MODEL-BASED ESTIMATION ON BUILDING ENVELOPE INFILTRATION

by

ZHENGWEN HAO

ZHENG O’NEILL, COMMITTEE CHAIR
KEITH WOODBURY
FEI HU

A THESIS

Submitted in partial fulfillment of the requirements for the degree of Master of Science in the Department of Mechanical Engineering in the Graduate School of The University of Alabama

TUSCALOOSA, ALABAMA

2019
ABSTRACT

Buildings consumed nearly 40% of total energy in the U.S. Air leaks through the building envelope are one of the factors increasing building energy consumption. The estimated energy use associated with infiltration loss through the building envelope within residential and commercial buildings in the United States for the year 2010 is 4 quads annually, which accounts for nearly 10% of the total energy use in buildings. The U.S Department of Energy published a building technologies program air leakage guide. In this air leakage guide, the DOE proposed five requirements for infiltration measurement method. However, two commonly used infiltration diagnostic approaches, blower door test and tracer gas method, are unable to meet the DOE requirement. Since existing infiltration diagnostic approaches do not meet with the DOE requirements, the building infiltration measurement is a challenge. To address the current challenges, a scalable and low-cost Building Infiltration Estimator with Ultrasonic Thermometry (BLAST) is proposed. The proposed method contains the physical measurements and a model-based estimation. This study is focusing on the model-based estimation part.

To estimate building infiltration, a building envelope heat transfer model has to be developed. Recent study shows that a low-ordered three resistance-two capacitance (3R2C) thermal network model is sufficient to describe the building envelope heat transfer. A customized 3R2C thermal network is developed to represent building envelope heat transfer. Based on the 3R2C thermal network, the energy balance equation for building envelope has been applied to get the state-space model. The state-space differential equation is one of the key points to determine the suitable
estimation method. This study uses an Extended Kalman filter (EKF) to inversely estimate the building infiltration using measurements of surface temperature and total heat flux, and a low-order state-space model. An EnergyPlus-based emulator is used to generate a virtual building and measurements to test the proposed estimation method. Nearly 80% estimated infiltration resistances are within the 20% error band compared to the calculated infiltration resistance from EnergyPlus. This preliminary study shows the EKF based estimator with the proposed measurements is promising for building infiltration estimation.
### LIST OF ABBREVIATIONS AND SYMBOLS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>HVAC</td>
<td>Heating, Ventilation, and Air Conditioner</td>
</tr>
<tr>
<td>RC</td>
<td>Resistance – Capacitance</td>
</tr>
<tr>
<td>EKF</td>
<td>Extended Kalman Filter</td>
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<td>UKF</td>
<td>Unscented Kalman Filter</td>
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<tr>
<td>DOE</td>
<td>Department of Energy</td>
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<tr>
<td>ASTM</td>
<td>American Society for Testing and Materials</td>
</tr>
<tr>
<td>UT</td>
<td>Unscented Transform</td>
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<tr>
<td>BLAST</td>
<td>Building Infiltration Estimator with Ultrasonic Thermometry</td>
</tr>
<tr>
<td>ABIMS</td>
<td>Acoustic Building Infiltration Measurement System</td>
</tr>
<tr>
<td>ARX</td>
<td>Autoregressive Exogenous</td>
</tr>
<tr>
<td>ARMAX</td>
<td>Autoregressive-moving-average model</td>
</tr>
<tr>
<td>VAV</td>
<td>Variable Air Volume</td>
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<tr>
<td>ODE</td>
<td>Ordinary Differential Equation</td>
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</table>
NOMENCLATURE

Nomenclature

T    Temperature
R    Thermal resistance
C    Thermal capacitance
q    Heat flux

Subscripts

is   Inside surface
os   Outside surface
in   In zone
inf  infiltration
win  window
ACKNOWLEDGMENTS

I would first like to thank my advisor Dr. Zheng O’Neill at the Department of Mechanical Engineering at the University of Alabama. She helped me go through my master program. She constantly answered me questions and guided me to the end.

I would also like to thank my thesis committee members, Dr. Keith Woodbury and Dr. Fei Hu. I appreciate their committed service, supports, ideas, and valuable comments on the thesis improvement.

My appreciation also extends to my colleagues at the University of Alabama High Performance Building Laboratory, both present and former, namely, Fuxin Niu, Yanfei Li, Liu Liu, and Zhihong Pang for their support and help in direct or indirect ways.

Financial support from the American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) through the ASHRAE 2017 Innovation Research Grant is greatly appreciated.

Finally, I must express my very profound gratitude to my parents for providing me with unfailing support. They offered me advice during the most difficult time, even though the tough time is endless. They brought me hopes when I felt depressed and helped me overcome countless difficulty during my study in this master program.

The accomplishment would not have been possible without the support and effort from them. Thank you.
## CONTENTS

ABSTRACT ..................................................................................................................................... ii

LIST OF ABBREVIATIONS AND SYMBOLS ........................................................................ iv

NOMENCLATURE ....................................................................................................................... v

ACKNOWLEDGMENTS ............................................................................................................. vi

LIST OF TABLES .......................................................................................................................... x

LIST OF FIGURES ....................................................................................................................... xi

1. INTRODUCTION ................................................................................................................... 1

2. LITERATURE BACKGROUND REVIEW ........................................................................... 5

  2.1 Ongoing Infiltration Diagnostic Research ................................................................. 5

  2.2 Building Models From The Past ...................................................................................... 6

    2.2.1 White Box Model ........................................................................................................... 6

    2.2.2 Black Box Model ............................................................................................................ 6

  2.3 Hybrid Model ........................................................................................................................ 8

    2.3.1 Energy Balance Model ................................................................................................... 8

    2.3.2 Resistance-capacitance (RC) model ............................................................................... 9

  2.4 Estimation Method .............................................................................................................. 13
2.4.1 Genetic Algorithm (GA) .................................................................................................................. 13
2.4.2 Discrete Kalman Filter ...................................................................................................................... 15
2.4.3 Extended Kalman Filter .................................................................................................................. 15
2.4.4 Unscented Kalman Filter ................................................................................................................ 16
3. METHODOLOGY AND APPROACH ........................................................................................................ 19
  3.1 Overall Description .............................................................................................................................. 19
  3.2 Building Model .................................................................................................................................. 19
    3.2.1 EnergyPlus Emulator .................................................................................................................... 19
    3.2.2 Overall RC model ......................................................................................................................... 23
    3.2.3 Test RC Model and Energy Balance Equation ............................................................................... 25
    3.2.4 Building Thermal Resistance .................................................................................................... 26
  3.3 Estimation Model .................................................................................................................................. 29
    3.3.1 State-space Equation ..................................................................................................................... 29
    3.3.2 Extended Kalman Filter ................................................................................................................. 31
    3.3.3 Filter Design .................................................................................................................................. 34
    3.3.4 Constraint ...................................................................................................................................... 37
4 RESULTS AND DISCUSSIONS .................................................................................................................... 39
  4.1 Simulated Results ................................................................................................................................. 39
    4.1.1 Solar Air Temperature ................................................................................................................... 39
    4.1.2 Infiltration Resistance .................................................................................................................. 40
4.2 Simulated Results ................................................................................................................ 42

4.3 Discussion ........................................................................................................................... 47

5 FUTURE WORK .................................................................................................................. 49

5.1 Use air temperature estimate building infiltration ............................................................. 49

5.2 Cases with windows and cases in different locations. ........................................................ 51

5.2 Unscented Kalman Filter .................................................................................................. 51

5.3 Unmeasured Disturbance .................................................................................................. 52

REFERENCES ............................................................................................................................. 56
LIST OF TABLES

Table 3-1 Wall material characteristic by layers ................................................................. 27
Table 3-2 Wall conduction thermal resistance by layers ....................................................... 28
Table 3-3 Wall thermal capacitance by layers ..................................................................... 29
Table 4-1 Statistic metric for infiltration resistance estimation ........................................... 46
LIST OF FIGURES

Figure 1–1 The flow chart for BLAST ........................................................................................................... 3
Figure 2–1 ABIMS infiltration method [6]........................................................................................................... 5
Figure 2–2 Heat transfer in a multi-zone system [13] ...................................................................................... 9
Figure 2–3 Schematic for heat balance through a wall [15] ........................................................................... 10
Figure 2–4 Thermal network for building [18] ............................................................................................... 11
Figure 2–5 RC model for building surface [19]............................................................................................... 12
Figure 2–6 Node placement of testing room [20] ........................................................................................... 12
Figure 2–7 An example of genetic algorithm procedure [16] ....................................................................... 14
Figure 2–8 An example of the schematic of the EKF estimator [16] .............................................................. 16
Figure 2–9 Jacobian-based and UT-based mean and covariance comparison [24] ........................................ 17
Figure 3–1 Building outlook in Google Sketchup .......................................................................................... 20
Figure 3–2 Floor plan of the simulated building ........................................................................................... 21
Figure 3–3 EnergyPlus Infiltration Object .................................................................................................... 22
Figure 3–4 The RC network for the building envelope .................................................................................. 23
Figure 3–5 The proposed 3R2C model for the testing wall ......................................................................... 25
Figure 3–6 Side view of the simulated building with the selected wall surface ........................................... 27
Figure 3–7 Extended Kalman filter update process .................................................................................... 33
Figure 4–1 Outside wall construction in the EnergyPlus ............................................................................. 39
Figure 4–2 Solar absorptivity of the outside layer .......................................................................................... 39
Figure 4–3 Calculated solar air temperature from the EnergyPlus................................. 40
Figure 4–4 Simulated whole year infiltration resistance ................................................. 41
Figure 4–5 Infiltration resistance used for this study....................................................... 42
Figure 4–6 Outside surface temperature estimation ....................................................... 43
Figure 4–7 Building infiltration resistance estimation..................................................... 44
Figure 4–8 Count of the estimation error........................................................................ 45
Figure 5–1 RC model with additional disturbance ......................................................... 53
Figure 5–2 estimation results comparison between conventional modeling, adding unmeasured disturbance modeling, and true value ................................................................. 54
1. INTRODUCTION

Buildings consumed 38.61% total energy consumed in the U.S. According to a recent Department of Energy (DOE) report, the estimated energy use associated with the infiltration loss through the building envelope in the U.S. for the year of 2010 is 4 quads annually, which accounts for nearly 10% of the total energy use in buildings [1]. Building infiltration increases total energy consumptions of both existing building and new construction. In general, air leaks through windows and opaque building envelope elements of building enclosures. Infiltration diagnostic technologies can be used to establish the extent of infiltration in an existing building or verify the performance of new construction. According to the DOE, the infiltration measurement techniques should meet the criteria as follows:

1. Suitable for all building types
2. Usable in occupied buildings
3. Accurate regardless of outdoor weather conditions
4. Low-effort for setup
5. Capability to quantify both the location and extent of infiltration [2].

Currently, there are two most commonly used infiltration measurement techniques, which are blower door testing and tracer gas method. Blower door testing mounts a fan into an existing exterior door. The purpose of that fan is to lower the inside air pressure. Due to the higher outside air pressure, the air flows through all unsealed leakages. A frame with a flexible panel,
which fits the doorway, a variable-speed fan, and a pressure gauge, which is used for measuring pressure differences, are three main components of the blower door infiltration techniques. The variable-speed fan pulls the air to the outside, which lowers the air pressure inside the building. Because the indoor air pressure is lower, the air will flow through the building envelope crack or the unsealed wall. The infiltration rate can be measured due to the imbalance of the air pressure between the inside and the outside [3]. It is easy to establish a blower door testing for small commercial buildings and homes. The blower door method mounted a fan into an existing door, and the large commercial building may have many exterior doors. It is hard to apply it to large commercial buildings and industrial facilities. Also, a blower door testing can only quantify air leakages, but cannot identify the location of the air leakage. Moreover, a blower door testing needs to set up at the building pathway, which may create some difficulties for the normal operations in the testing buildings.

The tracer gas method is a direct measurement of air infiltration. There are two types of tracer gas testing: single zone test and multi-zone test. The single zone test measures the whole house or building air exchanges, and the multi-zone test measures the air flow from the outside room by room. The gas used for such testing should be non-toxic, colorless, inert, and stable. As a result, the commonly used tracer gas includes Carbon Dioxide, Nitrous Oxide, Freon, Helium, and Sulfur Hexafluoride [4]. Several types of equipment are needed to accomplish the tracer gas test, which includes a zone mixing fan and a space heater. The decay rate of tracer gas is used to determine the infiltration rate. According to ASTM E741 [5], the air should be maintained well mixed, and heaters are used to control the room temperature during the test. The singe zone tracer gas test can measure the infiltration when the air handler is on, or the ventilation system is operated. Similar to the blower door method, the tracer gas testing also has some limitations. For
the single zone tracer gas test method, the testing result is heavily based on the test day weather condition, because the tracer gas properties may be affected by the weather condition. Also, since the pressure difference used to calculate the infiltration could be caused by the duct leakage, it is difficult to separate the infiltration from the duct leakage. The multi-zone tracer gas infiltration test will provide an overall view of the building infiltration together with the infiltration rate for individual zone under a specific operation or specific weather conditions.

Neither of blower door testing and tracer gas method meets five criteria listed by the DOE. Therefore, a new infiltration diagnostic method needs to be developed. The new infiltration measurement method should meet all five DOE criteria. In summary, measuring and quantifying building envelope infiltration is challenging. The existing infiltration diagnostic technologies normally required significant effort (e.g., blower door testing and tracer gas methods).

To address the preceding challenge, we propose a scalable and low-cost Building Infiltration Estimator with Ultrasonic Thermometry (BLAST) to detect and quantify the infiltration (quantitative location and extent) through the building envelope for residential and commercial buildings, empowered by inverse modeling coupled with ultrasonic thermometry. The flow chart for the proposed BLAST procedure is shown in Figure 1–1. The proposed approach consists of two steps: measurement and estimation. Total heat flux and outside surface temperature are measured using ultrasonic sensors. A model-based estimator then takes these two measurements to estimate the infiltration through data fusion using Extended Kalman Filter (EKF).

![Figure 1–1 The flow chart for the BLAST procedure](image)
This thesis focuses on the second step of a model-based estimation, which is the red box in Figure 1–1. An EnergyPlus-based emulator is used to generate simulation data to test the proposed estimator. The simulation output data is used as either known inputs or measurements to the estimator. For example, the building related data (e.g., geometry) from the EnergyPlus is used to construct the building envelope heat transfer model. This building envelope heat transfer model should be able to be represented in a form of differential equations. Differential equations then can be written in the form of state-space equations, which are governing equations of the model used in the proposed estimation method of EKF. Finally, the proposed model-based estimator of EKF will estimate some unknown states including the infiltration related variables.

This work used the whole building simulation program EnergyPlus as the emulator to generate inputs and virtual measurements. Two measurements are used in this work: total surface heat flux and outside surface temperature. A thermal resistance and capacitance (RC) network is used to develop a set of state-space equations, which is used as governing equations of the underlying model for the extended Kalman filter. The infiltration resistance is estimated by the extended Kalman filter. The estimated infiltration resistance is compared with the infiltration resistance calculated based on the EnergyPlus outputs as a validation.

This thesis is organized as follows: Chapter Two is the literature review for the related work, which covers ongoing infiltration diagnostics research, three generic modeling techniques (i.e., white-box model, black-box model, and gray box/hybrid model), and model-based estimation. Chapter Three is methodology and approach. The building model and the estimation method explored in this study are introduced in Chapter Three. Chapter Four introduces the preliminary results. Finally, the future work will be covered in Chapter Five.
2. LITERATURE BACKGROUND REVIEW

2.1 Ongoing Infiltration Diagnostics Research

There are some ongoing building envelope infiltration measurement methods such as acoustic building infiltration measurement system (ABIMS). Figure 2–1 shows the schematics of the ABIMS method.

![ABIMS infiltration method](image)

*Figure 2–1 ABIMS infiltration method [6]*

Unlike blower door and tracer gas methods which depend on the pressure difference between the indoor and outdoor environment, the ABIMS uses the acoustic leakage to estimate infiltration [6]. However, according to 2017 building technologies office peer review [6], the research team currently is still trying to address the following issues:

1) The current testing method cost is higher than what is expected
2) The current method is only able to measure infiltration in the testing lab, it cannot be applied when the building is occupied or under construction.

3) There is a need to derive a relation between the infiltration and the acoustic data.

2.2 Generic Building Models

There are three types of building models, white box model, black box model, and hybrid model. In this section, white box and black box models are briefly reviewed firstly. Both models are commonly used in the building research field, but they are not selected for this study focusing on the state and parameter estimations. Then, the underlying model used in this study, a hybrid model, is reviewed.

2.2.1 White Box Model

One of the traditional models is the white box model. The white box model is based on first principles. Inputs of white box models are actual building location, weather conditions, building physical descriptions, and building mechanical, energy component and system descriptions. For example, the whole building simulation program EnergyPlus is based on the white box model [7]. In the EnergyPlus, the output directly flows through the data simulation engine. Most white box models do not require any state-space equation. It is difficult to directly utilize this type model, such as EnergyPlus, as the underlying model with the selected estimation method of EKF, because it is hard to extract state-space equations from this type model.

2.2.2 Black Box Model

The black box model uses direct inputs and gives direct output like the white box model. However, the black box model in building area is data-driven, which required fewer inputs associated with building physical parameters. In general, it required less effort to construct the
black box model. However, like the white box model, most of the black box models do not require differential equations or state-space equation. An autoregressive-moving-average model (ARMAX) is an example of the black box model. The ARMAX model is capable of catching the relations between system input and output with the consideration of noise in linear systems. Unknown parameters are estimated using the data by the least square method [8]. Park et al. use the ARMAX to construct a building system and parameter identification model. The building system includes a well-insulated room and an electric heater. The system is modeled as a second-order resistance-capacitance (RC) model [9]. Parameters of linear autoregressive exogenous model (ARX) and ARMAX models were obtained and converted to physical parameters through a Laplace transformation. The simulated thermal energy for ARX and ARMAX models were identical to electrical energy used [10]. Lu and Viljanen proposed an ARX model to predict the indoor temperature and relative humidity. The nonlinear autoregressive with exogenous input (NARX) was used to search the optimal network structure for the indoor temperature and relative humidity predictions. The outdoor temperature, outdoor relative humidity, heating power, ventilation rate, and previous indoor air temperature and relative humidity were used as inputs. Input data were collected every 15 minutes for a 30-day period. The current indoor area temperature and relative humidity are variables to be predicted. The prediction showed that the NARX model was able to catch the indoor temperature with a good accuracy [11]. Jimenez used the ARMAX model to identify main building characteristics of building components [12]. The black box model is another example of a traditional model that can be used for building applications, which gives direct inputs and predicts direct outputs. The black box model is not chosen as an underlying model in this model-based estimation study.
2.3 Hybrid Model

However, in most cases, researchers are unable to acquire all building physical data, or unable to find enough data to train the model. Thus, a hybrid model is developed for combining advantages from both the white box model and black box model. In general, a hybrid model utilizes some building physics and limited data. In this study, we use a set of mathematical equations to estimate other unknown parameters. To get the set of mathematical equations, energy balance equations for the HVAC equipment and resistance-capacitance network for the building envelope are widely used.

2.3.1 Energy Balance Model

Liao and Dexter used energy balance equations to estimate average air temperatures in a multi-zone heating system. An overall structure of a multi-zone heating system includes three major parts: a boiler, a water distribution system, and the building [13]. One system can have more than one boiler. Water circulates through the building. The hot water is supplied from the boiler and distributed to the radiator, and cooled down and becomes the cold water. The cold water returns to the boiler for heating. The radiator maintains and controls air temperature in the building. The heat transfer process in one zone includes the convection from the hot water to the radiator, the convection from the radiator to the inner layer of the building envelope, the convective heat exchange between the inner layer of the building envelope and the air, the conduction between the inner and outer layer of the building envelope, the infiltration through windows, and the solar radiation. The schematic of heat transfer in a multi-zone system is shown in Figure 2–2.
2.3.2 Resistance-capacitance (RC) model

Another commonly used method is the resistance-capacitance (RC) model. The RC network is used to derive a set of first order differential equations. Braun and Chaturvedi developed a 3R2C thermal network model, which is able to estimate the building load. [14] The 3R2C building envelope model correlates the zonal air temperatures with ambient air temperature. In general, the resistances cover outside convection resistances, wall conduction resistances, inside convection resistances, and window thermal resistance. Zheng et al. applied a heat balance for the building envelope. The schematic of the heat balance is shown in Figure 2–3. [15]
Figure 2–3 Schematic for heat balance through a wall [15]

The heat balance on the building envelope can be represented as a 3R2C model. Unknown parameters in the 3R2C model can be estimated by a set of first order differential equations. O’Neill et al. used a 3R2C model to estimate the thermal internal load for a zone in a classroom and office building [16]. O’Neill and Narayanan also applied the RC network to a supermarket refrigeration system to estimate the cold room temperature [17]. Wang and Xu developed an RC network for both building envelope and internal mass. The schematics of a thermal network model for the building is shown in Figure 2–4 [18].
Ogunsola et al. used a 3R2C model combined with a 2R2C model to estimate a real-time thermal load. Two virtual thermal nodes were introduced to capture thermal mass effects. In their study, a new RC model for the building envelope was also introduced, which considers the solar radiation effect. Solar radiation affects building heat transfer through imposing the heat flux on the surface [19]. In addition, the convection and conduction heat transfer through the window and infiltration through the window are affected by the air temperature difference between the ambient and in-zone. The RC network used by Ogunsola et al. is shown in Figure 2–5.
Research shows that a higher quality model can give a better estimation accuracy. Kim and Park proposed a complex RC model to estimate a process disturbance in a building space [20]. The internal load was defined as the sum of instantaneous thermal loads excepted for the load caused by building envelope. As a result, the heat gain from people, equipment, lighting, heat gain/loss from infiltration by windows, and occupants, and heat gain from the air movement between zones are considered as internal loads. They proposed an RC structure with windows for estimating the building internal load, which can be used for infiltration estimation. The RC network describes an outside wall with a window as an 8 node RC system. Figure 2–6 shows the node for the testing room.
After models used for model-based estimations are developed, another crucial step for model-based estimation is an appropriated estimation method. Four estimation methods are introduced in the following section.

2.4 Estimation Method

2.4.1 Genetic Algorithm (GA)

Wang and Xu used a genetic algorithm to estimate the building internal mass. The GA is used to estimate the resistance and capacitance in the proposed 2R2C system. Resistances and capacitances of the 2R2C system are optimized parameters [18]. GA is generally robust in finding a global optimal solution. It starts with an initial estimate with four parameters under the assumed range. In GA, four parameters are the chromosome and the assumed ranges are search spaces. Initially, four parameters produce the initial population to start a GA run. The GA run will be terminated if the number of the current generation is equal to the maximum number of range [18]. Two GA runs are necessary to accurately estimate unknown variables. Figure 2–7 shows the GA procedure used this paper.
The GA estimator predicts the cooling load for all data points with an acceptable accuracy. This shows a simplified 2R2C thermal network with the genetic algorithm is capable of estimating the cooling load by using the short-term data. However, the genetic algorithm has some limitations. First, the evaluation function must reflect the system accurately. A wrong choice of evaluation function will be unable to estimate unknown variables, and unable to find the optimal parameters for the system. Secondly, the initial data affects the optimization process, and the genetic algorithm is sensitive for the initial data. An initial set which closes to true values will result in better optimal parameters. Finally, the quality of the genetic algorithm result is affected by the problem size. Using the genetic algorithm to estimate a larger problem will generally lead to a lower estimation accuracy.
2.4.2 Discrete Kalman Filter

Kim et al. used a Kalman filter to estimate the internal load in buildings. The Kalman filter uses a set of differential equations to estimate the state of the process [20]. The filter was developed to estimate the state $x \in \mathbb{R}^n$ of a discrete-time with the linear differential equation, inputs $u$, process noise covariance $w$, measurements, and measurement noise covariance $v$ [22]. The filter can estimate unmeasured states of the modeled system with given inputs and measurements. The estimation process can be divided into two processes: time update and measurement update. The time update equation estimates the a priori estimate for the next time step by using the current state and an error covariance. The measurement update equation is a feedback, which incorporates a new measurement into the a priori estimate to get a posteriori estimate [22, 23]. Kim proposed that an internal aggregated load is the sum of instantaneous loads except for the load due to the building envelope. The internal load covers the heat gain from people, equipment, lighting, and infiltration by operating windows [20].

2.4.3 Extended Kalman Filter

O’Neill et al. used the extended Kalman filter (EKF) to estimate the internal load in buildings. Unlike the discrete Kalman filter, which, can only handle a linear system, the EKF is capable of estimating the states and/or parameters of a nonlinear system [16]. The Jacobian matrix is used for the linearization process of the EKF. The schematic of the EKF estimator used by O’Neill et al. is shown in Figure 2–8:
The EKF uses a Jacobian matrix to linearize a nonlinear system. The Jacobian matrix is the first partial derivative of states. Compared to other advanced linearization processes, the Jacobian approach provides a linearization with sufficient accuracy. However, for some state-space matrices, it is impossible to get the first order Jacobian matrix. The Jacobian matrix is able to linearize a non-linear system with good accuracy, but there are better linearization processes available in the literature such as unscented transform.

2.4.4 Unscented Kalman Filter

Radecki and Hencey proposed using Unscented Kalman filter (UKF) to estimate thermal resistance and capacitance of a building wall. The EKF used a Jacobian matrix to linearize a nonlinear system [24]. The first order linearization process could introduce large errors in the posterior mean and covariance [25]. On the other hand, the UKF takes advantages of the
unscented transform (UT) to calculate the statistics of a random variable. The comparisons of mean and covariance propagation between a UT-based approach and a Jacobian-based approach are shown in Figure 2–9.

![Figure 2–9 Jacobian-based and UT-based mean and covariance comparison [25]](image)

The purple circle in Figure 2–9 shows that using the Jacobian linearized system creates a larger covariance and the mean is not the same as the original value. While, the unscented transform gives nearly the same mean and covariance compared to the original data, which is the green circle in Figure 2–9.

Unlike the EKF using the first order linearization through the Jacobian matrix, the UKF uses the unscented transform, which is a higher order differentiation. This gives the UKF a better
estimation accuracy. However, the underlying model must be well designed, otherwise, the filter will diverge during the update processes.
3. METHODOLOGY AND APPROACH

3.1 Overall Description

In this study, the underlying building envelope heat transfer model is a set of state-space equations which are derived from the thermal resistance and capacitance (RC) network. The RC network model is able to simulate the building dynamics better than the black models [26]. It has been recently used for simulating the transient building load prediction [14] and control algorithms testing [27]. An EKF based estimator using 3R2C thermal network model for infiltration estimation are presented. First, a low-order, state-space 3R2C model of building envelope dynamics and the underlying filter theory will be presented, then an EnergyPlus-based emulator will be used to generate simulation data to test the proposed estimator. The preliminary testing results illustrate the feasibility and acceptable accuracy of the proposed model-based estimation approach.

3.2 Building Model

3.2.1 EnergyPlus Emulator

An EnergyPlus-based emulator is used to provide the simulated data to test the proposed estimator. In this emulator virtual environment, measurements used in the estimator will come from the virtual sensor (i.e., outputs) from EnergyPlus instead of actual physical sensors.

For this study, the measurements are not come from physical measurement directly. Instead, measurements for this study come from the EnergyPlus emulator. The EnergyPlus
provides necessary measurements, building physical parameters, and other parameters used in model based estimation. For this study, a small commercial building contains three office rooms, one waiting room, on store, and one working room is constructed in Google Sketchup with OpenStudio plugin. The Google Sketchup with OpenStudio is a free software that allows researchers to construct building envelope and export to EnergyPlus IDF file for energy simulation. Figure 3–1 shows the building outlook in Google Sketchup.

![Building outlook in Google Sketchup](image)

*Figure 3–1  Building outlook in Google Sketchup*

Figure 3–2 shows the top view of the small commercial building and the building floor plan.
The infiltration of one exterior wall in the working room of a small commercial building is estimated in this study. In this study, inputs and outputs from EnergyPlus are used as the data source (e.g., virtual measurement) for the proposed estimator. This small commercial building has a variable air volume (VAV) system. For this VAV system, it has DX AHU with a gas burner. Each thermal zone has its own VAV box and its own thermostat. The thermostat with a dual-setpoint for the working room, where the testing wall is located, is set to 19.4 °C. The building infiltration is set as 0.4 air change per hour (ACH) for each zone. In EnergyPlus, the object “ZoneInfiltration: DesignFlowRate” has been used to add the infiltration for the emulated building. EnergyPlus has outputs for the infiltration associated to each wall. Figure 3–3 shows the infiltration object in the EnergyPlus.
As shown in Figure 3–3, the building infiltration is set as always on, which building has infiltration all the time.

The EnergyPlus provides the necessary output for this study. The following EnergyPlus variables are outputs for this study:

1) Zone mean air temperature
2) Outdoor air dry bulb temperature
3) Inside surface solar radiation heat gain rate
4) Inside face temperature
5) Outside face temperature
6) Outside face convection heat gain rate
7) Inside face solar radiation heat gain per area
8) Zone total infiltration heat gain energy
9) Zone total infiltration heat loss energy
10) Wall inside face convection heat transfer coefficient
11) Outside face convection heat transfer coefficient
12) Outside face solar radiation heat gain rate

13) Outside face solar radiation heat gain rate per area

Outputs (8) and (9) are used to calculate the building infiltration, output (2), (6), (11), and (13) are related to inputs for the estimator. Other outputs are used as references for the initial conditions. Chicago O’Hare TMY3 weather file was used for this simulation. The time step was set to be 2-minute. We are assuming that during a given hour, the weather data is the same.

3.2.2 Overall RC model

A low-order, state-space model of building envelope dynamics based on non-linear algebraic and differential equation formulation is used for estimating unknown parameters and states. For the building envelope, the resistance-capacitance (RC) network is shown in Figure 3–4

![Figure 3–4 The RC network for the building envelope](image)

The notation used in Figure 3–4 follows:

- $T_{os}$ is the outside surface temperature (°C)
- $T_{ls}$ is the inside surface temperature (°C)
- $T_{in}$ is the zone mean air temperature (°C)
- $T_{sol-air}$ is the solar air temperature (°C)
\( q_1 \) is the zone infiltration heat flux (W)

\( q_2 \) is the outside surface convection heat flux (W)

\( q_3 \) is the inside surface solar radiation heat flux (W)

\( q_{\text{total}} \) is the total heat flux of test wall (W)

\( R_1 \) is the inside surface convection thermal resistance (K/W)

\( R_2 \) is the total wall thermal conductivity (K/W)

\( R_3 \) is the outside surface convection thermal resistance (K/W)

\( R_{\text{inf}} \) is the zone infiltration thermal resistance (K/W)

\( C \) is the wall thermal mass (J/K)

\( R_{\text{win}} \) is the window thermal resistance (K/W)

In Figure 3–4, the RC model uses the solar air temperature, \( T_{\text{sol-air}} \), to consider the solar radiation impact on the outside surface, which is a widely used approach to simplify the procedure to calculate the solar radiation heat flux on the exterior surface. Therefore, the outside surface solar radiation flux is not included in this model. The heat is transferred to from the outside air to the outside surface through the convection resistance, \( R_3 \). Then, the heat is transferred from the outside surface to the inside surface through the wall conduction resistance, \( R_2 \). Last, the heat is transferred from the inside surface the zone air through the convection resistance \( R_1 \). The heat is transferred from the outside air to the zone air through the infiltration resistance, \( R_{\text{inf}} \), and window resistance, \( R_{\text{win}} \). The radiation flux on the interior surface, \( q_3 \), is hard to measure but can be estimated. O’Neill, et al. assumed all solar radiation that enters the room first hits the floor [16]. Then the floor diffusely reflects the radiation to the interior surface based on area-weighted solar distribution factors. The solar heat flux on the given interior surface is calculated by:
\[ q_3 = (1 - \varepsilon_{wall \ floor}) H_{tot} \sum_{m=1}^{N} \frac{\varepsilon_{wall} + \tau_{wall}}{A_m (\varepsilon_{wall} + \tau_{wall})} \]  

(1)

Where

\( \varepsilon_{wall \ floor} \) is the solar absorptivity of floor

\( H_{tot} \) is all solar radiation transmitted from the exterior to the room (W)

\( m \in n \) is the index for surfaces including wall, roof and windows

\( \varepsilon_{wall} \) is the solar absorptivity of wall

\( \tau_{wall} \) is the solar transmissivity of the wall

\( A \) is the wall surface area (m²)

In this study, a case without window was tested to verify the RC model with the solar-air temperature is the right modeling infrastructure for the estimation of building infiltration with the filters.

3.2.3 Test RC Model and Energy Balance Equation

For this specific problem, a 3R2C thermal network model is used. The 3R2C model is shown in Figure 3–5.

![Figure 3–5 The proposed 3R2C model for the testing wall](image-url)
The governing equation for the test wall can be described as:

\[
C \frac{dT_{os}}{dt} = \frac{T_{is} - T_{os}}{R_2} + q_2
\]  

\[
C \frac{dT_{is}}{dt} = \frac{T_{in} - T_{is}}{R_1} + \frac{T_{os} - T_{is}}{R_2} + q_3
\]  

\[
q_2 = \frac{T_{sol-air - T_{os}}}{R_3}
\]  

\[
q_1 = \frac{T_{sol-air - T_{in}}}{R_{inf}}
\]  

\[
q_{total} = q_1 + q_2
\]  

\[
C \frac{dT_{in}}{dt} = 0
\]  

Equations (4), (5), and (6), can be combined and plugged into Equation (2), then Equation (2) can be rewritten as:

\[
C \frac{dT_{os}}{dt} = \frac{T_{is} - T_{os}}{R_2} + q_{total} - \frac{T_{sol-air - T_{in}}}{R_{inf}}
\]  

3.2.4 Building Thermal Resistance

The selected wall, as shown in the red square in Figure 3–6, is an exterior wall of the working room, which has 20.07 square meters, located at the northwest corner of the test building. For this preliminary study, there are no windows in this working room.
Figure 3–6 Side view of the simulated building with the selected wall surface

The test wall is a three-layer composite wall with wood, foam, and concrete. Each wall layer’s physical parameters are listed in Table 3-1.

Table 3-1 Wall material characteristic by layers

<table>
<thead>
<tr>
<th></th>
<th>WOOD</th>
<th>FOAM</th>
<th>CONCRETE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific heat (J/kg-K)</td>
<td>900</td>
<td>1400</td>
<td>1000</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>530</td>
<td>10</td>
<td>1400</td>
</tr>
<tr>
<td>Thickness (m)</td>
<td>0.009</td>
<td>0.0615</td>
<td>0.1</td>
</tr>
<tr>
<td>Conductivity (W/m²-K)</td>
<td>0.14</td>
<td>0.14</td>
<td>0.51</td>
</tr>
</tbody>
</table>

The wall conduction thermal resistance is determined by surface material characteristics, the single layer conduction thermal resistance can be found by:

\[ R_{cond,i} = \frac{L}{kA} \]  

(9)

Where
$R_{cond,i}$ is single layer wall thermal resistance (K/W)

L is wall thickness (m)

k is wall layer thermal conductivity (W/m2-K)

The calculated conduction thermal resistance for each layer of the testing wall is shown in Table 3-2.

Table 3-2 Wall conduction thermal resistance by layers

<table>
<thead>
<tr>
<th></th>
<th>Wood</th>
<th>Foam</th>
<th>Concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conduction resistance (K/W)</td>
<td>0.0032</td>
<td>0.02189</td>
<td>0.0098</td>
</tr>
</tbody>
</table>

The testing wall is a three layers composite wall, the total wall thermal resistance follows:

$$R_2 = \sum_{i=1}^{3} R_{cond,i}$$

(10)

By equation (10), the total wall conduction thermal resistance is 0.03489 (K/W)

The wall thermal capacitance is part of the wall physical characteristics, which can be found by:

$$C_i = \rho C_p L A$$

(11)

Where

$C_i$ is single layer wall thermal capacitance (J/K)

$\rho$ is wall material density (kg/m$^3$)

$C_p$ is wall specific heat (J/kg-K)

The single layer wall thermal mass is shown in Table 3-3.
The total wall thermal mass can be calculated by

\[ C = \frac{1}{\sum_{i=1}^{3} C_i} \]  \hspace{1cm} (12)

The total thermal capacitance of the testing wall is 14,320 (J/K), the 3R2C model has two capacitances, which are assumed as the same. The thermal capacitance is 7,160 (J/K).

The inside surface convective thermal resistance can be calculated by:

\[ R_1 = \frac{1}{h_i A} \]  \hspace{1cm} (13)

Where

\( h_i \) is the inside surface convection heat transfer coefficient (W/m²K)

For this study, the inside surface convection resistance is considered as a constant. The inside surface convection heat transfer coefficient is set as the average of the heat transfer coefficient during the whole year. The inside surface convection resistance is calculated by equation (13) and the value is 0.02055 (K/W).

3.3 Estimation Model

3.3.1 State-space Equation
Most of the estimation methods required state-space equations. In this work, the state-space equation is used to represent a single wall energy balance equation. To get the state-space equation, we divide the thermal capacitance “C” for equation (3), equation (7), and equation (8) to get differential equations. For this study, the zone mean air temperature, thermal resistances, and thermal capacitances are set as constant. The estimation wall is a wall without windows, the inside surface solar radiation flux is set as a constant of zero. The thermal resistance and capacitance are assumed as constant, we have:

\[
\begin{align*}
\frac{dT_{os}}{dt} &= \frac{T_{is} - T_{os}}{R_2C} + \frac{q_{total}}{C} - \frac{T_{solar-air} - T_{in}}{R_{inf}C} \\
\frac{dT_{is}}{dt} &= \frac{T_{in} - T_{is}}{R_1C} + \frac{T_{os} - T_{is}}{R_2C} + \frac{q_3}{C} \\
\frac{dT_{in}}{dt} &= 0 \\
\frac{dR_{inf}}{dt} &= 0 \\
\frac{dq_3}{dt} &= 0 \\
\frac{dR_1}{dt} &= 0 \\
\frac{dR_2}{dt} &= 0 \\
\frac{dC}{dt} &= 0
\end{align*}
\] (14) (15) (16) (17) (18) (19) (20) (21)

From equation (14) to equation (21), each term on the right-hand side is defined as a state, we have eight states for this model:
Then equation (14) to equation (21) can be represented in the state-space form of:

\[
\dot{x} = f(x) = \begin{bmatrix}
\frac{x_2-x_1}{x_8} + \frac{q_{tot}}{x_8} - \frac{T_{solat}}{x_8} - x_3
+ \frac{x_4}{x_8} + \frac{x_5}{x_8}
+ \frac{x_6}{x_8}
+ \frac{x_7}{x_8}
+ \frac{x_8}{x_8}
\end{bmatrix}
\]

\[
\begin{bmatrix}
T_{os}
T_{is}
T_{in}
R_{inf}
q_3
R_1
R_2
C
\end{bmatrix}
\]

(22)

3.3.2 Extended Kalman Filter

The EKF has been commonly used for the non-linear estimation. The advantage of using Kalman Filter (KF) is that KF is capable of estimate unknown states with given model and inputs. For this study, an EKF is implemented because the underlying model is non-linear. The non-linear system dynamics for EKF can be represented by [24]:

\[
x(k) = F[x(k-1), u(k-1), w(k-1)]
\]

(24)

\[
y(k) = G[x(k), v(k)]
\]

(25)

Where

\( F \) is the state-space equation
x is the state to be estimated

u is the input

w is the process covariance noise

X is the EKF state-space equation

G is the measurement state-space equation

v is the measurement covariance noise

y is the EKF measurement equation

k is the current state

The EKF is a useful tool to estimate a non-linear system, which a linearization is required for the estimation. The linearization process takes the state-space equation F, and the measurement equation G, with only considerations of the first derivative, to linearize the system. The Jacobian matrix is commonly used in this approach, which is shown in equation (24) and equation (25):

\[ J(k|k - 1) = \frac{\partial F}{\partial x} \]  \hspace{1cm} (26)

\[ H(k|k - 1) = \frac{\partial G}{\partial x} \]  \hspace{1cm} (27)

Where

J is the Jacobian matrix for state-space equation

H is the Jacobian matrix for measurement equation

After the linearization, the system can be treated as a linear system, where state-space equation and its covariance can be represented, as shown in Figure 3–7.
As shown in Figure 3–7, after the initial state and its associated covariance, the linearized state-space equation, and covariance equation are chosen, the EKF uses the predicted state and the covariance to calculate a new Kalman gain. The last step of a single prediction is to update or correct the predicted state and the covariance by using the new Kalman gain. Then, the updated state and covariance become the new input parameters for the next state estimation. The detailed process follows:

1. The EKF updates the covariance matrix as follows:

\[
P(k|k - 1) = J(k)P(k - 1|k - 1)J^T(k) + Q
\]  
(28)

Where

P is the covariance matrix

Q is the standard deviation of process noise matrix

2. Based on the updated covariance matrix, the Kalman gain calculated by:

\[
K(k) = P(k|k - 1)H^T(k)(H(k)P(k - 1|k - 1)H^T(k) + R)
\]  
(29)

Where
3. The state-space equation updates by using the Kalman gain and measurement equation:

\[ X(k) = X(k|k - 1) + K(Y(k) - Y(k|k - 1)) \]  

(30)

4. Finally, the covariance is updated by:

\[ P(k) = (I_n - K(k)H(k))P(k|k - 1) \]  

(31)

Where

I is the identity matrix

3.3.3 Filter Design

The estimation accuracy not only depends on the fidelity and accuracy of the underlying model, but it also affected by the parameters used in the filter itself. Thus, a right filter design is a key factor for an accurate estimation. The filter design includes accurate inputs and an appropriate design of the covariance. To use the Kalman Filter for the estimation, the input and measurement for the EKF are required. The inputs may come from the direct field measurements or some calculations using measurements. In this study, there are two inputs, namely, solar air temperature and total surface heat flux. The total heat flux through an exterior wall consists of infiltration heat flux and outside convection heat flux, which follows equation (6). The infiltration heat flux is not a direct output from EnergyPlus, but it can be calculated by:

\[ q_1 = \frac{E_{inf}}{\ell} \]  

(32)

Where
\( E_{inf} \) is zone infiltration energy (J)

\( t \) is the time step (s)

For this study, the simulation time step is set to 120 seconds in the EnergyPlus, and infiltration heat transfer energy is from the EnergyPlus output (8) and (9).

The solar air temperature can be calculated from outdoor air temperature, the wall outside layer solar absorptivity, the total solar radiation incident on the surface, and the outside convection heat transfer coefficient [28]. The solar air temperature can be calculated by:

\[
T_{sol-air} = T_{oa} + \frac{\alpha q}{h_{os}}
\]  

(33)

Where

\( T_{oa} \) is the outdoor air temperature (°C)

\( \alpha \) is outside layer solar absorptivity

\( q \) is the outside surface incident solar radiation rate per area (W/m²)

\( h_{os} \) is the outside surface convection heat transfer coefficient (W/m²K)

Once the input for the EKF acquired, the state space equation (23) becomes:

\[
\dot{x} = f(x) = \begin{bmatrix}
x_2 - x_1 \\
x_7 x_8 \\
x_3 - x_2 \\
x_6 x_8 \\
x_1 - x_2 \\
x_7 x_8 \\
x_4 x_8 \\
x_8 
\end{bmatrix} + \begin{bmatrix} u_{qtot} \\
x_8 \\
x_1 - x_2 \\
x_7 x_8 \\
x_5 \\
x_8 \\
x_8 \\
x_8 \end{bmatrix} - \begin{bmatrix} u_{Tsol-air} - x_3 \\
x_8 \\
x_1 - x_2 \\
x_7 x_8 \\
x_5 \\
x_8 \\
x_8 \\
x_8 \end{bmatrix}
\]  

(34)

To use the EKF to estimate unknown states, it also requires a measurement equation. For this study, the outside surface temperature is the only measurement, which is the first state for this model. The measurement equation is as follows:
\[ y = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]X \] (35)

Please note both inputs (i.e., solar-air temperature and total surface heat flux) and one measurement (i.e., total surface heat flux) are from the EnergyPlus simulation.

To initiate the EKF, it also requires Jacobian matrices for state-space equation and measurement equation. The Jacobian matrix is the first derivative of the state-space equation, which can be found by equation (26) and equation (27). The Jacobian matrix for state-space equation \( X \) is:

\[
J = \begin{bmatrix}
-\frac{1}{x_7x_8} & \frac{1}{x_7x_8} & \frac{1}{x_4x_8} & \frac{u_{Tsolar}-x_3}{x_4^2x_8} & 0 & 0 & \frac{x_1-x_2}{x_7x_8} & \frac{x_1-x_2}{x_7^2x_8} & \frac{u_{qtot}}{x_8^2} + \frac{u_{Tsolar}-x_3}{x_4x_8^2} \\
\frac{1}{x_7x_8} & -\frac{1}{x_7x_8} & \frac{1}{x_4x_8} & 0 & \frac{x_2-x_3}{x_8} & \frac{x_2-x_3}{x_7^2x_8} & \frac{x_2-x_3}{x_8^2x_6} & \frac{x_2-x_3}{x_8^2x_7} & \frac{x_5}{x_8^2} \\
0 & 0 & 0 & 0 & \frac{x_1-x_2}{x_7^2x_8} & \frac{x_1-x_2}{x_7^2x_8} & \frac{x_5}{x_8^2x_6} & \frac{x_5}{x_8^2x_7} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{x_5}{x_8^2} & \frac{x_5}{x_8^2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} (36)

The Jacobian matrix for the measurement equation follows:

\[ H = 1 \] (37)

A Matlab Kalman Filter toolbox is used for the filter. The appropriate design of the covariance, the process noise, and the measurement noise is another important step for the proposed model-based estimator. In this study, the process noise covariance, \( W \), the measurement noise covariance, \( V \), and initial state error covariance matrix \( P_0 \) are assumed as follows:
\[
W = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 10,000,000 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 10,000,000 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]  
(38)

\[
V = 1
\]  
(39)

The state 6 of the indoor heat convection resistance is assumed as a constant, which is not always true. Therefore, a high process noise is given. The infiltration resistance is also highly an uncertain non-constant. A high process noise is used to compensate for this uncertainty from the model. Other process noise covariance is set as one. To improve the estimation accuracy for the initial states, a high initial state error covariance is given. The measurement is considered to be accurate. Thus, the measurement noise covariance is set to one. We are assuming that the room air temperature is in a quasi-steady-state with a relatively constant value during the given time step, which is shown in the state space equation (34).

3.3.4 Constraint

Constraints based on the physics are used in the filter such that the estimated values don’t violate the physics. For this study, the soft constraints with given bands are applied to the estimation model. The lower bound of all states of temperatures is set at \(-60 \, ^\circ C\), and the upper bound of temperatures is set at \(120 \, ^\circ C\). This constraint is applied to the outside surface
temperature, the inside surface temperature, and the zone mean air temperature. Another constraint is set for the infiltration resistance with a lower bound of 0.00001 K/W. This makes the estimated infiltration resistance to be always greater than 0. For this study, the testing wall is in the black room without windows. Therefore, a constraint of the inside surface solar heat flux \( q_3 \) is set to 0.00001 W to ensure this heat flux to be zero.
4 RESULTS AND DISCUSSIONS

4.1 Simulated Results

4.1.1 Solar Air Temperature

The solar air temperature is calculated by equation (30). The EnergyPlus output 2), 11), and 13) are used to compute solar air temperature. The solar absorptivity is found by the EnergyPlus outer layer wall material, which is 0.7. Figure 4–1 shows the outside wall construction, and Figure 4–2 shows outer layer wall material characteristics.

```
<table>
<thead>
<tr>
<th>Field</th>
<th>Units</th>
<th>Obj2</th>
<th>Obj3</th>
<th>Obj4</th>
<th>Obj5</th>
<th>Obj6</th>
<th>Obj7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outside Layer</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer 2</td>
<td>WoodBlock</td>
<td>0.7</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer 3</td>
<td>FoamPlus</td>
<td>0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Layer 4</td>
<td>Masterboard</td>
<td>0.7</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Layer 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Layer 6</td>
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<td></td>
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<td></td>
<td></td>
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<td>Layer 7</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer 8</td>
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<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Layer 9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

**Figure 4–1 Outside wall construction in the EnergyPlus**

```
<table>
<thead>
<tr>
<th>Field</th>
<th>Units</th>
<th>Obj11</th>
<th>Obj12</th>
<th>Obj13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td></td>
<td>Gypsum</td>
<td>Concrete</td>
<td>Foam</td>
</tr>
<tr>
<td>Roughness</td>
<td></td>
<td>Medium</td>
<td>Rough</td>
<td>Insulation</td>
</tr>
<tr>
<td>Thickness</td>
<td>m</td>
<td>0.04</td>
<td>0.1</td>
<td>0.0615</td>
</tr>
<tr>
<td>Conductivity</td>
<td>W/m-K</td>
<td>0.16</td>
<td>0.51</td>
<td>0.14</td>
</tr>
<tr>
<td>Density</td>
<td>kg/m³</td>
<td>784.5</td>
<td>1400</td>
<td>10</td>
</tr>
<tr>
<td>Specific Heat</td>
<td>J/kg-K</td>
<td>630</td>
<td>1000</td>
<td>1400</td>
</tr>
<tr>
<td>Thermal Absorptance</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Solar Absorptance</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Visible Absorptance</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
</tbody>
</table>
```

**Figure 4–2 Solar absorptivity of the outside layer**
Based on parameters and outputs from the EnergyPlus, the whole year solar air temperature for the surface 9 in the testing building in Chicago is shown in Figure 4–3.

![Figure 4–3 Calculated solar air temperature from the EnergyPlus](image)

4.1.2 Infiltration Resistance

The EnergyPlus unable to output infiltration directly, to calculate simulate infiltration resistance, the EnergyPlus output 8) and 9) are used. The known infiltration resistance from the EnergyPlus is defined as:

\[
R_{inf} = \left| \frac{T_{sol-air} - T_{in}}{q_1} \right|
\]  

(41)

Based on equation (38), Figure 4–4 shows the simulated whole year infiltration resistance
For this study, the first 5,400 data points of building infiltration resistance are used to test the EKF estimate building infiltration method. The first 5400 infiltration resistance data is shown in Figure 4–5.
4.2 Simulated Results

The estimation results for the testing black wall using the simulated data from the EnergyPlus are presented. The estimation starts on January 1\textsuperscript{st}, and the estimation period is set to be 5,400 time steps. The outside surface temperature estimations are shown in Figure 4–6.
The red dot is virtual measurements from EnergyPlus, and the blue line is the estimated outside surface temperature from the estimator. As shown in Figure 5, the estimated and measured outside surface temperature match with each other very well, which is expected due to the filter design. Because outside surface temperature is the only measurement for the EKF, the covariance is chosen to trust this measurement.

The EKF estimated infiltration resistance is shown as the blue line. This is compared with the actual known values from the EnergyPlus.

Figure 4–7 shows that the EKF estimated infiltration resistance has the same trend compared with true values. There are 21 time steps that the estimated infiltration resistance reached the upper bound of the constraint. This occurred at the end of January 2\textsuperscript{nd}. The reason is unknown currently.
and is being investigated. Most likely, this is due to relatively small infiltration resistances during that period, which causes the challenges for the filter design.

Figure 4–7 Building infiltration resistance estimation

Figure 7 gives an evaluation of the proposed estimator in terms of the count of the percentage error in a 10% bin.

For this estimation with a 2-minute sampling frequency, there are 5,400 estimation points. Figure 4–8 shows that 3,446 (63.81%) estimations are within the 10% error band, and 778 (14.41%) estimation errors are between 10% to 20%. 78.22 % estimations fall in the range of 20% error band. Another 873 (16.17%) estimation points are within the 20% to 30% error band. 106 out of 5,400 estimation points (1.96%) have more than 50% estimation error.
To better determine the estimation accuracy, statistic metric including the coefficient of variation of root-mean-square error (CV-RMSE), $R^2$, and mean based error (MBE) are used.

To find the CV-RMSE, the RMSE needs to be calculated by:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(x_{EKF}-x_{E+})^2}{n}}$$

(42)

Where

$RMSE$ is the root-mean-square-error

$x_{EKF}$ is the EKF estimated infiltration resistance

$x_{E+}$ is the EnergyPlus calculated infiltration resistance

$n$ is the size of the estimation sample
The CV-RMSE can be calculated by:

\[
CV - RMSE = \frac{RMSE}{\bar{x}}
\]  

(43)

Where

CV-RMSE is the coefficient of variation of root-mean-square error

\( \bar{x} \) is mean of infiltration resistance

The \( R^2 \) is calculated by:

\[
R^2 = 1 - \frac{\sum_{i=1}^{n} x_{E,EKF}^2}{\sum_{i=1}^{n} x_{E,+}^2}
\]  

(44)

Another statistic metric used to evaluate the estimation accuracy is MBE:

\[
MBE = \frac{\sum_{i=1}^{n} x_{E,EKF} - x_{E,+}}{n}
\]  

(45)

Where

MBE is the mean based error

The statistic metric for the infiltration estimation is shown in Table 4-1.

<table>
<thead>
<tr>
<th>CV-RMSE (%)</th>
<th>( R^2 )</th>
<th>MBE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1216</td>
<td>0.1083</td>
<td>0.1645</td>
</tr>
</tbody>
</table>

*Table 4-1 Statistic metric for infiltration resistance estimation*

This statistic metric analysis shows that the proposed model-based estimator gives a reasonable accuracy using the data from the EnergyPlus emulator.
4.3 Discussion

The following factors could contribute to the non-perfect estimation of the infiltration resistances:

1) The state-space equation can be improved further to better reflect the building envelope heat transfer. The current state-space model (i.e., 3R2C model) has been successfully applied to estimate the building internal loads and the wall physical parameters in the literature (Lee and Braun 2008). However, it may need some adjustments to estimate the building infiltration.

2) Using the solar-air temperature to consider solar radiation impacts on the exterior surface could cause some errors. The estimated wall conduction thermal resistance and capacity are nearly constants during the whole estimation period, which is as expected since the wall conduction thermal resistance and capacity are set as constant in the EnergyPlus emulator. These constant values are almost identical to the values used in the EnergyPlus, which demonstrates that the proposed EKF-based estimator performed well for the parameter estimation.

In summary, the proposed infiltration estimation method is a model-based infiltration estimator through data fusion using filters (e.g., Extended Kalman Filter). Fused data includes heat flux and surface temperatures measured from ultrasonic thermometry. This study is only focusing on the inverse modeling with data assimilation assuming that measurements from ultrasonic thermometry are known. This is totally different with the traditional forward infiltration model such as the infiltration model used in the EnergyPlus [29], which requires the inputs of infiltration air flow rate at the design condition and some specific coefficients. The infiltration air flow rate at design conditions often are not easy to obtain for a given building
unless a blower door or tracer gas method is used. The proposed inverse modeling approach with filters have been widely used in the aerospace and automobile industry, while little has been explored in building science. The beauty of the proposed approach stems from the combination of physics and measurement. The key to the proposed method is how to intrusively measure heat flux and surface temperatures on the external surface.
5 FUTURE WORK

Some future works are listed as follows:

5.1 Directly use outside air temperature to estimate the building infiltration

To consider the solar radiation impacts on building envelope heat transfer, the underlying model in the presented work is using the solar air temperature to approximate the solar radiation flux on the exterior surface. Another way to consider the solar radiation flux on the exterior surface is to directly impose the solar heat flux on the surface. We can apply the similar RC network model, as shown in Figure 5-1. In this configuration, the infiltration resistance is placed between the indoor air temperature and the outside air temperature, which is a more realistic scenario.

![Figure 5-1 A revised 3R2C model using outside air temperature](image)
Using this revised 3R2C model may help estimate building infiltration more accurately, but the remaining challenge is how to calculate or measure the outside surface solar radiation flux. From Figure 5-1, we can write following differential equations:

\[
\frac{dT_{os}}{dt} = \frac{T_{is} - T_{os}}{R_2 C} + \frac{T_{air} - T_{os}}{R_3 C} - \frac{T_{air} - T_{in}}{R_{inf} C} + \frac{q_{rad}}{C} \tag{46}
\]

\[
\frac{dT_{is}}{dt} = \frac{T_{in} - T_{is}}{R_1 C} + \frac{T_{os} - T_{is}}{R_2 C} + \frac{q_3}{C} \tag{47}
\]

Where,

- \(T_{air}\) is outside air temperature (°C)
- \(q_{rad}\) is outside surface incident solar radiation (W)
- \(R_{inf}\) is wall surface thermal infiltration resistance between indoor air temperature and outdoor air temperature (K/W)

Equations (46) and (47) are used to describe the dynamics of the outside surface and inside surface temperatures. By combining these two equations with Equation (16) to (21), it forms a new set of state-space equations to depict the building envelope heat transfer. These equation then can be integrated with the proposed EKF to estimate the unknown state (e.g., \(R_{inf}\)). For the further work, the estimator with this new set of equations will be further explored. Besides the new inputs of \(q_{rad}\), the other required measurements will be remained the same in this revised approach.
5.2 Cases with windows and cases in different locations.

The current testing wall does not contain any windows. A case with windows should be tested to prove the proposed method versatility. The RC model for the case with the windows shown in Figure 3–4.

The EKF covariance P, Q, and R are varied with different locations. In this study, the only tested location is Chicago, IL. Other locations in different climate zones should be tested.

5.2 Unscented Kalman Filter

Unlike the EKF using Jacobian, the UKF uses the unscented transform. The unscented transform gives nearly the same mean and covariance compared to the original data. The unscented transform uses a nonlinear transformation to calculate statistics of a random variable. Random variable $x$ has a mean of $\bar{x}$ and covariance $P_x$. The statistic of $y$ can be calculated by $2n+1$ sigma vectors $X_i$, which forms as a matrix $X$ [24, 25]. To generate sigma vectors, the following equations are used:

$$X_0 = \bar{x}$$

$$X_i = \bar{x} + (\sqrt{(n + \lambda)P_x})_i \quad i = 1, 2, ..., L$$

$$X_i = \bar{x} - (\sqrt{(n + \lambda)P_x})_{i-L} \quad i = 1, 2, ..., L$$

Where

$x$ is the random variable;

$\bar{x}$ is the mean of random variable;
\( P_x \) is the covariance of random variable;

\( \lambda \) is the scaling vector;

\( X_i \) is the sigma vector;

The weight of \( X_i \) can follows:

\[
W_0^{(m)} = \frac{\lambda}{n+\lambda} \tag{51}
\]

\[
W_0^{(c)} = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta) \tag{52}
\]

\[
W_i^{(m)} = W_i^{(c)} = \frac{1}{2(n+\lambda)} \quad i = 1, 2, \ldots, 2L \tag{53}
\]

Where

\( \lambda = \alpha^2(n + \kappa) - n \) is scaling parameter;

\( \alpha \) determine the spread of sigma points around \( \bar{x} \);

\( \kappa \) is the secondary scaling factor;

\( \beta \) is the incorporate to the distribution of \( x \);

\((\sqrt{(n+\lambda)P_x})_i\) is the \( i \)th row of the matrix square root

The UKF uses the unscented transform, which is a higher order differentiation. This gives UKF a better estimation accuracy.

5.3 Unmeasured Disturbance

Research shows that unmeasured disturbance such as an internal heat gain has large effects on building model [30]. By adding an estimator for an unmeasured disturbance will give a
better estimation accuracy. The RC model always has some unmeasured disturbances. Kim et al. proposed a model with considerations of unmeasured disturbance. The model is shown in Figure 5–2.

![RC model with additional disturbance](image)

*Figure 5–2 RC model with additional disturbance [29]*

The model is used to estimate the zone and wall thermal capacitance and resistance, and effective window area for the transmitted solar radiation.

With an appropriate design of the upper and lower bounds, the Levenberg-Marquardt algorithm produced a consistent estimation of parameters with inputs plus internal heat gains. To show the difference in the parameter estimation, the same procedure was conducted without considerations of the internal heat gain, which is denoted as conventional approach. Kim et al. estimated the cooling rate and the zone temperature in a close loop operation by adding measurement
disturbances. By adding unmeasured disturbance, the estimation result is much closer to the true value [29].

Figure 5–3 estimation results comparison between conventional modeling, adding unmeasured disturbance modeling, and true value [29]
The estimation comparison is shown in Figure 5–3. The green line is the estimation result from the model with unmeasured disturbance, the blue line is the estimation result from the model without disturbances, and the black line is the true value. In Figure 5–3, the model with disturbances has much better estimation results.

By adding unmeasured disturbance, the estimation accuracy increases significantly. This illustrates a possible way to improve infiltration estimation results in our study.
REFERENCES


