho_5: There will be no significant difference in improvement of achievement among students in the experimental and control groups.

This hypothesis was tested by using a Lindquist Type I design. The independent variables were treatment group and repeated measures of achievement. The dependent variable was the achievement score.

Hartley's *F*-max test of group variance was employed to test homogeneity of the experimental and control groups on pretest achievement scores. The variance for the experimental group was 50.77 while the variance for the control group was 43.57. The computed *F*-max was not significant, $F\text{-max}(2, 145) = 1.17$, $p > .05$. Hence, the experimental and control groups were homogeneous according to pretest achievement scores.

The pretest achievement means had to be equivalent for the experimental and control group before analysis. Inspection of these means (see TABLE 7) indicated that the means were essentially the same. Statistical verification of this was incorporated in the analysis.

TABLE 7

Achievement Means for Treatment Groups

Group	*n*	Pretest mean	Posttest mean
Experimental	133	19.87	19.74
Control	146	19.99	18.94

Note. Individual norms: $M = 29$ and $SD = 12$.

The analysis of variance (see TABLE 8) began with the interaction component. Since the F value for interaction was not significant, neither group changed in achievement more than the other. Neither of the main effects, treatment group and repeated measures, was significant. The nonsignificance verified the aforementioned assumption of equivalent pretest achievement means. By inspecting the means in TABLE 7, the lack of significance was obvious since only slight decreases occurred in the achievement means.

From these results, the null hypothesis of no improvement in achievement could not be rejected.

Summary

The method of data collection was reviewed in this chapter. Using these data, five null hypotheses were tested by using analysis of variance and correlational techniques. From the results of these analyses, none of the null hypotheses could be rejected. Therefore, the five experimental hypotheses previously stated were rejected.

TABLE 8

Analysis of Variance of Achievement
Scores for the Treatment Groups

Source	SS	df	MS	F
Between experimental and control	15.97	1	15.97	.21
Error between experimental and control	21,077.49	277	76.09	
Total between experimental and control	21,093.47	278		
Between pretest and posttest	51.79	1	51.79	3.18
Interaction	30.39	1	30.39	1.87
Error within subjects	4,512.82	277	16.29	
Total within subjects	4,595.00	279		

Note. $F(1, 200) = 3.89$, $p = .05$.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The first section of this chapter includes a summary of the literature related to attitudes and achievement and an overview of the study. The second section includes the conclusions that were drawn as a result of the study. The last section lists recommendations for further study.

Summary

The literature concerning mathematical attitudes and mathematical achievement indicated that, generally, there was a significant positive correlation between the two. However, no cause-effect relationship between mathematical attitudes and achievement has been found. The literature dealing with actual attempts to change attitudes was then reviewed. Curricular attempts to change attitudes, such as the School Mathematics Study Group program, have usually resulted in neutral or negative effects on attitude in comparison with the traditional curriculum. Similarly, attempts to change attitudes by using various instructional methods were found to have a neutral effect. Discovery, individualized, and laboratory methods have not changed student attitudes to any great extent. When supplementary materials were used to develop better attitudes, conflicting results were found. No advantage was found for using enrichment exercises, games, or films in mathematics classes. However, when the supplementary materials were related to student interests or when applications were used, attitudes toward mathematics improved. In addition, the effect of these supplementary materials on achievement has been favorable, with the experimental group generally attaining equivalent or significantly higher scores than the control group.

To further investigate the effect of supplementary materials on student attitudes and achievement, the present study tested the effectiveness of supplementary geometry units on mathematical attitudes and achievement of eighth-grade students. The primary concern was to determine if mathematical attitudes improved by relating geometry to art and to nature. Hence, 10 supplementary units were written from extant materials to demonstrate how geometric principles relate to artistic designs and natural phenomena. Five of the units relating geometry to nature included two soap film experiments and teacher presentations about honeycombs, snowflakes and networks. The other five units, which related geometry to art, included geometrical illusions, line designs, string art, tessellations, and shapes that may elicit different emotions.

To test the ten supplementary geometry units used in this study, eighth-grade general mathematics classes were selected. Five teachers were randomly assigned both experimental and control groups. The units were used during the geometry instructional period, which varied from three to five weeks. Each teacher administered both the attitude and achievement pretests and posttests. After the instructional period, all tests were mechanically scored. These data were used to test five experimental hypotheses:

H_1: There will be a significant difference in attitude improvement among students in the experimental and control groups.

H_2: When the experimental group is divided into negative, neutral, and positive attitude subgroups by pretest scores, there will be a significant improvement of attitudes within each subgroup.

H_3: When subjects are placed into low, average, and high achievement subgroups by pretest scores, there will be a significant difference in improvement of attitudes within each experimental and control subgroup.

H_4: There will be a significant correlation between attitude change scores and achievement change scores for all subjects.

H_5: There will be a signfiicant difference in improvement of achievement among students in the experimental and control groups.

The analysis of the data included correlational and analysis of variance techniques. The analyses for hypotheses 1, 4, and 5 revealed no significant differences or correlation. For hypothesis 2, the attitude score decreased significantly for those students in the experimental group who had positive pretest attitudes. For hypothesis 3, one significant F ratio was determined but was inconsequential in this study. Thus, on the basis of these results, all five of the experimental hypotheses were rejected.

Conclusions

From the analysis of hypothesis 1, it was obvious that no attitude changes occurred for either the experimental or control groups. Hence, the supplementary geometry units were ineffective in improving mathematical attitudes in the experimental group.

From the analysis of hypothesis 2, no improvement in attitudes was evident for any of the three attitude levels within the experimental group. In fact, a decrease in the mean attitude score was found for those experimental students in the positive-attitude group. However, similar investigation of the other treatment group revealed that attitudes decreased significantly for the positive-attitude control group as well. Therefore, conjectures concerning the effect of the units on the positive-attitude group must be made with caution.

From the analysis of the remaining hypotheses, it was obvious that the units were ineffective in producing attitude improvements even though various levels of achievement were considered. Also, there was no correlation between attitude and achievement change as a result of using the units. Finally, the units did not produce any improvement in achievement for the experimental group.

Recommendations

The following recommendations are made as a result of the knowledge gained from this study:

Mathematical attitude instruments should be constructed to reflect student attitudes toward a particular discipline, such as geometry, algebra, or trigonometry.

The length of time for future studies of attitude improvement should be extended to at least one semester.

Through a subjective evaluation of the effect of the units, the participating teachers in this study felt that attitudes did improve for some students. Therefore, further study is needed to determine which students benefit most from such materials. Future studies should concentrate on one specific subgroup of students, such as low achievers or those with neutral attitudes, to determine attitude improvement.

Achievement tests in future studies should test only the content presented during the instructional period.

Other units that relate mathematics to various disciplines should be constructed and tested.

APPENDIX A

ATTITUDE TOWARD MATHEMATICS SCALE

ATTITUDE TOWARD MATHEMATICS

INSTRUCTIONS: Place your name in the upper right hand corner of the answer sheet. Do not write on this booklet. Use pencil only.

This is to find out how you feel about mathematics. You are to read each statement carefully and decide how you feel about it. Then indicate your feeling by marking on the answer sheet as follows:

If you STRONGLY DISAGREE with a statement, blacken the letter A on the answer sheet for that item.
If you DISAGREE with a statement, blacken the letter B on the answer sheet for that item.
If your feeling about a statement is NEUTRAL, blacken the letter C on the answer sheet for that item.
If you AGREE with a statement, blacken the letter D on the answer sheet for that item.
If you STRONGLY AGREE with a statement, blacken the letter E on the answer sheet for that item.
(Note that the numbers on the answer sheet go across the page from left to right.)

		STRONGLY DISAGREE	DISAGREE	NEUTRAL	AGREE	STRONGLY AGREE
1. Mathematics often make me feel irritable and angry.	1.	A	B	C	D	E
2. I usually feel happy when doing mathematics problems.	2.	A	B	C	D	E
3. I think my mind works well when doing mathematics problems.	3.	A	B	C	D	E

		STRONGLY DISAGREE	DISAGREE	NEUTRAL	AGREE	STRONGLY AGREE
4. I avoid mathematics because I am not very good with numbers.	4.	A	B	C	D	E
5. When I cannot figure out a verbal problem, I feel as though I am lost in a mass of words and numbers and cannot find my way out.	5.	A	B	C	D	E
6. My mind goes blank and I am unable to think clearly when working mathematics problems.	6.	A	B	C	D	E
7. Mathematics is a stimulating and interesting subject.	7.	A	B	C	D	E
8. I sometimes feel like running away from my mathematics problems.	8.	A	B	C	D	E
9. I feel sure of myself when doing mathematics.	9.	A	B	C	D	E
10. When I hear the word mathematics, I have a feeling of dislike.	10.	A	B	C	D	E
11. I am afraid of mathematics.	11.	A	B	C	D	E
12. Mathematics is fun.	12.	A	B	C	D	E
13. I like anything with numbers in it.	13.	A	B	C	D	E
14. Mathematics problems often scare me.	14.	A	B	C	D	E

		STRONGLY DISAGREE	DISAGREE	NEUTRAL	AGREE	STRONGLY AGREE
15. I feel good toward mathematics.	15.	A	B	C	D	E
16. I usually feel calm when doing mathematics problems.	16.	A	B	C	D	E
17. I think about mathematics problems outside of class and like to work them out.	17.	A	B	C	D	E
18. Mathematics tests always seem difficult.	18.	A	B	C	D	E
19. I have always liked mathematics.	19.	A	B	C	D	E
20. I would rather do anything else than do mathematics.	20.	A	B	C	D	E
21. Trying to work mathematics problems makes me nervous.	21.	A	B	C	D	E
22. Mathematics is easy for me.	22.	A	B	C	D	E
23. I feel especially capable when doing mathematics problems.	23.	A	B	C	D	E
24. I dread mathematics.	24.	A	B	C	D	E
25. Mathematics class stimulates me to look for ways of applying mathematics to solving practical problems.	25.	A	B	C	D	E
26. Time drags in a mathematics lesson.	26.	A	B	C	D	E

APPENDIX B

SUPPLEMENTARY GEOMETRY UNITS

SOAP FILM I

I. Objective.

The teacher will conduct an experiment to illustrate to students the natural angles and minimum configurations formed by soap film in two-dimensional space.

II. Teacher Information.

Soap film and soap bubbles can be a source of fascination for any age group. Who would think that the soap bubbles many children use for entertainment are governed by mathematical principles? In fact, the complex mathematical and scientific principles underlying soap film configurations may not be understood by either the eighth-grade student or by the teacher. However, this should not prevent the teacher from exploring these principles. Therefore, the following basic principles concerning soap films will be investigated.

1. A system tends to stay in a formation with the least energy or least surface stress. In a soap film system, this condition is synonymous with least surface area. That is, a soap film stays in the formation with minimum area.
2. Soap film surfaces meet in only two ways:
    a. Three surfaces meet along a smooth curve or straight line at angles of 120 degrees with respect to one another.

b. Six surfaces, which form four curves or lines, meet at a single vertex. These four curves meet at the vertex at angles close to 109 degrees.

Before performing the actual experiment in class, try the experiment to make sure the soap solution is of the correct consistency. The soap solution itself can be bought commercially or it can be made by mixing equal portions of dishwashing detergent and water.

The two soap film units have been artificially separated only to accommodate class time. Since both experiments in one class period may take too much time, two separate experiments are suggested. SOAP FILM I emphasizes two dimensional concepts while SOAP FILM II readily illustrates their three dimensional equivalents.

III. Classroom Procedure.

Get students interested at first by simply blowing bubbles. Explain briefly what the demonstration is about. After students are gathered around the demonstration table, perform the following experiments:

1. Place three tacks, with points up, between the two glass panes so that an equilateral triangle is formed. Dip into soap solution, making sure all three tacks are joined by soap film. Observe the film formation from the top of the glass. Ask students why this particular figure developed. FIGURE 1 shows three possible ways of combining the three tacks. The

structure with the three-way joints (1A) uses less material than any other configuration, thus principle one is observed. Make sure to point out that the angles formed are 120 degrees.

2. Place three tacks in various positions between the panes to show that a three-way joint forms. Show also that the only exception to this rule is when the three tacks themselves form a 120-degree angle as in FIGURE 2.
3. Ask students how four tacks will be joined by the film. Place four tacks in a square between the panes and dip into the solution. Observe that two triple joints are formed since this figure requires less materials to join the four tacks than any other figure (see FIGURE 3; 3A requires less soap film to connect the four tacks than 3B or 3C). Distribute four tacks in irregular shapes; two triple joints emerge. The exception is when 120-degree angles are already formed by the tacks themselves (FIGURE 4). Notice that principle 2 is always being observed.
4. Place five tacks in a regular pentagon shape and note the triple joints. FIGURE 5 shows one possibility.
5. Place six tacks in a regular hexagon shape and note the triple joints. (This figure, while it does not represent the least material in joining six tacks, does require the least energy to maintain. In FIGURE 6, 6B requires less material than 6A. These

and more complicated figures are governed by networks that must meet the two constraints, directness in joining points and overall length. See Stevens' book for an excellent treatment of this phenomena.)

6. Place many tacks between the panes in an irregular fashion. Again, triple joints that require the least amount of energy to maintain are formed.

Conclude with the statement that the foregoing experiments show that mathematical principles are inherent in soap film systems.

IV. Materials Needed.

Two sheets of glass (approximately 15 x 15 cm), eight to ten thumb tacks, demonstration table, towels, and soap solution are needed.

V. Optional Student Activities.

Measure or calculate the length of the lines in FIGURES 1 and 3 to show that 1A and 3A are the least length.

VI. Sources.

Almgren, F., Jr., & Taylor, J. E. The geometry of soap films and soap bubbles. Scientific American, 1976, 235(1), 82-93.

Courant, R. Soap film experiments with minimal surfaces. American Mathematical Monthly, 1940, 47, 167-175.

Courant, R., & Robbins, H. What is mathematics? New York: Oxford University Press, 1941.

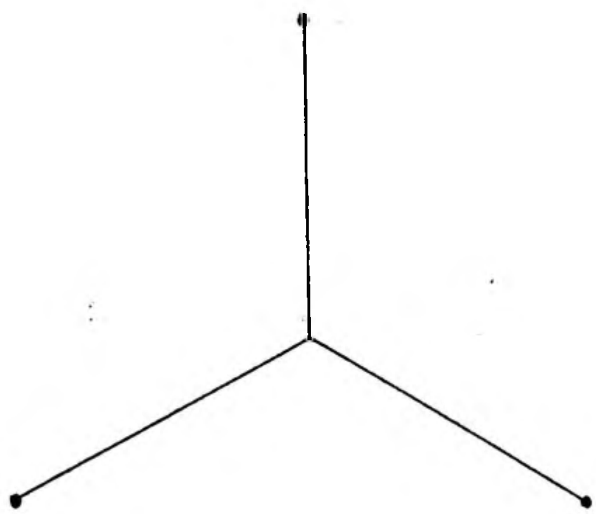
Ehrmann, R. M. Minimal surfaces rediscovered. Mathematics Teacher, 1976, 69, 146-152.

Moulton, J. P. Experiments leading to figures of maximum area. Mathematics Teacher, 1975, 68, 356-363.

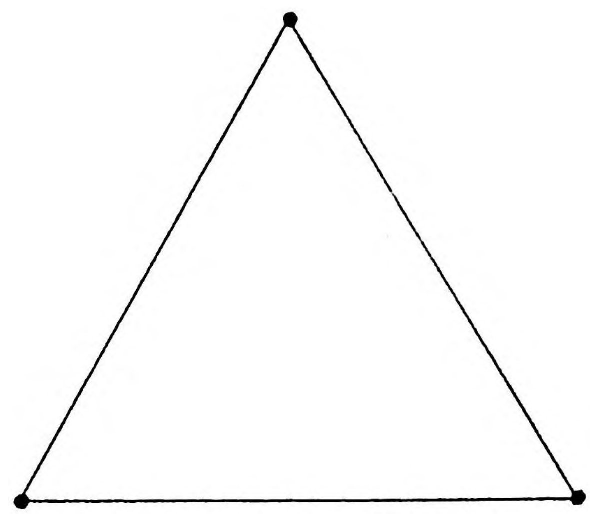
Stevens, P. S. Patterns in nature. Boston: Little, Brown, and Co., 1974.

SOAP FILM I

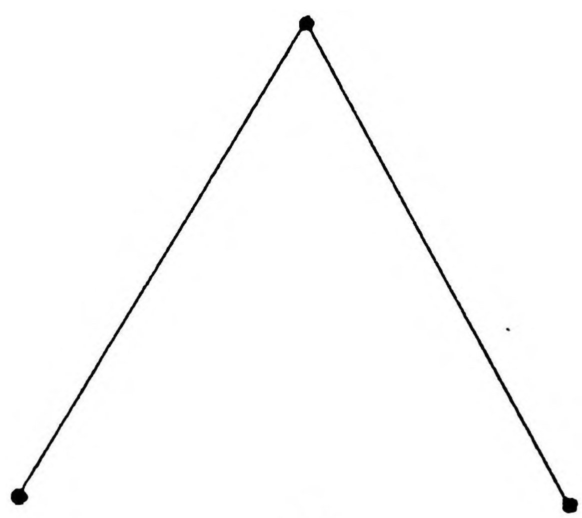
FIGURE 1



1A



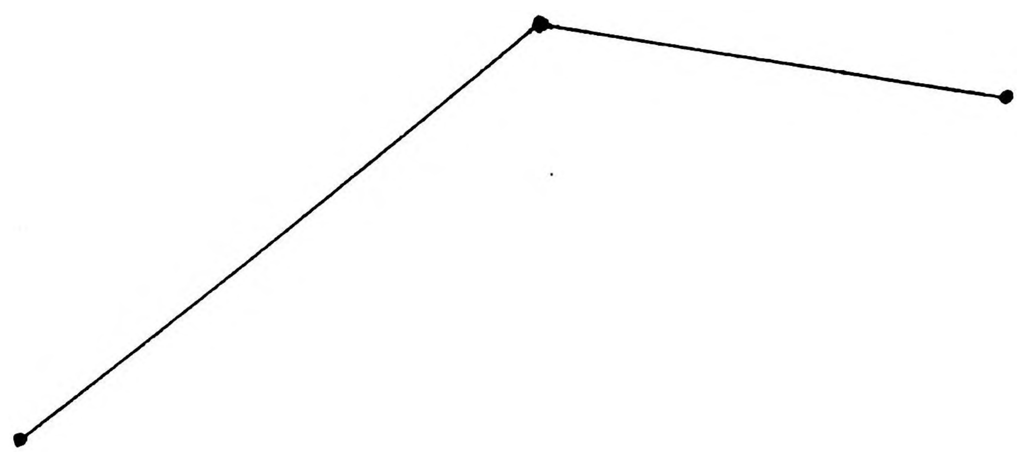
1B



1C

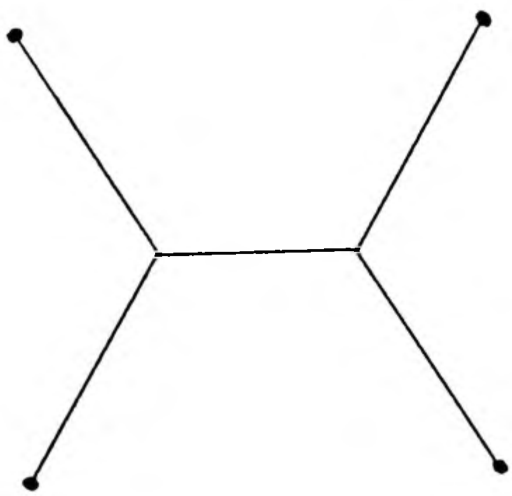
SOAP FILM I

FIGURE 2

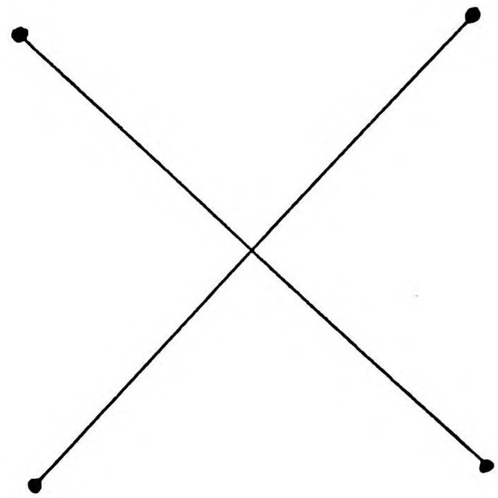


SOAP FILM I

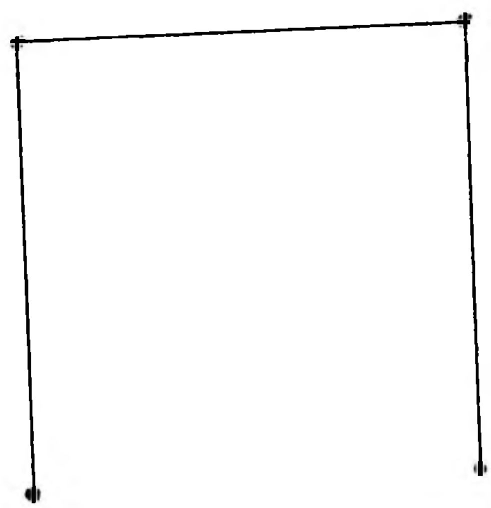
FIGURE 3



3A



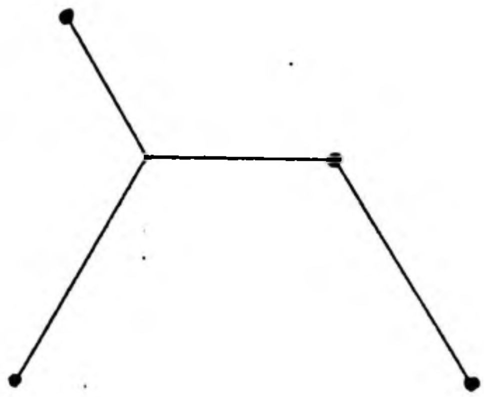
3B



3C

SOAP FILM I

FIGURE 4

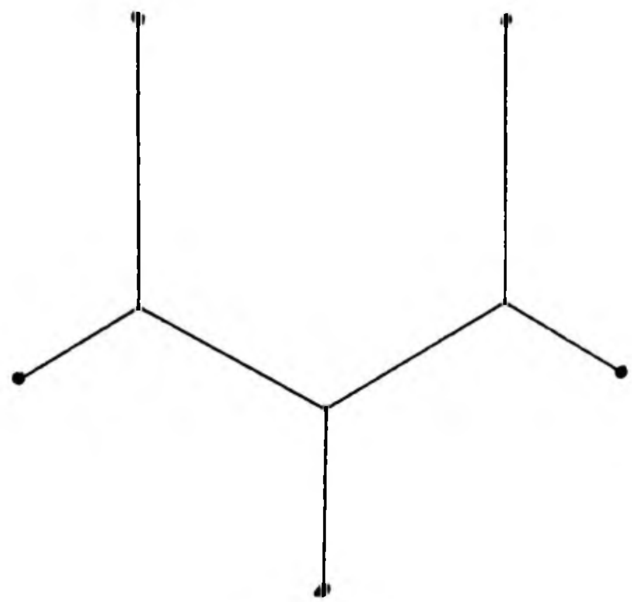


4A

4B

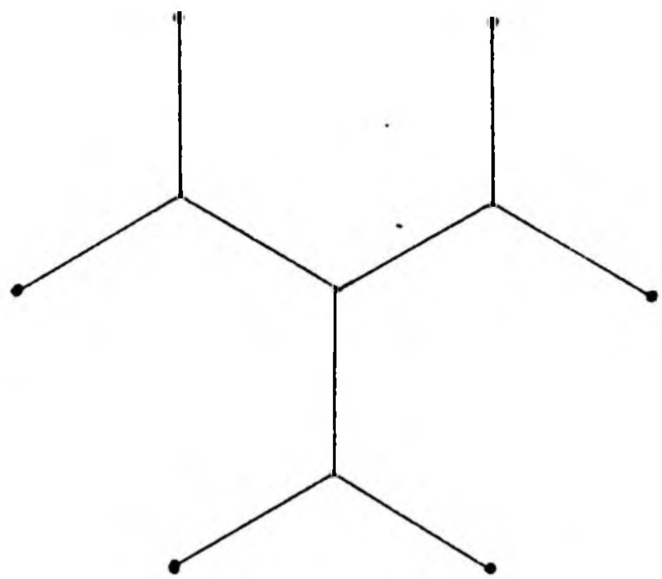
SOAP FILM I

FIGURE 5

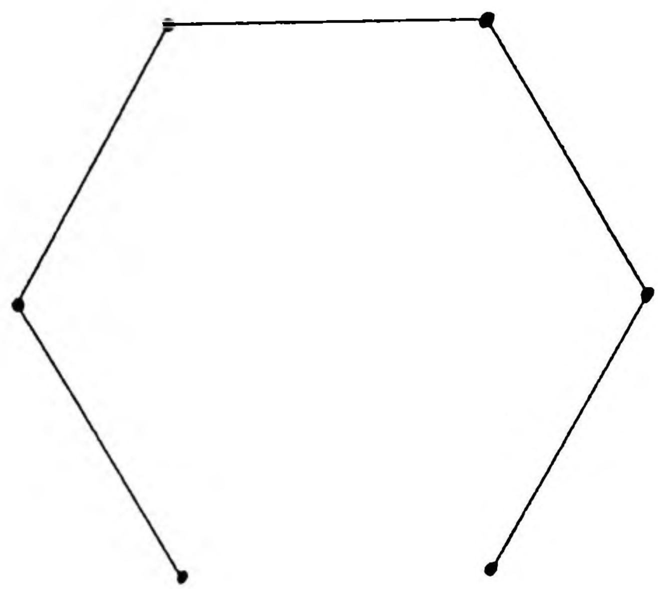


SOAP FILM I

FIGURE 6



6A



6B

SOAP FILM II

I. Objective.

The teacher will conduct an experiment to illustrate to students the natural angles and minimum configurations formed by soap film in three-dimensional space.

II. Teacher Information.

This second soap film exercise demonstrates the same laws as the first experiment. These laws are again listed for convenience.

1. A system tends to stay in a formation with the least energy or least surface stress. In a soap film system, this condition is synonymous with least surface area. That is, a soap film stays in the formation with minimum area.
2. Soap film surfaces meet in only two ways:
    a. Three surfaces meet along a smooth curve or straight line at angles of 120 degrees with respect to one another.
    b. Six surfaces, with form four curves or lines, meet at a single vertex. These four curves meet at the vertex at angles close to 109 degrees.

With the materials listed in section IV, make the following structures. Make a circle with thread and one with wire. Tie the thread circle to the wire circle with two pieces of thread as illustrated in FIGURE 1.

Make an egg-shaped structure with three wires forming approximately 120-degree angles with each other (see FIGURE 2). Make a tetrahedron with wire as shown in FIGURE 3. Make a cube with wire as illustrated in FIGURE 4. Be sure all wire structures can be completely submerged in the soap solution.

III. Classroom Procedure.

In the first soap film demonstration, it was observed that soap films maintain a figure that requires the least energy. The following experiments show that soap film systems tend toward a least area configuration.

1. Dip the circles into a soap solution. With a needle, prick the soap film inside the thread circle (see FIGURE 1). The film should burst so that the thread forms a perfect circle. Ask students why a circle is formed. One reason is that with the circular structure, the pressure at each point of the thread is equalized. Also, a circle has the maximum area for a given perimeter, thus minimizing the surface area of the soap film in observance of principle 1. Principle 1 also holds in experiments 2, 3, and 4 since in all cases soap film area (stress) is minimized.
2. Dip the egg-shaped wire structure in the soap solution. In observance of principle 2a, three soap film surfaces

meet in a curved or straight line. Inform the students that the angle formed by any two of the planes is 120 degrees.

3. Ask students to speculate as to the film formation for the tetrahedron. Dip the tetrahedron into the solution. In accordance with principle 2b, the six surfaces and four lines meet at one vertex. Tell the students that the angle formed by two lines is approximately 109 degrees. A bubble may form in the center of the tetrahedron but note that the same principle holds.

4. Again, ask students to speculate about formations for the cube. Then place the cube in soap solution. A rectangular bubble or a plane surface forms in the middle of the cube in adherance to principle 2b. This structure is not only a fascinating combination of all the foregoing soap film principles but is aesthetically pleasing as well.

End with a review of how mathematical principles are inherent in soap film systems.

IV. Materials Needed.

Demonstration table, towels, pliable wire, needle or pin, thread, pliers, and soap solution are needed.

V. Optional Student Activities.

Make other wire structures to show these same principles hold.

VI. Sources.

Almgren, F., Jr., & Taylor, J. E. The geometry of soap films and soap bubbles. Scientific American, 1976, 235(1), 82-93.

Courant, R. Soap film experiments with minimal surfaces. American Mathematical Monthly, 1940, 47, 167-175.

Courant, R., & Robbins, H. What is mathematics? New York: Oxford University Press, 1941.

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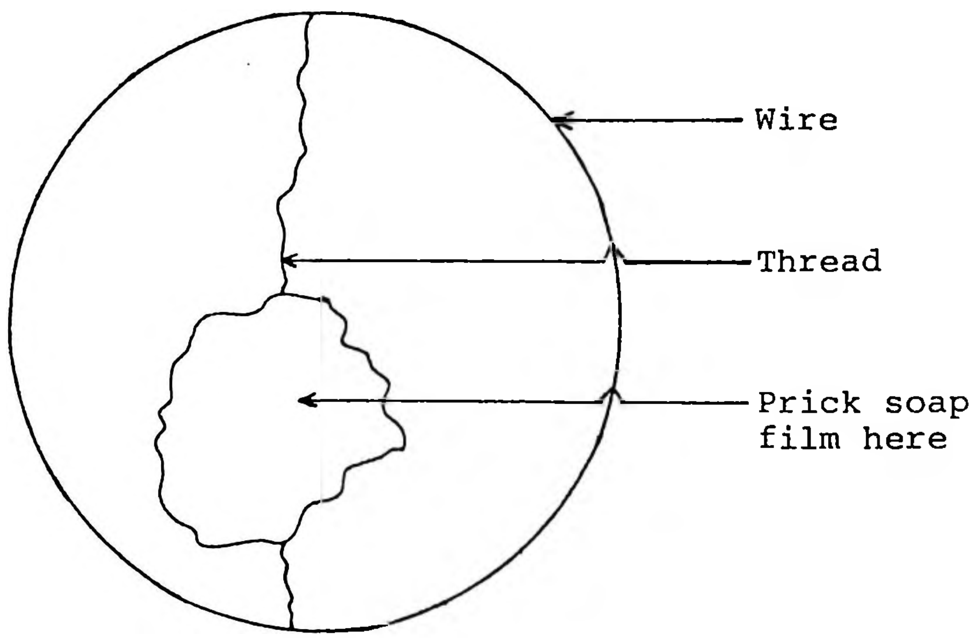
Moulton, J. P. Experiments leading to figures of maximum area. Mathematics Teacher, 1975, 68, 356-363.

Stevens, P. S. Patterns in nature. Boston: Little, Brown, and Co., 1974.

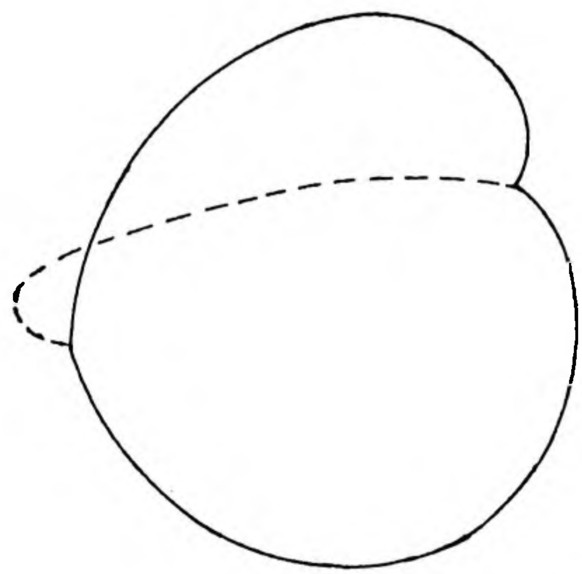
SOAP FILM II

FIGURE 1



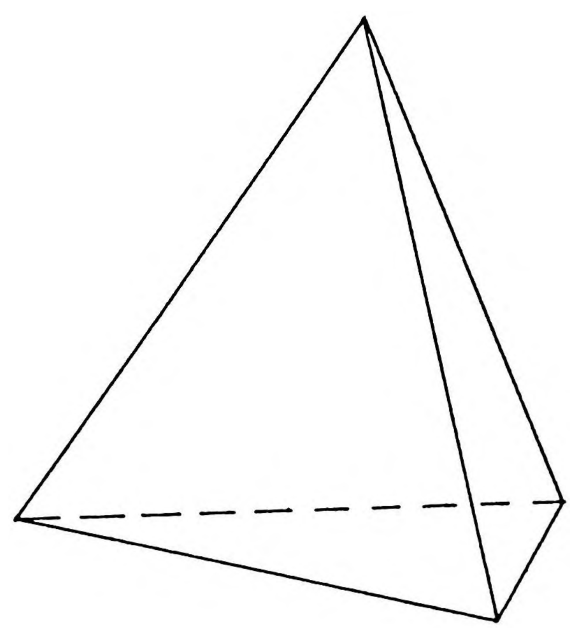
SOAP FILM II

FIGURE 2



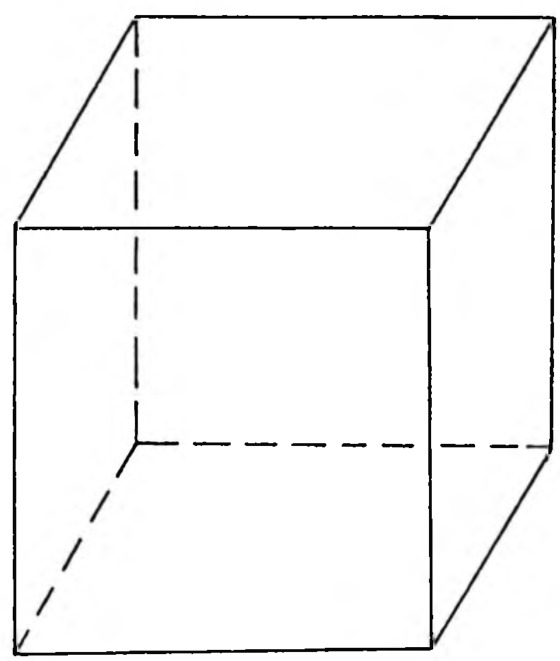
SOAP FILM II

FIGURE 3



SOAP FILM II

FIGURE 4



HONEYCOMBS

I. Objective.

The teacher will show students how the regular hexagon contributes to the efficiency and strength of the honeycomb.

II. Teacher Information.

Articles dating back to the nineteenth century attest to the fact that honeycombs have long been a source of fascination for man. The most controversial topic was whether the hexagonal pattern of the honeycomb was due to inherent intelligence of the bee or to natural forces acting on the structure. However, one can study the mathematical principles involved and appreciate the efficiency of the hexagonal patterns without debating the intelligence/nature problem.

The major problem is why the honeycomb is composed of regular hexagons. Why is it not composed of squares, circles, or some irregular figure? The two main principles involved in the explanation are close-packing and maximizing area. Both principles should be thoroughly explained to the students.

The suggested placement for this unit is after the study of area and perimeter of polygons.

III. Classroom Procedure.

Introduce the subject by stating that the honeycomb constructed by bees is composed of regular hexagons. Encourage students to propose reasons why the hexagon is used.

To begin the explanation of the advantages of the hexagonal shape in the honeycomb, consider the polygon that encloses the most area for a given perimeter. As an example, if the perimeter of a quadrilateral is 12 units, what quadrilateral will enclose the most area? The answer is a 3 x 3 square; any other quadrilateral with perimeter 12 will not have as much area. The major point is, for any given number of sides, the regular polygon contains the greatest area for a specified perimeter.

The next question is which regular polygon contains the greatest area for a given perimeter? The following figures help to answer this question.

Show FIGURE 1. This equilateral triangle has a perimeter of 12 units and an area of 6.9 square units.

Show FIGURE 2. This square has a perimeter of 12 units and an area of 9 square units.

Show FIGURE 3. This regular hexagon has a perimeter of 12 units and an area of 10.4 square units. Do not concentrate on how to find the area of these figures since students should already know this.

However, if explanation is needed for the regular hexagon, it may be simpler to find the area by first dividing the hexagon into triangles.

Show FIGURE 4. This circle (a polygon with n sides) has a perimeter of 12 units and an area of 11.5 square units.

Of these four figures with perimeter 12, which contains the greatest area? One would think that the bee would be well advised to build the honeycomb with circular cells, thus storing a maximum amount of honey while expending the least amount of wax to make the walls. However, a second principle has to be considered. Bees must construct many cells close together to store honey.

Show FIGURE 5. Packing the circles close together as in FIGURE 5A or FIGURE 5B will waste too much wax between the cells.

Now the question is what regular polygon can be closely packed together without wasting space between? With a little drawing and speculation, one may soon discover that there are only three regular polygons that can be packed together without wasted space between the figures.

Show FIGURE 6. Explain that equilateral triangles can be packed together with no wasted space.

Show FIGURE 7. Squares can also be packed together with no wasted space between figures.

Show FIGURE 8. This demonstrates that regular hexagons can be packed closely together.

Considering the first principle, the regular hexagon structure is the best choice for the honeycomb since it contains more area than the triangle or square for a given perimeter and it can be closely packed together. Hence, the bee has "selected" the most efficient figure in terms of maximizing honey storage and minimizing wax production.

As additional information, tell students that another property that adds strength to the honeycomb is that, at each vertex, there are only three walls meeting. The hexagon is the only figure for which this is true (see FIGURES 6, 7, and 8). This 3-way joint not only adds strength but is very common in other natural phenomena as well. On the other end of each honeycomb cell, the six sides meet in a configuration of three rhombuses. The cells interlock perfectly with the other side of the honeycomb adding further to its structural soundness.

IV. Materials Needed.

An overhead projector and transparencies of FIGURES 1 through 8 are needed.

V. Optional Student Activities.

Show that a regular pentagon with perimeter 12 contains less area than a regular hexagon of the same perimeter (this may be very difficult for eighth-graders but it will show that another triangle area formula may be needed).

Honeycomb cells end with 3 rhombuses that fit together perfectly with the other side of the honeycomb. Build several cells from wire and paper to show how they interlock to add to the structural stability.

VI. Sources.

Bleicher, M. N., & Toth, L. F. Two dimensional honeycombs. American Mathematical Monthly, 1965, 72, 969-973.

Moulton, J. P. Experiments leading to figures of maximum area. Mathematics Teacher, 1975, 68, 356-363.

Pettigrew, J. B. Design in nature. London: Longman, Green, and Co., 1908.

Siemens, D. F. The mathematics of the honeycomb. Mathematics Teacher, 1965, 58, 334-337.

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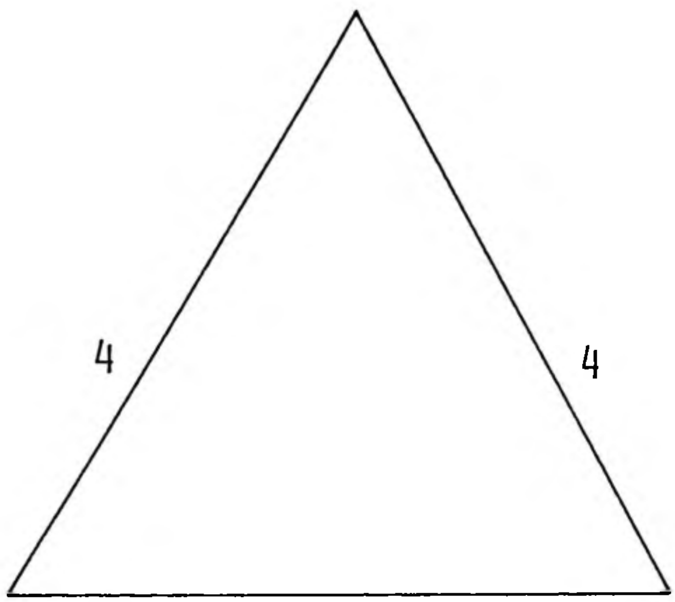
Toth, L. F. What the bees know and what they do not know. Bulletin of the American Mathematical Society, 1964, 70, 468-481.

Vogel, L. Construction of a honeycomb. School Science and Mathematics, 1937, 37, 386-387.

HONEYCOMBS
FIGURE 1

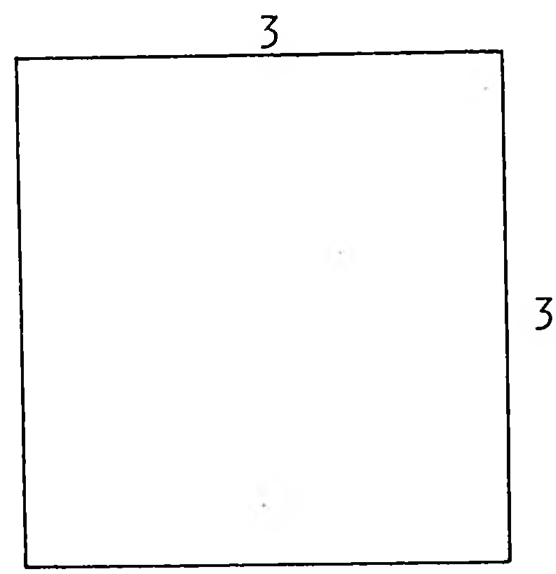


PERIMETER = 12

AREA = 6.9

HONEYCOMBS

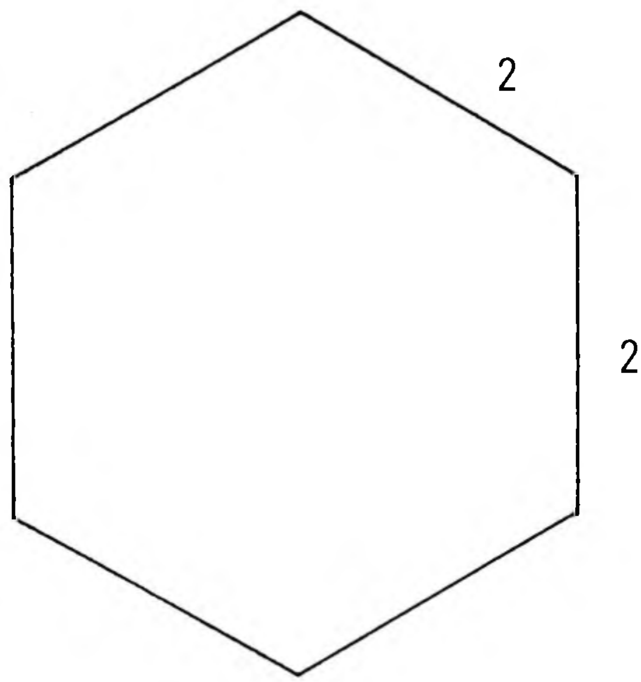
FIGURE 2



PERIMETER = 12

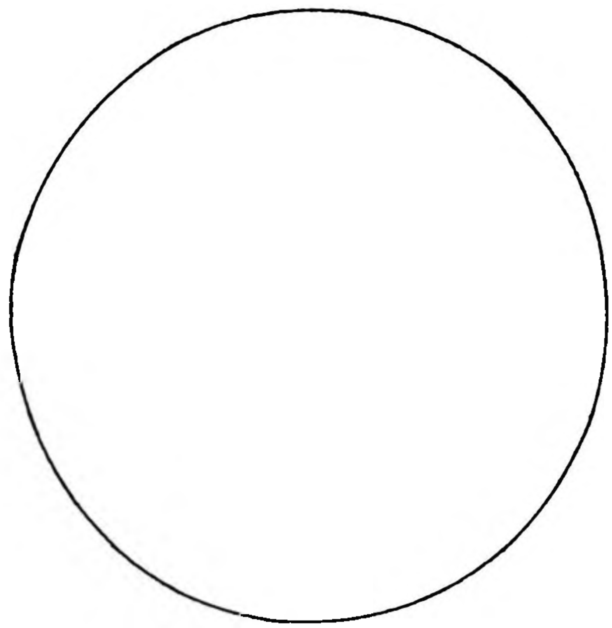
AREA = 9

HONEYCOMBS
FIGURE 3



PERIMETER = 12
AREA = 10.4

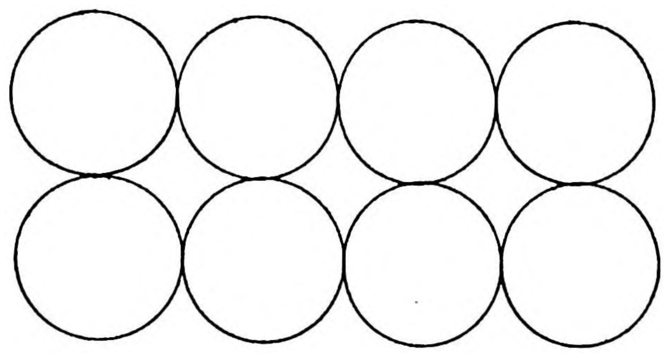
HONEYCOMBS
FIGURE 4



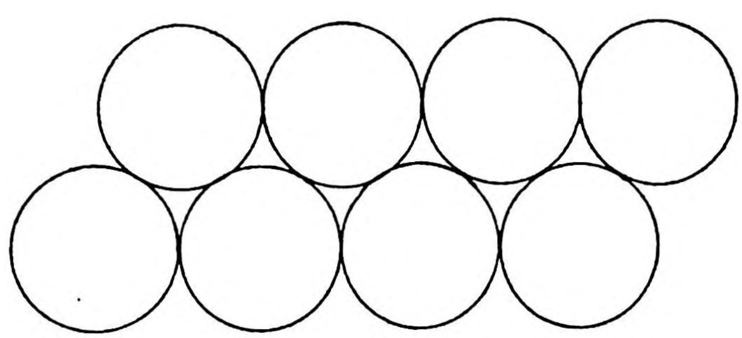
PERIMETER = 12
AREA = 11.5
RADIUS = 1.9

HONEYCOMBS

FIGURE 5



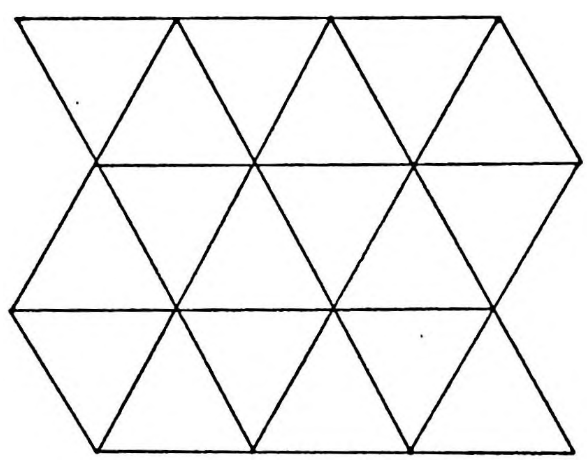
5A



5B

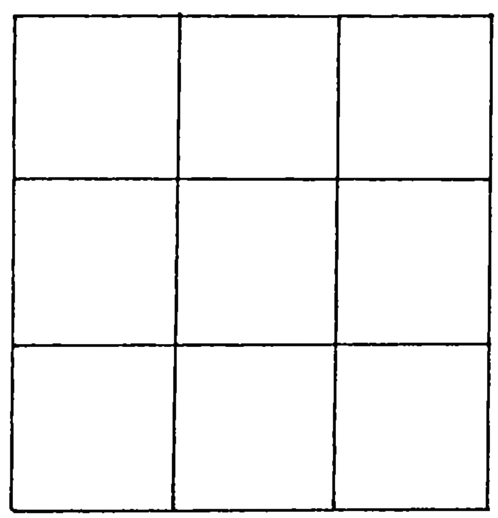
HONEYCOMBS

FIGURE 6



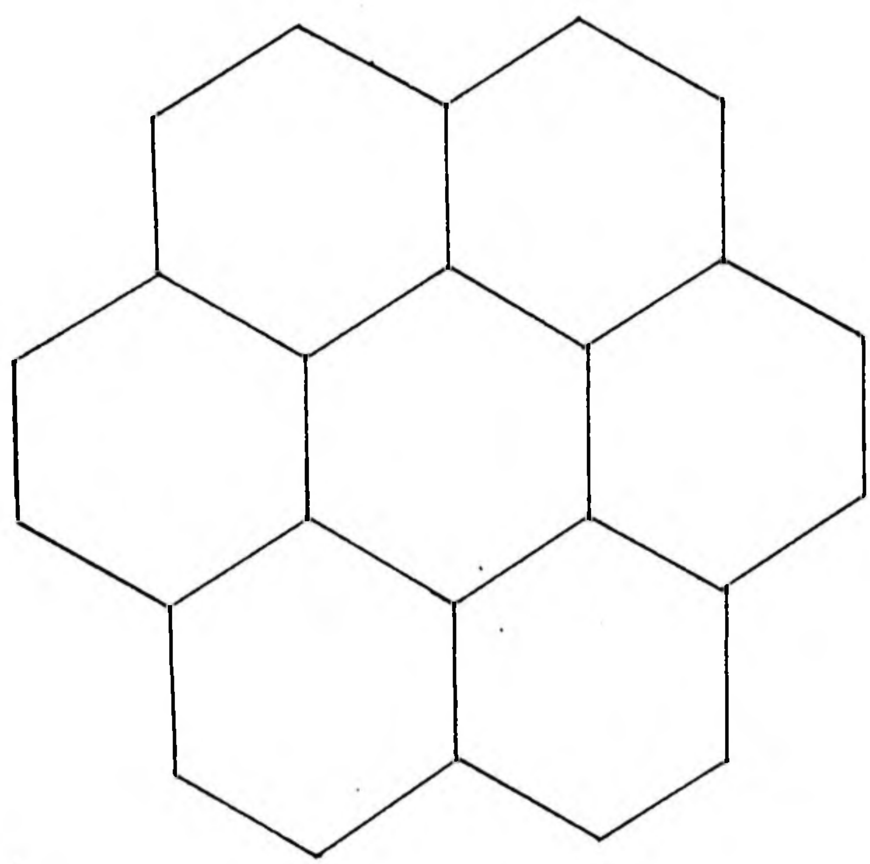
HONEYCOMBS

FIGURE 7



HONEYCOMBS

FIGURE 8



SNOWFLAKES

I. Objective.

The teacher will demonstrate to the students the hexagonal patterns and symmetry of snowflakes.

The shapes and patterns that exist in nature today tend to be the most efficient possible under the given circumstances. For example, the honeycomb is the most efficient form in which to store honey, given restrictions on stability and volume (see HONEYCOMBS for further explanation). The best possible configuration usually appears in nature only after many years of evolution. Thus, one might expect the more efficient patterns to occur repeatedly in natural phenomena. Indeed, such is the case. Spiral patterns, branching patterns, 120-degree joints, and pentagonal shapes occur quite frequently in nature.

Another figure commonly found in nature is the regular hexagon. The 120-degree joint of the regular hexagon tessellation is demonstrated in SOAP FILM I. The hexagon is also shown in the honeycomb unit. This unit on snowflakes is another example of the hexagon shape in nature.

Snowflakes develop in various forms: flat, as columns or long needles, or as compound structures. However, the flat snowflake or a cross section of a columnar flake is almost always hexagonal in shape. The main consideration in determining the form of the snowflake is air temperature. However, the hexagonal shape arises from the unique triangular arrangement of water molecules within the snowflake. Complex scientific principles governing the growth of crystals are involved but are not to be presented here.

The main points to be presented to the students are the characteristic hexagonal shape and the symmetry of the snowflake. In addition, the fact that no two flakes have been found to be exactly alike can be mentioned.

The suggested placement for this unit is after the study of polygons or as an introduction to symmetry. It may be beneficial to present this lesson in conjunction with the soap film and honeycomb units so that comparisons can be made.

III. Classroom Procedure.

Begin the presentation to students with a discussion of the most prevalent patterns in nature. Ask students if they have noticed any particular shape that occurs frequently in nature.

Show FIGURE 1A. Tell students that one of the most common figures in nature is the regular hexagon. The snowflake shown in 1A has the characteristic hexagonal shape. Turn the figure through 60 degrees to show that the original shape is maintained. Ask the students what mathematical property is demonstrated by the result of this rotation.

Show FIGURE 1B. Point out the hexagonal shape to students. Ask students what shape occurs if the figure is rotated 60 degrees.

Show FIGURE 1C. Stress the hexagonal shape. Mention the various forms of snowflakes (flat, columnar, or compound structures). Also, state that a very large number of different snowflakes designs is possible. While almost every snowflake has this hexagonal shape, no two snowflakes have ever been found to have exactly the same design.

Show FIGURE 1D. Emphasize the characteristic hexagonal shape and the symmetry. Ask students if there are any lines of symmetry or points of symmetry.

Hence, it is obvious from these examples that the regular hexagon is one of the most important polygons in nature.

IV. Materials Needed.

An overhead projector, paper, and a transparency of FIGURE 1 are needed.

F. Optional Student Activities.

Find out what line symmetry means if you do not already know. Draw all lines of symmetry possible for a regular hexagon. Determine the points of symmetry for the regular hexagon.

VI. Sources.

Bentley, W. A., & Humphreys, W. J. Snow crystals. New York: Dover Publications, 1962.

Knight, C., & Knight, N. Snow crystals. Scientific American, 1973, 228(1), 100-107.

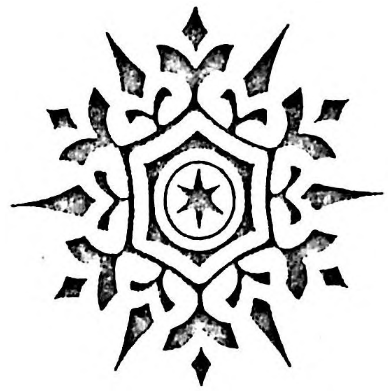
Mason, B. J. The growth of snow crystals. Scientific American, 1961, 204(1), 120-134.

Quander, D. D. Snowflakes--an introduction to symmetry. Science and Children, 1970, 8(4), 22-24.

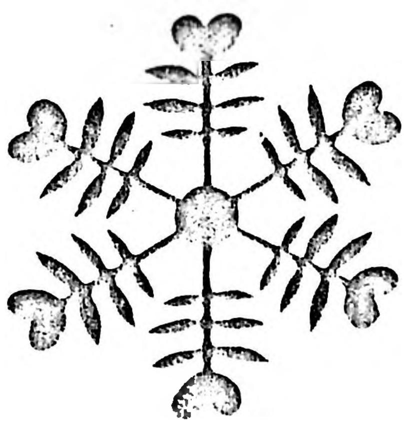
SNOWFLAKES

FIGURE 1

1A



1B



1C

1D

NETWORKS

I. Objective.

The teacher will show students how systems can be analyzed to predict the branching patterns in nature.

II. Teacher Information.

We observed one of nature's basic laws in the soap film experiments, namely that natural systems tend toward a least area configuration. In general, objects in nature tend toward a pattern with the least energy (e.g., least area, lowest altitude, tightest fit, least motion). The object of this unit is to try to predict mathematically, in a slightly abstract form, what patterns are most probable in nature. The basic problem is analogous to determining the best or most efficient way of connecting points. Since this unit is rather abstract, it should be presented <u>after</u> the other four units on nature.

This lesson is extremely simplified for incorporation into an eighth-grade classroom. An excellent and thorough treatment of this subject is given in Stevens' book, <u>Patterns in Nature</u>.

III. Classroom Procedure.

Begin by reminding students that soap film systems tend toward the figure with the least area or least length. Mathematically, is there a way to predict what configurations in nature may be preferred?

Consider the problem of trees. What is the most efficient way to get water from the trunk to the outermost leaves? That is, what pattern requires the least amount of wood and connects the wood to the leaves in the most direct way?

Show FIGURE 1A. Suppose the middle point is the trunk of the tree and the six outermost points are leaves. What is the best way to get water to all the leaves? How can the points be connected with the least distance and in the most direct way?

Place FIGURE 1B over 1A. This shows one way to connect the points. How much length (or how much wood) is required? Measure each line segment to show the length of each is 5 cm. Therefore, the total length of the system is 30 cm. This pattern of six branches connecting six leaves is the most <u>direct</u> way to get water to the leaves or to connect the system. However, it would not be appropriate for tree growth since a tree cannot sustain <u>each</u> of its leaves with a separate branch.

Are there any other ways to connect the points? Allow students to offer suggestions.

Remove FIGURE 1B and place FIGURE 1C over 1A. This shows another way to connect the points which is not as direct but requires the same length of 30 cm. In this figure, one branch sustains all six leaves.

Remove FIGURE 1C and place FIGURE 1D over 1A. This shows a more efficient way to connect the points. This branching pattern is composed of 120-degree angles. Each segment measures 3 cm, for a total distance of 27 cm. Therefore, this pattern requires less wood than the other two patterns and is reasonably direct. Three main branches sustain the six leaves. Even by increasing the number of points, the 3-way joints and 120-degree angles are the best choice in terms of connecting points directly and overall length.

One might think then that nature would be well advised to adopt this 3-way joint branching pattern. In fact, this pattern _is_ very common in nature, as seen in the soap film experiments and the honeycombs. Actual tree growth is very similar to this though the angles made in branching vary from 75 to 90 degrees (instead of 120) due to frictional resistance. However the 3-way joints in tree growth are quite prevalent. This 3-way branching also occurs in distribution of blood through the blood vessels, in lightning streaks, and in rivers.

Therefore, through the use of mathematics, figures can be analyzed to predict which patterns actually do prevail in nature.

IV. Materials Needed.

An overhead projector, a transparent ruler, and transparencies of FIGURES 1A through 1D are needed.

V. Optional Student Activities.

Make a square arrangement with four dots. Sketch the diagonals to locate the center point. Draw a figure so that 3-way joints connect the five points at 120-degree angles. Measure the total distance of the lines drawn. Is it shorter than the length of the two diagonals added together?

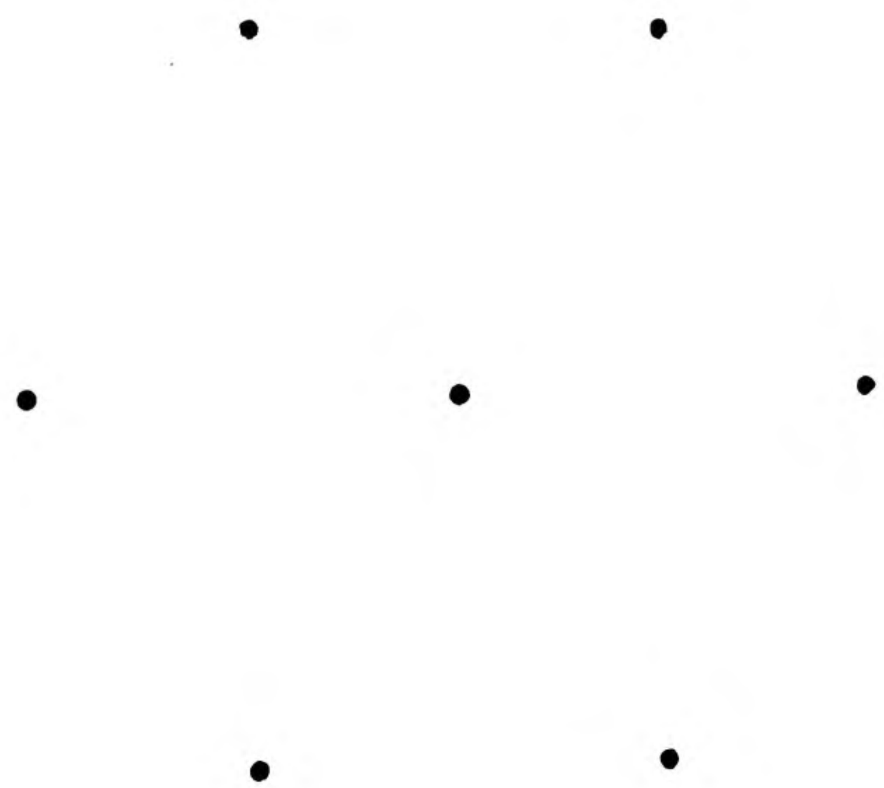
VI. Sources.

Smith, C. S. Structure, substructure, superstructure. In G. Kepes (Ed.), Structure in art and in science. New York: George Braziller, 1965.

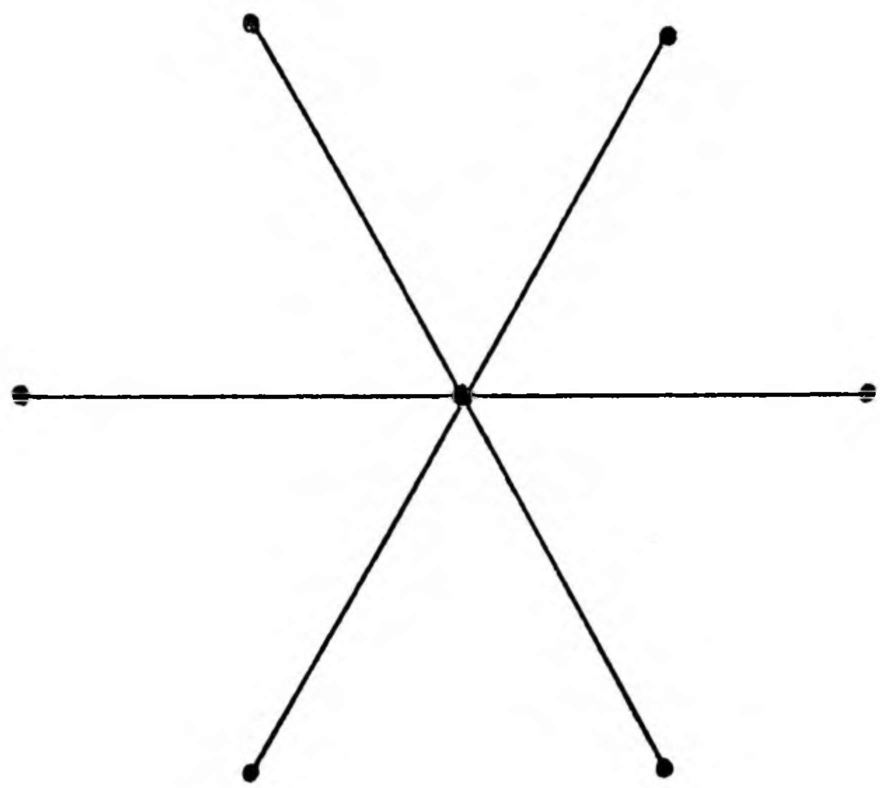
Stevens, P. S. Patterns in nature. Boston: Little, Brown, and Co., 1974.

NETWORKS

FIGURE 1A

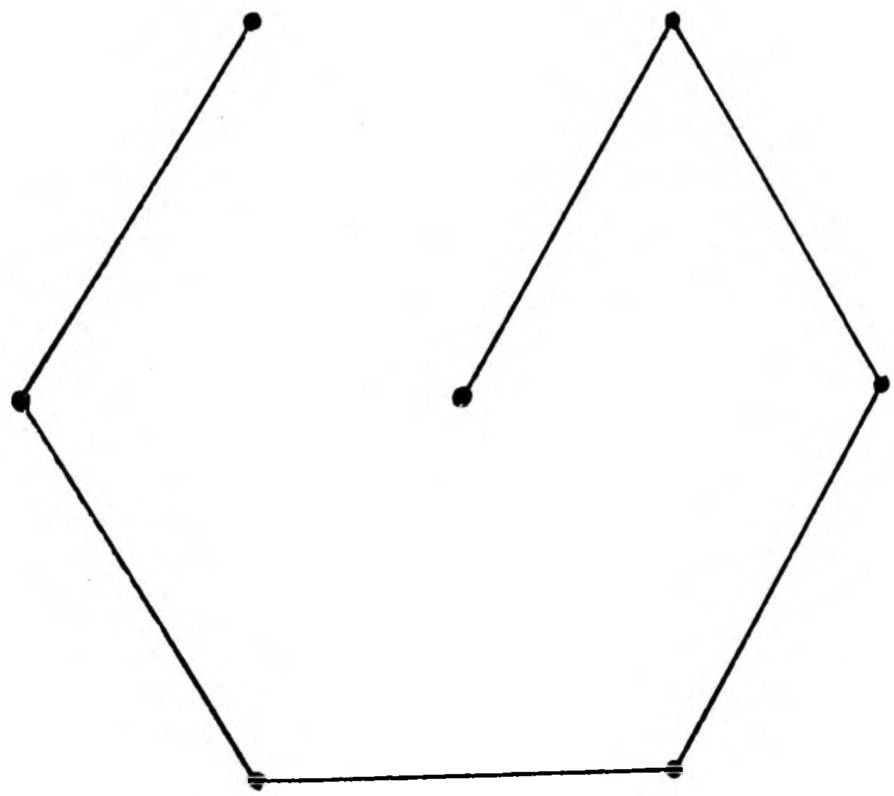


NETWORKS
FIGURE 1B



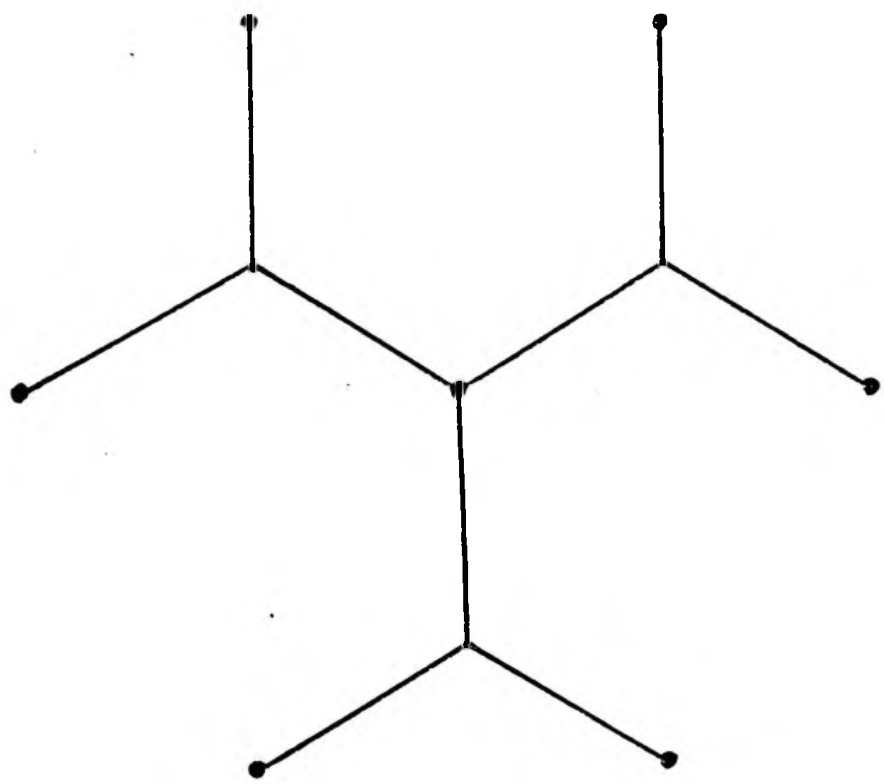
NETWORKS

FIGURE 1C



NETWORKS

FIGURE 1D



GEOMETRICAL ILLUSIONS

I. Objective.

The teacher will show students how certain geometric configurations alter perception and illustrate the necessity of accurate measurement.

II. Teacher Information.

People seem to have enough confidence in their optical judgments that the old adage "seeing is believing" is usually accepted. The major point of this unit is to refute this adage by showing we cannot rely solely on our judgments but must make accurate measurements and comparisons. The optical illusions included offer vivid proof that our visual estimates of area, angle, length, and curvature are often false.

One important property of our visual sense is that we assess the size and dimensions of an object in relation to its surroundings. Hopefully this introduction to simple geometric illusions will encourage students to think anew of the relationship between their environment and geometry.

III. Classroom Procedure.

Before beginning the unit, make sure there is no keystoning (distortion) with the overhead projector. Distortion will certainly destroy some of the illusions. Keystoning is prevented by projecting the image perpendicular to the screen.

Begin by stating that geometric configurations in certain settings can alter our perception of objects. The following examples illustrate the necessity of accurate measurement.

Show FIGURE 1. Which appears longer, the height or the width of the hat (AB or CD)? After allowing students to speculate, measure the distances AB and CD with a transparent ruler. While the height does indeed appear to be longer than the width, actually the height measures 7.5 cm and the width measures 8.8 cm.

Show FIGURE 2. Is the monument itself longer than the base on which it stands? While the monument appears longer than the base, careful measurement shows that in fact both are equal.

FIGURES 1 and 2 show that vertical distances usually appear to be greater than equal horizontal distances. Tell the students the following practical implications of this phenomena. An architect must be aware of this illusion so that if he desires the height of a monument to <u>appear</u> to be equal to the base, in actuality he will have to build the base longer than the height. Also, when purchasing canned products, a tall, skinny can may not necessarily contain more than a short, bulky can.

Show FIGURE 3. Are the four horizontal lines parallel? By using rulers, you can verify that the lines are parallel even though they do not appear to be.

Show FIGURE 4. Do the two center lines bulge in the middle? Though the lines seem to curve, they are parallel. In both of these figures, an illusion is created by the setting in which the lines appear.

Show FIGURE 5. Which of the three lines F, E, or C is a continuation of D? It appears as though F would be the correct choice. However, careful placement of a straight-edge reveals that E is the correct choice.

Show FIGURE 6. Which drawing is larger? Although equal in area, the bottom picture appears larger since it is positioned close to the narrow part of the top figure.

Show FIGURE 7. Allow students time to interpret what they see. Is it a cube in the corner of a room? Is it a small cube attached to the front corner of a large cube? Is it a large cube with a chunk removed from the front part? Focus of attention determines perception in this case.

Show FIGURE 8. This is a drawing of a bridge or sidewalk fading into the distance (perspective drawing). Which of the three towers is the tallest? Only through precise measurement can you see that the towers are equal, though the third tower appears to be the largest. Since artists use perspective in paintings, they must be aware of this size distortion.

Thus, it can be seen that the setting, positioning, and focus of attention alter our perception of objects. Only through accurate measurement and comparison can false judgment be avoided. From these illusions, one can see the important role that geometry plays in the perception of our surroundings.

IV. Materials Needed.

An overhead projector, a transparent ruler, paper, and transparencies of FIGURES 1 through 8 are needed.

V. Optional Student Activities.

Draw a vertical line, call it A. Perpendicular to the base of the line, draw a horizontal line (B) bisected by A so that the horizontal line appears to be equal to the vertical line. Now measure the lines accurately to check your estimates.

How do you judge the size of three dimensional objects on the TV screen? How can a particular setting in a TV show or commercial affect your interpretation of the size of objects?

VI. Sources.

Brandes, L. G. Optical illusions: A presentation for high school mathematics students. School Science and Mathematics, 1954, 54, 557-566.

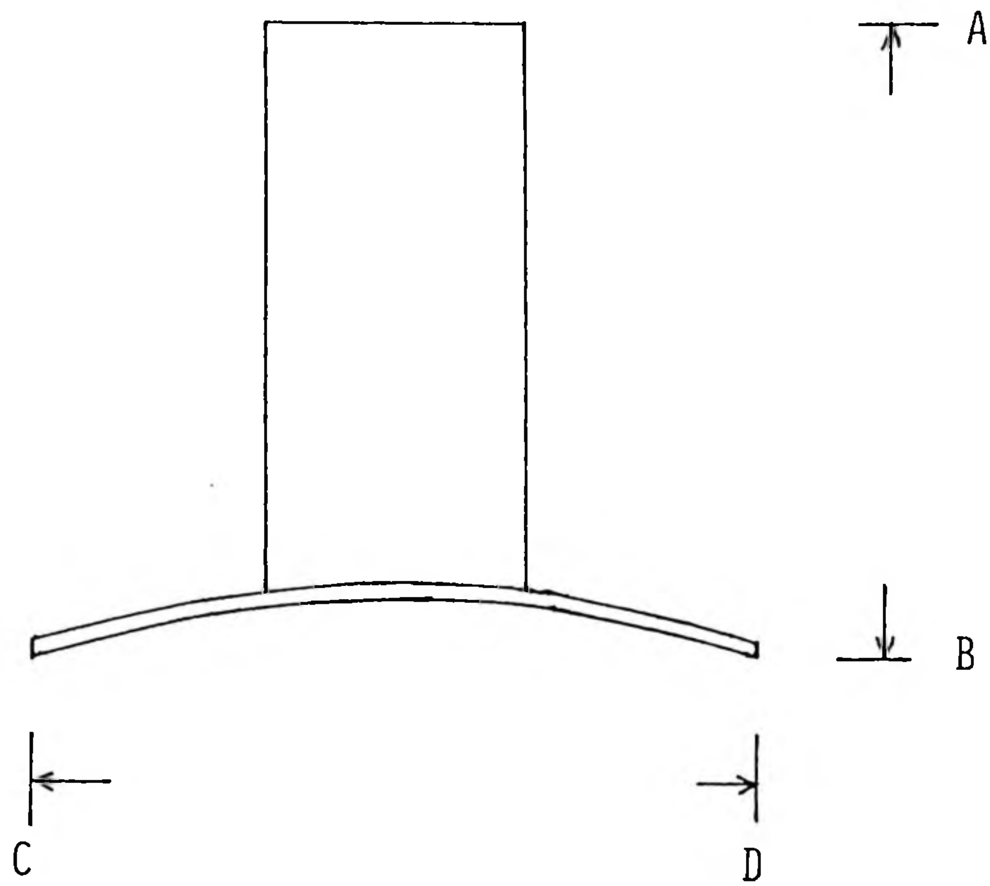
Brandes, L. G. An introduction to optical illusions. Portland, Maine: J. Weston Walch, 1956.

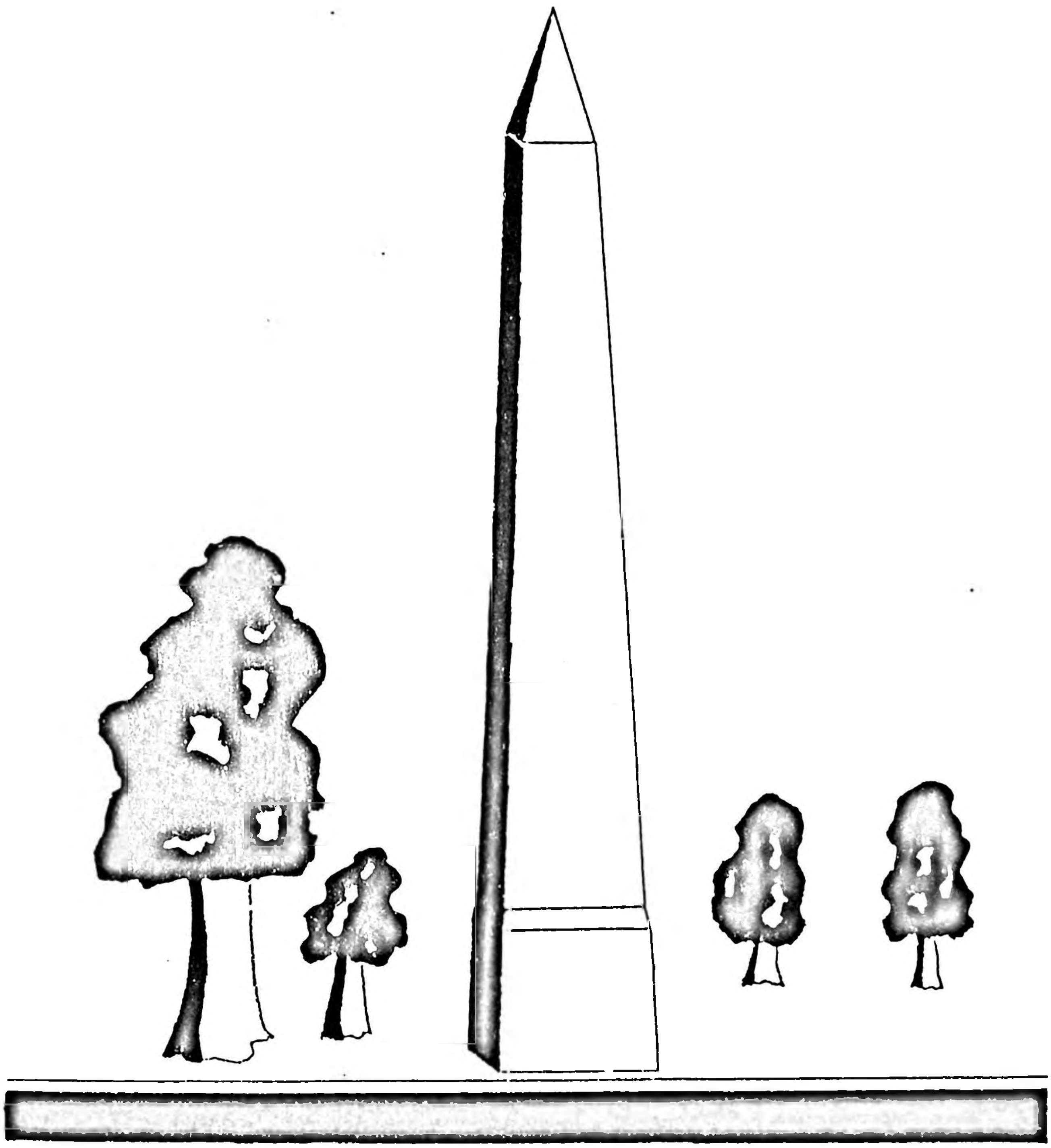
Luckiesh, M. _Visual illusions: Their causes, characteristics, and applications_. New York: D. Van Nostrand Co., 1922.

Tolansky, S. _Optical illusions_. New York: MacMillan, 1964.

GEOMETRICAL ILLUSIONS

FIGURE 1

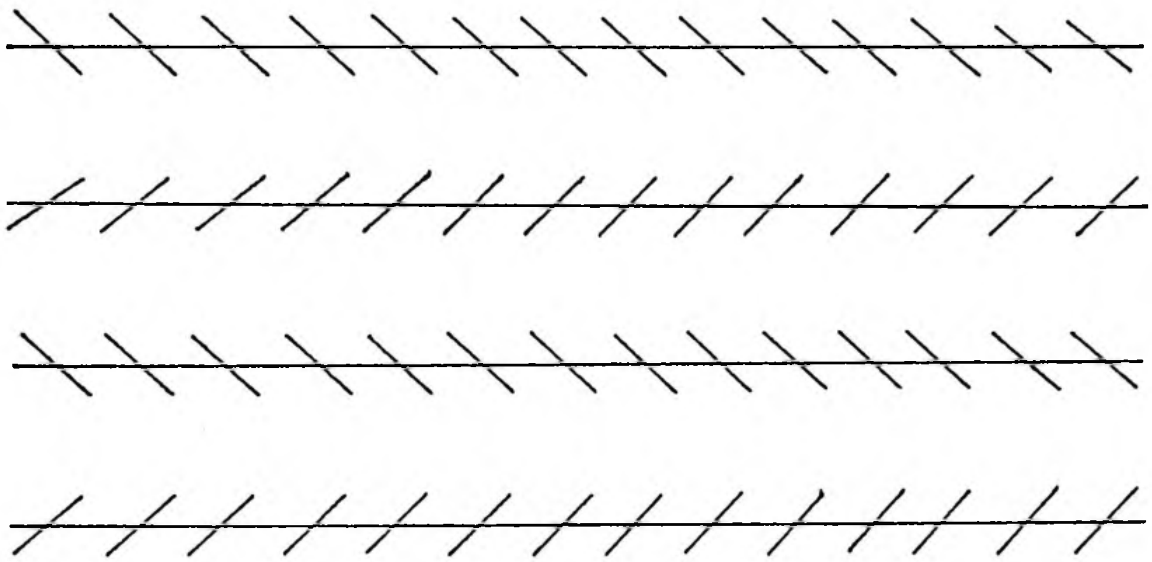




GEOMETRICAL ILLUSIONS
FIGURE 2

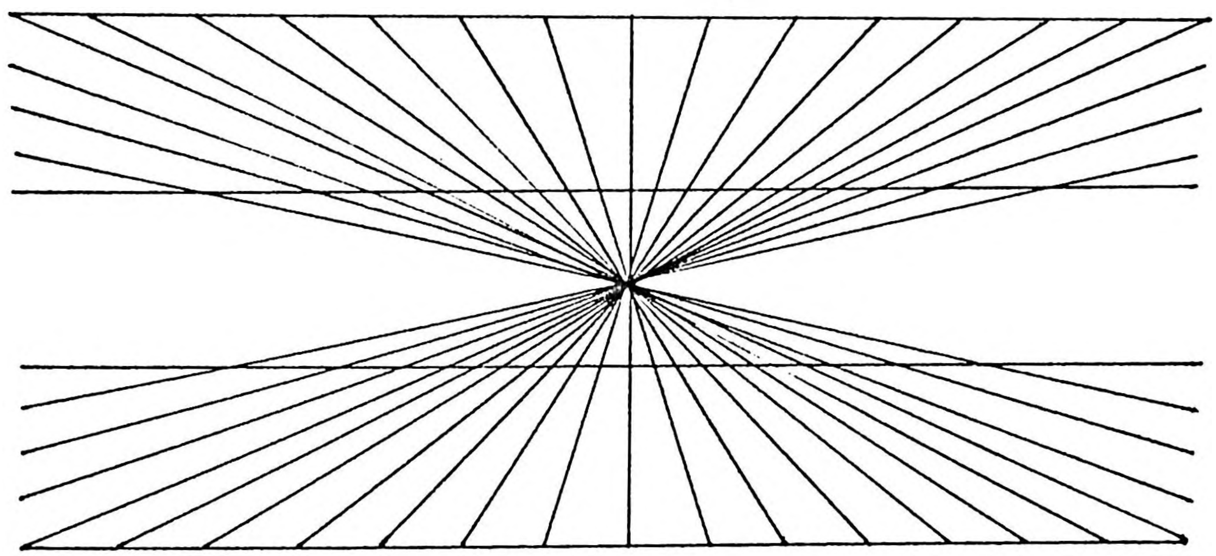
GEOMETRICAL ILLUSIONS

FIGURE 3



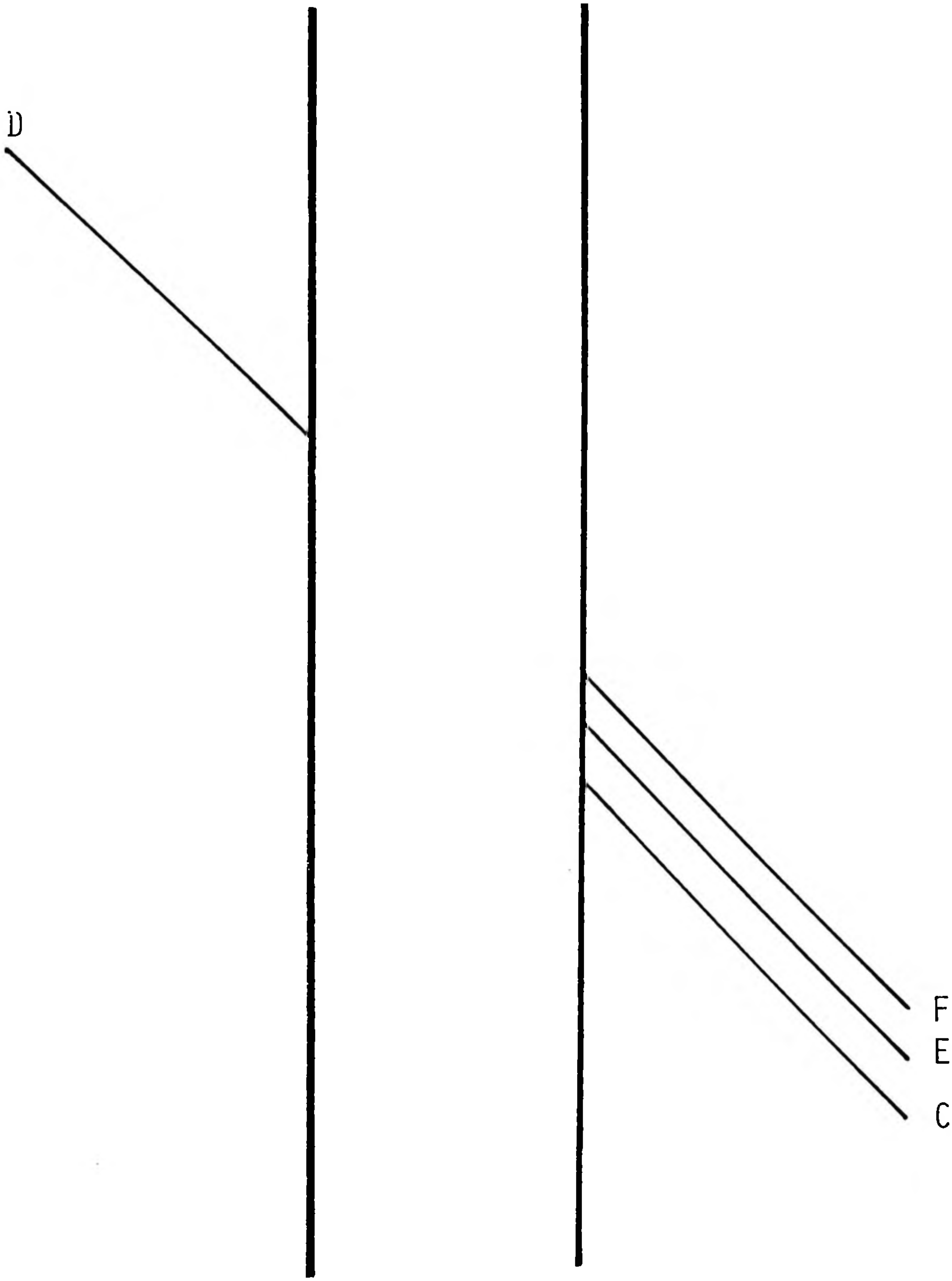
GEOMETRICAL ILLUSIONS

FIGURE 4



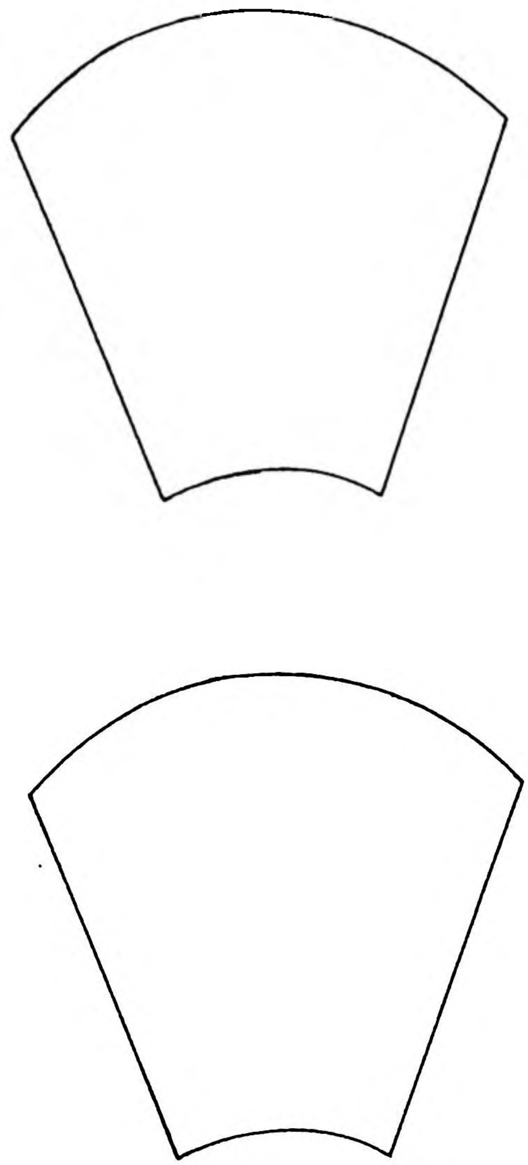
GEOMETRICAL ILLUSIONS

FIGURE 5



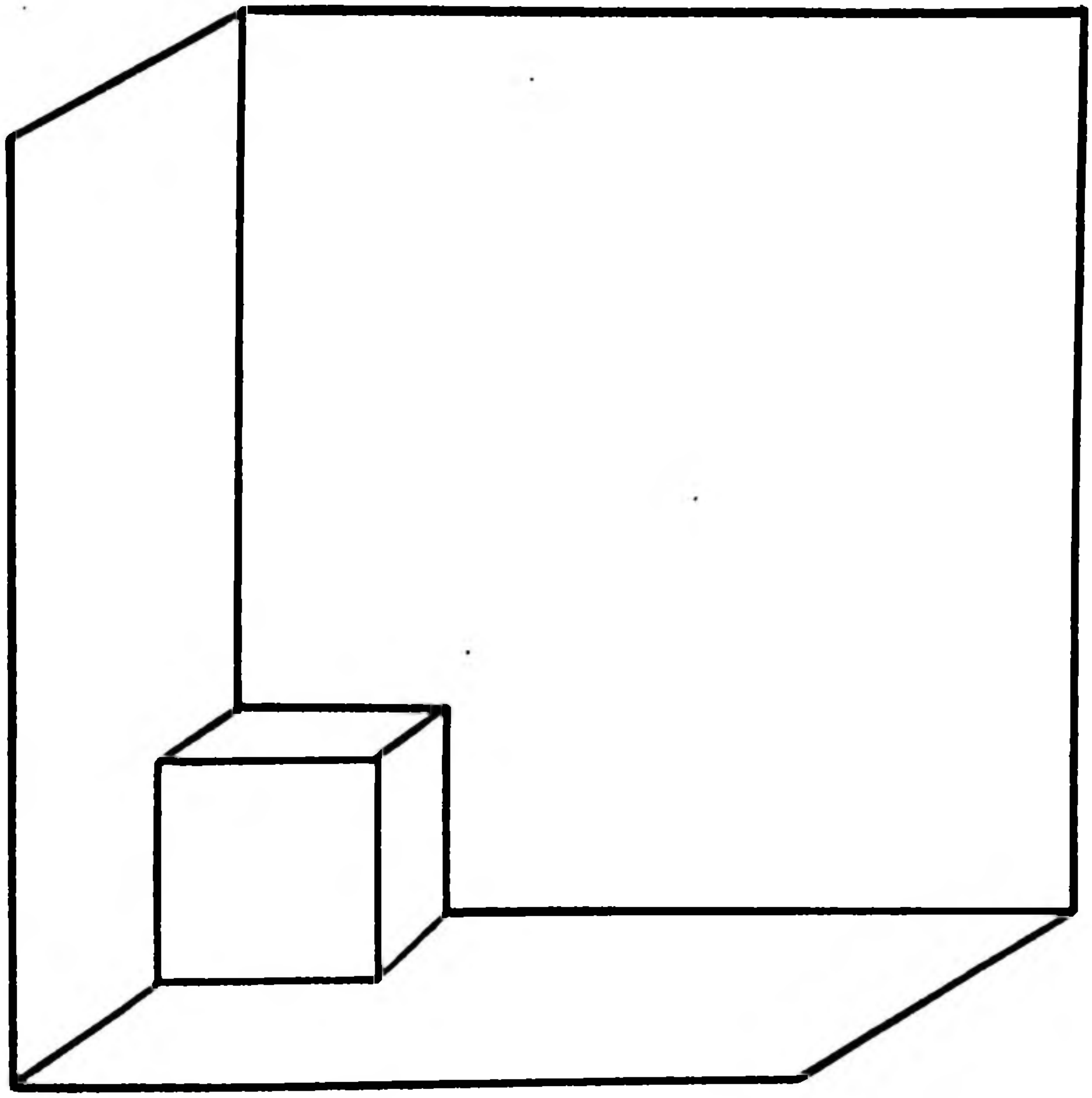
GEOMETRICAL ILLUSIONS

FIGURE 6



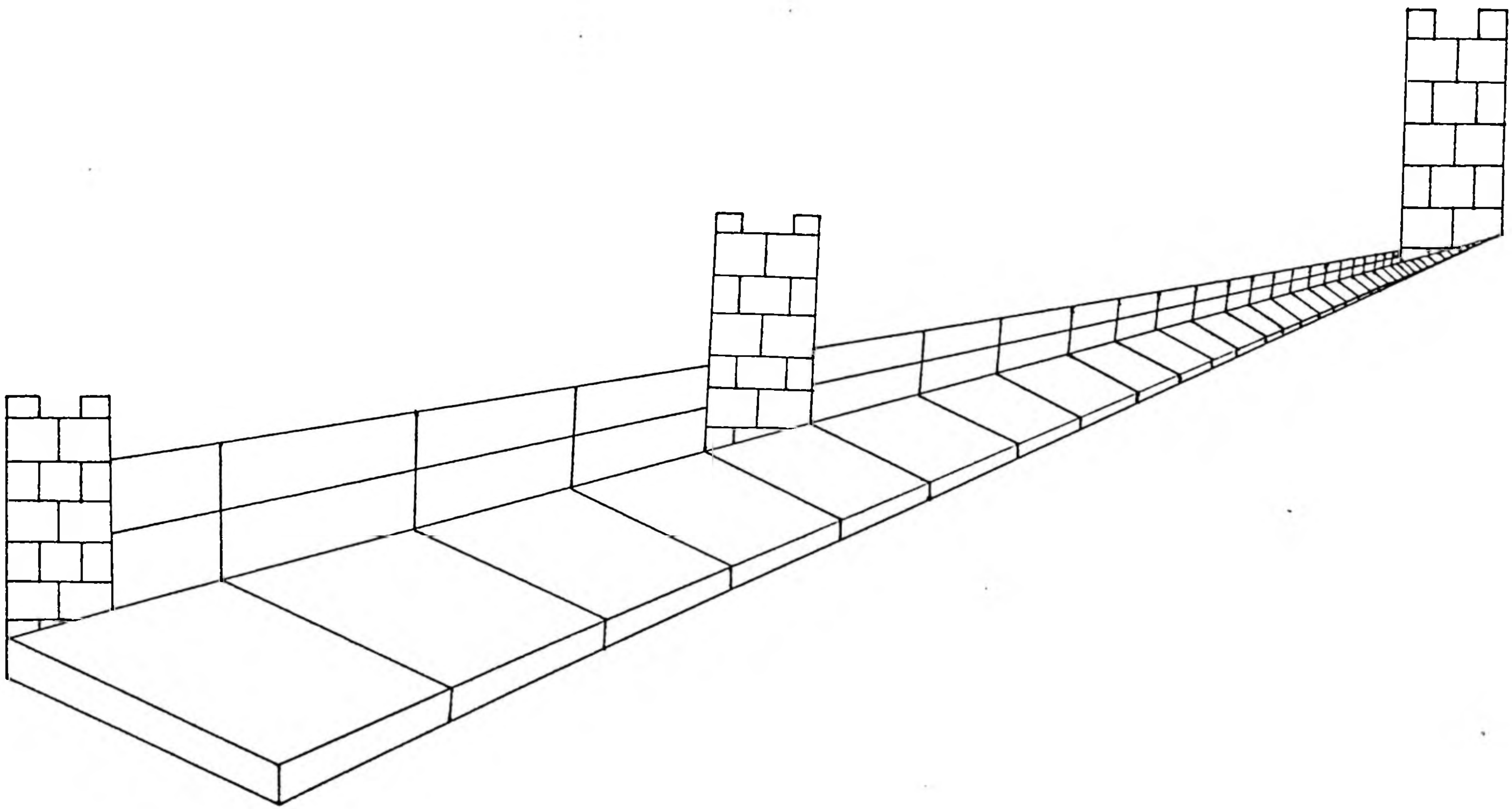
GEOMETRICAL ILLUSIONS

FIGURE 7



GEOMETRICAL ILLUSIONS

FIGURE 8



SHAPES AND FEELINGS

I. Objective.

The teacher will demonstrate to the students the geometric shapes that may elicit certain emotional reactions.

II. Teacher Information.

Geometric shapes surround us in our everyday living. The most obvious example of this is in the architecture of our man-made environment. Buildings, monuments, and other physical structures usually form some basic geometric figure. In our natural environment, hexagons, pentagons, and spirals are among the most common shapes.

One purpose of this unit is to show some of the simplest geometric figures that occur in nature. However, the main objective is to show how three of the shapes in our environment (straight lines, angles, and curves) influence our emotions. For example, sharp angles are common in our natural surroundings, especially in mountainous regions. What feelings or emotions develop while looking at sharp angles? Usually, angles elicit feelings of violence, unrest, ruggedness, and perhaps even masculinity.

The second geometric shape in this unit is smooth, horizontal lines, which may be either straight or curved. What emotion is elicitied when sand dunes, grassy meadows, rolling hills, or other gradual curvatures are encountered in nature? The feeling usually elicited is one of contentment, restfulness, and ease.

Finally, what emotions are elicited by looking at a spiral design? Curves that continually change direction usually stimulate one to be active and give a feeling of motion.

Students might well be hesitant to participate when asked to display reactions to the pictures provided. However, encourage students to communicate any ideas or emotions they may have.

Since this unit includes the most elementary of geometric designs, it may serve as a good introduction to geometry.

III. Classroom Procedure.

Ask students what geometric shapes surround them in their classroom and in their home. Ask if they have associated any feeling with any particular shape that is common in their everyday life.

Show FIGURE 1A. Ask students if any particular emotion comes to mind. Try to get as many reactions as possible from various students. After ample time is allowed for responses, tell students what emotions are usually elicited with sharp angles.

Show FIGURE 1B. This picture shows one example of sharp angles as they occur in nature. Ask students if the emotions discussed with the previous picture are actually elicited here. Ask for other examples in the environment where sharp angles can be seen.

Show FIGURE 2A. Ask if any feeling comes to mind from looking at smooth curves such as those in 2A. After student suggestions are made, tell what emotions are usually elicited when smooth curves are observed.

Show FIGURE 2B. This picture of smooth, rolling hills shows one example in nature. Ask if the emotions discussed earlier are actually elicited here. Ask for further examples in our natural environment where gradually changing curves can be seen.

Show FIGURE 3. Ask students what emotional reactions they have to the spiral pattern. After suggestions from students, tell what feelings are usually elicited when spirals are observed. Ask for examples of spirals in our natural environment. Galaxies, sea shells, tornadoes, and whirlpools may serve as examples.

Summarize the lesson and discuss how other changes in the environment may elicit emotions.

IV. Materials Needed.

Pictures of mountain and farm scenery, sketches of angles and curves, and a drawing of a spiral are needed.

V. Optional Student Activities.

How might an interior decorator use angles and curves in the design of a room to create certain moods?

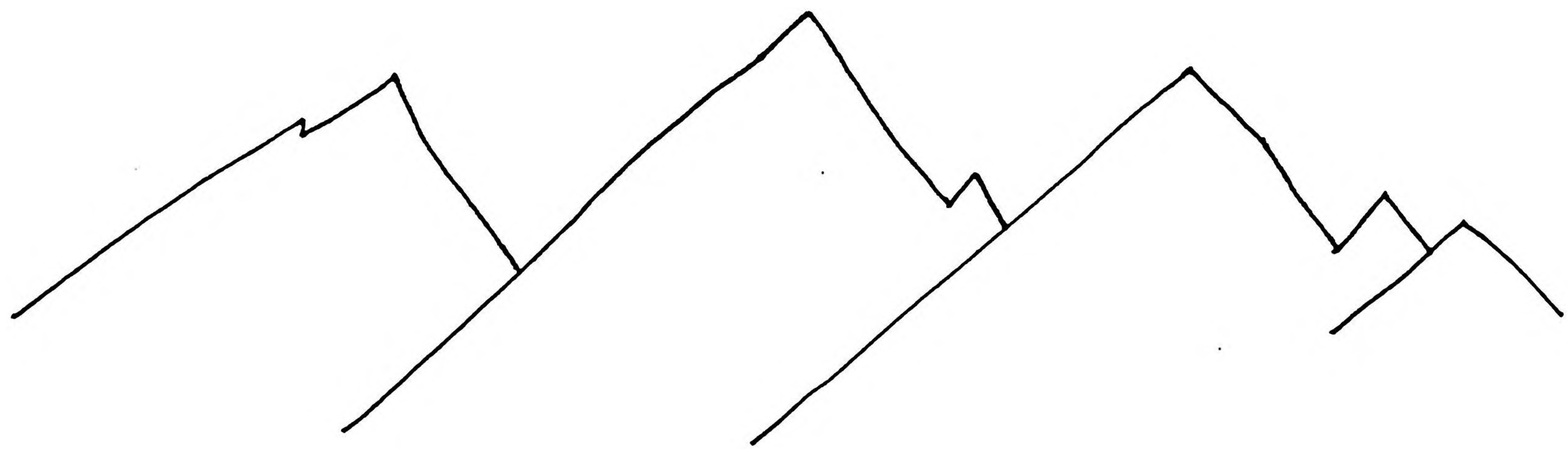
What other spiral patterns are in your home or environment?

VI. Sources.

Ravielli, A. Adventure in geometry. New York: Viking Press, 1957.

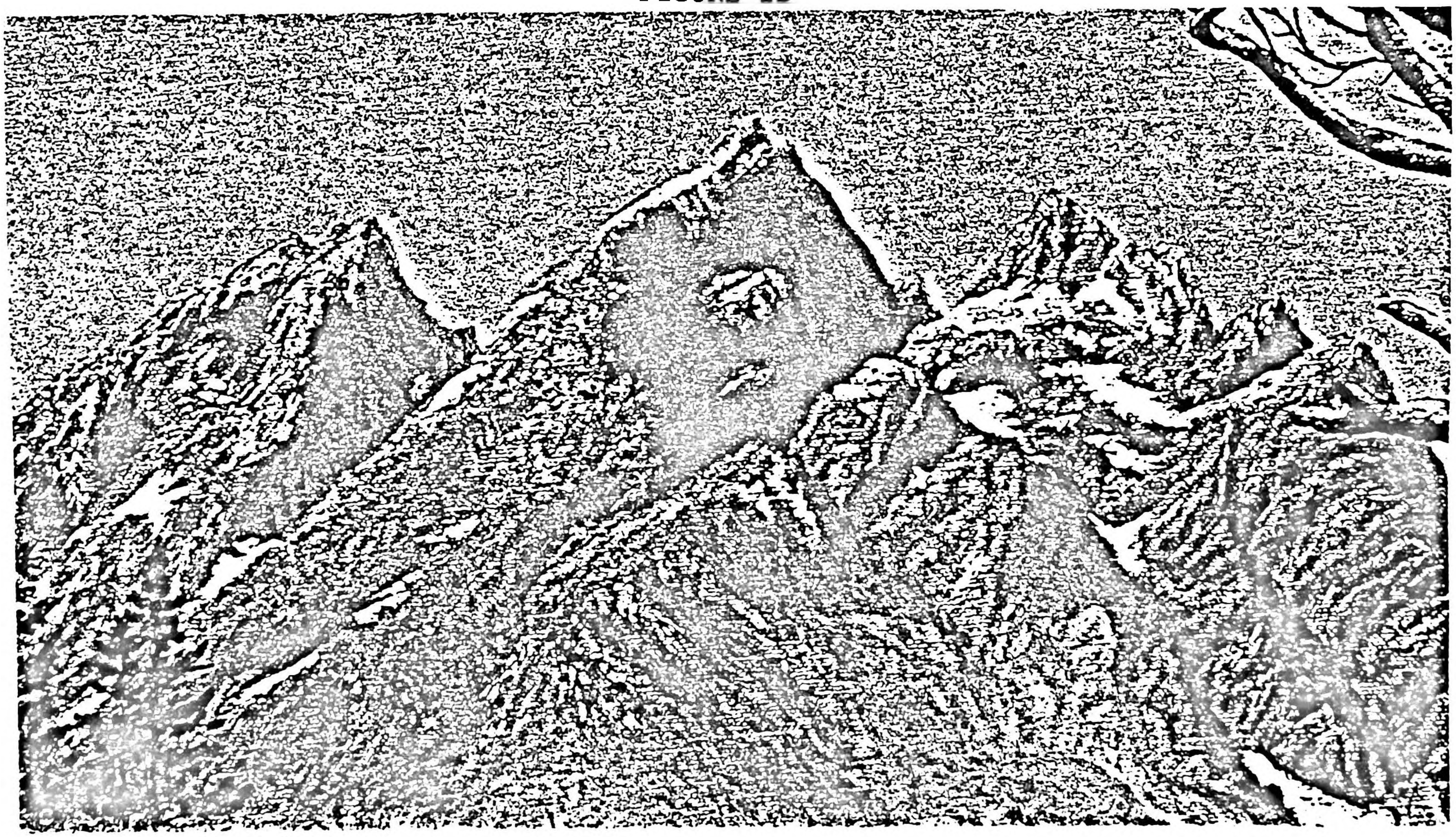
SHAPES AND FEELINGS

FIGURE 1A



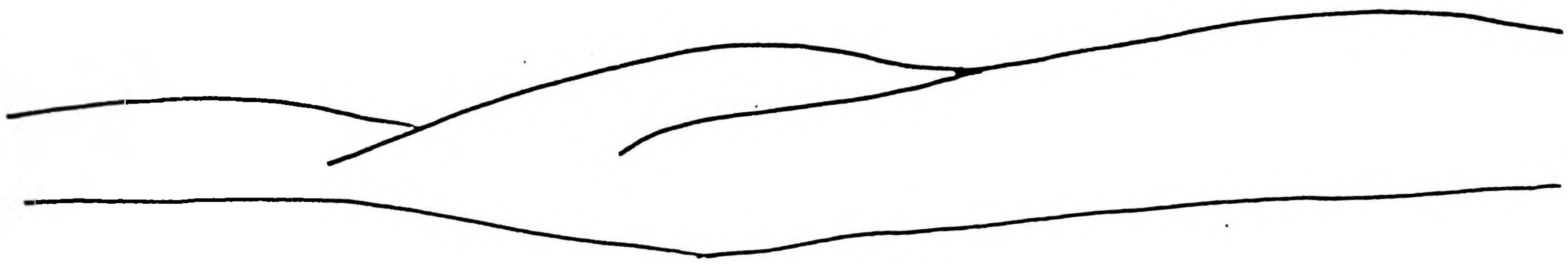
SHAPES AND FEELINGS

FIGURE 1B



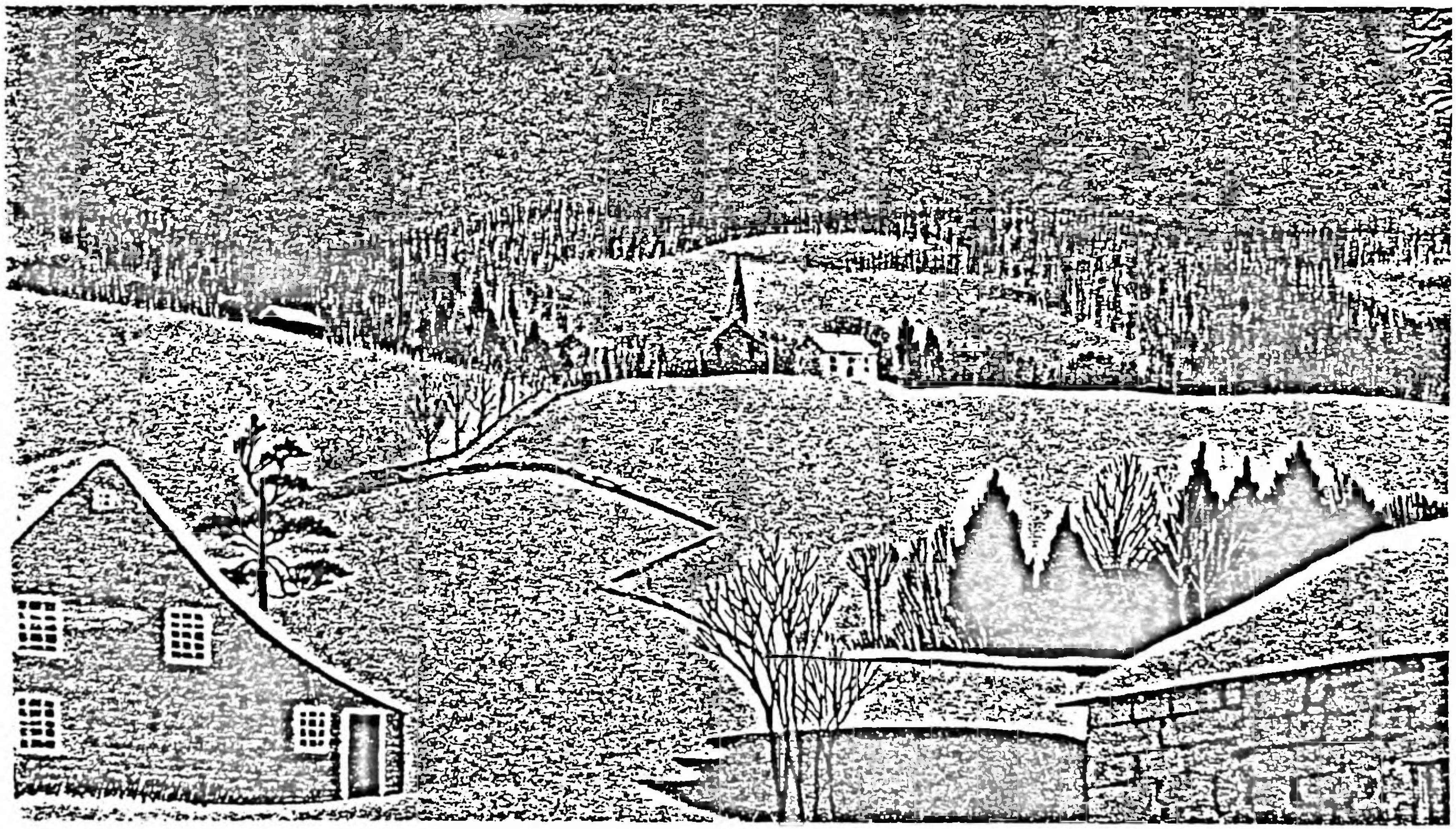
SHAPES AND FEELINGS

FIGURE 2A



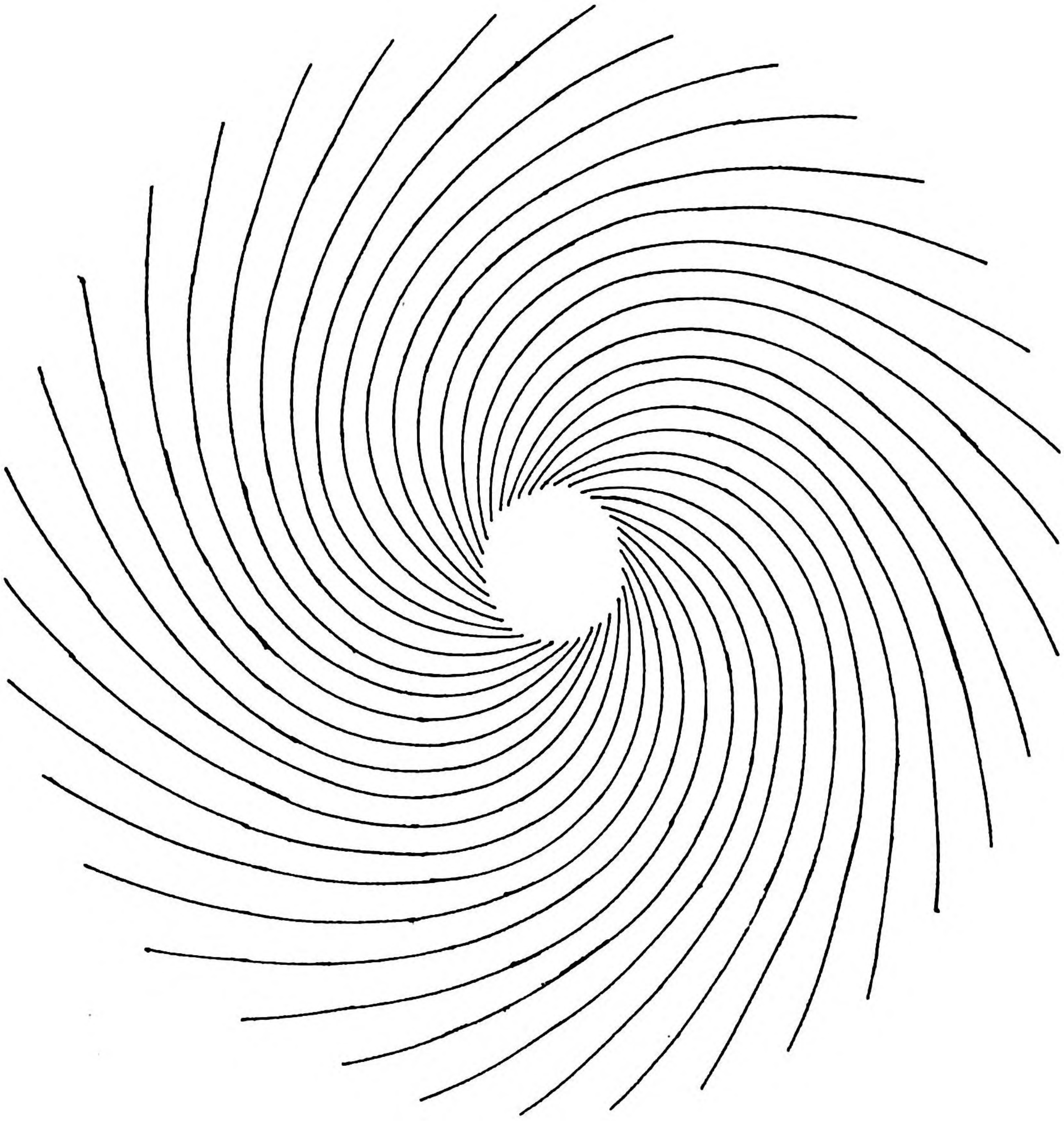
SHAPES AND FEELINGS

FIGURE 2B



SHAPES AND FEELINGS

FIGURE 3



LINE DESIGNS

I. Objective.

The teacher will demonstrate to students how aesthetically pleasing designs can be produced by using only straight lines.

II. Teacher Information.

By using a simple geometric figure, the line, and by following a minimum number of rules, artistic figures can be drawn. Even non-artists can create original designs easily by using this method. The basic idea is to create a curved figure by using straight lines only.

Since all line designs follow the basic pattern shown in FIGURE 1, make sure students understand how this figure is made. This unit may be given at any point during the instructional period.

III. Classroom Procedure.

Begin by stating that one of the most basic concepts in geometry, the line, can be used to create complex designs. State that the process does not require artistic talent; all that is needed is a pencil and ruler.

Show FIGURE 1. In this figure, only straight lines are used to create a curve. How can you develop a curved figure from straight lines? First, draw any size angle with equal sides. Divide both sides into the same number of equal parts. Number the points as shown in

FIGURE 1. Connect equal numbers with a line segment and a curve appears. The curve can be altered by the size of the angle, the number of equal parts, and the size of each division. Tell students that they must understand this procedure since all other designs follow this basic pattern.

Show FIGURE 2A. Three equal segments are divided into 14 equal parts. By following the same rule as before, make three curves. Connect 2 or 4 points with line segments as shown before to demonstrate the procedure. Place FIGURE 2B over FIGURE 2A. This shows the final figure of three curves.

Show FIGURE 3. Starting with an equilateral triangle, mark off 16 equal parts on each side. If the midpoint of each side is considered the starting point, three curves can be drawn as shown. Explain this by showing how one of the curves is constructed.

Show FIGURE 4. Begin with a square with the diagonals drawn. Each side and each diagonal is divided into 20 equal parts. Make a curve in each of the eight angles formed around the outside of the figure. For easier explanation to students, cover the figure so that only one angle is revealed. After students see how one curve is constructed, reveal the entire figure to show all eight curves completed.

Show Figure 5. How many angles are involved in this figure? How is the overlapping illusion created? Which figures were drawn first? This is an advanced example so you need not explain the tedious step-by-step process to the students. Students will be able to draw this type of figure with practice.

Thus, artistic designs can be created with simple geometric figures. The basic idea is simple but by using different geometric figures, backgrounds, and colors, striking art works can be created.

IV. Materials Needed.

An overhead projector, transparent ruler, paper, and transparencies of FIGURES 1 through 5 are needed.

V. Optional Student Activites.

Create other line designs using various polygons as the base.

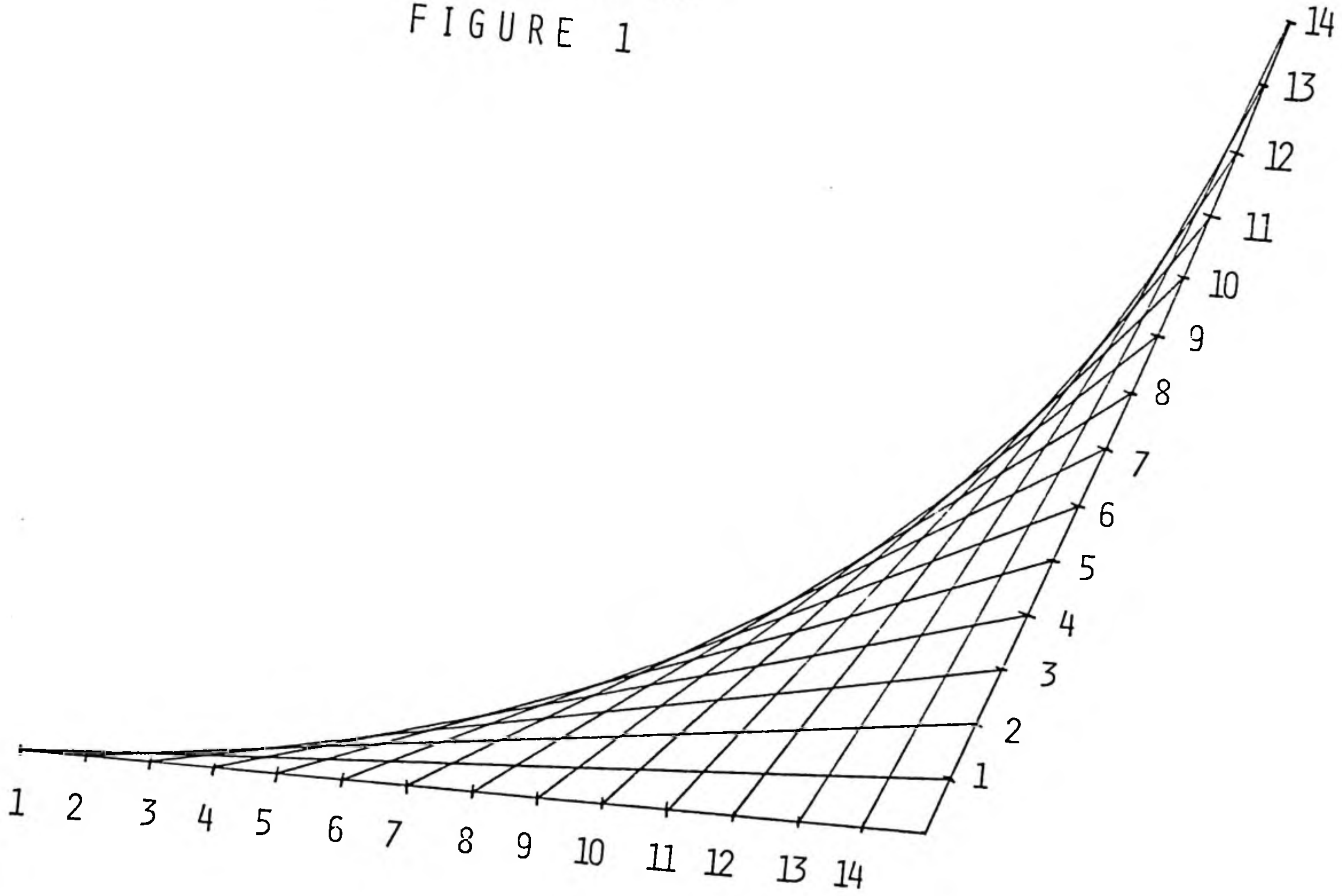
VI. Source.

Ogletree, E. Geometry: An artistic approach. Arithemtic Teacher, 1969, 16, 457-461.

Seymour, D., Silvey, L., & Snider J. Line designs. Palo Alto, California: Creative Publications, 1974.

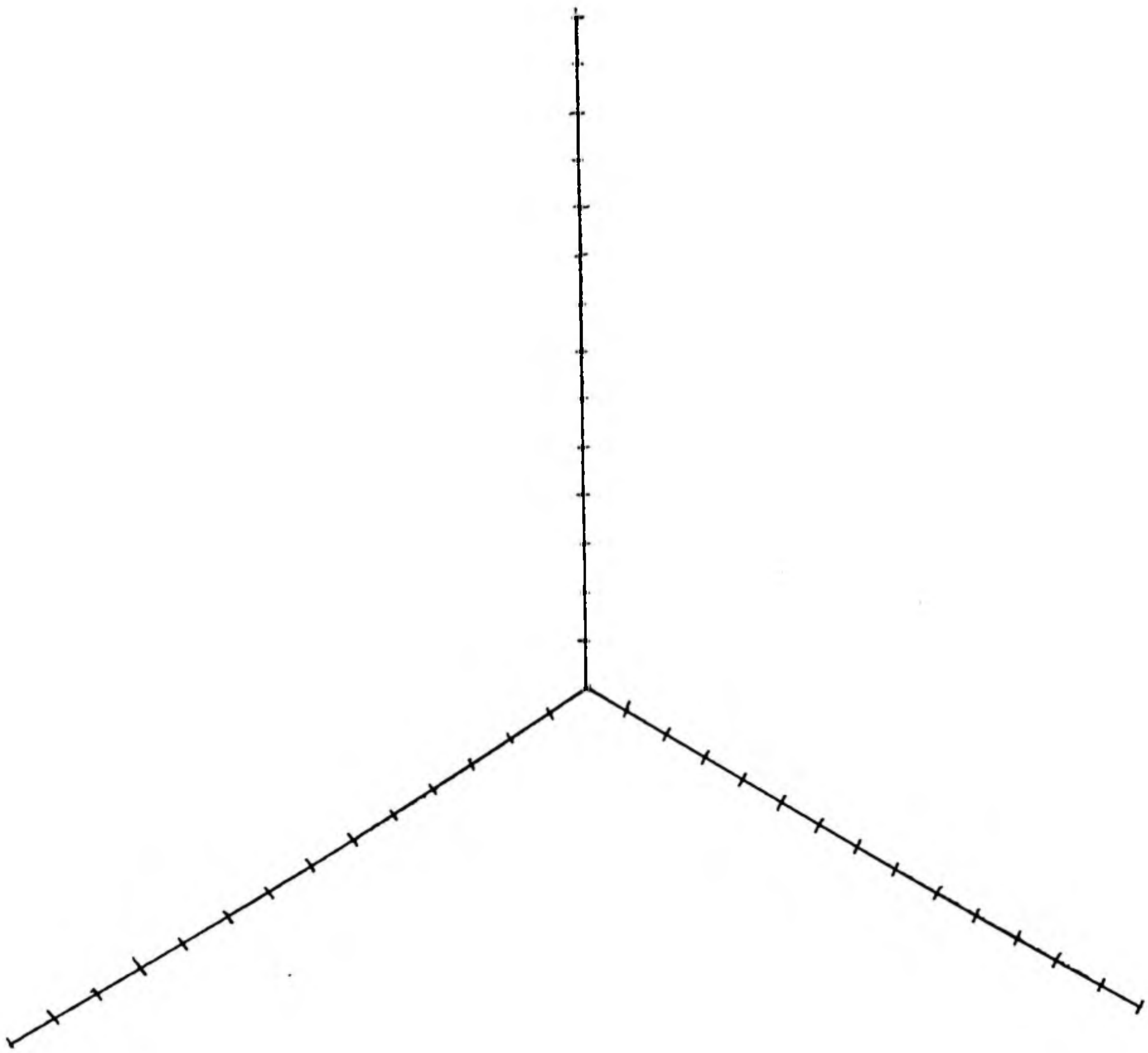
LINE DESIGNS

FIGURE 1



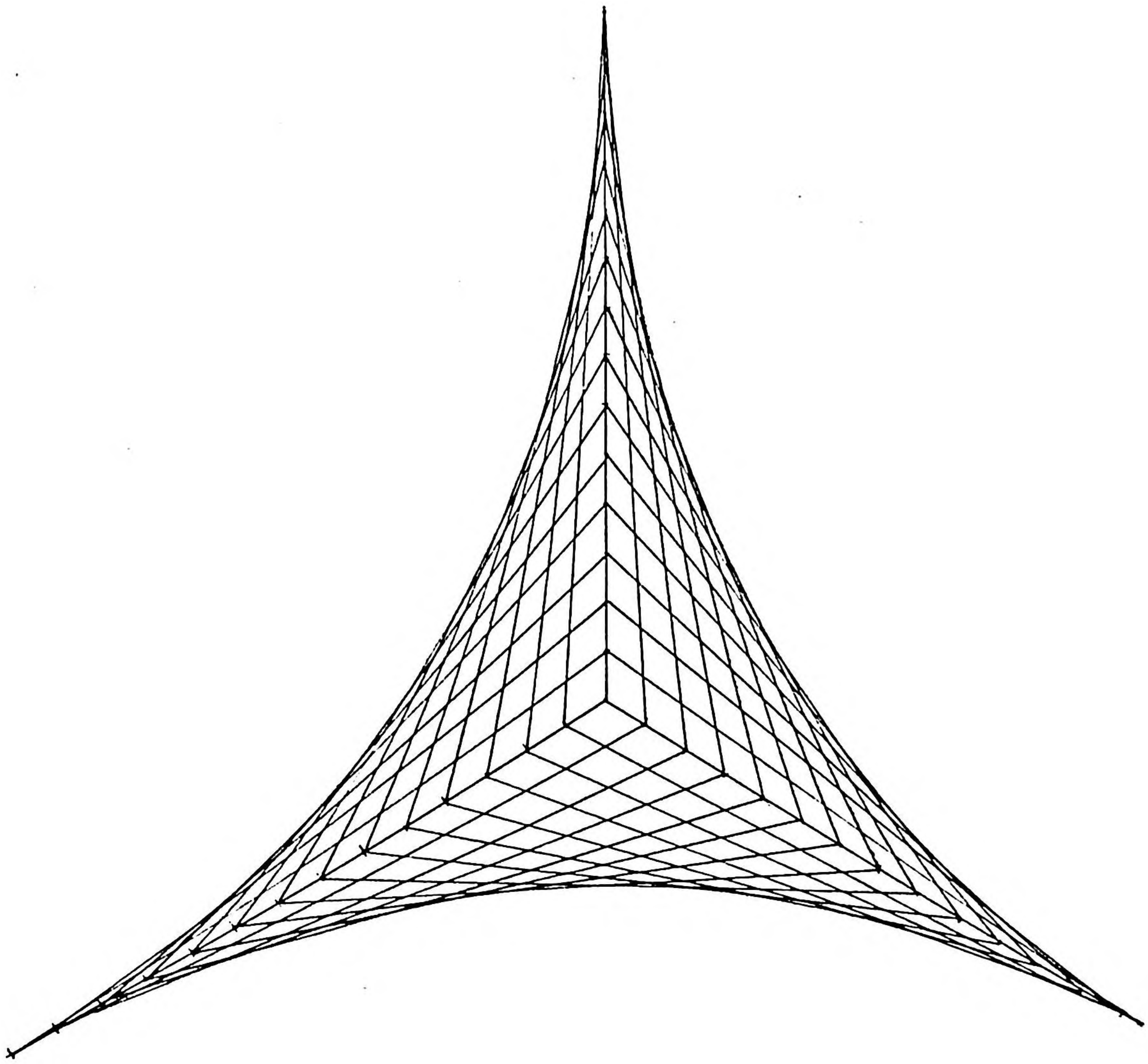
LINE DESIGNS

FIGURE 2A

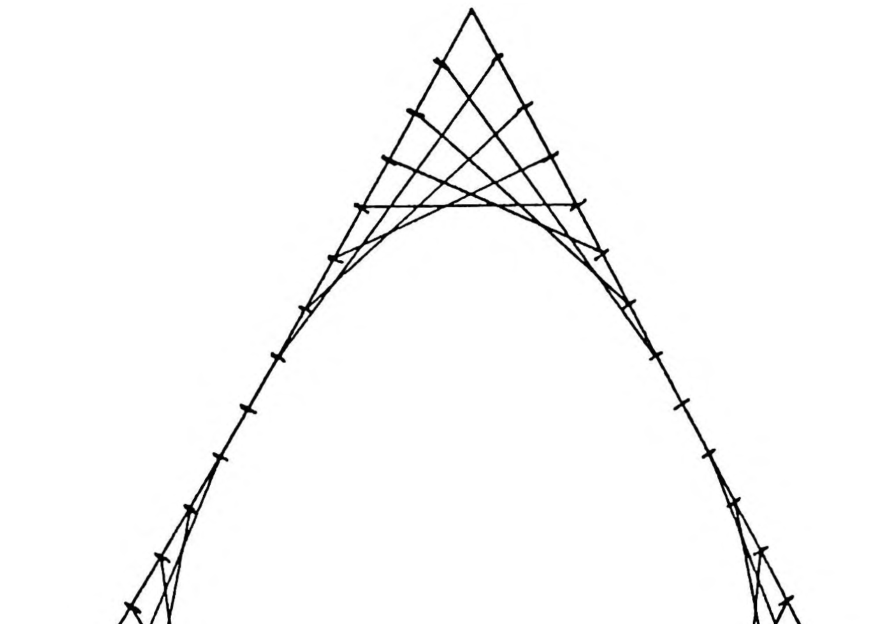


LINE DESIGNS

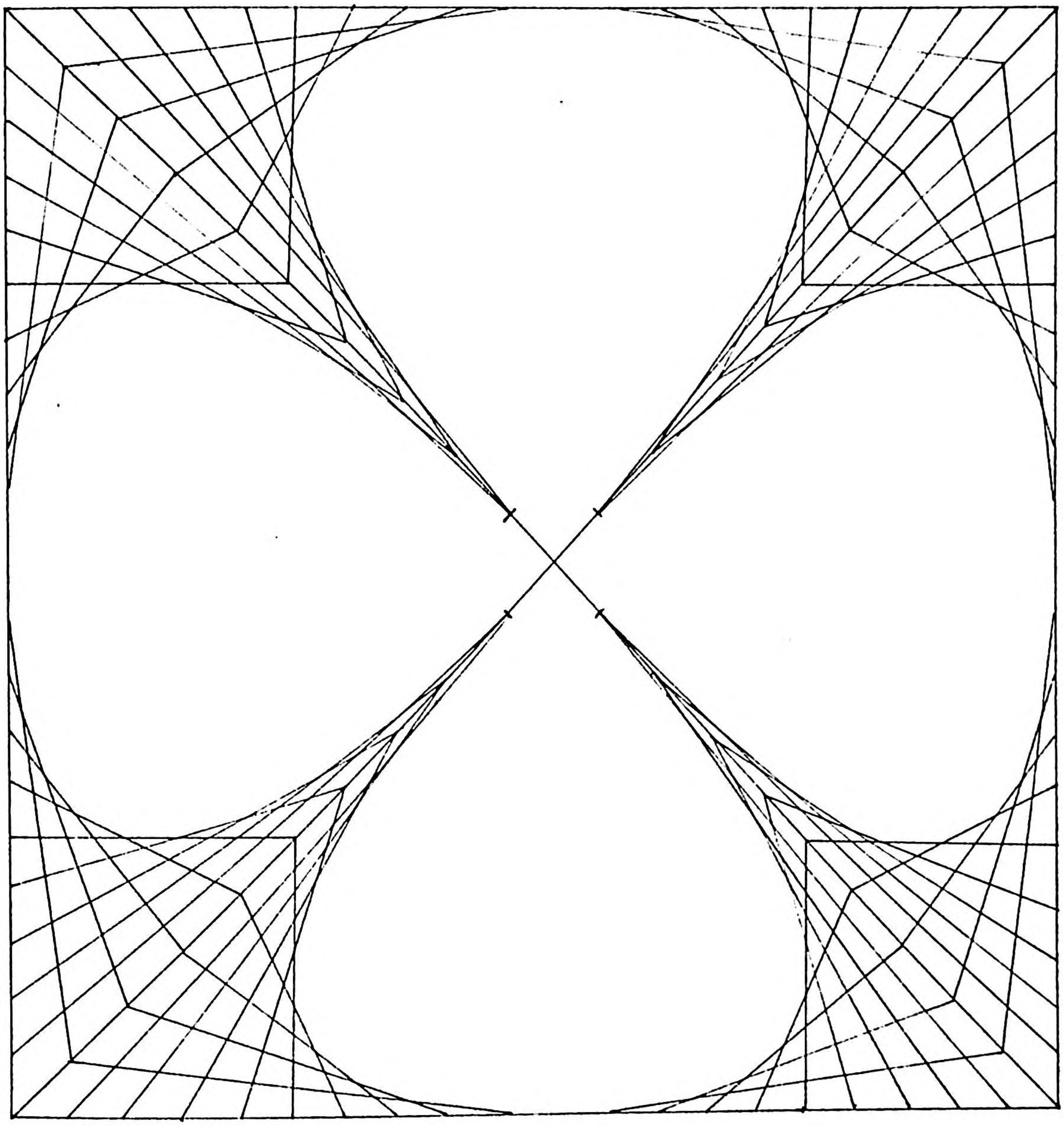
FIGURE 2B

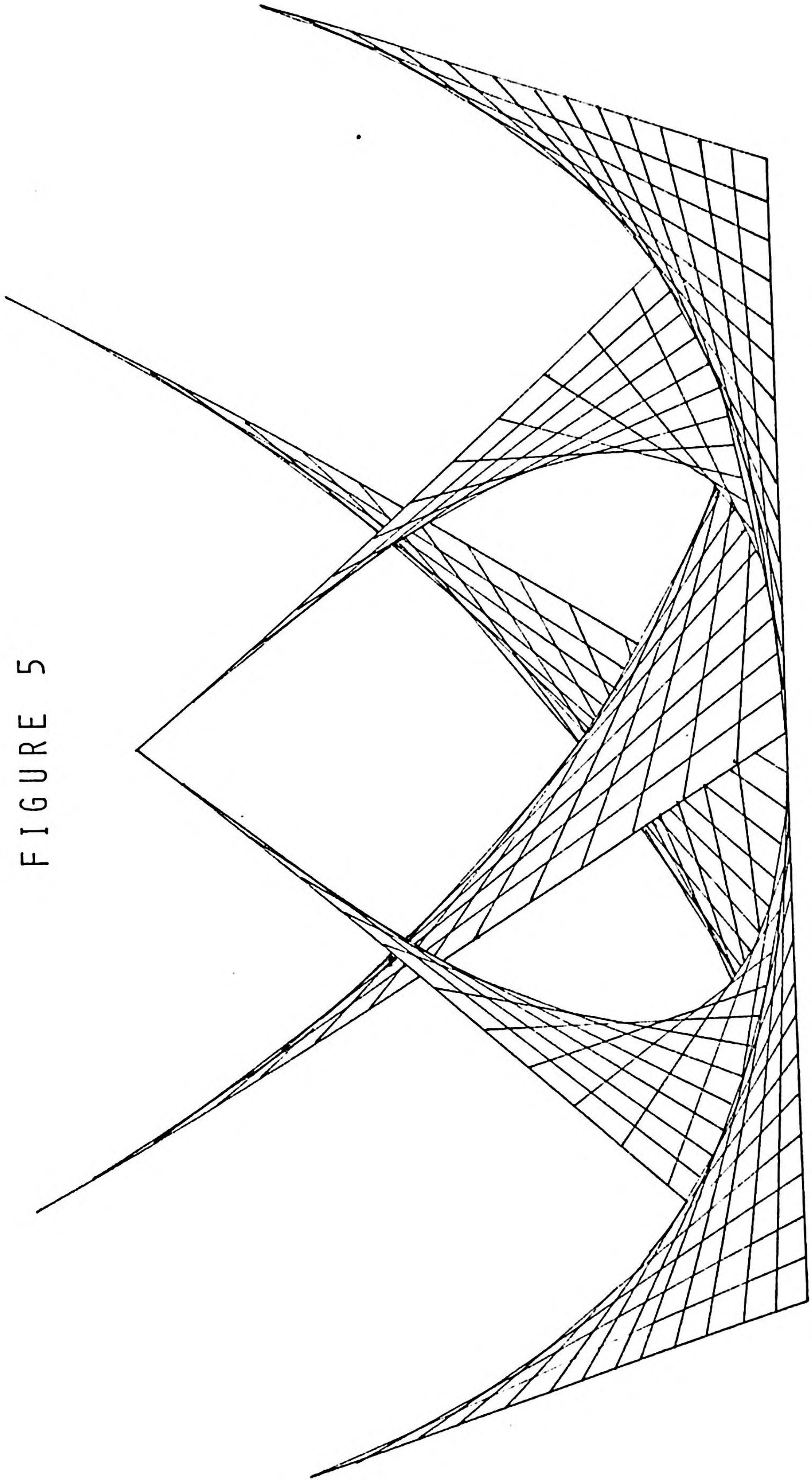


LINE DESIGNS
FIGURE 3



LINE DESIGNS
FIGURE 4





LINE DESIGNS
FIGURE 5

STRING ART

I. Objective.

The teacher will show students how aesthetically pleasing designs can be made by using a simple geometric figures.

II. Teacher Information.

This unit is an extension of the line design unit, hence, it should be presented after the line designs. While line designs are basically concerned with two-dimensional figures, string art allows the construction of three-dimensional objects using the same principles. By using multicolored thread, more vivid designs are possible. Since actual construction of these figures may involve a great deal of time, only the end product will be presented to the students. String art examples vary in construction and a step-by-step explanation for each design is a lengthy, tedious process. Therefore, a brief explanation of the basic construction is left to the individual teacher.

The teacher will have to construct the string designs before presenting the lesson to the class. They may be constructed by drawing outlines on cardboard and stitching the pattern with needle and thread. Another variation is to make a design in a piece of plywood with nails and wind the thread around the nails. The books listed in section VI give detailed instructions for complex designs.

Some string figures can be constructed by using only the principles in the line designs unit. For example, FIGURE 1 shows how the square can be used as a basic outline, while FIGURE 2 shows a pentagon.

The directions given in the next section are necessarily general, since specific instructions vary according to the particular string designs constructed.

III. Classroom Procedure.

Review the basic procedure of how to construct line designs. Add that more complex, multicolored designs can be constructed with thread.

Show a string art design that is similar to the one shown in FIGURE 1. Explain the basic construction to the students.

Show a string art design that is similar to the one shown in FIGURE 2. Stress that the same principles are used as in constructing line designs.

Show a string design similar to the one shown in FIGURE 3. Tell students that only straight lines are used to form the circles inside the perimeter.

Show a string art structure similar to the one shown in FIGURE 4. For complex string designs such as this, stress the aesthetic qualities but not the tedious step-by-step construction.

Any other string art designs similar to those in FIGURES 1 through 4 can also be shown.

From these examples, one can see that geometrical figures can be used to create beautiful artistic designs.

IV. Materials Needed.

String art designs similar to those shown in FIGURES 1 through 4 are needed.

V. Optional Student Activities.

Create a three-dimensional string art design.

VI. Sources.

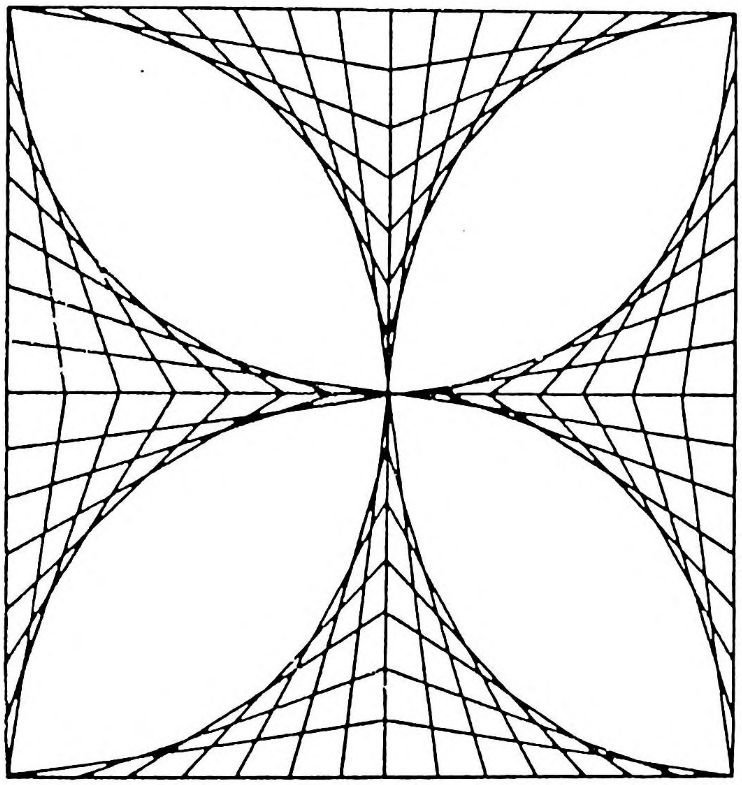
Ogletree, E. Geometry: An artistic approach. Arithmetic Teacher, 1969, 16, 457-461.

Seymour, D., Silvey, L., & Snider, J. Line designs. Palo Alto, California: Creative Publications, 1974.

Winter, J. String scultpure. Palo Alto, California: Creative Publications, 1972.

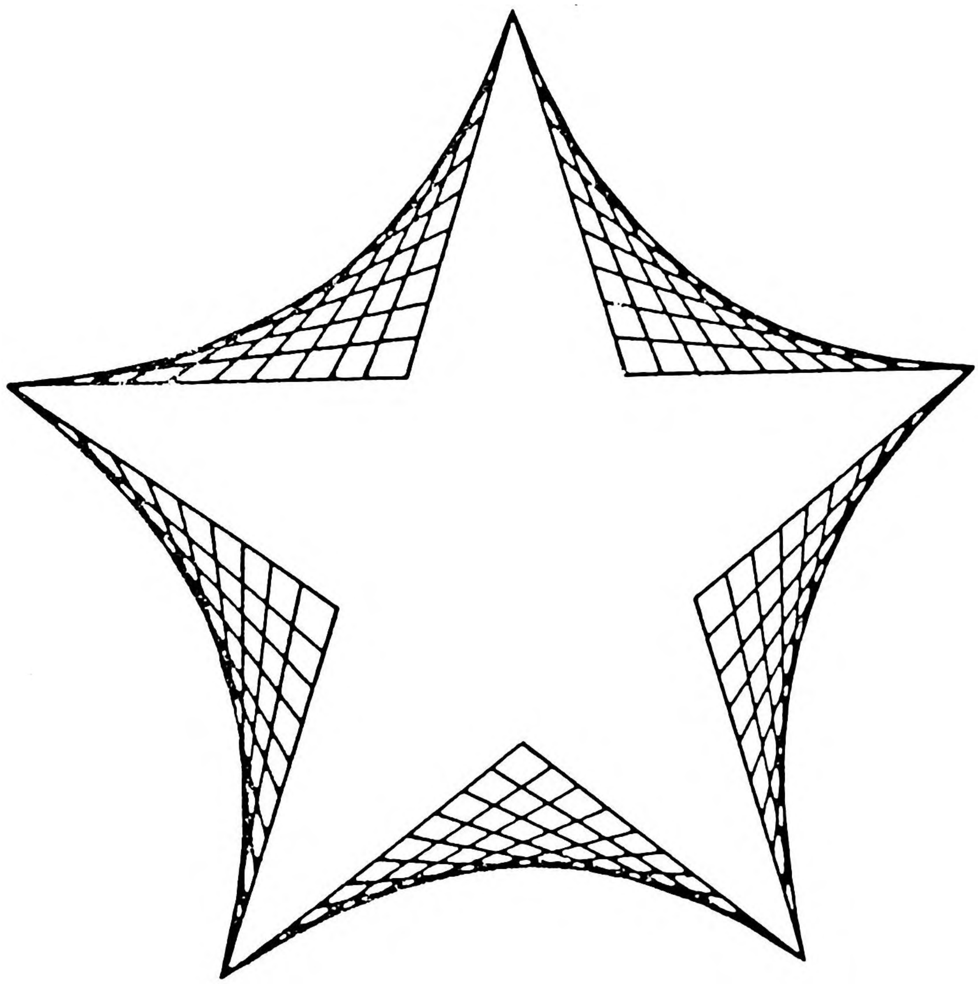
STRING ART

FIGURE 1



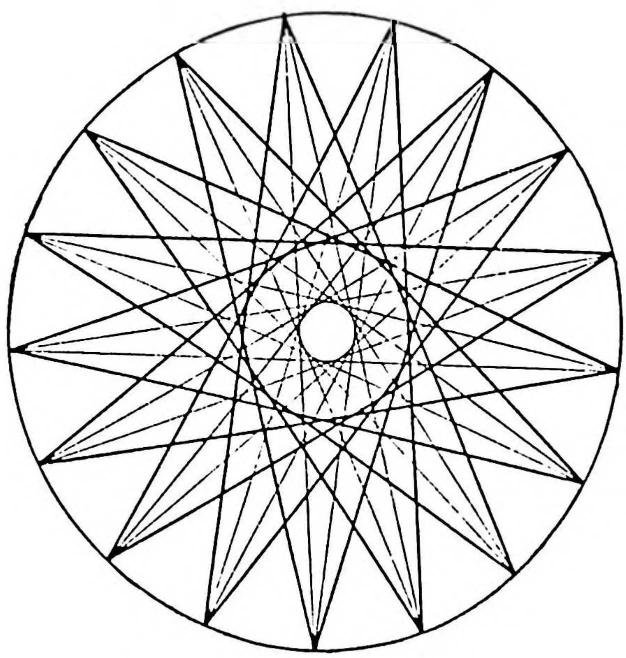
STRING ART

FIGURE 2



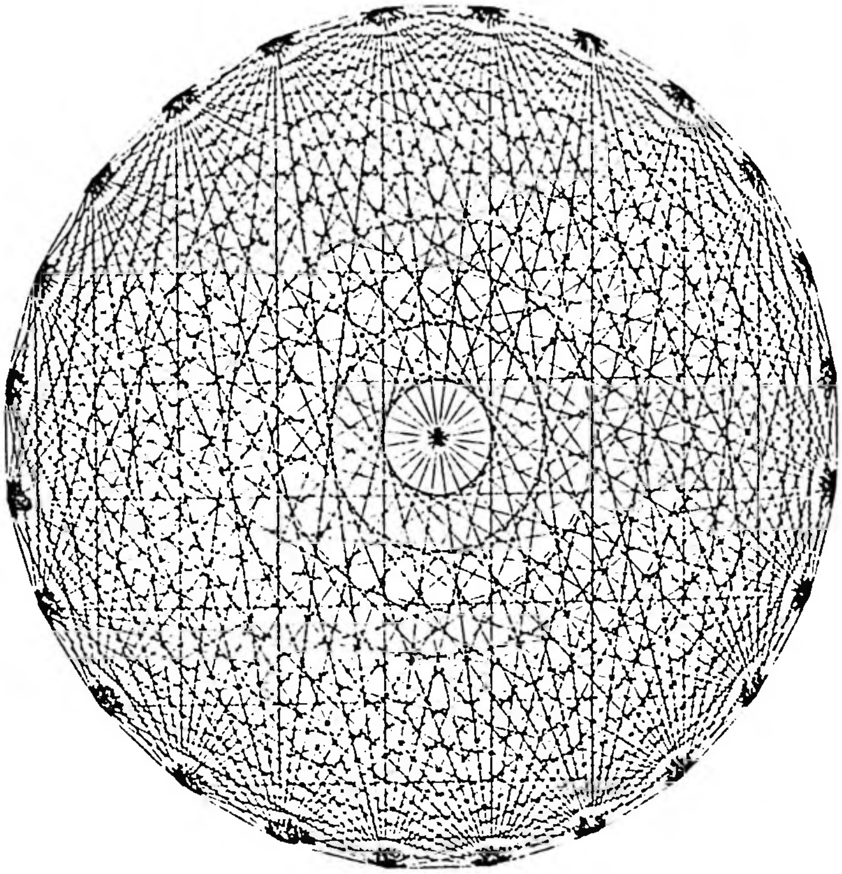
STRING ART

FIGURE 3



STRING ART

FIGURE 4



TESSELLATIONS

I. Objective.

The teacher will show students how to draw tessellations using only one polygon.

II. Teacher Information.

A tessellation is a polygonal design that completely fills a space with no overlapping. One example is the honeycomb structure mentioned in the unit entitled HONEYCOMBS. The honeycomb is a tessellation of regular hexagons positioned so that none of the figures overlap. In addition, there are no spaces between the individual hexagons. Another example of a tessellation is the tile floor found in most classrooms. Square tiles form a tessellation that covers a complete space (the floor) with no overlapping.

Tessellations do not have to be made from only one figure. For example, a tessellation can be made from squares and triangles, hexagons and triangles, or many other combinations. However, for simplicity, only tessellations made from one polygon are included in this unit.

Tessellations are used frequently to form original artistic designs. Stained-glass windows are quite often in the form of a tessellation. Probably the best known artistic use of tessellations can be found in the works of M. C. Escher. His use of symmetry to form complex works of art is unique.

The object of this lesson, then, is to show how one polygon can be arranged to form artistic designs. The unit can be used either during or after the study of polygons. Present the lesson after the honeycomb unit so that the honeycomb can be used as an example of a tessellation.

Before beginning the unit, cut a paper triangle exactly the size of the small triangle in FIGURE 1A. Then cut a quadrilateral exactly the size of the small one shown in FIGURE 2A. These will be used as templates when the lesson is presented to the students.

III. Classroom Procedure.

Begin by telling students that artistic designs can be made from simple geometric figures. For example, the following designs are made from triangles and quadrilaterals.

Show FIGURE 1A. Demonstrate the procedure for constructing this array by tracing additional triangles with the triangle template previously constructed. Emphasize to students that only one triangle is used to complete the design. With a magic marker, color every other triangle. The result should be similar to the design in FIGURE 1B.

Tell the students that any type of figure can be drawn inside the triangles to create a unique mosaic.

As an example, use the following combination of overlays.

Place FIGURE 1C over FIGURE 1A. This, along with the added color, makes an original design.

Show FIGURE 2A. Demonstrate the method for drawing this figure by tracing additional quadrilaterals with the template. Stress that even though the quadrilateral is not regular, it can still be used to completely fill a space. With a magic marker, color every other quadrilateral to form a colorful mosaic similar to the one shown in FIGURE 2B.

Original sketches can be made in the quadrilaterals to form another mosaic. As an example, combine overlays of FIGURES 2C and 2A.

Place FIGURE 2C over 2A. This, along with the color added previously, makes an original design.

Thus, as seen in these examples of tessellations, original artistic designs can be drawn from very simple geometric figures.

IV. Materials Needed.

Scissors, paper, magic markers, an overhead projector, and transparencies of FIGURES 1A through 1C and 2A through 2C are needed.

V. Optional Student Activities.

Create other designs by selecting other polygons. Encourage students to draw more complex figures by using more than one polygon.

VI. Sources.

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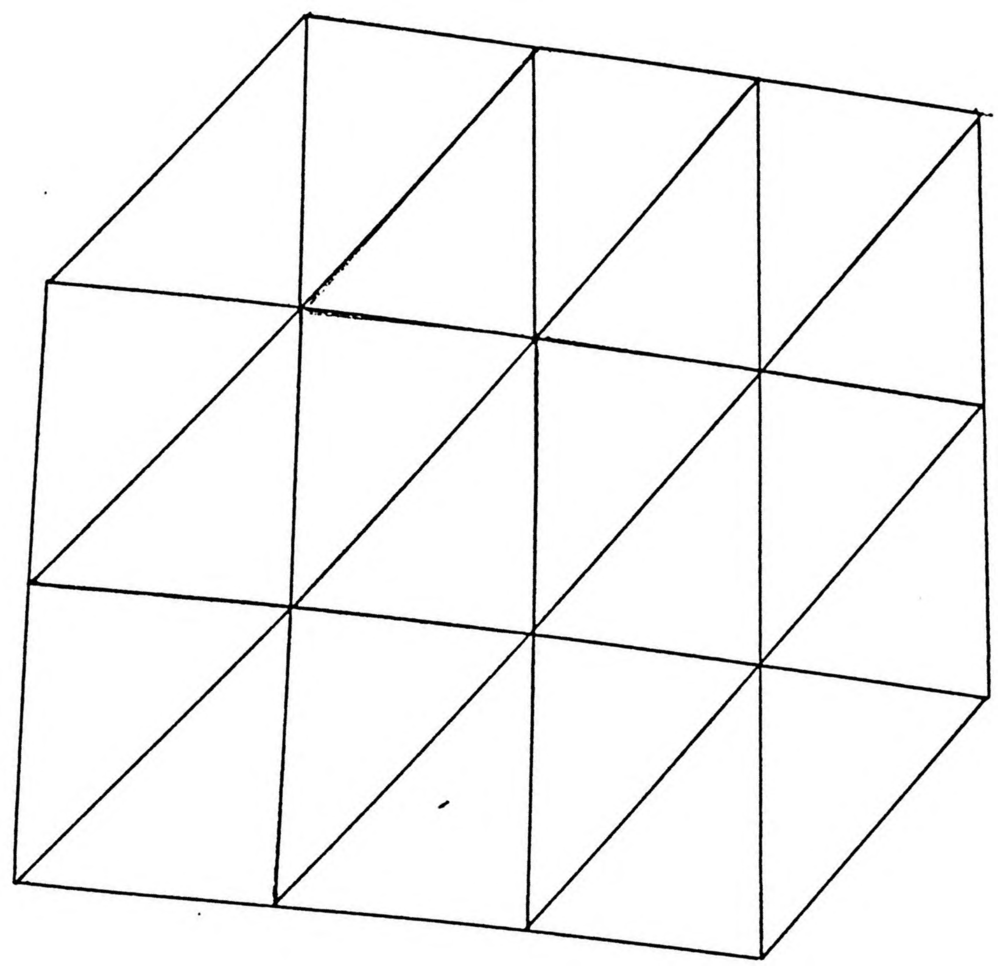
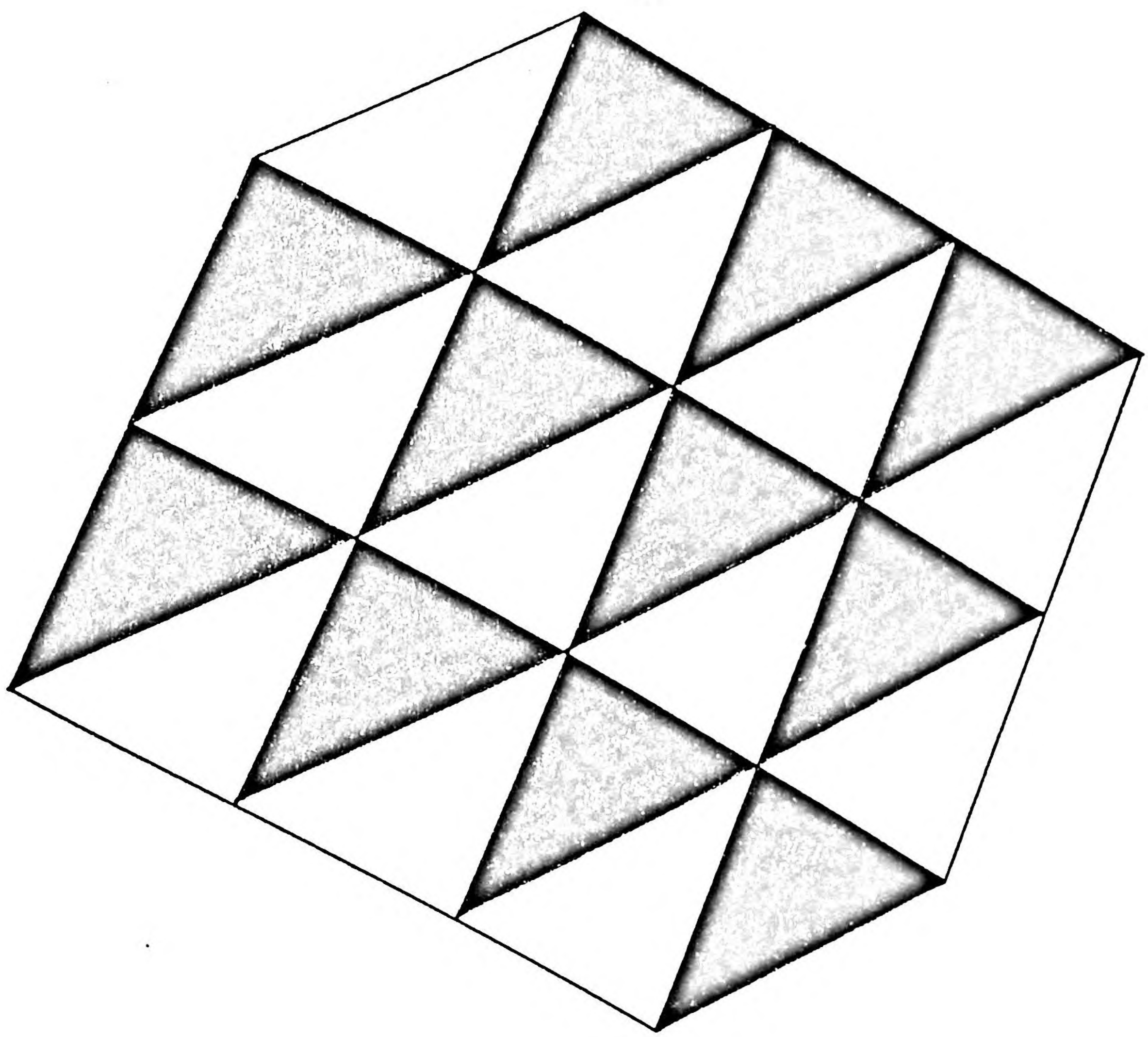
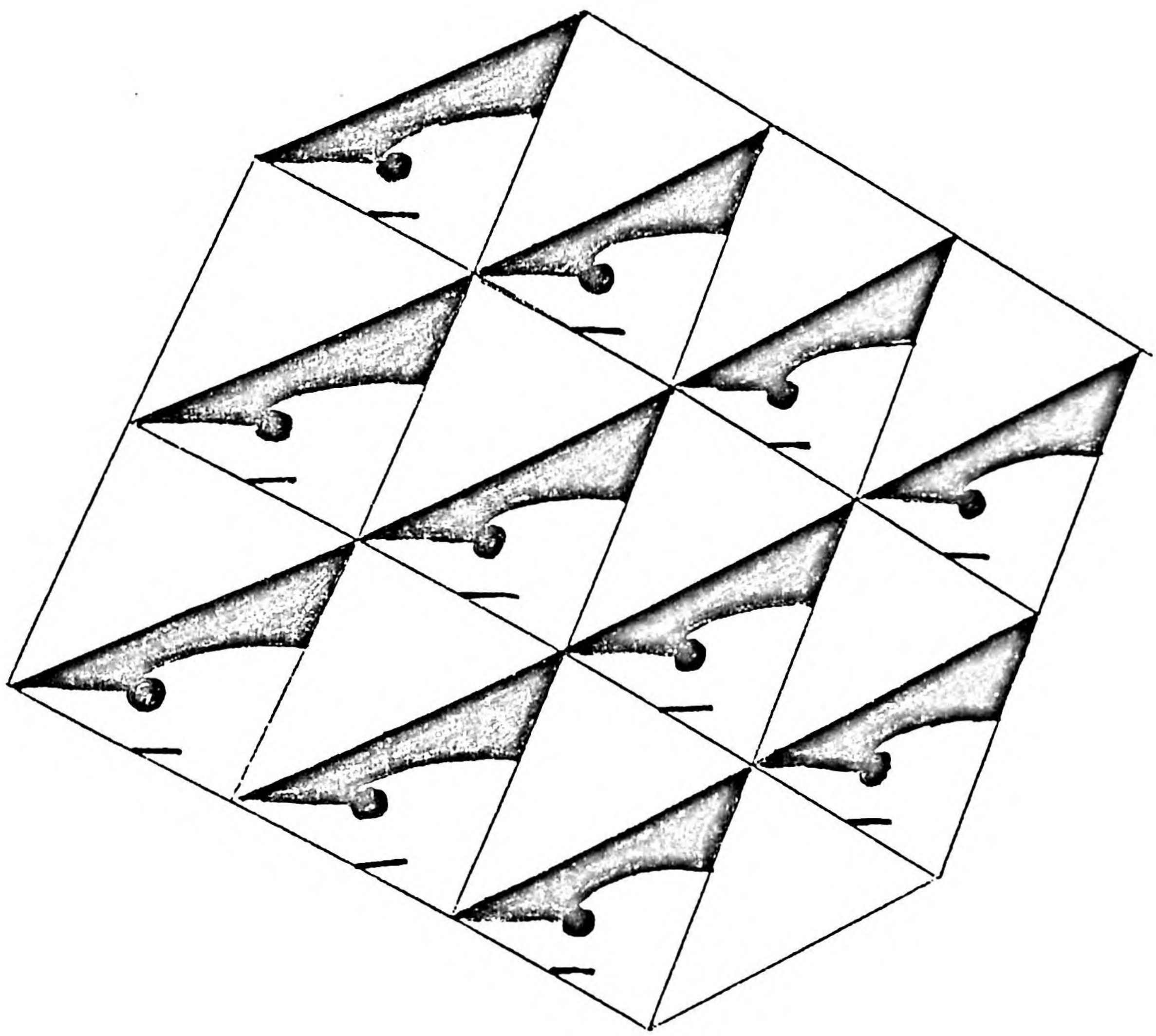
TESSELLATIONS
FIGURE 1A



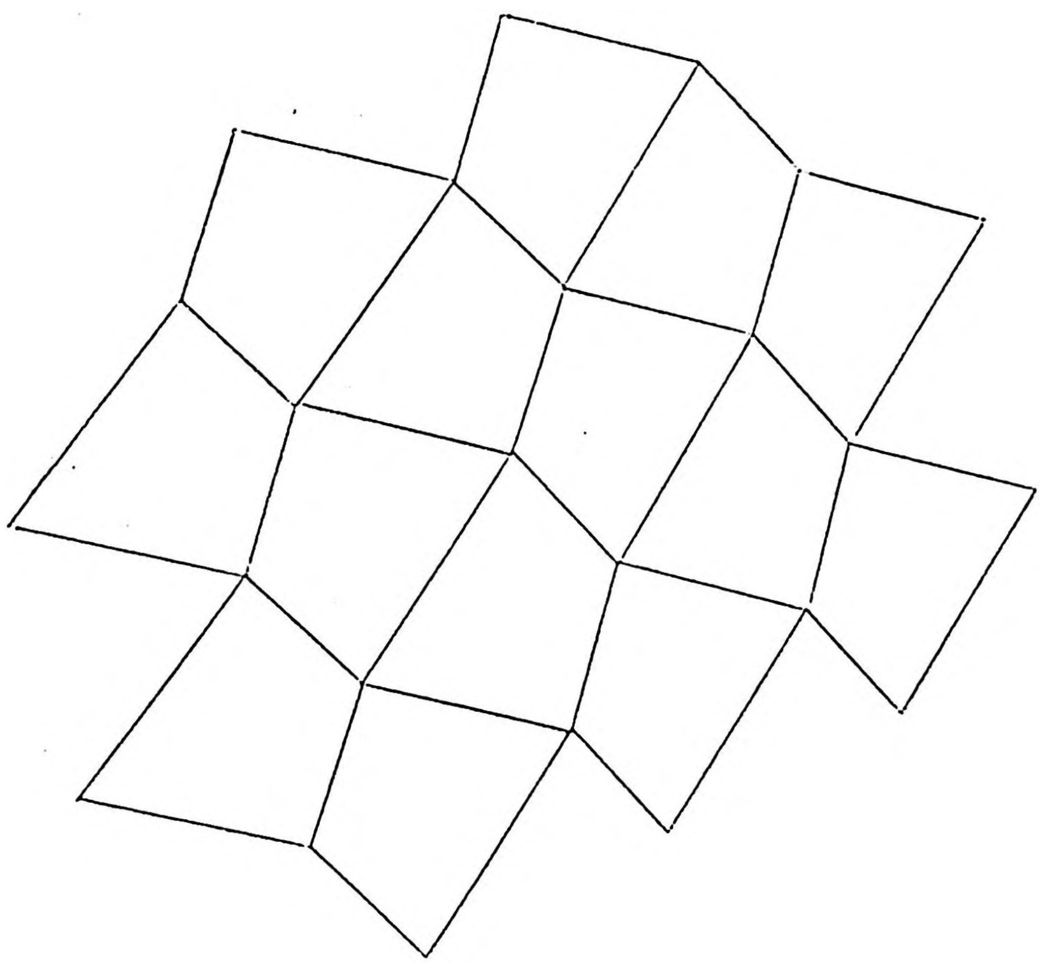
TESSELLATIONS
FIGURE 1B



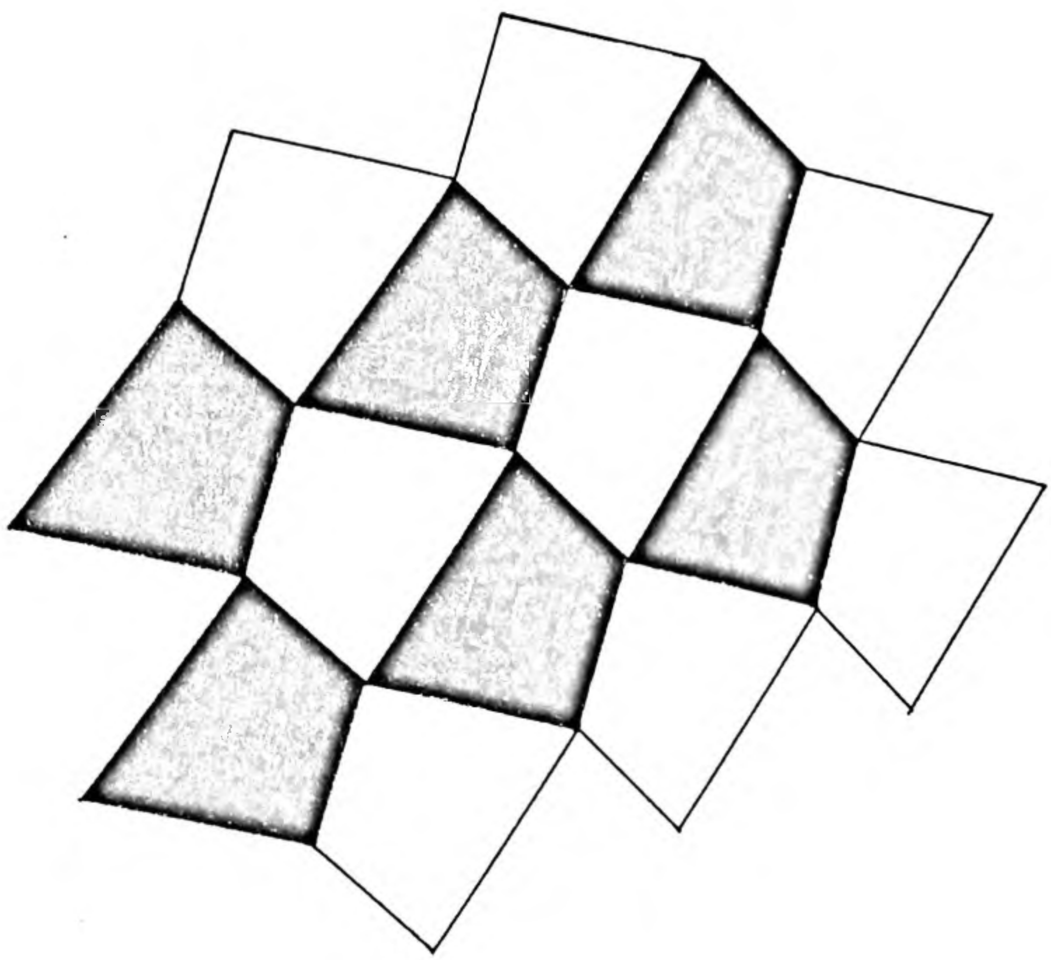
TESSELLATIONS
FIGURE 1C



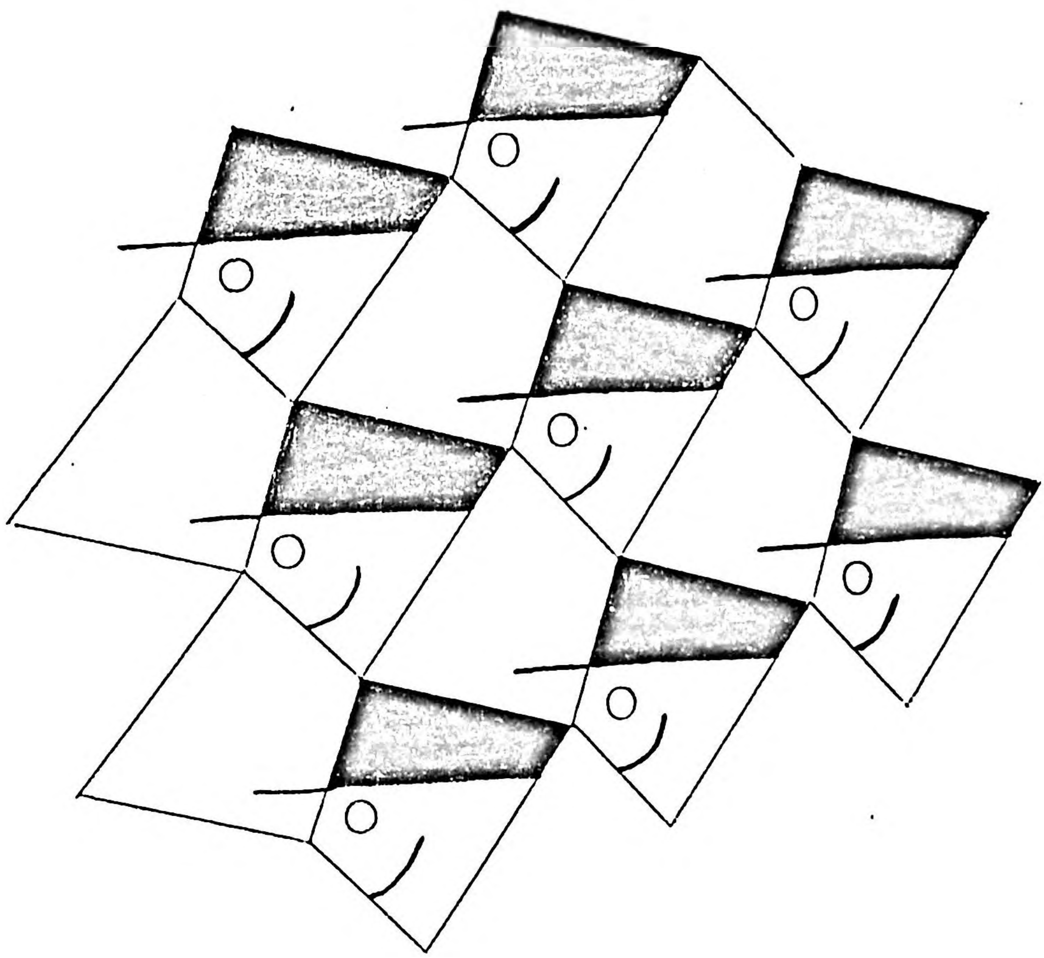
TESSELLATIONS
FIGURE 2A



TESSELLATIONS
FIGURE 2B



TESSELLATIONS
FIGURE 2C



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