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INNER RING STRUCTURES IN GALAXIES AS DISTANCE INDICATORS. II. CALIBRATION OF INNER RING DIAMETERS AS QUATERNARY INDICATORS

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ABSTRACT

The linear diameters D_r of the inner ring (r) and pseudo-ring (rs) structures in spiral galaxies of different morphological types and stage Sab to Sd $(2 \leq T \leq 7)$ in the revised Hubble system are calibrated by means of ~ 150 galaxies which have the best determined distance moduli $\mu_0^w(\Lambda_c)$ and $\mu_0^w(V_M)$ previously derived from tertiary indicators (B_T^0, D_0) via the corrected luminosity index Λ_c and from the 21 cm line widths via revised versions of the Tully-Fisher relations.

The logarithm of the linear diameter D_r is shown to be a linear function of the family (F) and stage (T) parameters, and of the corrected luminosity class (L_c), if available. Two- and three-parameter calibration formulae are derived, and the zero point, i.e., the linear diameter of the ring in an average galaxy of type SAB(r)bc II $(F=0,\ T=4,\ L_c=3)$, is determined to be $\langle D_r \rangle \approx 4.1$ kpc.

Possible dependences of the zero point on variety (r vs. rs), axis ratio (i.e. inclination), and redshift (i.e., distance) are investigated and found to be negligible in the distance modulus interval $29 < \mu_0 < 34$.

The calibration is extended to early-type spirals and lenticulars $(-2 \leq T < 2)$ by means of ~ 50 objects which are probable members of groups including spirals whose distance moduli are known from radio or optical indicators.

Formulae for the derivation of distance moduli from apparent diameters of rings reduced to standard values of F, T (and, where available, L_c for $T \geq 2$) are given. The accidental mean errors of the distance moduli so derived are expected to be $\sigma(\mu_0^r) \approx 0.6$ mag (three-parameter formula) and 0.7 mag (two-parameter formula) in well-observed cases, somewhat poorer than the tertiary indicators, but still useful, particularly in view of the potential range of applicability $(\Delta \leq 100$ Mpc).

The zero point of the ring distance scale is consistent with the basic zero point defined by the primary and secondary indicators within the internal m.e. of 0.2 mag or better, and has an external mean error from all sources of ~ 0.25 mag.

In an appendix the ring diameters in our catalog are compared to those recently published by Kormendy and by Pedreros and Madore; all three sets are in excellent systematic agreement (with a few exceptions), and their external mean errors are calculated. The distances of several hundred ringed galaxies derived from all available measurements and the present calibration will be reported in Paper III.

Subject heading: galaxies: structure

I. INTRODUCTION

Inner ring structures are common features of spiral and lenticular galaxies which are located in the intermediate zone between the central bulge and the disk. In the (r) variety of the revised Hubble classification system the ring is complete and closed, while in the transition variety (rs) between the pure ringed (r) and pure spiral (s) varieties incomplete or open "pseudo-rings" are present (de Vaucouleurs 1959, 1963). Altogether, nearly 50% of all spiral and lenticular galaxies possess an (r) or (rs)-type ring structure; in fact, the ring phenomenon is present at various levels of detectability throughout the continuum of varieties from (r) to (rs) to (s), where it vanishes.[1]

In a recent paper (de Vaucouleurs and Buta 1980*a*, Paper I), we showed on the basis of a dimensionless analysis that the diameters of both pure and pseudo-

[1] In many early-type spirals and lenticulars, rings are near the edge of a lens. However, lens diameters are more poorly defined (edge of diffuse feature) than rings which are defined by the ridge line of the brightness distribution. How rings are related to lenses is immaterial for this study. For measurements of lenses see our catalog (de Vaucouleurs and Buta 1980*b*). For a discussion of lenses see Kormendy (1979).

inner rings can be used as geometric distance indicators if the type, family, and luminosity class of a galaxy are known. The dimensionless logarithmic parameters $X = \log D_0/D_r$ and $Y = \log A_e/D_r$, where D_0 is the isophotal galaxy diameter, A_e is the "effective" aperture diameter, both taken from the *Second Reference Catalogue of Bright Galaxies* (RC2) (de Vaucouleurs, de Vaucouleurs, and Corwin 1976), and D_r is the diameter of the (r) or (rs) structure, taken from the catalog of de Vaucouleurs and Buta (1980b), were shown to have significant dependences on stage $T(a, ab, b\ldots)$ and family $F(A, AB, B)$, but only weak or insignificant dependences on variety (r vs. rs) and corrected luminosity class L_c. From this analysis we inferred the form of the calibration at least for spirals later than Sa. In the present paper we proceed to determine the constants of the correlation of the linear diameters of inner rings with F, T, and L_c by using two different tertiary distance scales: one based on B_T^0, $\log D_0$, and luminosity index Λ_c (de Vaucouleurs 1979a, b) and a second one based on B_T^0, $\log D_0$, and the maximum rotation velocity V_M derived from the 21 cm line width, i.e., revised forms of the Tully-Fisher relation (Bottinelli *et al.* (1980) (§ III). Because these two indicators apply only or mainly to spirals of types $T \geq 2$ (Sab or later), we will extend the calibration of rings to earlier types $T < 2$ (lenticulars and early spirals) by means of distances inferred from group membership (§ IV). In a third paper, the present calibrations will be used to determine distances to several hundred spiral and lenticular galaxies of the (r) and (rs) varieties.

II. GENERAL CONSIDERATIONS

Before proceeding with the calibration, we would like to comment briefly on Kormendy's (1979) remarks about the use of inner ring diameters as distance indicators. Kormendy showed that the diameters D_r of inner rings in SB galaxies are correlated with galaxy absolute magnitude; because the relation is of the form $M_T^0 + 5 \log D_r$ = constant, for a given type, he concluded that inner rings could not be used as distance indicators since the correlation implies that at a given apparent magnitude, an intrinsically faint galaxy nearby would have the same apparent ring diameter as an intrinsically bright galaxy far away. However, his conclusion would be correct only if we had no distance-independent way of distinguishing an intrinsically bright galaxy from an intrinsically faint one. This, of course, is not the case as van den Bergh (1960a, b) showed long ago with his concept of galaxy luminosity classes, recently refined by combining luminosity class (corrected for inclination) with Hubble stage to form a corrected luminosity index Λ_c (de Vaucouleurs 1977).[2] Moreover, the quantity $M_T^0 + 5 \log D_r$, which is an index of average surface brightness, depends not only on type but on family as well. This can be shown by combining the X-parameter solution (eq. [7a], Paper I; as revised in § IIId of this paper) with the linear calibration formulae (de Vaucouleurs 1979a) for logarithmic isophotal diameter $\log D_0$ and total absolute blue magnitude M_T^0 of the galaxy. The result is that

$$M_T^0 + 5 \log D_r = -0.95 + 0.75F - 0.20T, \qquad (1)$$

where T = stage index on the RC2 scale, F = "family index," i.e., -1 for SA, 0 for SAB, and $+1$ for SB galaxies (see § IIIc), and D_r is in parsecs; this can be reduced to a relation between apparent quantities,

$$B_T^0 + 5 \log d_r = 16.73 + 0.75F - 0.20T, \qquad (2)$$

if d_r is in the units of 0.'1. The approximate validity of equation (2) was checked by calculating mean values of the surface brightness index $B_T^0 + 5 \log d_r$ for 298 RC2 galaxies in our sample (Fig. 1). The mean surface brightness index decreases with type and increases with family as predicted, except possibly for the SA galaxies, where the scatter is large and the statistics are poor. However, the fact that this correlation is distance independent does not prove that the rings cannot be used as distance indicators. It only means that ring diameters, like many other physical characteristics of galaxies, are not "pure" distance indicators, i.e., capable of being used by themselves independently of all other properties of a galaxy; they depend in part on the total mass and structure of the parent galaxy, as indicated by its Hubble stage, absolute luminosity, and family. We will show that given auxiliary information on the structure of a galaxy via its classification parameters F, T, and L_c, the ring diameters can be reduced to a standard diameter and used as distance indicators over a potentially large distance range. Pure distance indicators, such as RR Lyrae variables, novae, supernovae, and, possibly, brightest M supergiants, are, unfortunately, rare, and, except for supernovae, can be detected at present only in the nearest galaxies. Most secondary and tertiary indicators require additional information which is certainly not reason enough to reject them, as indeed their successful applications demonstrate. It is the purpose of this paper

[2] To dispel any possible confusion we specify that the luminosity classes L used here are exclusively those in the original van den Bergh (1960a, b) system, as coded in RC2, and corrected for inclination effects (de Vaucouleurs, de Vaucouleurs, and Corwin 1978; de Vaucouleurs 1979a). No use is made of the luminosity classes in the revised Shapley-Ames catalog (Sandage and Tammann 1981), which, according to Tammann, Yahil, and Sandage (1979), do not correlate well with absolute magnitude. The external agreement between L estimates from different sources is 0.6–0.8 L step, corresponding to 0.3–0.4 mag (de Vaucouleurs, de Vaucouleurs, and Corwin 1978), and the mean error of the absolute magnitudes derived from the corrected luminosity index Λ_c is $\sigma \approx 0.45$ mag (exclusive of zero point; de Vaucouleurs 1979a). For further discussion of errors, see § V.

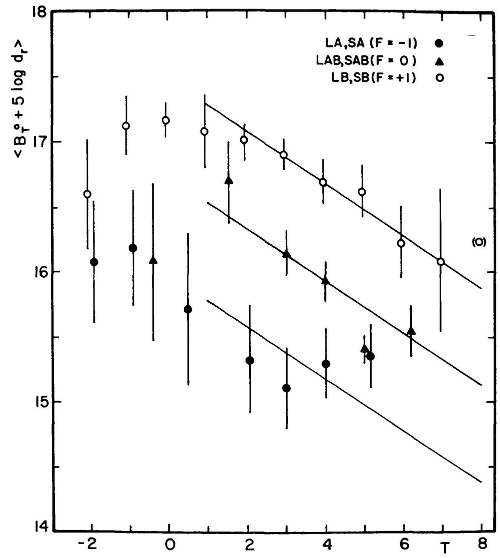

FIG. 1.—Mean surface brightness index $\langle B_T^0 + 5 \log d_r \rangle$ as a function of stage T and family F for 298 RC2 spiral and lenticular galaxies. The straight lines over the range of spiral types, $1 \leq T \leq 7$, are based on eq. (2). The mean surface brightness is seen to be a strong function of both family and stage.

to establish the absolute calibration of ring diameters as distance indicators.

III. THE RING DIAMETER CALIBRATION

a) Statistical Properties of the Ringed Galaxy Sample

The sample of galaxies used for the calibration of ring diameters has been taken from the catalog of de Vaucouleurs and Buta (1980*b*). This catalog is a compilation of measurements of the dimensions of several kinds of distinct structural components of galaxies (nucleus, lens, and ring structures) which were initially given in the notes to the (First) *Reference Catalogue of Bright Galaxies* (RC1; de Vaucouleurs and de Vaucouleurs 1964). Measurements of 250 inner rings (r) and 225 pseudo-rings (rs) are included in the catalog, of which 423 were used in Paper I. By comparison of 43 objects in common with Kormendy (1979), we showed that ring diameter measurements have relative mean errors of only ~5%, so that measuring errors will not be a major cause of dispersion in the calibration (see also the Appendix).

An indication of the completeness of our ringed galaxy sample may be obtained from Table 1. Here N is the number of galaxies of the (r) or (rs) varieties in the RC2 within the indicated intervals of logarithmic isophotal diameter $\log D_0$, while n is the number of these galaxies whose (r) or (rs) diameters are given in our catalog. The ratios $n(r)/N(r)$ and $n(rs)/N(rs)$ are shown in Figure 2*a*. Apparently, our sample is more than 80% complete with regard to ring measurements only for galaxies having $D_0 > 4'.5$. The completeness decreases rapidly with decreasing apparent diameter and drops to 50% near $D_0 \approx 3'$ for (rs) types and $D_0 \approx 2'$ for (r) types. The incompleteness is due to two factors. First, a larger number of galaxies are included in the RC2 than in RC1. Thus, many rings simply have not yet been measured. Second, there will be cases where the ring is a very weak, uncertain, or inferred feature [e.g., in edge-on objects such as NGC 4762, classified as LB(r)O+? in RC2] and could not be accurately measured on the plate material used for the classification. This would be especially true for galaxies of the (rs) variety, and it probably explains why the number ratios in Table 1 are noticeably lower for the (rs) galaxies than for the (r) galaxies. Evidently, the observer measured

TABLE 1

STATISTICAL COMPLETENESS OF THE RINGED GALAXY SAMPLE[a]

	PURE RINGS			PSEUDO-RINGS			ALL RINGS
$\log D_0$ RANGE	$N(r)$	$n(r)$	$\frac{n(r)}{N(r)}$	$N(rs)$	$n(rs)$	$\frac{n(rs)}{N(rs)}$	$\frac{n(r,rs)}{N(r,rs)}$
>1.85	7	7	1.00	21	15	0.71	0.79
1.84–1.75	11	9	0.82	16	14	0.88	0.85
1.74–1.65	15	14	0.93	24	19	0.79	0.85
1.64–1.55	42	32	0.76	62	28	0.45	0.58
1.54–1.45	60	43	0.72	90	54	0.60	0.65
1.44–1.35	80	46	0.58	108	40	0.37	0.46
1.34–1.25	71	33	0.46	87	31	0.36	0.41
1.24–1.15	47	16	0.34	61	13	0.21	0.27
1.14–1.05	23	3	0.13	26	8	0.31	0.22
<1.05	14	3	0.21	10	2	0.20	0.21

[a]N = number of galaxies in RC2 classified as r or rs. n = number of galaxies for which the dimensions of the r or rs structure are given in the catalog of de Vaucouleurs and Buta (1980*b*).

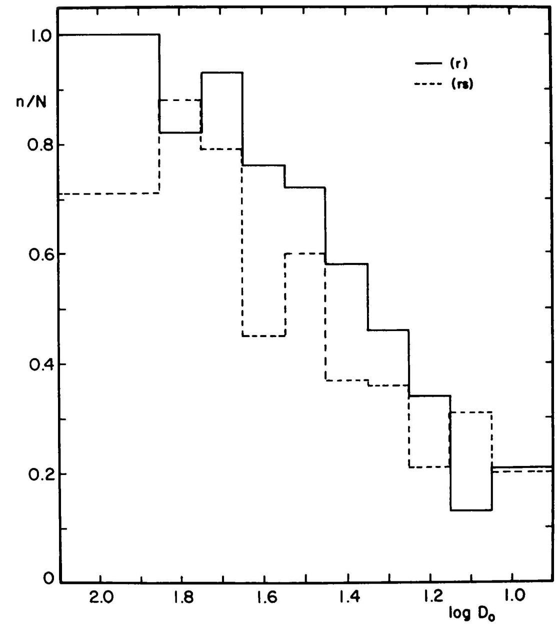

FIG. 2*a*

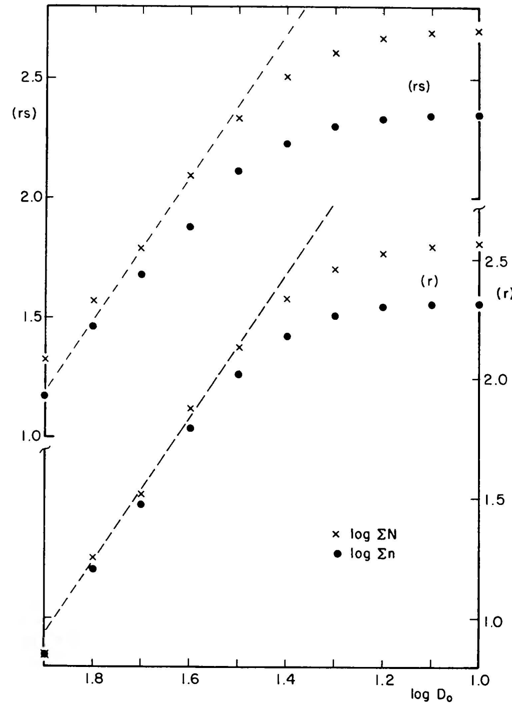

FIG. 2*b*

FIG. 2.—(*a*) Histogram of the ratios of the number of galaxies with measured (*r*) or (*rs*) diameters in the catalog of de Vaucouleurs and Buta (1980b) to the total number of galaxies classified as (*r*) or (*rs*) in the RC2, within equal intervals of logarithmic isophotal diameter log D_0. The 50% completeness level is reached near log $D_0 \approx 1.5$ ($D_0 \approx 3'$). (*b*) Logarithmic plots of the cumulative numbers of galaxies of the (*r*) variety (*below*) and (*rs*) variety (*above*) as functions of log D_0. Crosses represent all RC2 galaxies of the (*r*) or (*rs*) varieties, while dots represent only those galaxies included in the catalog of ring diameter measurements. Dotted lines have a slope of -3, which represents the cumulative distribution for a uniform density distribution. Incompleteness sets in rapidly for $D_0 < 3'$ in the RC2 sample.

well-defined (*r*) rings more readily than broken or ill-defined (*rs*) rings in small galaxies.

The cumulative frequency functions plotted in Figure 2*b* show that the RC2 is approximately complete with respect to inclusion of ringed galaxies only for log $D_0 \gtrsim 1.5$ ($D_0 > 3'$). In both the (*r*) and (*rs*) samples the cumulative distribution has a slope ~ -3 over this range, consistent with a roughly uniform density distribution, if log D_0 is an approximate indicator of distance. The rapid systematic deviation of both samples from the line of slope -3 for log $D_0 < 1.5$ indicates that the RC2 is quite incomplete either with regard to inclusion or identification of ringed galaxies among galaxies of small angular diameter. We also see from the graphs that our diameter catalog is increasingly incomplete with respect to the smaller ringed galaxies in RC2.

b) *The Primary Calibration Sample*

The primary sample of galaxies which we have used for the actual calibration of ring diameters consists of the subset of galaxies that have distances from tertiary indicators (B_T^0, log D_0, Λ_c, and V_M) and whose family, stage, and luminosity class (parameters F, T, and L) are given in RC2. Weighted mean distance moduli, $\mu_0^w(\Lambda_c)$, obtained via the corrected luminosity index Λ_c from an enlarged version of Table 1 of de Vaucouleurs (1979*b*), are available for 180 galaxies in the sample. Of these, 150 also have weighted mean moduli $\mu_0^w(V_M)$, based on the H I line width recalibration of the Tully-Fisher relation after Bottinelli *et al.* (1980). Comparison of the two systems of distance moduli for the "best observed" galaxies (i.e., with the best input data) shows that there is no scale error between the two scales and that they are of approximately equal precision, with $\sigma(\mu_0) \approx 0.4$ mag (Bottinelli *et al.* 1980); only a small zero point difference (<0.25 mag) may exist between them. The use of both distance scales allows a useful check on the calibration formulae; this is especially important because a calibration based on $\mu_0^w(\Lambda_c)$ alone will necessarily suffer correlation effects since $\Lambda_c = (T + L_c)/10$ and the ring diameters depend on T and L_c. Such effects will not arise in the $\mu_0^w(V_M)$ calibration because the Tully-Fisher relation is substantially independent of T and L_c (de Vaucouleurs *et al.* 1982).

Tables 2A and 2B give the distribution of galaxies in two calibrating samples with respect to several parameters. The distributions of the corrected radial velocities V_0 are given in Table 2C. The most frequent variety is *rs*, the mean stage is Sbc ($T = 4$), and the mean corrected

TABLE 2A

STATISTICAL PROPERTIES OF THE RING CALIBRATION SAMPLE: THREE-PARAMETER FORMULA

Calibrating Moduli	$\mu_0^w(\Lambda_c)$	$\mu_0^w(V_M)$
Number of galaxies N	178	149
$N(\Lambda_c \leq 1.2)$	166	141
$N(\Lambda_c > 1.2)$	12	8
Sample used in calibration ($\Lambda_c \leq 1.2$):		
Distribution with family (RC2):		
N(SA)	23	21
N(SAB)	88	76
N(SB)	55	44
Distribution with variety (RC2):		
$N(r)$	63	53
$N(rs)$	103	88
Distribution with stage T:		
2	14	7
3	32	25
4	53	52
5	53	45
6	13	11
7	1	1
$\langle T \rangle$	4.13	4.22
Distribution with corrected luminosity class L_c:		
0.5–1.50	42	38
1.51–2.50	17	16
2.51–3.50	71	59
3.51–4.50	19	15
4.51–5.50	12	9
5.51–6.50	2	1
6.51–7.50	3	3
$\langle L_c \rangle$	2.85	2.79

TABLE 2B

STATISTICAL PROPERTIES OF THE RING CALIBRATION SAMPLE: TWO-PARAMETER FORMULA[a]

Calibrating Moduli	$\mu_0^w(\Lambda_c)$	$\mu_0^w(V_M)$
Number of galaxies N	200	192
Distribution with family (RC2):		
$T<2$: N(LA)	9	10
$T<2$: N(LAB)	7	8
$T<2$: N(LB)	23	24
$T\geq 2$: N(SA)	22	27
$T\geq 2$: N(SAB)	82	75
$T\geq 2$: N(SB)	57	48
Distribution with variety (RC2):		
$T<2$: $N(r)$	22	24
$T<2$: $N(rs)$	17	18
$T\geq 2$: $N(r)$	59	50
$T\geq 2$: $N(rs)$	102	100
Distribution with stage T:		
−2	5	6
−1	14	14
0	9	9
1	11	13
2	13	11
3	30	32
4	47	47
5	52	42
6	14	14
7	5	4
$\langle T \rangle$	3.35	3.20

[a]Excluding six members of the Virgo E cluster (see Table 6) whose distance modulus is not based on either Λ_c or V_M.

TABLE 2C

DISTRIBUTION OF CALIBRATING GALAXIES WITH RADIAL VELOCITY

	TABLE 2A		TABLE 2B	
V_0 RANGE	$\mu_0^w(\Lambda_c)$	$\mu_0^w(V_M)$	$\mu_0^w(\Lambda_c)$	$\mu_0^w(V_M)$
0–999	30	26	34	33
1000–1999	64	56	89	88
2000–2999	44	38	47	42
3000–3999	13	10	15	16
4000–4999	11	8	11	9
5000–5999	3	3	3	3
≥6000	1	0	1	1

luminosity class is $L_c \approx 3$ (i.e., LC=II). Thus, the average ringed galaxy in our sample has $\Lambda_c = 0.7$, corresponding to an absolute magnitude $M_T^0 \approx -20.0$. The range of velocities is from 500 to 7000 km s^{-1}, although the sample is reasonably complete over the useful range $0.3 < \Lambda_c < 1.5$ only for $V_0 \leq 3000$ km s^{-1}. At velocities $V_0 > 3000$, there is a serious deficiency of low-luminosity galaxies ($L_c \geq 4.0$) compared to the lower velocity group. The possible bias introduced in our calibration solutions by inclusion of the high velocity galaxies will be considered in detail in §§ III*d*, *e*.

c) The Form of the Calibration Formula

In Paper I we inferred a simple formula for the dependence of inner ring diameters on F, T, and L_c. At least for spirals later than Sa ($T \geq 2$),

$$\log D_r = a_0 + a_1 F + a_2(T-4) + a_3(L_c - 3) \quad (3)$$

adequately represents the dependence of the ring diameters on the structural properties of the galaxy. Here F is the "family index," introduced in Paper I and coded as in § II above.

We also showed in Paper I that ringed galaxies classified by de Vaucouleurs (1963) with the refined symbolism S$\underset{\sim}{\text{A}}$B ($F = -0.5$) and SA$\underset{\sim}{\text{B}}$ ($F = +0.5$) had dimensionless parameters X and Y which were on the average intermediate between those for galaxies classified simply as SA, SAB, and SB. In the present calibration we will largely neglect these intermediate steps because such refinements are not coded and in most cases are not available for galaxies in the RC2. We will, however, take them into account when available in the determination of distances to be compiled in Paper III.

Although equation (3) with constant coefficients was suggested as a suitable formula in Paper I, the residuals from this relation have revealed that the coefficient a_3 may depend slightly on family. We will, therefore, consider a more complicated version of equation (3) to account for this effect. At the same time, the requirement that L_c be available restricts the number of galaxies to which the calibration could be applied to essentially those listed in the Shapley-Ames catalog (van den Bergh 1960*c*). However, because T and L are correlated (de Vaucouleurs 1977, 1979*a*), a two-parameter formula, i.e., with $a_3 = 0$, is also a useful representation applicable to galaxies whose luminosity class is still unknown.

Finally, we checked in two ways that the linear dependences of $\log D_r$ on F, T, and L_c expressed by equation (3) are satisfactory within the scatter of the data. First, linear diameters were calculated from the $\mu_0^w(\Lambda_c)$ and $\mu_0^w(V_M)$ distance moduli for the rings of all or most of the ~180 galaxies in our sample having both $T \geq 2$ and L_c. In addition, approximate distances were calculated from the redshift V_0 and the apparent mean Hubble ratio H^* in different directions (de Vaucouleurs and Bollinger 1979). For each of the three different sets of distance moduli, the numerical mapping technique of Jones *et al.* (1967) was used to determine a six-term polynomial of the form

$$\log D_r^0 = \alpha_1 + \beta_2 T + \gamma_3 L_c + \delta_4 T^2 + \varepsilon_5 T L_c + \zeta_6 L_c^2,$$

where $\log D_r^0 = \log D_r - 0.15F$ (see § III*d*) is the logarithmic ring diameter reduced to the standard family $F=0$ (SAB). For the $\mu_0^w(\Lambda_c)$ and $\mu_0^w(V_M)$ calibrations, only the first three coefficients of the linear terms of the polynomial were found to be significant; plots of the standard deviation e_k of the residuals versus the number of coefficients k showed no improvement with the inclusion of the nonlinear terms in T and L_c. With the $\mu_0(H^*)$ calibration, there was some slight improvement in the standard deviation with the inclusion of the nonlinear terms, but the improvement was only marginal and could merely reflect departures from linearity in the local velocity field. Hence, the validity of the simple linear expression adopted in equation (3) is confirmed. This, of course, does not rule out a dependence of the ring diameters on some other parameters, such as variety (§ III*d*).

A second check of the validity of equation (3) was obtained from the least-squares solutions themselves. Plots of the reduced diameters $\log D_r' = \log D_r - a_1 F - (a_2 - a_3)T$ versus corrected luminosity index Λ_c showed a linear relationship (Fig. 3) except for a small curvature perhaps due to incompleteness at $\Lambda_c > 1.2$. Two effects may contribute to the apparent curvature. First, the cutoff in the apparent diameters of inner rings that were measured in the original RC1 survey (the smallest are 0$'$.1 to 0$'$.2 in apparent diameter) must lead to a deficiency of intrinsically small rings; this naturally biases the calibration for low luminosity galaxies ($\Lambda_c > 1.2$). Second, the selection effect is enhanced by the slight but possibly significant difference in the coefficient a_3 between SAB and SB galaxies (see § III*d*). To avoid any possible bias, we excluded those galaxies having $\Lambda_c > 1.2$.

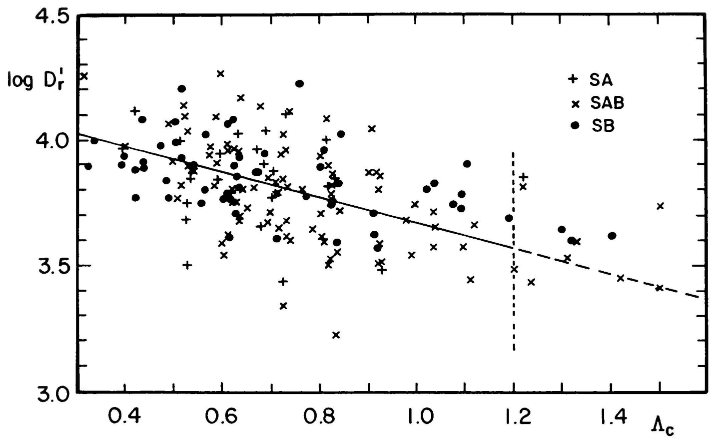

FIG. 3.—Reduced ring diameter, $\log D_r' = \log D_r - 0.15F + 0.05T$, vs. Λ_c for 178 galaxies in the full sample, based on $\langle \mu_0^w \rangle$. The vertical dashed line shows the adopted cutoff at $\Lambda_c = 1.2$.

TABLE 3

COEFFICIENTS OF EQUATION (3)

Calibrating Moduli	a_0 m.e.	a_1 m.e.	a_2 m.e.	a_3 m.e.	N	σ	Remarks
			Full Sample				
$\mu_0^w(\Lambda_c)$	3.630	0.165	−0.105	−0.058	156	0.123	(1)
	±0.011	±0.015	±0.009	±0.008			
$\mu_0^w(V_M)$	3.591	0.156	−0.070	−0.040	134	0.185	(2)
	±0.017	±0.024	±0.016	±0.012			
			Subsample (1)				
$\mu_0^w(\Lambda_c)$	3.617	0.178	−0.104	−0.059	92	0.130	(3)
	±0.015	±0.020	±0.013	±0.011			
$\mu_0^w(V_M)$	3.578	0.137	−0.097	−0.052	66	0.162	(4)
	±0.023	±0.029	±0.022	±0.017			
			Subsample (2)				
$\mu_0^w(\Lambda_c)$	3.620	0.167	−0.102	−0.053	80	0.122	(5)
	±0.015	±0.020	±0.014	±0.010			
$\mu_0^w(V_M)$	3.600	0.100	−0.107	−0.041	57	0.147	(6)
	±0.023	±0.028	±0.021	±0.016			
Adopted.......	3.61	0.15	−0.10	−0.05			
	±0.02	±0.02	±0.015	±0.01			

NOTE.—Full sample consists of all the spirals in the initial sample with $\Lambda_c \leq 1.2$. Subsample (1) includes only the "best observed" galaxies, as defined in the text. Subsample (2) is identical to (1) except that $r\underline{s}$ types are rejected. Divisions by family were coded according to RC2 types ($F = 0, \pm 1$ only).

REMARKS.—(1) After 2σ rejection of NGC 2701, 3177, 3310, 4051, 4389, 4689, 5172, 5248, 5660, and 7217; (2) after 2σ rejection of NGC 1073, 1187, 2815, 2889, 3642, 5660, 6814; (3) after 2σ rejection of NGC 3310, 5248, and 5660; (4) after 2σ rejection of NGC 1073, 3310, 5660, 7184; (5) after 2σ rejection of NGC 3310 and 4725; (6) after 2σ rejection of NGC 1073, 3310, and 7184.

This leaves a well-defined linear relation between the reduced diameter and Λ_c, hence between $\log D_r$ and T and L_c.

d) *Least-Squares Solutions*

The calibration of the ring diameters was performed by means of a standard multivariate linear regression program. Only the $\mu_0^w(\Lambda_c)$ and $\mu_0^w(V_M)$ distance moduli, which have typical mean errors of 0.4–0.5 mag, were used in the analysis. Moduli based on H^* are not used because nearly half of the ringed galaxies in our sample lie in the supergalactic equatorial belt (regions A, B, and C in de Vaucouleurs and Bollinger 1979, Fig. 1) where the velocity-distance relation is anomalous. Table 3 gives the coefficients a_i separately for the Λ_c and V_M calibration sets for the full sample and also for the best observed galaxies only (subsample 1). By "best observed" we mean galaxies for which (i) T or L are not marked "*" or "$" in the RC2; (ii) the galactic extinction, A_B, is ≤ 0.50 mag; (iii) the two components of the modulus based on B_T^0 and $\log D_0$ agree within 1.0 mag; (iv) for the V_M calibration only, the corrected 21 cm line width W^c is well determined, with $\sigma(\log W^c) < 0.10$; and (v) the inclination i derived from $\log R_{25}$ and T (Bottinelli *et al.* 1980) is $\geq 20°$. In addition, the solutions were restricted to $\Lambda_c \leq 1.2$, and in some solutions $r\underline{s}$ rings (de Vaucouleurs 1963) were rejected (subsample 2).

Table 3 shows that the coefficients are remarkably similar for the two different sets of calibrating moduli. After 2σ rejection, $\langle a_2 \rangle = -0.101$ (subsample 1) and -0.105 (subsample 2), while $\langle a_3 \rangle = -0.056$ (subsample 1) and -0.047 (subsample 2). We adopt $a_2 = -0.10$ and $a_3 = -0.05$; the difference between the two coefficients is apparently significant and shows why a residual effect of type was present in the dimensionless analysis of Paper I.

The family coefficient a_1 is somewhat unstable in these solutions. The two different sets of moduli give values of a_1 which only barely agree within the mean errors. The solutions after 2σ rejection give $\langle a_1 \rangle = 0.158$ (subsample 1) and 0.134 (subsample 2). The instability arises mainly in the V_M solutions and is probably due to the small number of nonbarred (SA) galaxies in our sample. It is not due to a dependence of the Tully-Fisher relation on family (de Vaucouleurs *et al.* 1982). On the other hand, the value of a_1 is stable in the Λ_c solutions, with $a_1 = 0.17$ on the average, although the value of 0.13 suggested by the dimensionless analysis of Paper I lies within the range of solutions.

As an aid to deciding on the value of a_1, the coefficients a_{1X} and b_{0X} of equation (6) in Paper I for the X parameter were recalculated over the sample of calibrating galaxies only (those of known L_c). The resulting coefficients, $a_{1X} = -0.15 \pm 0.02$ and $b_{0X} = +0.040 \pm 0.010$, were quite stable not only with respect to 2σ rejection but also in the various subsamples. We believe that the smaller value, 0.13, found in Paper I is affected by a selection effect in the diameter distribution for SA(r) galaxies, which have the smallest rings. The calibration sample, on the other hand, is restricted mainly to the nearer ringed SA types and is not seriously affected by this bias. On this basis, we adopt $a_1 = 0.15$ as a reasonable compromise to reduce the ring diameters to a standard or average galaxy, SAB(r)bcII ($F=0$, $T=4$, $L_c=3$), by the formula

$$\log D_r^c = \log D_r - 0.15F + 0.10(T-4) + 0.05(L_c - 3), \tag{4}$$

which is a good first-order representation.

To check whether any other parameters or selection effects are affecting the ring diameter data, the residuals from equation (4) were expressed as a linear function of logarithmic axis ratio $\log R_{25}$, variety v, and radial velocity V_0:

$$\log D_r^c = a_0 + a_4(\log R_{25} - 0.20) + a_5 v + a_6(V_0 - 2000), \tag{5}$$

where the variety index $v = -1$ for (r) and 0 for (rs). The variety is according to RC2, except in subsample (2), where $r\underline{s}$ types after de Vaucouleurs (1963) are rejected. Table 4, giving the coefficients of equation (5) for the same samples as in Table 3, shows that, on the average, $a_6 \approx 0$, i.e., there is no significant dependence of the reduced ring diameter D_r^c (calculated according to eq. [4]) on radial velocity (and hence distance). This shows that our calibration solution is not significantly affected by a Malmquist-type bias toward larger rings among more distant galaxies. Table 4 also shows that whereas a possibly significant a_5 term (variety) occurs for the $\mu_0^w(\Lambda_c)$ solutions, no such term arises in the $\mu_0^w(V_M)$ solutions. The only term which may be significant in Table 4 is the coefficient a_4 of $\log R_{25}$. Although the coefficient is only barely significant, it has the same sign in both solutions in the sense that the rings tend to appear larger in highly inclined galaxies. This is probably a selection effect, due to the difficulty of detecting small rings in highly inclined galaxies.[3] However, it is possible that the large value of a_4 derived from the V_M calibration may be partly reflecting residual errors in the V_M distance scale, particularly for low inclination galaxies. Because of this we do not believe that introduction of a $\log R_{25}$ term is warranted in the present calibration.

Next the coefficients a_2 and a_3 were tested for any dependence on family by making separate least-squares solutions to the data for each family. No significant dependence was found for the coefficient a_2, but for a_3 a definite trend is present. Figure 4 shows that SB rings are nearly independent of luminosity class (with $a_3 = -0.02 \pm 0.015$), while SAB rings have a stronger dependence ($a_3 = -0.065 \pm 0.015$). This trend may extend to the SA rings (where $a_3 \approx -0.10$ or larger), but the result has a low statistical significance due to the small number of such galaxies in the sample. Nevertheless, the sense of the trend is probably significant, especially since the two independent sets of calibrating distance moduli were found to be in very good agreement on the magnitude of the effect. This indicates that the dependence does *not* arise from a residual error in the Λ_c distance scale.

[3]Note that the largest value of $\log R_{25}$ in our final sample is only 0.50, corresponding to an axis ratio $b/a = 0.3$ and that $\langle \log R_{25} \rangle = 0.16$, compared with 0.22 for all spirals in RC2.

TABLE 4

COEFFICIENTS OF EQUATION (5)

Calibrating Moduli	a_0 m.e.	a_4 m.e.	a_5 m.e.	$10^5 a_6$ m.e.	N	σ	Remarks
			Full Sample				
$\mu_0^w(\Lambda_c)$	3.658	0.179	+0.041	−1.40	157	0.121	(1)
	±0.013	±0.083	±0.020	±0.80			
$\mu_0^w(V_M)$	3.618	0.403	+0.040	+2.06	137	0.194	(2)
	±0.022	±0.153	±0.034	±1.49			
			Subsample (1)				
$\mu_0^w(\Lambda_c)$	3.646	0.132	+0.039	−1.63	90	0.119	(3)
	±0.017	±0.110	±0.026	±1.03			
$\mu_0^w(V_M)$	3.594	0.429	−0.003	+1.54	68	0.173	(4)
	±0.029	±0.197	±0.043	±2.11			
			Subsample (2)				
$\mu_0^w(\Lambda_c)$	3.656	0.096	+0.051	−1.27	79	0.114	(5)
	±0.018	±0.112	±0.026	±1.03			
$\mu_0^w(V_M)$	3.609	0.359	+0.014	+1.89	58	0.155	(6)
	±0.028	±0.187	±0.041	±1.99			

NOTE.—See note to Table 3 for explanation of sample and subsamples.

REMARKS.—(1) After 2σ rejection of NGC 2701, 3177, 3310, 4536, 4689, 4725, 5172, 5660, and 7217; (2) after 2σ rejection of NGC 2889, 3642, 5660, and IC 1953; (3) after 2σ rejection of NGC 3177, 3310, 4725, 5248, and 5660; (4) after 2σ rejection of NGC 3310 and 5660; (5) after 2σ rejection of NGC 3310, 4725, and 5016; (6) after 2σ rejection of NGC 1073 and 3310.

A redetermination of the coefficients a_1 to a_3 in equation (3) with a term of the form $a_3 = a_{30} + a_{31}F$, accounting for the F dependence of a_3, verified the trend suggested by the separate solutions. Solutions based on $\langle \mu_0^w \rangle$ only are summarized in Table 5. From this the reduced ring diameter corrected for this second-order effect could be defined as

$$\log D_r^c = \log D_r - 0.17F + 0.10(T-4) + (0.065 - 0.045F)(L_c - 3), \qquad (6)$$

but it produces only a very slight improvement on equation (4).

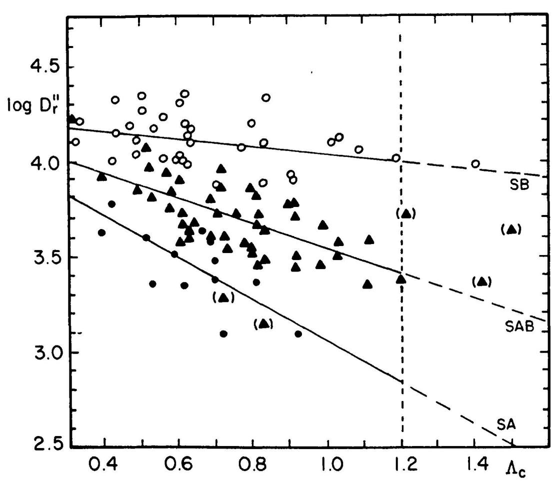

FIG. 4.—Reduced ring diameter, $\log D_r'' = \log D_r + 0.035T + 0.045FT$, as a function of Λ_c for subsample (1). The graph is constructed in a manner similar to Fig. 3, and illustrates the family dependence of the coefficient a_3.

The results of the residual analysis are summarized in Figure 5 for the V_M and Λ_c calibrations separately. It shows the standard deviation $\sigma(\log D_r)$ of the residuals from the solutions as each additional term F, T, L_c, v, V_0 or $\log R_{25}$ is added to the analysis. The two curves show conclusively that only the first three terms, F, T, and L_c, significantly improve the residuals in both cases. The other three terms make only marginal or no significant improvement.

Lastly, the coefficients a_1, a_2, and a_3 were tested for possible dependence on variety (r vs. rs) or radial velocity V_0. No dependence was found in either case, which confirms that a single relation is applicable to both varieties and that the coefficients are not seriously affected by the selection effects that can arise in a magnitude-limited sample.

e) Zero Point and Errors of the Three-Parameter Formula

The logarithms of the linear ring diameters reduced to $F=0$, $T=4$, and $L_c=3$ via equation (4) are plotted in

TABLE 5

COEFFICIENTS OF EQUATION (6)[a]

Sample	a_0	a_1	a_2	a_{30}	a_{31}	N	σ	Remarks
Full	3.608 ±0.012	0.169 ±0.017	−0.104 ±0.011	−0.064 ±0.009	0.044 ±0.013	156	0.138	(1)
(1)	3.597 ±0.016	0.178 ±0.022	−0.107 ±0.014	−0.068 ±0.012	0.044 ±0.015	93	0.135	(2)
(2)	3.606 ±0.016	0.147 ±0.021	−0.113 ±0.014	−0.060 ±0.012	0.036 ±0.015	80	0.122	(3)
Adopted ...	3.61 ±0.015	0.17 ±0.02	−0.10 ±0.015	−0.065 ±0.012	0.045 ±0.015			

[a]Based on the weighted average of the $\mu_0^w(\Lambda_c)$ and $\mu_0^w(V_M)$ moduli, for the same samples as in Table 3. See text for explanation of the weighting scheme.

REMARKS.—(1) After 2σ rejection of N237, 2701, 2815, 3310, 3893, 4689, 5172, 5660, and 7217; (2) N3310,5660 rejected; (3) N3310,7184 rejected.

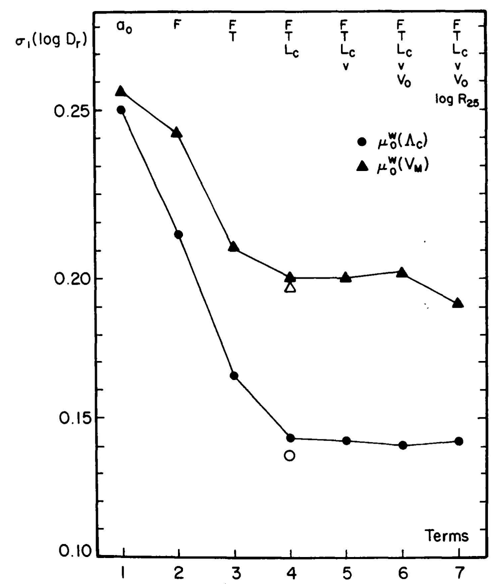

FIG. 5.—Standard deviation of the residuals of the log D_r calibration vs. number of terms used in least-squares solutions. Subsample (1) was used for this analysis, and the results for the two different tertiary distance scales, $\mu_0^w(\Lambda_c)$ and $\mu_0^w(V_M)$, are illustrated separately. The open symbols show the improvement in the solutions with F, T, and L_c only when the F dependence of a_3 is accounted for. Both curves show that only the terms in F, T, and L_c are significant.

Figure 6 for subsample (1) as a function of mean distance modulus $\langle\mu_0^w\rangle$ based on V_M and Λ_c. With the exception of three discrepant objects (NGC 3177, 3310, and 5660),[4] the reduced ring diameters are essentially constant over the 4.7 mag interval of distance moduli. This is verified by a least-squares solution, which for the illustrated sample yields

$$\log D_r(0,4,3) = \underset{\pm 0.014}{3.611} - \underset{\pm 0.014}{0.000}\,(\langle\mu_0^w\rangle - 31.2), \quad (7)$$

with a standard deviation $\sigma = 0.132$ for $N = 92$ objects. In this solution double weight was assigned to galaxies for which both $\mu_0^w(V_M)$ and $\mu_0^w(\Lambda_c)$ are available, while single weight was given to those with only $\mu_0^w(\Lambda_c)$. Restriction to subsample (2) yields a slightly lower value, 3.604, for the zero point, while use of all of the data with $\Lambda_c \leq 1.2$ (no further restrictions) yields 3.616. Adopting 3.61 as a reasonable compromise, the (logarithmic) mean reduced ring diameter is $\langle D_r^c\rangle = 4.07 \pm 0.13$ kpc, and the distance modulus is given simply by

$$\mu_0^r = 35.73 - 5\log d_r^c, \quad (8)$$

where $\log d_r^c \equiv \log d_r(0,4,3)$ is the logarithm of the *apparent* diameter (units of 0′.1) of the ring reduced according to equation (4).

An estimate of the precision of individual distance moduli calculated from equation (8) may be obtained by comparing μ_0^r with $\mu_0^w(V_M)$, which does not depend on F, T, or L_c. As will be shown in Paper III, there is no scale error between μ_0^r and $\mu_0^w(V_M)$ so that the errors can be approximately derived from $\sigma(\Delta\mu_0)$, where $\Delta\mu_0 = \mu_0^r - \mu_0^w(V_M)$. The well-defined rings (i.e., excluding $r\underline{s}$ types) with the best input data (no * or $ on T or L_c), as compared to the best V_M moduli (as previously defined), give $\sigma(\Delta\mu_0) = 0.74$ (after rejection of NGC 1073, 3310, and 7184). If $\sigma[\mu_0^w(V_M)] \approx 0.4$ mag, then $\sigma(\mu_0^r) = 0.62$ mag. For the same subsample of galaxies, but including only those with uncertain types (* or $ on T or L), $\sigma(\Delta\mu_0) = 1.1$, indicating that $\sigma(\mu_0^r) \approx 1.0$ mag. A

[4]NGC 3310 may not be a normal ringed galaxy. Recent studies (Balick and Heckman 1981, and references therein) suggest that it is a merger remnant, and that the ring may be only a transient phenomenon.

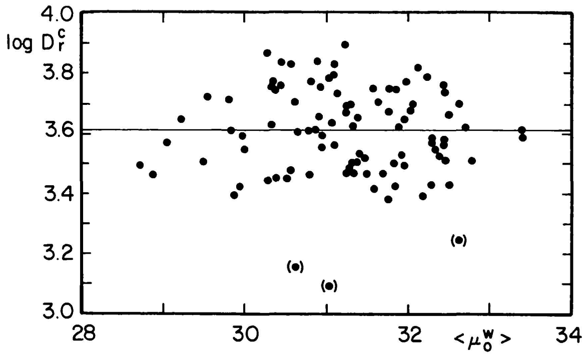

FIG. 6.—Standard ring diameter, $\log D_r^c = \log D_r(0,4,3)$, reduced via eq. (4), as a function of mean distance modulus $\langle \mu_0^w \rangle$ for galaxies in subsample (1). There is no significant dependence of the constant on distance, indicating that there is no serious Malmquist bias in the sample.

more detailed analysis of the errors from all sources is presented in § V.

IV. VARIATION OF RING DIAMETER WITH STAGE T ALONG THE SPIRAL SEQUENCE

Because the luminosity class of many galaxies has not yet been evaluated, it is useful to have a calibration involving F and T only. This is possible not only because T and L are correlated, but also because it is important to tie in the diameters of rings in early-type galaxies ($T<2$) to those in later-types ($T \geq 2$). For the later types we can still use $\mu_0^w(V_M)$ and $\mu_0^w(\Lambda_c)$, but for the early types there is as yet no reliable distance scale based on the same zero point. For this reason, we had to calibrate ring diameters in early-type galaxies by means of ringed galaxies which appear to be members of nearby groups that include other (spiral) galaxies having V_M and Λ_c moduli. This will insure that our distance calibration has the same zero point across the entire Hubble sequence.

a) *Distances to Ringed Galaxies Having $T<2$ ($S0^0$ to Sa)*

The distances to early-type ringed galaxies were determined by their association with spirals in well-documented nearby groups. Of the 114 galaxies with $T<2$ in our primary sample (used in Paper I), 50 had enough information to be assigned to one of about 30 groups, including many from the 54 groups of de Vaucouleurs (1975). Additional information on possible group membership was obtained from the extensive analysis by Corwin (1967). For each ringed galaxy suspected to be in a group, the redshift was used as the principal criterion of probable membership. When a ringed galaxy with $T<2$ could be assigned to a group with a reasonable degree of confidence, we then checked whether any of the other probable members had a $\mu_0^w(\Lambda_c)$ or $\mu_0^w(V_M)$ distance modulus. In many cases several of the spiral members had such moduli available so that reasonably precise mean distance moduli could be obtained. These are summarized in Table 6 which lists the name of the ringed galaxy, its radial velocity V_0, the group name, the mean radial velocity of the group and number of redshifts, the average Λ_c modulus $\langle \mu_0^w(\Lambda_c) \rangle$, the average V_M modulus $\langle \mu_0^w(V_M) \rangle$, and the weighted mean modulus $\langle \mu_0^w \rangle$. Weights were assigned as follows: for the Λ_c moduli, $w=1/2$ when either T or L was marked * or $ in the RC2, $w=1/4$ if both were so marked; otherwise unit weight was assigned. In the V_M moduli, only galaxies which had $\sigma(\log W^c)<0.10$ and $i \geq 20°$ were used with unit weight. In both cases, galaxies for which $\mu_0(B_T^0)$ and $\mu_0(\log D_0)$ differed by more than 1.00 mag were rejected. A few galaxies in the V_M set which had only $\log D_0$ moduli were assigned $w=1/3$. The total weights used to calculate the mean modulus $\langle \mu_0^w \rangle$ are given in Table 6 (note that the redshift was used only as evidence of group membership, but not to evaluate the distance).

b) *Two-Parameter Calibration in Terms of F and T*

Figure 7 shows the mean logarithmic ring diameter $\langle \log D_r^0 \rangle$ (reduced to $F=0$) derived from $\langle \mu_0^w \rangle$ for all types $-2 \leq T \leq 7$. The sample involved is summarized in Table 2B. Because the distance moduli derived for the early-type galaxies are essentially on the same zero point as those for the spirals with $T \geq 2$, the maximum between $T=2$ and 3 and the falloff for $T \leq 2$ must be real. This falloff in the diameters of inner rings parallels the behavior of absolute magnitudes and linear diameters of galaxies; both quantities show a similar maximum in the intermediate spiral stages (although perhaps closer to $T=4$; see de Vaucouleurs 1977, Fig. 2). This could partly explain why the reduced dimensionless ratios X' and Y' from Paper I could be well represented by a straight line over the same range of types, $-2 \leq T \leq 7$. The only difference in our present analysis is that the data now seem to warrant a family term for the lenticulars, whereas we neglected such an effect in Paper I because it appeared that the dimensionless ratios for the lenticulars were not significantly sensitive to family. We have since determined that this was partly caused by the inclusion in the sample of some LA galaxies where the feature measured as an inner ring is actually an outer ring. This is almost certainly the case for the galaxies NGC 3267, 3269, and 4429, all of which have low X values and rings which are large and diffuse. Because outer rings are considerably larger than normal inner rings in nonbarred galaxies, the inclusion of such rings acts to bias the diameter distribution for LA(r) galaxies and mask a family effect. Each galaxy in the present calibration sample was inspected on the Sky Survey plates, and except for NGC 4429, each was verified to have a true inner ring. In the case of NGC 4429, we are

TABLE 6

DISTANCE MODULI FOR RINGED GALAXIES EARLIER THAN Sab

Galaxy Name	T	V_0	Group Name and Number	$\langle V_0 \rangle$ m.e.	$\sigma_1(V_0)$ N	$\langle \mu_0^w(\Lambda_c) \rangle$ W	n	$\langle \mu_0^w(V_M) \rangle$ W	n	$\langle \mu_0^w \rangle$ m.e.	Remarks
NGC 0936	−1	1350	Cetus I	1426	204	30.80	5	30.58	3	30.71	
			G15	55	14	4.00		3.00		0.12	
NGC 1201	−2	1630	Eri I	1538	190	31.01	9	30.48	5	30.80	
NGC 1302	0	1626	G31	36	28	7.50		5.00		0.18	
NGC 1371	+1	1397									
NGC 1461	−2	1357									
NGC 1317	+1	1918	For I	1489	282	...		31.26	1	31.26	
NGC 1326	−1	1167	G53	66	18			1.00		...	
NGC 1437	+1	1078									
NGC 1358	0	3878	Eri II	4059	215	32.18	3	32.67	2	32.43	
				57	14	2.00		2.00		0.20	
NGC 1433	+1	802	NGC 1433	807	143	29.56	1	30.06	2	29.89	
NGC 1512	+1	558	G21	43	11	1.00		2.00		0.20	
NGC 1553	−2	1064	NGC 1566	1013	219	...		29.16	1	29.16	
			G16	66	11			1.00		...	
NGC 2217	−1	1243	NGC 2217	1514	162	30.80	2	31.03	4	30.95	(1)
				72	5	2.00		4.00		0.15	
NGC 2646	−2	3704	NGC 2523	3793	160	32.62	2	32.59	1	32.61	
				72	5	1.25		1.00		0.01	
NGC 2681	0	760	NGC 2841	602	115	30.06	3	30.20	3	30.13	
			G6	47	6	3.00		3.00		0.19	
NGC 2811	+1	2260	Hya IB	2277	112	31.70	1	32.57	1	32.40	(2)
NGC 3081	−1	2146	Hya IB	35	10	0.25		1.00		0.35	
NGC 2844	+1	1477	NGC 2712	1695	159	33.10	1	32.19	2	32.44	(3)
				80	4	0.50		1.33		0.36	
NGC 2855	0	1660	Hya IA	1859	98	32.08	3	32.06	2	32.07	(2)
NGC 2983	−1	1752		30	11	2.00		2.00		0.15	
NGC 2859	−1	1657	NGC 2964	1491	128	31.87	3	30.96	3	31.23	
			G42	45	8	1.25		3.00		0.33	
NGC 2950	−2	1451	NGC 2768	1605	180	32.20	2	31.58	2	31.85	
			G41	64	8	1.50		2.00		0.25	
NGC 3185	+1	1147	NGC 3190	1243	134	32.46	2	31.72	2	32.04	
			G47	45	9	1.50		2.00		0.42	
NGC 3489	−1	577	NGC 3627	619	76	29.83	1	29.34	2	29.44	
			G9	34	5	0.50		2.00		0.17	
NGC 3626	−1	1363	NGC 3607	1171	218	31.52	2	30.86	2	31.19	
			G49	82	7	2.00		2.00		0.21	
NGC 3637	−2	1649	NGC 3672	1550	159	31.06	1	31.30	1	31.18	
NGC 3892	−1	1532	G23	71	5	1.00		1.00		0.12	
NGC 3729	+1	1117	UMa IN	1081	117	31.25	5	30.92	5	31.09	
			G34	32	13	5.00		5.00		0.15	
NGC 3945	−1	1340	NGC 4125	1413	97	31.48	1	...		31.48	(4)
				49	4	1.00				...	

TABLE 6 —*Continued*

Galaxy Name	T	V_0	Group Name and Number	$\langle V_0 \rangle$ m.e.	$\sigma_1(V_0)$ N	$\langle \mu_0^w(\Lambda_c) \rangle$ W	n	$\langle \mu_0^w(V_M) \rangle$ W	n	$\langle \mu_0^w \rangle$ m.e.	Remarks
NGC 4138	−1	1090	UMa IS	934	195	31.07	7	30.66	9	30.77	
NGC 4220	−1	1052	G32	44	20	3.50		9.00		0.13	
NGC 4245	0	882	Coma IA	784	92	31.46	3	30.75	2	31.14	(5)
NGC 4314	+1	879	G13	29	10	2.50		2.00		0.23	
NGC 4324	−1	1602	Vir II X_A	1311	300	30.74	10	30.29	7	30.53	(6)
NGC 4580	+1	1187	G26	83	13	8.25		7.00		0.20	
NGC 4643	0	1234									
NGC 4385	−1	2015	Vir II X_B	2188	197	30.99	4	32.35	1	31.29	(6)
			G26	74	7	3.50		1.00		0.45	
NGC 4454	0	2220	Vir II V	2482	311	31.89	4	32.08	3	32.00	
				127	6	2.25		3.00		0.11	
NGC 4340	−1	844	Vir E	1000	400	...		...		30.50	(7), (8)
NGC 4371	−1	898	G18	60	42						
NGC 4429	−1	1029									
NGC 4459	−1	1039									
NGC 4596	−1	1882									
NGC 4608	−2	1758									
NGC 5448	+1	2100	NGC 5322	2148	127	32.28	2	31.48	2	31.70	
				37	12	0.75		2.00		0.25	
NGC 5544	0	3292	NGC 5395	3586	258	32.92	1	33.27	4	33.22	
				86	9	0.50		3.33		0.15	
NGC 5701	0	1579	Vir III	1623	157	31.02	7	30.86	11	30.91	
				38	17	5.25		11.00		0.14	
NGC 7020	−1	2915	Pavo-Indus	2889	56	31.90	2	...		31.90	(9)
			(3000)	19	9	2.00				0.54	
			G45								
NGC 7576	−1	3729	NGC 7585	3683	217	...		32.11	1	32.11	
				109	4			0.50		...	
IC 5240	+1	1476	Grus	1580	159	31.59	4	31.38	2	31.51	
			G27	39	17	3.50		2.00		0.26	

REMARKS.—(1) Tabulated as a member of the NGC 2207 group (G36) by de Vaucouleurs (1975), this galaxy could instead be the brightest member of a group in the foreground. There are two redshift groups among the members listed by Corwin (1967), one at $\langle V_0 \rangle = 1514 \pm 162$ and another at $\langle V_0 \rangle = 2466 \pm 83$. We have assigned NGC 2217 to the nearer group, although its membership is still somewhat uncertain. (2) There are at least four redshift groups among the members listed by Corwin (1967). We refer to Hya IA as the group with the lowest $\langle V_0 \rangle$, and Hya IB as the group with the second lowest $\langle V_0 \rangle$. (3) Doubtful group. (4) Inner pseudo-ring measurement listed by de Vaucouleurs and Buta (1980*b*) was uncertain. Revised dimensions of $1'.58 \times 1'.22$ based on a photograph given by Kormendy (1979) are preferred. (5) Two redshift groups are present among the members listed by de Vaucouleurs (1975) and Corwin (1967), one at $\langle V_0 \rangle = 784 \pm 92$ and another at $\langle V_0 \rangle = 1128 \pm 94$. Coma IA refers to the lower redshift group, while Coma IB refers to the higher redshift group. Note that the Λ_c and V_M distance moduli give no indication that the high redshift group is more distant than the low redshift group. (6) Members as listed by Corwin (1967) occupy two redshift groups, one at $\langle V_0 \rangle = 1311 \pm 300$, and the other at $\langle V_0 \rangle = 2188 \pm 192$. V_M and Λ_c moduli confirm that the higher redshift group is more distant than the nearer one. (7) Members were assigned by de Vaucouleurs and de Vaucouleurs (1973). The distance modulus is an average of values obtained by de Vaucouleurs (1977) (30.4), Visvanathan (1978) (30.6), Hanes (1979) (30.7), and Michard (1979) (30.5). (8) NGC 4429—the feature tabulated by de Vaucouleurs and Buta (1980*b*) as an inner ring is probably an outer ring. A possible true inner ring, of the kind associated with a dark dust crescent, is visible on an image tube plate taken by J. Wray. The dimensions of this feature are $0'.55 \times 0'.26$. (9) The Pavo-Indus Cloud (de Vaucouleurs 1956) is a complicated region with several redshift groups. The ringed lenticular NGC 7020 is associated with the subgroup at $\langle V_0 \rangle \approx 3000$.

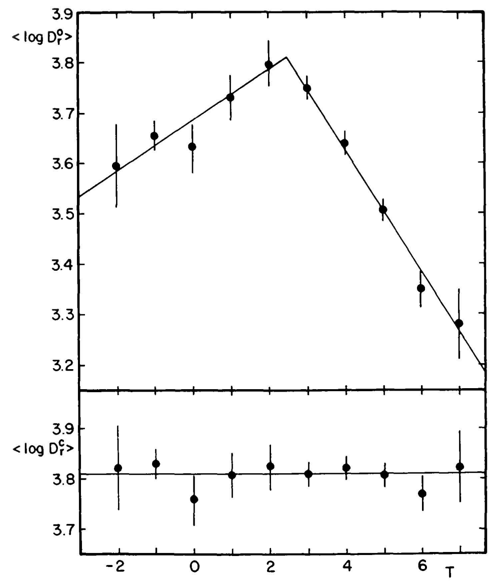

FIG. 7.—*Upper section*: mean values of the family reduced ring diameter, $\log D_r^0 = \log D_r - 0.15F$, as a function of stage for $-2 \leq T \leq 7$. *Lower section*: mean values of the fully reduced diameter, $\log D_r^c = \log D_r(0, 2.5)$, reduced via eq. (10), over the same range of types.

certain now that the feature tabulated as (r) in the RC1 and in our measurement catalog is actually an outer ring (R). On the other hand, there does seem to be a true inner ring in this galaxy which is associated with a dark dust crescent in the bulge; its dimensions are given in the notes to Table 6. With the data in Table 6, a significant family effect is present at stage $T = -1$. For the LB galaxies, $\langle \log D_r \rangle = 3.82 \pm 0.03$, while for the LA galaxies, $\langle \log D_r \rangle = 3.53 \pm 0.08$ (after rejection of NGC 4459). The difference of nearly 0.3 indicates a similar value for the family coefficient in equation (3), $a_1 = 0.15$, as was found for the spirals. For this reason, we have applied this correction across the entire range $-2 \leq T \leq +7$, although at $T = -2$ the data are insufficient to conclusively prove that the effect is still present.

As shown in Figure 7 the variation of $\log D_r^0$ with Hubble stage is linear and of the form

$$\log D_r^0 = c_0 + c_1(T - 2.5) \tag{9}$$

in each of the two type intervals $-2 \leq T \leq 2$ and $3 \leq T \leq 7$ with a common zero point at $T = 2.5$. Table 7A gives values of c_0 and c_1 for a subsample of well-observed galaxies (defined as before). The solutions were weighted according to the sum of the weights used to calculate $\langle \mu_0^w \rangle$, except for the early types ($T \leq 2$), where a maximum weight of 2 was assigned in order not to bias the solutions by a few points with excessive group weights. Table 7A shows that the zero points (at $T = 2.5$) for the two different type intervals agree very closely within the mean errors: for $3 \leq T \leq 7$, $c_0 = 3.818 \pm 0.027$, while for $-2 \leq T \leq 2$, $c_0 = 3.804 \pm 0.044$. With $c_0 = 3.81$ and fixed at this value, the slope c_1 was rederived for each interval of types. The solutions (Table 7B) show that ring diameters in the early types have a similar scatter to those in the late types. The relation

$$\log D_r^c(0, 2.5) = \log D_r - 0.15F - c_1(T - 2.5), \tag{10}$$

where $c_1 = 0.05$ for $-2 \leq T \leq 2$ and -0.12 for $3 \leq T \leq 7$, reduces the diameters to constant type and family.

Figure 8 shows the logarithmic ring diameters reduced to $F = 0$ and $T = 2.5$ via equation (10). As expected from the neglect of the L_c term, the scatter is

TABLE 7[a]

COEFFICIENTS OF EQUATIONS (9) AND (10)

T Range	c_0 m.e.	c_1 m.e.	N	σ	2σ Rejections
		A. Coefficients of Equation (9)			
$-2 \leq T \leq 2$ ...	3.804	+0.043	64	0.169	NGC 1358, 2681, 4459, and 4580
	±0.044	±0.016			
$3 \leq T \leq 7$ ...	3.818	−0.124	162	0.159	NGC 470, 1068, 3177, 3310, 3455,
	±0.027	±0.012			5172, 5660, 7137, and I239
		B. Coefficient c_1 of Equation (10)			
$-2 \leq T \leq 2$ ...	3.81	+0.049	63	0.160	NGC 1358, 2681, 4459, 4580, 7020
		±0.007			
$3 \leq T \leq 7$ ...	3.81	−0.122	162	0.156	NGC 237, 470, 1068, 3177, 3310,
		±0.006			3455, 5172, 5660, 7137, and
					IC 239

[a]Sample of galaxies involved in the $2 \leq T \leq 7$ range was restricted to those with known B_T^0, $A_B \leq 0.50$ mag, $\sigma(\log w^c) < 0.10$, $i \geq 20°$, and $|\mu_0(\Lambda_c \text{ or } V_M, B_T^0) - \mu_0(\Lambda_c \text{ or } V_M, \log D_0)| \leq 1.00$ mag. Similar restrictions were applied in the calculation of the mean group moduli used for the range $-2 \leq T < 2$.

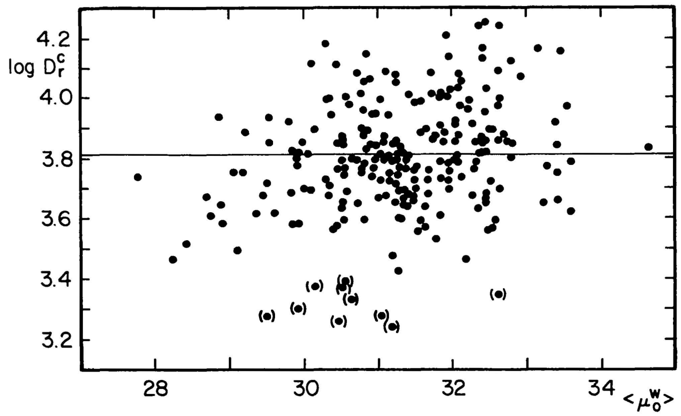

FIG. 8.—Corrected ring diameter, $\log D_r^c = \log D_r(0,2.5)$, reduced according to eq. (10), as a function of $\langle\mu_0^w\rangle$ for 240 spiral and lenticular galaxies. The 10 aberrant galaxies with $\log D_r^c < 3.40$ are NGC 1068, 2681, 3177, 3310, 3455, 4459, 4580, 5660, 7137, and IC 239.

somewhat larger than in Figure 6, which is based on the three-parameter solution. The scatter is also larger because the sample of galaxies in Figure 7 includes those with uncertain or doubtful types (it was necessary to relax this restriction in the calculation of the mean Λ_c group moduli). The most conspicuous feature of this graph is the set of 10 galaxies with unusually low values of $\log D_r^c$. These include a few peculiar objects, such as NGC 1068 and 3310, both of which have abnormally small inner rings; in several others the rings are not well defined. The presence of these objects shows that extreme cases do exist and that inner rings as classified in RC2 are not entirely homogeneous. If rings are in general related to galactic resonances, as is suggested by most recent theoretical work (e.g., Duus and Freeman 1975; Schwarz 1979; Huntley 1980), then these extreme cases may reflect unusual pattern speeds or atypical mass distributions. In any case, these galaxies require further detailed study.

Figure 8 also shows that there is only a very slight dependence of the reduced ring diameter on distance. This is indicated by a least-squares solution, which, after 2σ rejection of the 15 objects listed in Table 7B (including NGC 1659), yields

$$\log D_r^c(0,2.5) = \underset{\pm 0.010}{3.810} + \underset{\pm 0.010}{0.030}\,(\langle\mu_0^w\rangle - 31.25). \tag{11}$$

The positive sign of the slope is such that the reduced ring diameter increases slightly with distance, as might be caused by a magnitude selection effect (Malmquist bias). As a check on the sign and magnitude of the effect, the coefficients in equation (11) were recalculated using distances determined from the velocity-distance relations in different directions (de Vaucouleurs and Bollinger 1979) but with $\log D_r^c$ calculated again from $\langle\mu_0^w\rangle$. This insures that there are no strong regression effects between $\log D_r^c$ and the corresponding distance for a given galaxy. The result (after rejection of the same galaxies),

$$\log D_r^c(0,2.5) = \underset{\pm 0.010}{3.810} + \underset{\pm 0.009}{0.014}\left[\mu_0(H^*) - 31.33\right], \tag{12}$$

shows a considerably weaker and nonsignificant dependence of the zero point on distance. The slight residual trend could be due to a combination of the neglect of the L_c term in the calibration for the spirals and the fact that L_c depends slightly on distance in our calibration sample, for which $\langle L_c\rangle = 4.10$ for $\langle\mu_0^w\rangle < 30.5$ ($N = 31$) and 2.46 for $\langle\mu_0^w\rangle \geq 32.30$ ($N = 37$). Because T and L are correlated (de Vaucouleurs 1979*a*), part of the neglected L_c dependence is actually absorbed in the coefficient of stage T, but apparently not enough to remove the trend in Figure 8 altogether. Since the adopted zero point, 3.81, is appropriate for the middle range of the distances ($\langle\mu_0^w\rangle \approx 31.5$), we retain it as a compromise. Then the distance modulus to a ringed galaxy with only F and T available will be given by

$$\mu_0^r = 36.73 - 5\log d_r^c, \tag{13}$$

where $\log d_r^c = \log d_r(0,2.5)$ is the apparent logarithmic ring diameter reduced according to equation (10).

As with the three-parameter formula, an estimate of the precision of individual distance moduli calculated from equation (13) may be obtained by comparing them with corresponding values of $\mu_0^w(V_M)$. Considering the range $2 \leq T \leq 7$ only, where a direct comparison can be made, a comparison of distance moduli from the best defined rings (excluding *r<u>s</u>* types) and input stages (no * or $ on T) with the best V_M moduli as defined in § III*e* gives $\sigma(\Delta\mu_0)=0.79$, implying that $\sigma(\mu_0^r)\approx0.68$ mag. For the exact same set of galaxies used to define subsample (2) in the three-parameter solution, $\sigma(\Delta\mu_0)=0.80$, or $\sigma(\mu_0^r)\approx0.70$. When the analysis is repeated for the separate families (with restrictions, no * or $ on T, no $r\underline{s}$ types), then $\sigma(\Delta\mu_0)=0.89$, 0.81, and 0.69 mag, corresponding to $\sigma(\mu_0^r)=0.80$ for 16 SA, 0.70 for 42 SAB, and 0.56 for 38 SB galaxies, respectively. Thus, the two-parameter formula is in general poorer than the three-parameter formula, especially for SA and SAB galaxies where the effect of luminosity class may be most important.

V. PROPAGATION OF ERRORS IN THE RING CALIBRATION

The errors in individual distance moduli calculated from equations (8) and (13) can also be estimated from the errors in the input parameters. Thus,

$$\sigma^2(\mu_0^r)=\sigma^2(A)+25\left[\sigma^2(\log d_r)+(0.15)^2\sigma^2(F)+(0.10)^2\sigma^2(T)+(0.05)^2\sigma^2(L_c)\right]$$

for equation (8), while

$$\sigma^2(\mu_0^r)=\sigma^2(A)+25\left[\sigma^2(\log d_r)+(0.15)^2\sigma^2(F)+(0.12)^2\sigma^2(T)\right]$$

for equation (13) (spirals with $T\geq2$ only), where A is the zero point of the ring distance scale. A discussion of the errors in the T and L parameters has been given by de Vaucouleurs (1979*a*). In the best cases, estimates of the T parameter have $\sigma(T)\approx0.7$, while estimates of the L parameter have $\sigma(L)$ in the range 0.6 to 0.8 (de Vaucouleurs, de Vaucouleurs, and Corwin 1978). Since the error due to the inclination correction to L is small, we take $\sigma(L_c)=0.7$ as representative. The errors in T and L then contribute errors of 0.35 and 0.18 mag, respectively, and 0.39 mag together, in the moduli derived from equation (8); the error in T correspondingly contributes an error of 0.42 mag in the moduli derived from both equations (8) and (13) is 0.30 mag. Apart from questions of interpretation, there are at least two sources of error in the apparent ring diameter d_r. The first is due simply to measuring errors, and estimates of these errors in several different sets of independent ring diameter measurements are given in the Appendix. From that analysis we adopt, as representative of the combined data to be used in Paper III, $\sigma(d_r)=0\overset{'}{.}10$, which corresponds to $\sigma(\log d_r)\approx0.04$ for a ring $\sim1'$ in diameter, the approximate average diameter of inner rings in our diameter catalog. The second source of error in d_r is due to the neglect of the possible ellipticity of inner rings, particularly in SB galaxies where it may reach a maximum of 0.2 (axis ratio = 0.8); therefore, the maximum error expected from this source is $\pm10\%$ in d_r or 0.04 in $\log d_r$. The rms error due to neglect of this effect may then be $\sigma(\log d_r)\approx0.015$, corresponding to 0.07 mag in the distance moduli. The total accidental mean errors (exclusive of zero point errors) of the distance moduli are then 0.54 mag from equation (8) and 0.56 mag from equation (13).

The mean error of the zero point of equation (8) is $\sigma_0=5\sigma(a_0)\approx0.10$ mag, exclusive of the zero point error $\sigma_0=0.20$ mag of the tertiary calibrators, or 0.22 mag inclusive. For equation (13), the mean error of the zero point for the spirals is $\sigma_0=5\sigma(c_0)\approx0.15$ mag, exclusive of the zero point errors of the tertiary calibrators, or 0.25 mag inclusive. Therefore, the expected external total mean errors of individual distance moduli are $\sigma(\mu_0^r)\approx0.58$ mag from equation (8) and 0.61 mag from equation (13). Both values agree closely with those (0.62 mag for eq. [8] and 0.68 mag for eq. [13]) determined from the comparison with $\mu_0^w(V_M)$ in §§ III*e* and IV*b*. We conclude that $\sigma(\mu_0^r)=0.6$ mag is a realistic estimate of the true mean error of the distance moduli to be derived in Paper III, for the best observed galaxies.

VI. CONCLUSIONS

We have established that inner ring diameters are useful distance indicators which are only slightly less precise than the main tertiary indicators. Because of the intermediate scale of the reduced ring diameter ($D_r^c=$ 4 kpc), rings can be detected to distances in excess of 40 Mpc, and the larger SB rings could, in principle, be detected out to the distance of the Hercules cluster (A2151) at $\Delta\approx100$ Mpc (Corwin 1971, 1977). We have shown that SB rings are better distance indicators than SA or SAB rings. Of particular interest is the indication that SB rings depend more weakly on luminosity class than SA and SAB rings, which increases their value as distance indicators. We do not have a good explanation as to why the coefficient a_3 in the calibration with luminosity class depends on family F; however, it is clear that the trend is not caused by residual errors in the tertiary distance scales used since the Λ_c and V_M moduli are in agreement on the magnitude of the effect.

When F, T, and L_c are available, ring diameters yield distance moduli which have errors $\sigma(\mu_0^r)\approx0.6$ mag in the best cases. When only F and T are available, the typical errors will be $\sigma(\mu_0^r)\approx0.7$ mag. Much larger errors may occur in some objects because of identification uncertainties. The strong dependence of ring diameters on family F constitutes the most serious drawback of the rings as distance indicators, since it requires that this qualitative parameter be estimated as accurately as possible. Although the calibration formulae apply strictly to RC2 types, they can be used without change of coefficients to refined families ($F=\pm0.5$) when available. The distance scales derived from the rings and from other indicators will be compared in Paper III.

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APPENDIX A

COMPARISON OF INDEPENDENT RING MEASUREMENTS

A recent study by Pedreros and Madore (1981) has made available new, independent diameter measurements of inner rings in nearly 150 galaxies, about two-thirds of which are in common with those in the catalog of de Vaucouleurs and Buta (1980*b*). In addition, both sets of data have galaxies in common with those measured by Kormendy (1979). The availability of three independent sets of ring diameter measurements allows a triangular comparison which leads to a direct determination of the mean errors in each set. To estimate the combined errors, it is sufficient to determine the standard deviation from the mean regression lines. Preliminary solutions of the form $D_1 = e_0 + e_1 D_2$ showed that $e_0 = 0$ within the mean errors and that the errors are independent of D; hence, the relation between the two systems is merely one of proportionality. Solutions of the form $D_1 = e_1 D_2$ give:

$$D_{\mathrm{P}} = \underset{\pm 0.011}{1.005}\, D_{\mathrm{V}} \qquad (N=92),$$

with $\sigma_{\mathrm{PV}} = 0'.137$, after rejection of NGC 1659, 2347, and 7217;

$$D_{\mathrm{P}} = \underset{\pm 0.017}{1.005}\, D_{\mathrm{K}} \qquad (N=18),$$

with $\sigma_{\mathrm{PK}} = 0'.099$; and

$$D_{\mathrm{K}} = \underset{\pm 0.013}{1.007}\, D_{\mathrm{V}} \qquad (N=42),$$

with $\sigma_{\mathrm{KV}} = 0'.119$, after rejection of N1353, where P=Pedreros, V=de Vaucouleurs, and K=Kormendy.

In all three cases the coefficient does not depart significantly from unity. Since the measuring errors in each set are independent, the combined errors σ_{ij} give immediately the individual errors

$$\sigma_{\mathrm{P}} = 0'.085, \quad \sigma_{\mathrm{V}} = 0'.108, \quad \text{and} \quad \sigma_{\mathrm{K}} = 0'.051;$$

the latter is in good agreement with the accuracy claimed by Kormendy (1979). The ring diameters rejected from the above solutions were determined to refer to different features in the galaxies. For example, the ring feature in NGC 7217 measured by Pedreros is not the true inner ring of this galaxy but is instead associated with a lens formation between the inner and outer rings. The same kind of disagreement can be ascribed to confusion between the ring and lens in NGC 2347, while in NGC 1659 and 1353 the ring features measured by Pedreros and Kormendy may refer to false rings mimicked by the outer spiral structure. When these few discrepant cases are rejected, the errors are satisfactorily small for all three sets but do differ significantly from set to set. It is obvious that the RC1 ring diameter measurements which were merely by-products of large galaxy classification surveys are less precise than those of Kormendy and of Pedreros who made special efforts to measure as precisely as possible small selected samples of rings. Weighted means of separate measures where available will be used for the final determinations of distances to be compiled in Paper III.

APPENDIX B

FAMILY INDEPENDENCE OF ISOPHOTAL DIAMETERS

In Paper I, Table 13, we presented data suggesting equality of mean isophotal diameters D_0 in the three families SA, SAB, SB. In that table the values of $\log D_0(1) = \log D_0 + 0.60(\Lambda_c - 1)$ were calculated using $\mu_0^w(\Lambda_c)$ from Table 1 of de Vaucouleurs (1979*b*), where $\mu_0^w(\Lambda_c)$ is in part derived from the apparent value of $\log D_0(1)$; this circular argument weakens the evidence. It is preferable to use an independent distance indicator, such as the velocity-distance relation in different directions (de Vaucouleurs and Bollinger 1979). We have, therefore, repeated the calculations, using values of the Hubble ratio H^* appropriate to different sky areas. Excluding the Virgo cluster and galaxies with $V_0 < 500$ km s^{-1}, we find $\langle \log D_0(1) \rangle = 4.130 \pm 0.015$ for 119 SA galaxies, with a dispersion $\sigma_1 = 0.164$, 4.167 ± 0.014 for 135 SAB ($\sigma_1 = 0.158$), and 4.129 ± 0.016 for 120 SB ($\sigma_1 = 0.177$), respectively, after 2σ rejection of a few aberrant objects in each case. The result confirms our conclusion that isophotal galaxy diameters are substantially independent of family, but the errors are more realistic.

APPENDIX C

SYMBOLS AND THEIR DEFINITIONS

For the convenience of the reader, we provide here in tabular form a listing of definitions of the main symbols used in this paper.

A_B	The galactic extinction correction in blue light, taken from RC2.
A_e	The effective aperture which transmits half the total blue light flux, also from RC2.
$a_0,\ldots,a_3$	Least-squares coefficients in the three-parameter fit.
B_T^0	Total (or asymptotic) B magnitude, corrected for galactic extinction, internal absorption, and redshift.
c_0, c_1	Least-squares coefficients in the two-parameter fit.
D_0	Isophotal diameter at the $\mu_B = 25.0$ mag arcsec^{-2} level, corrected for galactic extinction and inclination.
$D_{\rm K}$	Ring diameter measurement from Kormendy (1979).
$D_{\rm P}$	Ring diameter measurement from Pedreros and Madore (1981).
$D_{\rm V}$	Ring diameter measurement from de Vaucouleurs and Buta (1980*b*).
D_r	Linear ring diameter in pc.
$D_r^c(0,4,3)$	Ring diameter in pc corrected to $F=0$, $T=4$, and $L_c=3$ via equation (4).
$D_r^c(0,2.5)$	Ring diameter in pc corrected to $F=0$ and $T=2.5$ via equation (10).
$d_r^c(0,4,3)$	Apparent ring diameter in units of $0\overset{\prime}{.}1$ corrected to $F=0$, $T=4$, $L_c=3$ via equation (4).
$d_r^c(0,2.5)$	Apparent ring diameter in units of $0\overset{\prime}{.}1$ corrected to $F=0$ and $T=2.5$ via equation (10).
D_r^0	Ring diameter in pc reduced to $F=0$ only.
D_r'	Ring diameter in pc reduced to $F=0$ and corrected partly for type such that the residual dependence is a function of Λ_c, via equation (4).
D_r''	Same as D_r', except that equation (6) is used and family dependence is not removed.
F	The family index, i.e., -1 for SA, 0 for SAB, and $+1$ for SB galaxies.
L_c	Luminosity class corrected for inclination via the logarithmic axis ratio $\log R_{25}$ (see de Vaucouleurs, de Vaucouleurs, and Corwin 1978).
M_{T^0}	Total absolute blue magnitude.
R	Symbol for outer ring structure.
R_{25}	Isophotal axis ratio at $\mu_B = 25.0$ mag arcsec^{-2} level, taken from RC2.
r	Pure inner ring.
rs	Pseudo-inner ring.

T	Hubble stage coded in numerical form (see RC2).
V_0	Radial velocity corrected for solar motion with respect to the velocity centroid of the Local Group (from RC2) (IAU Commission 28 convention).
V_M	Maximum rotation velocity as determined from the 21 cm line width.
v	The variety index, i.e., -1 for (r) and 0 for (rs) galaxies.
W^c	21 cm line width corrected for turbulence.
Λ_c	Corrected luminosity index defined as $(T+L_c)/10$.
μ_0	Denotes a true (fully corrected) distance modulus.
$\mu_0(H^*)$	Distance modulus determined from the ratio V_0/H^*, where H^* is the value of the Hubble parameter appropriate to different sky directions (de Vaucouleurs and Bollinger 1979).
μ_0^r	The ring diameter distance modulus, calculated from equation (8) or (13).
$\mu_0^w(\Lambda_c)$	Weighted mean distance modulus from tertiary indicators B_T^0, $\log D_0$, and luminosity index Λ_c (de Vaucouleurs 1979*b*).
$\mu_0^w(V_M)$	Weighted mean distance modulus from B_T^0, $\log D_0$, and V_M (Bottinelli *et al.* 1980).
$\langle\mu_0^w\rangle$	Mean of both $\mu_0^w(\Lambda_c)$ and $\mu_0^w(V_M)$, when available for a given galaxy.
σ	Used exclusively to denote mean error. For example, $\sigma(\mu_0^r)$ is the mean error of a single distance modulus based on the diameter of an inner ring and the morphological classification of the galaxy.

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