ELEMENTARY MATHEMATICS INSTRUCTIONAL COACHES:
QUALITIES OF QUALIFIED MATHEMATICS SPECIALISTS

by

NICOLETTE I. NALU

DIANE C. SEKIERES, COMMITTEE CHAIR
CYNTHIA V. SUNAL, COMMITTEE CO-CHAIR
RANDALL CHARLES
JIM GLEASON
DENNIS W. SUNAL

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ABSTRACT

This mixed methods study analyzed the content knowledge, pedagogical knowledge, and leadership knowledge and skills of elementary mathematics specialists (EMSs); in addition, the study examined EMSs’ ability to select and implement a high-quality mathematical task.
DEDICATION

This dissertation is dedicated first and foremost to my Lord, Jesus Christ and my family. Without either of them, I would not be able to say that I am finally, Dr. Nicolette Ingrid Nalu. I want to thank my mom and dad for always pushing us to be the best and always making sure we had everything we needed to be the best version of us that we could be. I want to thank my family, Jennifer, Bill, my little Poohbear Winston, Patrick, Katie, and my precious babies John Paul, Peter, Isabella, Oliver, and Mary for your prayers and for making me so proud to be your sister and Aunt Nicky.
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CHAPTER 1
INTRODUCTION TO THE STUDY

The Association of Mathematics Teacher Educators (AMTE, 2013) identified three areas of expertise for a mathematics specialist: (a) content knowledge for teaching mathematics, (b) pedagogical knowledge for teaching mathematics, and (c) leadership knowledge and skills (AMTE, 2013, p. 4). AMTE (2013) described content knowledge for teaching mathematics as a deep understanding of the mathematics content developed in Grades K-8, whereas pedagogical knowledge for teaching mathematics was defined as knowledge of learners, learning, teaching, curriculum, and assessment. Leadership knowledge and skills were described as (a) having a broad view of many aspects and resources to support and facilitate effective instruction and professional growth in mathematics; (b) the ability to take on nonevaluative leadership roles; and (c) being able to plan, develop, implement, and evaluate professional development programs. When an Elementary Mathematics Specialist is equipped with all three suggested areas of expertise, according to AMTE (2013), he/she is considered a well-rounded, qualified EMS.

Conceptual Framework

Standards from AMTE (2013) were used as the conceptual framework for describing the knowledge and abilities of EMSs. The three parts of the AMTE (2013) standards are shown in Figure 1.
Figure 1. Standards for elementary mathematics specialists (AMTE, 2013, p. 4).

Research Questions

Because this study was grounded in the AMTE (2013) qualifications necessary to be
certified a certified EMS, the research questions were:

Research Question 1. What is the relationship between the Elementary Mathematics
Specialists’ ability to select a high-quality mathematical task and their
implementation of a mathematical task?

Research Question 2. What is the relationship between the Elementary Mathematics
Specialists’ ability to select a high-quality mathematical task and their content
knowledge for teaching elementary mathematics?

Research Question 3. What are the relationships between the Elementary Mathematics
Specialists’ leadership knowledge and skills, their beliefs and practices about
teaching and coaching mathematics with conceptual understanding, and their
instructional practices pertaining to mathematics?
Statement of the Problem

Elementary mathematics specialists are defined as “teachers, teacher leaders, or coaches who are responsible for supporting effective mathematics instruction and student learning at the classroom, school, district, or state levels” (AMTE, 2013, p. 1). According to AMTE (2013) and the National Council of Teachers of Mathematics (NCTM, 1981, 1984), there should be certified EMSs in schools or districts to appropriately support teachers in mathematics and ultimately promote gains in students’ mathematical understanding and achievement (Dossey, 1984; Fennell, 2006; NMAP, 2008; Reys & Fennell, 2003). Students perform better when an EMS, with the needed requirements for the specific certification of the EMS title, is part of the instructional team (AMTE, 2013; Ferrini-Mundy & Johnson, 1997).

For over three decades, research on EMSs has shown them to be a “suggested solution to ensure a strong mathematics teaching practice and higher student achievement in mathematics” (McGatha & Rigelman, 2017, p. 16). A second area of research highlights the importance of EMS qualifications. Even so, many EMSs are coaching outside of their own subject area of expertise; they are often education generalists (AMTE, 2013; NCTM, 2000; NMAP, 2008; NRC, 1989) or former reading or literacy coaches in elementary schools (McGatha & Rigelman, 2017). EMSs without a mathematics-specific EMS certification may not have the required expertise to be most effective in a school or for a district. As a result, EMSs without the specific certification may not be adequately qualified to support elementary teachers with improving their mathematics instruction (Greenberg & Walsh, 2008; Reys & Fennell, 2003). Campbell and Malkus (2009) stated, “Simply allocating funds and filling the position of an elementary mathematics coach in a school will not yield increased student achievement” (p. 22). Thus, the
topic for this research focused on determining the qualifications, or knowledge and skills of EMSs who were in an instructional leadership position during the time of the study.

**Purpose of the Study**

The purpose of this study was threefold: First, the study analyzed the relationship between EMSs’ ability to select a high-quality mathematical task and their implementation of a mathematical task. Second, the study examined the relationship between the EMSs’ ability to select a high-quality mathematical task and their content knowledge for teaching elementary mathematics. Third, the study examined the relationships between EMSs’ leadership knowledge and skills, their beliefs and practices about teaching and coaching mathematics with conceptual understanding, and their instructional practices pertaining to mathematics. The study began with the premise that EMSs should have the qualifications of a certified EMS proposed by AMTE (2013).

**Significance of the Study**

This study has the potential to inform and support state qualification guidelines for a certified EMS. It may also inform research interested in the possible relationship of EMSs’ beliefs about high-quality mathematical task selection, the implementation of high-quality mathematical tasks, and the EMSs’ beliefs and practices related to instructional coaching and conceptual understanding in mathematics. In addition, this study contributed to the existing literature regarding mathematics coaching and the preparation of EMSs.

**Limitations**

This study began in the spring of the 2016-2017 school year, continued through the summer of 2017, and concluded in the fall of the 2017-2018 school year. The study took place in one southeastern state in the United States. The participants for this study were drawn from
educators in the role of an EMS in public schools across the state. The current study included one phase during which participants completed survey data; followed by a second phase of data collection from a smaller, convenience sample of participants who completed reflective journal entries. I observed the smaller subsample teaching mathematics through their own selection and implementation of two high-quality mathematical tasks. The subsample also participated in an informal, semistructured, individual interview following the second observation of the task implementation.

**Assumptions**

This study was based on the following assumptions: (a) The sample was a good representation of the EMS population in one southeastern state in the United States; (b) the instruments selected captured the three suggested qualifications of a qualified EMS; and (c) the EMSs were truthful in their responses to the survey items, journal prompts, and interview questions (AMTE, 2013).

**Positionality**

I am an elementary education specialist in the state in which I collected the data. I have been in this position and employed by a state university and the state’s department of education for 10 years. I began my teaching career in the same county where I attended a private college for my undergraduate and master’s degrees. I taught elementary school for 6 years prior to accepting a position as an elementary mathematics specialist on behalf of the state’s Department of Education personnel.

Having worked in this position for approximately 10 years, I connected with people in education not only in my geographical region, but also in other regions of the state where I facilitated numerous mathematics professional development sessions and training for regional in-
service teachers and college preservice teacher candidates. I am a friend and resource to many teachers, curriculum specialists, special education teachers, administrators, central office employees, and the state’s Department of Education personnel across the state. It is possible that my previous or current relationships with other professionals in my field may have skewed the data. I am aware of this and attempted to reduce this bias.

I did not collect data or select any participants from the area in which I worked during the study (i.e., in-service region four) to prevent any ethical issues from arising. I did not wish to jeopardize any on-going professional or personal relationships I had with the various educators in my region. Because of my role, my expectations of educators in a mathematics-coaching role are high. I anticipate that changes are needed in mathematics education and that is where my passion lies: in improving mathematics coaching, teaching, and learning in this state.

Throughout the study, I kept a paper journal of all my thoughts and reflections, which I recorded as often as necessary to clarify my thinking and prevent bias from clouding the research process.

**Definition of Terms**

The following terms are defined as presented for the purposes of this study.

*Beliefs* are a person’s personal judgments about mathematics formulated from experiences in mathematics, including beliefs about the nature of mathematics, learning mathematics, and teaching mathematics (Raymond, 1997).

*Belief systems* are important influences on the ways individuals conceptualize tasks, learn from experiences, and play a role in defining teaching tasks and organizing the knowledge and information relevant to those tasks (Nespor, 1987); the connections between a learner’s beliefs and their outcome behaviors (Rivera, 2012).

*Coaching* is the art of helping teachers with teaching (Feger, Woleck, & Hickman, 2004).
Coaching requires questioning skills and demonstration techniques, along with interpersonal skills to maintain a comfortable, collaborative relationship with the teacher (Campbell, Ellington, Haver, & Inge, 2013).

*Common mathematics content knowledge* is the mathematical content knowledge and skills required for various aspects of work and everyday life, as opposed to classroom mathematics (Ball, Thames, & Phelps, 2008).

*Content knowledge for teaching mathematics* is a deep understanding of mathematics for grades K-8 and further specialized mathematics knowledge for teaching (AMTE, 2013).

*Elementary mathematics specialists* are teachers, teacher leaders, or coaches, who are responsible for supporting effective mathematics instruction and student learning at the classroom, school, district, or state levels (AMTE, 2013; Ball, 1988). Other common terms for EMSs include, but are not limited to: (a) instructional coach, (b) instructional partner, (c) mathematics specialist, (d) curriculum coach, (e) mathematics coach, (f) literacy coach, (g) intervention coach, (h) elementary mathematics coach, (i) elementary mathematics instructional leader, (j) mathematics resource teacher, (k) mentor teacher, and (l) teacher leader (Fennell, 2011; McGatha & Rigelman, 2017; Reeder & Utley, 2017).

*High-quality task* is a problem that involves some aspect of a new mathematics idea to be learned and in which; students must productively struggle to devise a solution. The process of solving this kind of a task enables students to connect previously learned ideas with new ideas, and that process of making connections builds understanding.

*Journal* is keeping a log or personal notebook so teachers can write down any thought related to their mathematics learning. Journaling can also be a writing activity in which one can go beyond the recording of events and explore personal thoughts. Journal writing in
mathematics class consists of reflections on material learned in class, reactions to readings or lectures, or responses to open-ended prompts (Borasi & Rose, 1989).

**Modeling** is where the specialist may either teach the students in a teacher’s classroom while the teacher is observing, or the specialist and teacher may visit another teacher’s classroom to observe skillful instruction that all three staff members can discuss later (Campbell et al., 2013).

**Pedagogical knowledge for teaching mathematics** is the knowledge of learners and learning, teaching, and curriculum and assessment within the field of mathematics (AMTE, 2013).

**Worthwhile task** is a problematic problem that poses questions for students, there’s no prescribed or memorized rules or methods, there is no one correct solution (multiple exit points), has a high cognitive demand, and has multiple entry points (Van de Walle, Karp, & Bay-Williams, 2015).

**Overview of the Following Chapters**

Chapter 2 offers a review of the literature, including popular curriculum reports, curriculum standards, and federal legislation in education. The chapter expands on the importance of teaching for conceptual understanding through problem-solving and high-quality mathematical tasks. It also describes some mathematical task models. The last major section in Chapter 2 focuses on EMS job responsibilities, history, and factors affecting EMS effectiveness.

The literature review concludes with descriptions of EMSs’ mathematical knowledge and a discussion of how their beliefs may affect mathematics teaching and coaching. As student achievement in this state has not drastically improved over time, even with the presence of EMSs
in schools, this study posits that EMSs are not sufficiently prepared to positively impact students’ conceptual learning in mathematics.

Chapter 3 describes the research methodology utilized in the study, which includes the research design, the research instruments, the data collection methods, the research setting, the participants, and the timeline of the study. Chapter 3 concludes with the data analysis procedures, triangulation, and the trustworthiness of this research study. Chapter 4 contains the qualitative and quantitative results of the study. Chapter 5 includes a reporting of the results from the researcher’s perspective and implications of the study and concludes with recommendations for similar future studies.
CHAPTER 2
LITERATURE REVIEW

Introduction

This chapter presents a review of major curriculum reports in mathematics, curriculum standards, and federal legislation in education. It examines the importance of teaching for conceptual understanding through high-quality mathematical tasks and problem-solving, while also describing a few of many research-based models for implementing mathematical tasks that develop conceptual understanding and mathematical reasoning. Additionally, this chapter elaborates on research that investigates EMSs’ abilities to select high-quality, exemplar mathematical tasks with embedded problem-solving strategies that promote critical thinking and develop students’ conceptual understanding. The last main section in this chapter focuses on studies and discussions of EMSs in general: what EMSs do, what they are responsible for in a school, the history of EMSs, their positive impact in education, and factors that inhibit their effectiveness in school settings. The literature review concludes with an explanation of the literature on types of knowledge needed for teaching mathematics conceptually, the impact of EMSs’ beliefs about mathematics, and what good teaching and mathematics coaching should look like according to the research.

Curriculum Reports and Standards

The history of and current trends in mathematics education are key to understanding why EMSs can improve mathematics instruction. Over the last 25 years, national bodies of research
and federal and state legislation have sought to improve mathematics instruction through mandated standards and assessments.

**Federal legislation.** Since 2001, two major initiatives from the federal government have sought to improve the mathematics, reading, and science performance of all students: the No Child Left Behind Act and the Every Student Succeeds Act (2015). No Child Left Behind (NCLB, 2001) was introduced in 2001 and President George W. Bush signed it into law in January 2002. The No Child Left Behind Act (2001) required states to report their annual summative assessment data for reading and mathematics achievement. It also addressed the learning needs of students of all races, socioeconomic status, zip code, disability, or background. NCLB documented gaps in mathematics achievement between minority, low-income, and White students in the United States (NCLB, 2001, p. 69). The legislation called for “students to be taught the basic foundational skills necessary in preparing them as the next generation of consumers, contributing citizens, and innovators of the years to come” (p. 69). According to NCLB, schools were required to provide students with “highly qualified teachers” (para. 23), meaning that teachers must be able to demonstrate competency in their subject content areas. In addition, teachers needed to understand mathematics content and pedagogy at levels that enabled them to represent mathematics in multiple ways and use various instructional strategies so that students had a variety of ways to understand the concepts taught (Ball, 1990). NCLB (2001) prompted many school districts to take a more critical look at implementing EMSs as part of an effort to increase student achievement as well as support teachers in preparing for the summative assessments needed for reporting purposes (McGatha & Rigelman, 2017).

addressed the same goals as NCLB but added further support for the goal of fully preparing all students for future successes in their education and in their future careers. ESSA also called for a high-quality Pre-K program focused on Science, Technology, Engineering, Mathematics (STEM), and literacy. ESSA and NCLB were responses to societal changes, greater demands for improved mathematics education, higher stakes testing results, and the desire for national academic competitiveness with other countries (Fowler, 2001).

**Major reports.** In 1989, the National Research Council (NRC) reviewed research to determine the conceptual and procedural knowledge students need to be proficient in mathematics. NRC defined mathematical proficiency as five interwoven and interdependent strands:

- **Conceptual understanding** is the comprehension of mathematical concepts, operations, and relations.

- **Procedural fluency** is the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.

- **Strategic competence** is the ability to formulate, represent, and solve mathematical problems.

- **Adaptive reasoning** is the capacity for logical thought, reflection, explanation, and justification.

- **Productive disposition** is the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. (NRC, 1989, p. 5)

*Everybody Counts* (1989), a report also written by the NRC, identified four conditions needed for students to become mathematically proficient:

- Mathematics should be learned through students’ invention.

- Teachers’ professional development must be of high quality, as well as ongoing and engaging in the area of sustained efforts for improved mathematics instruction.
- The curriculum, resources, instruction, professional development, and improvement in school effort must be aligned to develop mathematically proficient students.

- Students’ learning should be evaluated cumulatively over time (NRC, 1989, pp. 10-14).

As these reports show, instruction is a key component in effective education. With respect to instruction, *Everybody Counts* (NRC, 1989) recommended the following points of focus: (a) a substantial amount of class time should be spent in developing mathematical ideas and conceptual learning as opposed to procedural learning; (b) quality questioning is key in eliciting students’ thinking and enabling students to articulate, either verbally or in writing, their understanding of a concept; (c) students must be able to construct feasible arguments while commenting on others’ justifications; and (d) any type of technology, such as a calculator or computer, must be used in ways that are most beneficial to teach mathematically proficient students.

*Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014), in concert with a national push to confirm the effectiveness of teaching strategies, connects research-based teaching in mathematics to practice. The book includes information to help classroom teachers, curriculum leaders in the school, administrators, parents, and board of education personnel understand the Common Core State Standards for Mathematics (CCSSM, NRC, 2010). *Principles to Actions* recommends research-based actions to all stakeholders in education in part to change professionals’ understanding of what type of teaching produces high levels of mathematics learning. *Principles to Actions* has six Guiding Principles that reported over a decade of research about successful mathematics programs as well as exposed negative beliefs that continue to compromise improvements in mathematics education. The Teaching and Learning Guiding Principle is the largest section in *Principles to Actions* (NCTM, 2014) and is most relevant to this study. The remaining five principles support the Teaching and Learning
Principle. The Teaching and Learning Principle explains how complex teaching mathematics is because it requires teachers to have not only a deep understanding of mathematics content but also a clear understanding of the grade-to-grade progression of how students learn mathematics (NCTM, 2014). The Teaching and Learning Principle also calls for teachers to be equipped with pedagogical knowledge, knowledge needed for teaching mathematics effectively to all students (NCTM, 2014). The following Mathematics Teaching Practices are also part of the Teaching and Learning Principle and should be “consistent components of every mathematics lesson” (NCTM, 2014, p. 3):

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build on procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

The Executive Summary of Principles to Actions (NCTM, 2014) suggests that in order to ensure mathematics success for all students, superintendents, central office personnel, principals, coaches, specialists, other school leaders, teachers, and even policy and lawmakers in districts or states, need to focus on productive beliefs, new instructional practices in mathematics, and implementation of supporting elements in order to move past the obstacles that hinder positive progress in mathematics achievement. It is important to understand the Mathematics Teaching Practices. This study’s focus on EMSs’ foundational beliefs and practices, paired with their
content and pedagogical knowledge in mathematics, is the foundation of what effective instruction should look like in classrooms.

**Standards related to elementary school mathematics education.** The National Council of Teachers of Mathematics (NCTM, 1989) published the first set of content standards to give teachers a structure for improving students’ conceptual understanding and learning. The standards were designed to establish clear expectations for how mathematics teaching should take place in schools. Prior to these standards, many mathematics teachers across the country taught in traditional, teacher-centered classrooms where teachers often told the students the information they needed to know while the students completed problems on a workbook page or from the book to practice the new skill (Schank & Menachem, 1991). Because of the 1989 NCTM Standards, mathematics instruction began to shift from an emphasis on procedures to a focus on blending procedural knowledge with conceptual understanding (NCTM, 1989). The NCTM *Curriculum and Evaluation Standards for School Mathematics* (1989) challenged the traditional methods of teaching and promoted a problem-solving-based form of instruction. This attempted to improve student learning by allowing students multiple opportunities to (a) work with problem-solving questions or scenarios, (b) strategize a situation, and (c) demonstrate their ability to reason mathematically and critically to solve problems (NCTM, 1989).

Two years after the *Curriculum and Evaluation Standards for School Mathematics* (1989) was published, the *Professional Standards for Teaching Mathematics* (1991) was printed to focus on teaching practices. Four major core dimensions of the Professional Standards were (a) tasks, (b) discourse, (c) environment, and (d) analysis. Tasks were defined as questions, exercises, and high-quality problems that were used to build conceptual understanding while engaging students in meaningful mathematics learning (NCTM, 1991). Discourse in
mathematics refers to different ways of communication: verbally, or through one’s thinking that needs to take place when students are engaged in a high-quality task (NCTM, 1991). The classroom environment represented the setting in which learning takes place. The environment is key in building conceptual understanding because it is the context in which tasks and strategically encouraged discourse are implanted in the learning experience (NCTM, 1991). Lastly, analysis was described as facilitator-led reflection during instruction or after the lesson ended (NCTM, 1991). The analysis process also examined the relationship between what the teacher was doing, what the students were doing, as well as the level of depth of learning.

Four years after the *Professional Standards for Teaching Mathematics* (1991) was introduced, NCTM decided to move toward a specific focus on mathematics assessments. In 1995 NCTM released the *Assessment Standards for School Mathematics*, one of the many responses to the call for reform in teaching and learning mathematics. The *Assessment Standards for School Mathematics* (1995) revolved around three major terms: curriculum, evaluation, and standards. Curriculum is the plan for instruction that describes what students needed to know, how they would achieve their goals, and what the teacher’s role was in aiding the students with their conceptual understandings in mathematics. Also, curriculum is the context in which student learning and instructional decisions took place (NCTM, 1995). The evaluation section focused on the value of gathering information about student growth, through formative or summative assessments, while also evaluating curricular programs. The standard can be used to evaluate the quality of the curriculum or methods of evaluation. Five years later in 2000, NCTM introduced yet another set of standards in mathematics education.

In 2000, NCTM released *Principles and Standards for School Mathematics* (PSSM). These updated standards provided both content and instructional expectations for high-quality
mathematics classrooms and programs. An emphasis of PSSM (2000) was on the need to prepare and support teachers. PSSM also acknowledged the importance of reflecting on and assessing students’ learning, and on examining an instructional program’s efficacy. The *Curriculum Focal Points* (NCTM, 2006) are recommended major content areas of focus in mathematics for grades prekindergarten through eighth grade. The focal points are made up of foundational skills and concepts for mathematical understanding including the ability to apply mathematical knowledge to future mathematics learning.

The most recent blueprint for reform at the classroom level was the Common Core State Standards for Mathematics, CCSSM, developed under the leadership of state governments to improve mathematics education (NGA, 2010; Schmidt & Burroughs, 2013). As of 2015, CCSSM were adopted by 42 of 50 states and the District of Columbia. The Common Core Standards are informed by the highest, most effective standards from states across the United States and countries around the world. The standards define the knowledge and skills students should gain throughout their K-12 education to graduate high school prepared to succeed in entry-level careers, introductory academic college courses, and workforce training programs. (NGA, 2010, About the Common Core State Standard, p. 1, para. 4)

With the new reforms in education and the push for the CCSSM to be adopted by many states, teachers were expected to teach to rigorous standards (Rowland, 2015). According to the CCSSM, the standards required mathematics teachers to “significantly narrow and deepen the way time and energy are spent in the classroom” (NGA, 2010, introduction, p. 1) instead of focusing on covering as many topics as possible over the course of the school year.

For many years the curriculum has been “a mile wide and an inch deep” (Schmidt, Houang, & Cogan, 2002); the focus was on coverage of content rather than depth of understanding. The CCSSM (NGA, 2010) focused on three key instructional shifts: (a) focus,
(b) coherence, and (c) rigor. Focus referred to teaching in depth over breadth. Coherence meant, “the standards were designed around coherent progressions from grade to grade. Student learning was carefully connected across the grades so that students could build their new understandings on the foundations built in previous years” (NGA, 2010, introduction, p. 1). Lastly, rigor referred to “deep, authentic command of mathematical concepts, but not making the mathematics harder, nor introducing new topics at earlier grades” (NGA, 2010, introduction, p. 1).

In 2011 NCTM published The Administrator’s Guide: Interpreting the Common Core State Standards to Improve Mathematics Education. The Administrator’s Guide was one resource that helped administrators understand recommendations that were made over the last 10 years in mathematics education with an emphasis on the Common Core State Standards for Mathematics (CCSSM, NGA, 2010). This document features key ideas to keep in mind when looking at high-quality mathematics programs to purchase for districts.

**Standards for mathematical practice.** The CCSSM focused on developing higher-level thinking and reasoning skills in the mathematics classroom by outlining both standards for mathematics content and standards for mathematical practices. The Standards for Mathematical Practice (SMPs) were derived from “NCTM’s processes and proficiencies” (NGA & CCSSO, 2010, p. 6). The NCTM processes were “problem solving, reasoning and proof, communication, representation, and connections” (NGA & CCSSO, 2010, p. 6). The strands of proficiencies were (a) conceptual understanding, (b) procedural fluency, (c) productive disposition, and (d) adaptive reasoning (NRC, 2001; NGA & CCSSO, 2010, p. 6). According to the CCSSM, the Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. The SMPs focus on what
the students are doing in the classroom during mathematics time and provide guidance for what it means for students to understand mathematics through application (NGA, 2010, Standards for Mathematical Practice, p. 1, para. 1).

The following are the eight SMPs:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Standards for Mathematical Practice describe what mathematics educators should pursue in developing their students’ knowledge and understanding. The SMPs focus on student behaviors or habits that should be exhibited when solving high-quality tasks through problem solving. For this study, the SMPs are indicators that signal evidence of teaching and learning through high-quality tasks. The SMPs were some of the look-for indicators when using the observation instrument in data collection for this study.

**National and international studies in mathematics education.** National and international studies further support the need for reform in mathematics education in the United States. To understand teaching practices that effect students’ performance, two major studies are important: (a) the National Survey of Science and Mathematics Education (Banilower, Smith, Weiss, Malzahn, Campbell, & Weis, 2013) and (b) the Trends in International Mathematics and
Science Study (TIMSS, 1995, 1999, 2003, 2007, 2011, 2015). The National Survey of Science and Mathematics Education was designed to provide current information about trends in the areas of (a) teacher background and experience, (b) curriculum and instruction, and (c) the availability and use of instructional resources (Banilower et al., 2013, p. 1). A total of 7,752 participants made up a clustered and stratified sample of K-12 science and mathematics teachers in 50 states and the District of Columbia (Banilower et al., 2013). Banilower and colleagues found a disturbingly low percentage of elementary teachers reported that they felt well prepared to teach the following: Number and Operations, 77%; Measurement, 56%; Geometry, 54%; and early Algebra, 46%. A positive trend for mathematics instruction that the 2012 National survey data found was that the use of subject matter specialists for pullout instruction for enrichment and/or intervention was more established in mathematics than in science.

Like the National Survey of Science and Mathematics Education, the Trends in International Mathematics and Science Study (TIMSS, 1995, 1999, 2003, 2007, 2011, 2015) also identified global gaps in national achievement scores in mathematics and science. The TIMSS study in 2003 determined that there were three common characteristics among the highest achieving countries in mathematical content: (a) focus, (b) coherence, and (c) rigor (Schmidt, Wang, & McKnight, 2005). In the 2011 TIMSS study, the highest performing countries in fourth grade were Singapore, Korea, Hong Kong SAR, Chinese Taipei, and Japan (p. 39). There were only two benchmarking states that performed similarly to these top countries: Florida and North Carolina. The United States showed slight improvement each year in this report but was still not showing significant gains on the top-of-the-chart education systems such as Singapore, Hong Kong-CHN, Korea, Chinese Taipei-CHN, Japan, Northern Ireland-GBR, Russian Federation, England-GBR, Kazakhstan, and Florida-USA (TIMSS, 2011, results, p. 11). Only 4
years later, the 2015 TIMSS report indicated that on average, fourth- and eighth-grade students in the United States did show long-term gains between 1995 and 2015, but were still not competitive with the highest-ranking countries.

All the previously mentioned reports, standards documents, and studies clearly communicate the importance of developing students’ conceptual understanding as well as procedural knowledge in mathematics to promote high-level mathematical competence. If EMSs are to adequately support teachers developing students’ conceptual understanding, EMSs themselves need deep conceptual understanding of mathematics. One purpose of this study was to determine if there were relationships between EMSs’ level of conceptual understanding and their abilities in and beliefs about mathematics by analyzing their data from the Coaching Skills Inventory, Coaching Knowledge Survey, journal entries about the EMSs’ beliefs about mathematics instruction, and responses to some of the interview questions.

**Conceptual Understanding**

Students understand an idea in mathematics (i.e., conceptual understanding) when they know how that idea relates or connects to other ideas in mathematics (Hiebert & Carpenter, 1992). Understanding is a web or net of connected ideas. Therefore, new skills and concepts must be connected to students’ prior knowledge and understanding (Hiebert & Carpenter, 1992; Skemp, 1976, 1987, 1999). Lester and Charles (2003) stated that conceptual learning happens when relevant mathematics concepts and skills are integrated into the process of teaching through solving problems. When students learn through problem solving it helps them move beyond isolated ideas and more toward making connections and modifying their thinking to integrate prior knowledge (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Lambdin, 2003).
Students who understand mathematics concepts see mathematics as more than a set of memorized procedures (Webb, 2002). When students can make connections between mathematical representations or related mathematical ideas, they have conceptual understanding (Hiebert & Carpenter, 1992). Knowing what conceptual understanding is and what it looks like are major components in this study because the EMSs’ beliefs and practices related to instructional coaching and conceptual understanding in mathematics were documented and analyzed to look for deficits in their understanding and practice while implementing a high-quality task.

Students may have difficulty applying their knowledge unless they have a solid conceptual understanding coupled with procedural fluency (Tottossy, 2013). Procedural skills are evident when students show speed and accuracy in making calculations and solving problems (Webb, 2002). Application in this context refers to the students’ understanding of how to accurately use mathematical reasoning in situations that require mathematical knowledge, which is also known as procedural fluency (Webb, 2002). Principles to Actions (NCTM, 2014) defines procedural fluency as the ability to apply procedures accurately and efficiently and to have flexibility in one’s thinking and reasoning when realizing that one strategy or procedure is more appropriate than another. Students must also have multiple experiences building on their foundational ideas of conceptual understanding, strategic reasoning, and problem solving to develop procedural fluency (NCTM, 2000, 2014; NGA Center & CCSSO, 2010). “Developing conceptual understanding,” coupled with “procedural proficiency is crucial in becoming mathematically proficient” (Van de Walle, Karp, & Bay-Williams, 2015, p. 25).

Hiebert and Carpenter (1992) described procedural knowledge, as the knowledge gained when one knows how to solve or answer a problem with a basic procedure such as an algorithm.
or a rule. They stated that when students are only exposed to procedural learning situations, they are more susceptible to having misconceptions than are students who were taught more conceptually (Hiebert & Carpenter, 1992). Procedural knowledge is the knowledge gained from formal language or using algorithms, formulas, and procedures, but without concurrent conceptual understanding, there is not a deep level of connecting prior understanding with new knowledge (Carpenter et al., 2005; Hiebert et al., 2000).

Whether teaching for conceptual understanding or procedural skills during a mathematics lesson, teaching skills or concepts in isolation many times hinders the thinking process for students; it is more difficult to make connections between concepts (Hiebert & Carpenter, 1992; Skemp, 1976, 1987, 1999). Van de Walle et al. (2014) stated that, “The ineffective practice of teaching procedures in the absence of conceptual understanding results in a lack of retention and increased errors…and inefficient strategies” (p. 25).

Baggett and Ehrenfeucht (1997) posited that despite the many changes in mathematics education, the mathematical content in the elementary grades has remained virtually the same. They stated that “arithmetic skills should be de-emphasized and instead children should learn ‘general problem solving methods’ that are sufficient in modern technology society” (Baggett & Ehrenfeucht, 1997, p. 1). Contrary to the beliefs of many educators, mathematicians, and what current research says to support this idea, there is growing opposition from traditionalists who want to go back to the basics and learn mathematics procedurally. This method of teaching by telling (i.e., procedurally) and learning about specific topics in isolation without connections generally falls short because students are not given the opportunities to see how the topics overlap to build their conceptual knowledge (Baggett & Ehrenfeucht, 1997).
There are many benefits of learning mathematics with understanding. Students who understand mathematics see mathematics as more than a set of procedures to be memorized. Students who understand mathematics are better able to apply concepts and procedures to solve problems. Students who understand mathematics are motivated to learn. And, students who understand mathematics are better able to learn new mathematics (Lambdin in Lester & Charles, 2003).

**Teaching Through High-Quality Mathematical Tasks**

A teacher can tell students about mathematical concepts, but we know from research that this does not develop understanding (Hiebert & Carpenter, 1992; Skemp, 1976, 1987, 1999). Rather, conceptual understanding is best developed when new concepts and procedures are introduced through the process of solving problems in which the new ideas and procedures are embedded (Lester & Charles, 2003). This process is called teaching/learning through problem solving. When students learn through problem solving it helps them move beyond isolated ideas and more toward making connections with prior knowledge (Carpenter, Franke, Jacobs, Fennema, Empson, 1998; Lambdin, 2003). Developing conceptual understanding through solving problems begins with students solving high-quality mathematics problems, often called high-quality tasks.

A mathematical task is defined as a problem that involves some aspect of a new mathematics idea to be learned and in which students must productively struggle to devise a solution (NCTM, 2000). The process of solving this type of high-quality task enables students to connect previously learned ideas with new ideas, and that process of making connections builds understanding. Cognitive demand is the complexity of the thinking that students need to do to solve the problem or task. Research shows that the high cognitive demand of a task, paired with
students’ engaging in productive struggle, promotes students making stronger connections among ideas and concepts (Van de Walle et al., 2015); that is, conceptual understanding.

Stein, Grover, and Henningsen (1996) classified tasks between four different levels, depending on the cognitive demand required by the students. Tasks that are considered a low level of cognitive demand, low-level tasks, involve memorization or the application of procedures without any connection or understanding to any mathematical concept as seen in Table 1 (Stein et al., 1996). Low-level tasks are appropriate if the teacher expects students to memorize a formula or practice a procedure. However, if the instructional goal is understanding, then the instructional task should provide high cognitive demand: a high-level task (NCTM, 2000; Stein & Lane, 1996). High-level tasks generally have multiple entry and exit points that allow students to approach the task in different ways based on their prior knowledge of problem solving and mathematical reasoning as well as have various ways to express the solution (Van de Walle et al., 2015). High-level tasks can feature multiple representations and provide opportunities for mathematical discourse (Stein et al., 1996).

High-quality mathematical tasks are important vehicles for developing students’ capacity to think deeply and critically and develop true understanding when solving basic mathematics problems (Cai, Moyer, Nie, & Wang, 2006). Cai et al. (2006) investigated the effects of teachers using the Connected Mathematics Program (CMP) with middle school students versus teachers who were not using the same curriculum. They found that when teachers used the CMP curriculum and specific strategies, paired with high-quality tasks from the text, students were more engaged in the learning process while also exhibiting the ability to make connections and develop a mathematical lens for critical thinking (Cai et al., 2006; Doyle, 1983).
The National Council of Teachers of Mathematics (NCTM) has played a critical role in defining mathematical tasks and describing ways they should be implemented in classroom instruction to develop understanding. *Principles to Actions* (NCTM, 2014) and *Principles and Standards for School Mathematics* (NCTM, 2000) recommended that students need more work with complex mathematical tasks that allow students to explore mathematical concepts and relationships (Stein, Smith, Henningsen, & Silver, 2009). Stein et al. (1996) considered mathematical tasks intentional and enhanced classroom activities that allow students to focus on a specific mathematical concept. Cai et al. (2006) further described high-quality mathematical tasks as paramount for students to have the opportunities for critical thinking and reasoning, but also for students to figure out ways to process their information. Students having the opportunities for reflection and communication with one another are an essential opportunity for learning mathematics with conceptual understanding (Hiebert et al., 1997).

Some tasks “have the potential to engage students in complex forms of thinking and reasoning, whereas others focus on the memorization or use of rules or procedures” (Stein et al., 2009, p. 4). The current study used the Task Analysis Framework Guide (Table 1) to help provide a clearer explanation of what it means to “do mathematics” or what a rigorous or high-quality task experience includes (Stein et al., 2009).
### Table 1

**The Task Analysis Guide (Stein & Smith, 1998, p. 16)**

<table>
<thead>
<tr>
<th>Lower-Level Demands</th>
<th>Higher-Level Demands</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Memorization Tasks</strong></td>
<td><strong>Procedures With Connections Tasks</strong></td>
</tr>
<tr>
<td>Involves either producing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory. Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. Have no connection to the concepts or meaning that underlies the facts, rules, formulae, or definitions being learned or reproduced.</td>
<td>Focus students’ attention on the use of procedures for developing deeper levels of understanding of mathematical concepts and ideas. Suggest pathways to follow (explicitly or implicitly) that are broad, general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures to successfully complete the task and develop understanding.</td>
</tr>
</tbody>
</table>

**Procedures Without Connections**
Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. Have no connection to the concepts or meaning that underlies the procedure being used. Are focused on producing correct answers rather than developing mathematical understanding. Require no explanations, or explanations that focus solely on describing the procedure that was used.

**Doing Mathematics**
Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships. Demands self-monitoring or self-regulation of one’s own cognitive processes. Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task. Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.
Solving high-quality, worthwhile (Van de Walle et al., 2015) tasks requires problem-solving skills or knowledge that provides an entry point to solving a task. Van de Walle, Karp, and Bay-Williams (2015) proposed three approaches to teaching problem solving: (1) teaching for problem solving, (2) teaching about problem solving, and (3) teaching through problem solving. Teaching for problem solving is the traditional approach of having a few word problems at the end of a unit to see if students can apply what they know (Van de Walle et al., 2015). Teaching about problem entails students learning how to problem solve; that is, learning strategies, such as drawing illustrations, that help students in problem solving (Van de Walle et al., 2015). Teaching through problem solving means that students learn mathematics through inquiry-based learning (e.g., real-world contexts, problems, situations, models) (Van de Walle et al., 2015). Teaching through problem solving acknowledges what students know about mathematics and can explain what it means to do mathematics. This type of focused teaching and learning emphasizes students’ engagement in true inquiry (Van de Walle et al., 2015). When teaching through problem solving is the instructional approach used, students are more apt to develop conceptual understanding (NRC, 2001; Van de Walle et al., 2015). However, there are several challenges teachers face to appropriately teach mathematics through problem solving.

Stein, Grover, and Henningsen (1996) conducted a study using a random sampling of 144 mathematical tasks used with a reform-based instruction platform. In their study, a mathematical task was defined as a set of complex problems surrounding a specific mathematical idea on which the students’ attention is focused (Stein et al., 1996). Derived from the narrative summaries and field notes collected during 3 days of observations in three different math classrooms over a period of 3 years, the findings suggest that teachers were selecting and setting up tasks that enabled students’ thinking capacity to reach a high level. However, the cognitive
demand of the high-quality tasks, as well as some additional factors, tended to decline when the
task transitioned from the set up to the implementation phase (Stein et al., 1996). The study by
Stein et al. supported this dissertation study in terms of the selection and implementation levels
of high-quality mathematical tasks and the high level of conceptual understanding that must take
place for students to truly understand mathematics.

**Mathematical task models.** Van de Walle (2004) stated that the shift to instruction
focused on problem solving occurred to give all learners a chance to solve mathematics problems
at their appropriate developmental level. Van de Walle also stated that problem solving should
be part of the learning process by connecting thinking and mathematics content to real-world
contexts. Many researchers found relationships between various task-related variables and
students’ learning outcomes (Henningsen & Stein, 1997, p. 528; Smith et al., 1996; Stein &

From the analysis of the recorded lessons, the researchers developed the Mathematical
Tasks Framework (Stein et al., 1996) to model what the progression of a task should look like
during daily, mathematics instructional time. This model focused on teachers being intentional
about how they selected and implemented a mathematical task in their respective classrooms.
Figure 2 shows the task level set up, the level of the task implementation, and the level of student
learning.

*Task set up* is the task the teacher gives to the students to complete (Smith et al., 1996).
The task level can range from a low-level task students complete through verbal cues from the
teacher to a high-level task that requires students to apply prior knowledge and new knowledge
to solve the task (Smith et al., 1996). *Task implementation* is how the student works on the task,
based on how the teacher proposes the task to be completed per the teacher’s instruction (Smith et al., 1996).

<table>
<thead>
<tr>
<th>Task Set Up</th>
<th>Task Implementation</th>
<th>Student Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Level</td>
<td>High Level</td>
<td>High Level</td>
</tr>
<tr>
<td>Low Level</td>
<td>Low Level</td>
<td>Low Level</td>
</tr>
<tr>
<td>High Level</td>
<td>Low Level</td>
<td>Moderate Level</td>
</tr>
</tbody>
</table>

*Figure 2. Patterns of set up, implementation, and student learning (Stein & Lane, 1996).*

The QUASAR results also show that students who performed at the highest levels were in classrooms where the high-leveled tasks were set up and enacted at the highest cognitive demand that teachers intended (Stein & Lane, 1996). Stein and Lane noted in their results that when instruction was characterized using tasks featuring high-level cognitive demands, multiple entry points, multiple types of representations, and paired with opportunities for mathematical discourse, there were greater gains in student learning than in classrooms that did not demonstrate these characteristics. One of the greatest benefits in knowing about this model is
that it clarifies that although a task is selected with the intent of being a high-quality task, incorrect implementation may reduce cognitive demand, rendering the task a lower level task. Factors that are associated with the maintenance and decline of a high level of cognitive demand are the following: (a) a shift from the emphasis on making meaning or understanding concepts to simply having a correct answer, (b) limiting students’ time to have the opportunities for productive struggle for the high level of thinking and reasoning required to grapple with a high-quality task, (c) a lack of effective classroom management enabling students to be off task or disruptive, (d) selecting a task that is either too high or too low-level for the specific group of students, or (e) the teacher failing to have high standards or levels of accountability for the students’ products or thought processes (Stein et al., 1996). High-level cognitive demand tasks selected by the teacher help students’ learning in some of the following ways: (a) assist in the scaffolding of student reasoning, (b) allow the teacher to push hard for justifications from the students through questioning, (c) allow the process of selecting specific tasks that build on students’ prior knowledge, and (d) allow a sufficient amount of exploration time for the students to solve the problem (Stein et al., 1996). If the cognitive demand of the task is maintained and enacted at a high level, students generally exhibited higher student achievement (Boaler & Staples, 2008; Stein & Lane, 1996).

The previously described research demonstrates that teaching mathematics through problem solving is an effective instructional model to develop conceptual understanding (Van de Walle, 2015). The initial step in providing students with the opportunity to solve high-demand mathematical tasks begins with the EMS or teacher being able to discern what a high-quality task is and implementing it in certain ways. If EMSs are to effectively help teachers develop
students’ conceptual understanding in mathematics, then EMSs need to be able to identify and select high-quality mathematical tasks and implement them effectively.

**Elementary Mathematics Specialists**

This section outlines the history and roles of EMSs as well as the positive impact they can have on teachers and students. In addition, the section summarizes the types of knowledge possessed by EMSs who effectively support teachers in developing their students’ conceptual understanding. Lastly, the section discusses how EMSs’ beliefs about teaching and coaching relate to the types of knowledge they need to successfully support teachers.

**History of elementary mathematics specialists.** Mathematics educators have advocated for elementary mathematics specialists for many years (Fennell, 2006; Lott, 2003; Reys & Fennell, 2003). Several researchers have been at the forefront of examining the impact of EMSs on the mathematics achievement of students (Campbell, 2007; Campbell & Malkus, 2009, 2011; Erchick et al., 2007; Rowan & Campbell, 1995). In 1981, NCTM recommended that there should be a teaching credential endorsement for each state for EMSs (Fennell, 2011). Since 1981, the idea of mathematics specialists developed at a fast pace. A few years later the President of NCTM, John Dossey, wrote about the importance of a mathematics specialist in elementary schools (Dossey, 1984). Since 1984, there have been several NCTM presidents (Dossey, 1984; Fennell, 2006; Gojak, 2013; Lott, 2003) who have promoted the need for EMSs.

In 2008, the National Mathematics Advisory Panel stated, “research should be conducted on the use of full-time mathematics specialists in elementary schools” (p. xxii). Their statement was recommended based on the Panel’s findings relative to the importance of teacher content knowledge—that most preservice elementary teacher education programs do not address the teaching and learning of mathematics in enough depth to be effective teachers of mathematics.
A study by Gerretson, Bosnick, and Schofield (2008) showed that EMSs were needed to help teachers: (a) prepare students more effectively, (b) prepare students to attain higher performance on standardized testing, and (c) assist students to transition their knowledge to the real world. Grounded in best practices, the study by Gerretson et al. (2008) identified factors associated with the growing use of teacher specialists in elementary schools, primarily in mathematics. The study took place in multiple elementary schools in a large metropolitan school district in northeastern Florida. The focus of the study was to identify factors based on the use of teacher specialists (i.e., EMSs) in the capacity of mathematics instruction. They found that the district encouraged the elementary schools to use a modified specialist model based on a team teaching approach. Roughly 59% of the elementary schools in the study that used teacher specialist were considered traditional schools (Gerretson et al., 2008, p. 307). The findings also indicated that there was a need for teachers to improve their instructional practices to increase student achievement. The schools that made use of the specialist model indicated that the benefits that faculty members expressed were centered on the teachers having more time to plan effective instruction and focus on the implementation of the content.

According to Fennell (2011), nearly every state offered a certification for reading or literacy specialists in 2001, yet fewer than 15 states had certified elementary mathematics specialists in their schools. As of 2011, Arizona, California, Georgia, Maryland, Michigan, North Carolina, Ohio, South Dakota, Texas, Utah, and Virginia were the only states to have EMS guidelines for certifications (Fennell, 2011). Effective as of January 17, 2014, New Hampshire required a master’s degree in mathematics, education, or a related field. The Virginia Department of Education gave their approval for the EMS and Middle Education (ME) specialist degree on September 21, 2007. They also required a master’s level program in mathematics or
mathematics education, or a master’s degree from an approved mathematics specialist program.

Milestones of EMS, was originally created by Fennell (2011), but has since been updated by McGatha and Rigelman in 2017 (see Table 2).

Table 2

<table>
<thead>
<tr>
<th>Date</th>
<th>Milestones</th>
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<tbody>
<tr>
<td>1984</td>
<td>NCTM Recommends State Certification Endorsement for Elementary Mathematics Specialists</td>
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<tr>
<td>1987</td>
<td>Exxon Foundation supports K-3 Mathematics Specialists.</td>
</tr>
<tr>
<td>1989</td>
<td>Everybody Counts supports specialization in mathematics at the elementary school level.</td>
</tr>
<tr>
<td>2000</td>
<td>NCTM’s Principles and Standards for School Mathematics suggests exploration of models for elementary mathematics specialists and teacher leaders</td>
</tr>
<tr>
<td>2001</td>
<td>CBMS’ The Mathematical Education of Teachers recommends that mathematics in the middle grades should be taught by mathematics specialists, starting at least in the 5th grade</td>
</tr>
<tr>
<td>2001</td>
<td>Adding it Up reiterates the need for mathematics specialists</td>
</tr>
<tr>
<td>2001</td>
<td>PL 107-110 the No Child Left Behind Act</td>
</tr>
<tr>
<td>2003</td>
<td>NCTM President’s Message: The Time Has Come for Pre-K-5 Mathematics Specialists (Johnny Lott)</td>
</tr>
<tr>
<td>2006</td>
<td>NCTM President’s Message: We Need Mathematics Specialists NOW! (Francis [Skip] Fennell)</td>
</tr>
<tr>
<td>2007</td>
<td>Virginia Commonwealth University, Virginia Science and Mathematics Coalition: Virginia Mathematics Specialist Project</td>
</tr>
<tr>
<td>2009</td>
<td>Elementary Mathematics Specialists and Teacher Leaders Project established at McDaniel College</td>
</tr>
<tr>
<td>2010</td>
<td>AMTE Standards for Elementary Mathematics Specialists released (Revised in</td>
</tr>
</tbody>
</table>
2013 to be more reflective of *The Mathematical Education of Teachers II* and the *Common Core State Standards for Mathematics*.

**2010**

**2010**
- *Common Core State Standards for Mathematics* released (CCSS-M)

**2011**
- 45 States and the District of Columbia Transition to the CCSS-M and consider PARCC and SMARTER Balanced consortia assessments.

**2012**
- CBMS’ *The Mathematical Education of Teachers II* revised to provide recommendations for content background of elementary teachers to more reflective of content domains and standards of the CCSS-M. Varied references to the use and potential of elementary mathematics specialists provided.

**2012**
- NCTM/NCATE (now CAEP) *Standards for Elementary Mathematics Specialists* released. Program providers (campus and online) now use these standards for accreditation of advanced programs (endorsements and/or MS degree programs), which provide certification for elementary mathematics specialists provided.

**2013**
- NCTM President’s Message: *It’s Elementary! Rethinking the Role of the Elementary Classroom Teacher* (Linda Gojak)

**2015**
- NCTM Research Brief (updated from 2009) *The Impact of Mathematics Coaching on Teachers and Students* (Ed: McGatha, Davis Stokes)

**2015**
- Brookhill Institute of Mathematics: AMTE EMSs Research Conference

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- PL 114-95 the *Every Student Succeeds Act*

As an interest in EMSs grew across the nation, many states in the United States began to take notice of the benefits of having a qualified EMS in each elementary school. Currently in 2017, 20 states (Arizona, California, Georgia, Idaho, Kentucky, Louisiana, Maryland, Missouri, New Hampshire, North Carolina, Ohio, Oklahoma, Oregon, Pennsylvania, Rhode Island, South Dakota, Texas, Utah, Virginia, and West Virginia) offer certification or endorsement programs for elementary mathematics specialists (AMTE, 2013; ems&tl, 2011, 2017). There are also nine states that are in the process of, or in the final stages of seeking state level approval to adopt a
mathematics specialist certification (ems&tl, 2017; McGatha & Rigelman, 2017). Although only 20 states currently offer an EMS license, certificate, or endorsement, experienced teachers in almost every state are called upon to fill specialized roles related to mathematics teaching in elementary grades, often without any type of special training or certification (ems&tl, 2011). *Maryland’s Keys to Math Success: A Report from the Maryland Commission* (Maryland State Department of Education, 2001) used recommendations from several resources, *Principles and Standards for School Mathematics* (NCTM, 2000), *Adding it Up* (NRC, 2001), and *Mathematical Education of Teachers* (CBMS, 2001), to help make a push that their state pursue recommendations of an EMS certification (McGatha & Rigelman, 2017). As states and provinces move to more widespread implementation of EMSs, additional research is needed to document the positive impact on the teaching and learning of mathematics (Becker & McGatha, 2008; Brosnan & Erchick, 2010; Campbell, 1996; Campbell, 2007; Campbell & Malkus, 2009, 2011; Gerretson et al., 2008; McGatha, 2009; Meyers & Harris, 2008; Race, Ho, & Bower, 2002).

**Definitions, functions, and roles.** For the purposes of this study, research findings were used to (a) define EMS (Feger, Woleck, & Hickman, 2004; McGatha, 2009; Sutton, Burroughs & Yopp, 2009), (b) identify what roles and responsibilities are typical for an EMS, and (c) see how existing research was related to the data compiled from various quantitative and qualitative instruments during this current study. McGatha (2009) stated that a mathematics coach works with teachers, whereas a mathematics specialist’s main work is with students. According to Sutton et al. (2009) a coach can be defined as a specific educator in the school who collaborates with teachers in order to fine tune their (a) teaching practices, (b) content knowledge of the
subject, and (c) pedagogy of teaching; Feger et al. (2004) agreed that coaching was designed to focus on the art of helping teachers with classroom instruction.

The word *coaching* has many context-dependent definitions. As defined by the National Mathematics Advisory Panel (2008), coaching is a popular approach in the teaching field to aid teachers with improving elementary mathematics instruction; Showers (1985) agreed that coaching takes place in a systematic way, and along the way, teachers develop higher efficacy for adopting new instructional strategies. Coaching overall should be a relationship-based collaboration used to move teachers in the right direction of implementing new instructional strategies and making instructional shifts while at the same time encouraging professional growth (Showers, 1985). Showers suggested the following purposes that coaching encompasses: (a) building communities of teachers who continuously engage in the study of their craft, (b) developing the shared language and set of common understandings necessary for the collegial study of new knowledge and skills, and (c) providing a structure for the follow-up to training that is essential for acquiring new teaching skills and strategies (p. 19).

Neufeld and Roper (2003) defined an EMS’s primary roles as engaging teachers in collaborative practices, while at the same time leading them to use instructional practices out of their normal comfort zones. EMSs are then able to go deeper in their practices with the teachers and see shifts in thinking while engaging students in meaningful learning. Neufeld and Roper (2003) stated that in one of their promising districts, employing full-time or part-time coaches in schools who could provide ongoing, sustained support to principals and teachers, would positively impact classroom instruction and student achievement (p. iii).

Similar to Neufeld and Roper (2003), Wolpert-Gawron (2016) described an instructional coach as being a mentor, someone who organizes or conducts professional development;
someone who focuses on learning innovative instructional practices while also serving as a curator of the wisdom with content and pedagogy; being an advocate for publicity in the community; or being a change agent in education by collaborating with other instructional coaches. According to NCSM (2016), coaching suggests the collaboration of the expert in the field working beside the teacher in the classroom to provide the best balance between helping the teacher grow professionally while increasing student achievement in the mathematics classroom through purposeful and intentional instruction, based on research-based conceptual strategies, pedagogy, and understandings.

In his book, Instructional Coaching, Knight (2007) affirmed that in general, specialists should be the ones who provide ongoing support for teachers based on the professional development sessions they have attended, and the lessons learned from previous classroom instruction experiences. The presence of qualified EMSs in all schools should provide general elementary teachers with deeper content and pedagogical knowledge for teaching mathematics through effective coaching and lesson planning (Campbell & Malkus, 2011; Hill et al., 2005). Care must be taken that with the wide range of functions and roles that EMSs can have in their schools, this broad scope doesn’t lead EMSs away from a sole focus on mathematics instruction through coaching teachers.

In addition to serving as an instructional coach, an EMS also has many functions in a school or district (i.e., collaborator, resource, professional development provider, administrator, and teacher support). One function of a mathematics specialist is to support teachers and student learning through a variety of efforts such as facilitating book studies, establishing professional learning communities, or facilitating mathematics workshops or professional development (Campbell & Malkus, 2009; Fennell, 2011). Fennell (2011) suggested that mathematics
specialists and school-based specialists are responsible for sharing information about both the content standards and the Standards for Mathematical Practice:

Mathematics specialists share information about (a) how to help parents learn about the new math; (b) how teachers can communicate the new strategies with parents; (c) what type of professional development will be needed in order for a smooth transition for the teachers, students, and administrators; and (d) answers to questions about standardized testing. (Fennell, 2011, p. 55)

Campbell (1996) studied the implementation of hiring mathematics specialists in relation to specific professional development models like the teacher enhancement model. The teacher enhancement model involved the following: (a) an in-service program for all mathematics teachers that took place over the summer, (b) an EMS at each school, (c) manipulatives at each school, as well as (d) teacher planning and instructional problem solving each week during their common grade level planning time. (Campbell, 1996, p. 459)

In the Silicon Valley Mathematics Initiative, SVMI (2007), Becker studied coaching cycles and developed three main roles of a content specialist: (a) specialist as a collaborator, (b) specialist as a model, and (c) specialist as a leader. The specialist as a collaborator focused on being a resource to teachers by helping them improve their mathematics instruction through intensive professional development in the classroom. The specialist as a model focused on modeling lessons based on best instructional practices and helping the teachers understand the most important skills and concepts that they needed to teach the students. The role of the specialist as a leader was guiding the teacher in the most effective ways through pedagogy and instruction. The role of a leader also casts the EMS as a guide. As a guide, the EMS directs and redirects the teacher to take the most effective instructional and pedagogical approaches.

Mudzimiri et al. (2014) stated that regardless of the many different and unique roles that mathematics specialists can take, there must be a consistent expectation of what a specialist’s
daily work must entail to positively influence classroom instruction. The study attempted to gain a better description of the roles of an EMS, based on one, full-day observation of seven specialists in five different districts (Mudzimiri et al., 2014). The selected participants in the study were from a pool of specialists affiliated with the Examining Mathematics Coaching (EMC) Project (Sutton et al., 2011) and had established trust with project personnel. The findings of this study suggest that “the roles and responsibilities assumed by elementary mathematics specialists, the activities they engage in, and the content of their sessions with teachers vary widely in response to the shifting contexts, audiences, and tasks that arise throughout the day” (Mudzimiri et al., 2014, p. 19). Evidence from the observations showed that specialists worked on whatever needed to be done at that exact time, which was constructed by the teacher or administrator requests, which in turn consequently modified their daily routines (Mudzimiri et al., 2014).

The study of the model, Project IMPACT, Increasing the Mathematical Power of All Children and Teachers, consisted of 1,375 K-3 student participants from three urban schools. The study also used three additional comparable school sites with approximately 925 kindergartens through third-grade students. Four years later in 1994, the study included about 1,000 fourth- and fifth-grade students. Campbell found that 40% of the kindergarten through third grade elementary teachers in the study made significant shifts in their instructional practices by engaging students in the conceptual understandings of foundational mathematical concepts. NCTM confirmed the important roles that EMSs have in schools:

Teacher-leaders can have a significant influence by assisting teachers in building their mathematical and pedagogical knowledge. Teacher-leaders’ support on a day-to-day basis ranges from conversations in the hall to in-classroom coaching to regular grade-level and departmental seminars focused on how students learn mathematics—can be crucial to a teacher’s work life. (NCTM, 2000, p. 375)
Considerations need to be set for those individuals who are placed in mathematics-coaching roles. Mizell (2006) stated that having specialists in schools is a positive way to provide job-embedded, adult, professional learning in the everyday school day routine. In conclusion, the results collected through analyzing and interpreting the EMSs’ current coaching instructional practices, roles, and responsibilities, align with multiple pieces of data collected in this study.

**Coaching history and definitions.** The history and role of EMSs can be traced back for decades beginning with the concept of *instructional coaching*, which appeared in literature almost 80 years ago (Cassidy, Garrett, Maxfield, & Patchett, 2010). Educational researchers have developed a variety of coaching models such as cognitive coaching, peer coaching, specific content coaching, and instructional coaching (Hasbrouck & Denton, 2007; Knight, 2007; Knight & Cornett, 2009; Showers & Joyce, 1996). According to Adey, Hewitt, Hewitt, and Landau (2004), some coaching models may include demonstration lessons, observations with reflective feedback, a co-teaching model, forms of peer coaching, or even video-based feedback. Teachers that become specialists must learn how to transition from a teaching mind frame or teaching lens to a coaching mind frame or lens (Feger et al., 2004). Specialists need to be expert teachers in the same subject area as the teachers they serve and be able to provide specific support to teachers (Feger et al., 2004). With the many roles and responsibilities of an EMS, their experience as a mathematics teacher does not properly prepare them for what they need to be able to do (McGatha & Rigelman, 2017). EMSs could possibly be asked to design effective professional development for teachers, conduct classroom observations, assist teachers in analyzing data and determining instructional next steps, plan and provide intervention for struggling teachers, and organize effective and research-based resources for teachers (Chval, et
al., 2010; Campbell & Malkus, 2011). Feger et al. (2004) maintained that the main topics of coaching fall under the umbrella of coaching knowledge and skills, which are comprised of interpersonal skills, content knowledge, pedagogical knowledge, knowledge of the curriculum, awareness of coaching resources, and having the knowledge of the practice of coaching.

Bay-Williams et al. (2013) described five coaching skills needed for supporting teachers’ growth and professional development:

1. Building trust
2. Establishing rapport
3. Listening
4. Paraphrasing
5. Posing questions (p. 1)

These five coaching skills align with what AMTE (2013) described as aspects of leadership knowledge and skills (Bay-Williams et al., 2013). To discover participants’ enactment of leadership roles and responsibilities, the current study collected data from observations, journal entries, and interview questions, analyzing the EMSs’ responses pertaining to their coaching knowledge skills, their ideas and perceptions of successful mathematics coaching and coaches, in regard to working with teachers and modeling instruction with students in classrooms. McGatha and Rigelman (2017) stated that EMSs should focus on developing trust and establishing confidentiality with the teachers they work with and support, but also keep their relationships with administrators at the forefront. Some other things that McGatha and Rigelman (2017) suggested that EMSs may be asked to do in terms of their leadership role are the following: (a) attend professional development sessions or conferences, (b) enroll in classes, or (c) take on learning opportunities by possibly joining professional organizations (p. 29).
Coaching models. Specific content-focused coaching is a professional development model designed to promote student learning and achievement (Staub, 2001; Staub, West, & Miller, 1998; West & Staub, 2003). In content-focused coaching, the specialist and teacher plan, enact, and reflect collaboratively on the modeled and co-taught lessons (Staub et al., 1998; Staub, 2001). The teacher can also be the one teaching the lesson, subsequently reflecting with the specialist shortly afterward (Staub et al., 2003). Content-focused coaching is specific to a subject area, so the specialist can focus on one area of expertise. This coaching model supports the idea of having certified and qualified EMSs working in their areas of expertise and should match their job description, title, role, and responsibilities in their schools or districts. For the purposes of this current study, a specific coaching model was not observed or analyzed, but there were several instruments that asked the EMSs questions pertaining to their mathematics coaching, that is, skills, practices, knowledge, experiences, and beliefs.

Positive impact of elementary mathematics specialists. Existing research supports the fact that EMSs have a positive impact on teachers and students (AMTE, 2010; Campbell, 1996; ems&tl, 2009, 2011, 2016; Fennell, 2011; McGatha, 2009; Polly 2012; Utley & Reeder, 2016). The complexities of teaching and learning mathematics dictate that educators need more access to a mathematics specialist in every elementary school (Fennell, 2011). Several studies described positive changes in teachers’ practices because of interacting with an EMS professional, including actively engaging students, emphasizing reasoning and problem-solving over skills-based lessons, using students’ work to inform instruction, and effectively planning lessons (AMTE, 2013).

Specific content specialists are necessary to aid teachers with planning their instruction, preparing students more effectively, and improving students’ performance on standardized
assessments (Gerretson et al., 2008). Gerretson et al. conducted a study using data from the Florida Comprehensive Assessment Test (FCAT). They claimed that supporting effective mathematics instruction through continued access to a content specific specialist has a positive impact on student achievement and overall performance on the test. EMSs must have a deep and conceptual understanding of the elementary mathematics progression of topics across all domains and grade levels through the middle school level and utilize their knowledge while working with teachers (ems&tl, 2009, 2011; NCSM, 2008).

Campbell and Malkus (2011) conducted a 3-year randomized control study in five urban, and urban-edge school districts in Virginia. They found that specialists positively affected student achievement in the upper elementary Grades 3, 4, and 5. The EMSs in their study focused on engaging in high levels of professional coursework that concentrated on mathematical content, pedagogy, and coaching practices prior to and during their first year of placement. The participants came from five school districts with urban and urban-edge/rural-fringe communities. The study used two different cohorts of coaches, as well as utilizing a control group in the study. Two cohorts of coaches participated in a funded teacher-enhancement that facilitated professional development around mathematics content, pedagogy, and leadership courses. The 36 schools were assigned a number using a randomizing technique, and then one school was selected from each of the 12 triples. The coaches in the treatment schools were ranked based on experience as an elementary teacher. The first cohort of 12 coaches completed five content courses and one leadership course specializing in coaching prior to their placement. Following the first placement, they completed a second leadership-coaching course during their first year as an assigned coach. Over the 3-year study, the student data collected in the study was based on the annual state standardized test: Standards of Learning
Assessment (SOL). The students in this study who were enrolled in the schools with an EMS had significantly higher scores on their state’s annual achievement tests in Grades 3-5 than did the students in the control schools without the EMSs. It was only after 3 years of working with the specialists that the students were more focused in mathematical content experiences. These results further support the need for qualified EMSs in a school to increase student learning. Although there was no student data collected in this current research study, the findings confirm the need for EMSs to be properly equipped and supported with ongoing support and knowledge in content, pedagogy, and leadership.

In his book, *Unmistakable Impact: A Partnership Approach to Improving Instruction*, Knight (2011) emphasized the importance of specialists following up with teachers for the change in instructional practices to carry over to the classroom and to build sustainability. In his article, he stated that during a professional workshop, teachers should feel a sense of reassurance that the specialist will remember the key information to follow up with the teachers when collaborating, observing, reflecting, and modeling lessons.

Becker (2001) and McGatha (2008) both found comparable positive results in qualitative studies they conducted while working with mathematics coaches and classroom teachers’ instructional practices. Becker (2001) used qualitative methodology such as field notes, interviews, observations, and classroom evidence while studying six mathematics specialists and 14 elementary teachers in a professional development program and found positive changes in the teachers’ instructional shifts while working with the specialists. The three major components of Becker’s study were the following: (a) an intensive 3-week summer professional development focusing on mathematics content, PCK, and leadership skills, (b) summer laboratory school for children run by the participants, with staff support, provide professional development for team
teachers who teach the classes, and (c) comprehensive follow-up activities including workshops with leading national and international mathematics educators (Becker, 2001, p. 2).

The coaches in Becker’s study (2001) were classified into three groups based on patterns of how they interacted with their teachers (e.g., coach as collaborator, coach as model, or coach as a leader). Whether it was through cooperative lesson planning, modeling instruction, or working with the teacher about how to utilize the idea of reflection, the goal was to improve mathematics instruction by coaching teachers based on their individual needs. Some of the changes that came from the direct support were teachers’ increased emphasis on students’ problem-solving abilities as opposed to skill-based procedural instruction (Becker, 2001). Another major adjustment was that teachers focused on the big ideas of mathematical concepts rather than learning concepts and skills in isolation with no connection to prior knowledge. These adjustments were immense improvements from the teachers’ interactions with the EMS that positively affected student learning.

To learn more about mathematics coaches, McGatha (2008) conducted a case study with two elementary mathematics coaches researching their levels of engagement and relationship building with a teacher they coached in their school. The study focused on ways to improve coaching abilities while helping strengthen teachers’ instructional practice. Over a 7-month period and through reflective analysis, McGatha found positive changes in instructional shifts from the teachers who worked with mathematics specialists in a professional development program. The data collected were from the coaches’ reflective journals, pre- and postsurveys, postinterviews, and meetings with the coaches, teachers, and researcher (McGatha, 2008). The data were also analyzed using a framework from cognitive coaching that describes three main support functions of a coach: consulting, collaborating, and coaching. Some of the significant
findings in the research showed the shifts of allowing students to become independent thinkers, while also finding positive shifts in teachers’ abilities to analyze student work to help inform their instruction (McGatha, 2008). Both Becker (2001) and McGatha (2008) found empirical evidence supporting the positive impact of a mathematics specialist on teachers’ instructional practices.

Like the studies cited above, Campbell (1996) also found empirical evidence supporting the positive results from effective mathematics coaching. Campbell studied the implementation of mathematics specialists with specific professional development models. Project IMPACT was a school-based teacher enhancement model to foster student understanding and to support teacher change in predominately minority elementary schools. Data from primary grades reveal that there were significant gains in mathematics achievement scores for the children in the IMPACT schools. The students in the study were assessed by midyear and exit exams created by the project staff. Scripted interviews evaluated Kindergarten and first-grade students, whereas written tests administered in small groups (4 to 7 children) and scripted individual interviews evaluated the older students. Over the 5-year project implementation, the tests were administered either in a 3-week period during December for second grade, or a 5-week period in January and early February for the remaining grades. The exit tests were all administered during a 6-week period in May and early June for all elementary grades. She found that 40% of teachers from the 118 schools in the study made significant shifts in their instructional practices by engaging students in the conceptual understandings of foundational mathematical concepts. Campbell also (1996) stated that teachers would not be able to easily make shifts in their instructional practices without having the support of a mathematics specialist in the school.
Edwards and Newton (1994) found that teachers who participated in cognitive coaching were more satisfied with the teaching profession than teachers who were not part of a coaching community. When teachers were more comfortable with their curricular content, they felt as if they were not alone and tended to enjoy the partnership of learning when working with a specialist (Edwards & Newton, 1994). Time and time again, effective and research-based mathematics instruction facilitated through consistent access to a qualified content specific specialist showed positive impacts on student achievement and test scores (AMTE, 2010; Campbell, 1996; ems&tl, 2009, 2011, 2016; Fennell, 2011; Gerretson et al., 2008; McGatha, 2009; Polly 2012; Utley & Reeder, 2016). Several studies have reported growth in mathematics teaching and learning related to an increase in problem solving and reasoning, student achievement, and formative assessment by the teachers, all due to the work of EMSs in schools (Brosnan & Erchick, 2010; Campbell 1996; Campbell & Malkus, 2011; McGatha, 2009; Race, Ho, & Bower, 2002). Although many positive research studies have been conducted about qualified EMSs’ work with teachers or students in schools, there are also some factors that inhibited the effectiveness of EMSs.

**Factors that inhibit elementary mathematics specialists’ effectiveness.** In 2000, the National Survey of Science and Mathematics Education found that only 60% of the participating teachers in elementary schools felt that they were qualified to teach mathematics (Banilower, Smith, & Weiss, 2002). Mathematics tends to be an area in which many elementary teachers feel unprepared to teach in depth, in addition to not feeling comfortable talking about mathematics in general (Banilower et al., 2002). As is often the case, the problem with many classroom teachers not being adequately prepared to teach mathematics is exacerbated when these teachers become an EMS in their own school or district (Banilower et al., 2002).
In 2012 Horizon Research conducted a study of U.S. teachers. In their data collection and analysis, they found that only 4% of elementary teachers in the representative sample reported that they had a degree in mathematics or mathematics education. In the same study by Horizon Research, “only 10% of the teachers reported having completed coursework in all five areas recommended by NCTM (i.e., numbers and operations, algebra, geometry, probability, and statistics” (McGatha & Rigelman, 2017, p. 23). Despite the lack of coursework in specific mathematics content courses, a staggering 77% of the teachers surveyed stated that they felt prepared to teach mathematics. When taking a deeper look into specific content strands in mathematics, only half of the teachers reported that they felt prepared to teach measurement and data, geometry, and early algebra (McGatha & Rigelman, 2017). As stated in McGatha and Rigelman, the Horizon Research team suggested the use of EMS-certified teachers to provide the support for classroom teachers to meet the needs of all students in the classroom.

Specialists in elementary schools are often compensated through different funds than general or special education classroom teachers (Fennell, 2011), which in turn can determine the EMS’s responsibilities (McGatha & Rigelman, 2017). This is an important factor to note for this current research study because of the roles and responsibilities that the EMS in the current study took on in their schools or district. Fennell (2011) stated that if the EMS’s job entailed working with Title I or special education students, the EMS was generally funded through Title I funding. Title I paid specialists’ roles generally include co-teaching in a teacher’s room with Title I students or students with individualized education plans (IEPs) (Fennell, 2011; McGatha & Rigelman, 2017). Title I funded EMSs are generally responsible for organizing and implementing specific “pull out” programs for students with particular needs based on their IEP (Fennell, 2011; McGatha & Rigelman, 2017). Other EMSs in the schools may have the
responsibility of working with the teachers and giving them support wherever there is a need but did not work with any students.

The push for content specific instructional specialists is necessary for instruction to be conceptual and rigorous to better prepare the students for standardized testing and, more importantly, transitioning to apply their knowledge to real-world situations (Gerretson et al., 2008). Because students are not performing well on standardized tests (TIMSS 2008, 2011), this reality may be a direct or indirect result of not having a true EMS in the school that supports the development of reform-oriented instructional practices with classroom teachers.

One factor that inhibits the effectiveness of an EMS is if they are chosen to be an EMS just because they are a “good teacher.” AMTE (2013) stated that in most cases, elementary teachers are generalists. In 1989, the National Research Council noted the following:

The United States is one of the few countries in the world that continues to pretend—despite substantial evidence to the contrary—that elementary school teachers are able to teach all subjects equally well. It is time that we identify a cadre of teachers with special interest in mathematics and science who would be well prepared to teach young children both mathematics and science in an integrated, discovery-based environment. (p. 64)

The average, generalist, elementary teacher simply does not have the comprehensive, mathematical, conceptual understanding to successfully accomplish the role of a true and certified EMS (AMTE, 2013). Wu (2009) described the situation:

The fact that many elementary teachers lack the knowledge to teach mathematics with coherence, precision, and reasoning is a systemic problem with grave consequences. Let us note that this is not the fault of our elementary teachers. Indeed, it is altogether unrealistic to expect our generalist elementary teachers to possess this kind of mathematical knowledge. (p. 14)

AMTE (2013) proposed an alternative that focused on having an elementary teacher specialize in mathematics only, such as having EMSs in each school (NMAP, 2008; Reys & Fennell, 2003). Giving elementary specialists the responsibility of supporting teachers in all
content areas generally tends to be detrimental in the long run (ems&tl, 2009, 2011; Reys & Fennel, 2003). This can be detrimental because most specialists are not properly equipped to be experts in all content areas; nor are they properly trained in supporting the teachers in all areas (ems&tl, 2009, 2011; Reys & Fennel, 2003). Some challenges of EMSs are the sustainability of retaining them, the recruitment of EMSs, and their content, pedagogical and leadership backgrounds (McGatha & Rigelman, 2017). Thus, the topic for this study focused on the qualifications of the EMSs in an instructional leadership position during the time of their participation in the study. The data collected and analyzed may support the research that EMSs need to be properly equipped to make positive shifts in teachers’ pedagogical knowledge while implementing rigorous high-quality tasks to support students’ conceptual understanding in mathematics.

**Types of Knowledge Needed to be an Effective Elementary Mathematics Specialist**

The National Council of Supervisors of Mathematics (NCSM, 2008) stated that pedagogical knowledge and mathematical content knowledge are both central and vital components to successful mathematics teaching and coaching. EMSs must have a multitude of practices and educational experiences to support teachers with formative assessment and differentiating instruction for all learners (NCSM, 2008). Ball and colleagues (2008) stated that mathematical knowledge that is needed for everyday life is important, but an “unpacking” of mathematical ideas that is needed to make “features of particular content visible to and learnable by students” (p. 400), is essential for teaching mathematics. This type of knowledge for teaching is described by Ball and others (2008) as things such as, but not limited to, understanding how certain words have different meanings in a mathematics context, knowing and understanding the
difference between mathematical justifications versus explanations, and determining whether particular problem-solving strategies work in general.

Similarly, Feger and others (2004) described that most types of coaching fell under the two headings: coaching knowledge and skills. Some examples of coaching knowledge and skills are interpersonal skills, content knowledge, pedagogical knowledge, knowledge of the curriculum, awareness of coaching resources, and having the knowledge of the practice of coaching (Feger et al., 2004). Many of the most recently adopted mathematics standards require a deeper understanding of mathematical content knowledge by the EMS, the teacher, and ultimately the students (Rowland, 2015). EMSs are expected to show preparedness to teach 21st century mathematics skills. Paired with leadership knowledge and skills, EMSs must also be equipped with content and pedagogical knowledge for teaching mathematics (AMTE, 2013). EMSs must also have rich, quality professional development opportunities to grow professionally in mathematics content areas. Wu (2009) and NMAP (2008) proposed that putting a bigger focus on EMSs’ content and pedagogical knowledge could be an alternative to the problem of increasing the content knowledge of elementary classroom teachers and students alike. The current study sought to understand the qualifications of the EMSs in this southeastern state regarding their content knowledge, pedagogical knowledge, and leadership knowledge and skills (AMTE, 2013).

EMSs must be equipped with the mathematical content knowledge of the standards to effectively facilitate the learning of others. Ball et al. (2008) described content knowledge as “knowledge that is used in the work of teaching in ways common with how it is used in any other professions or occupations that also use mathematics” (p. 377). Content knowledge, specifically for teaching mathematics, was described by AMTE (2013) as “having a deep
understanding of mathematics for grades K-8 and further specialized mathematics knowledge for teaching” (p. 4).

Having knowledge of the CCSSM (2010) is only one of the components of the overall knowledge needed to be an effective EMS. To be considered effective, an EMS must make sure teachers are also experts in mathematics content and pedagogy to teach students the standards to attain procedural fluency and conceptual understanding (Campbell, Ellington, Haver, & Inge, 2013). According to AMTE (2013), EMSs were expected to have the habits of mathematical thinkers or mathematicians, in addition to applying and understanding the Standards for Mathematical Practice (SMP).

McGatha and Rigelman (2017) expressed the breadth of knowledge needed by EMSs in terms of knowledge of content and teaching: EMSs need to be able to (a) select and implement a mathematical task to students (Jackson, Shahan, Gibbons, & Cobb, 2012), (b) facilitate mathematical discourse (Smith & Stein, 2011), (c) prompt and promote student collaborative mathematical thinking (Franke et al., 2009; Webb et al., 2014), and (d) provide suitable scaffolding for students’ critical problem solving (Baxter & Williams, 2010). For this current study, some of the data collected from the EMSs will evaluate their pedagogical knowledge, specifically how they select and implement a high-quality mathematics task, how the EMS prompts the students to have mathematical discourse with their peers, how the EMS promotes mathematical reasoning, and how the EMS provides scaffolds to aid the students in their mathematical reasoning when problem solving. Also, in this current study, during the interview and with some of the journal entries, the EMS had the opportunities to express their understanding of the mathematics content standards and the importance of learning progressions.
McGatha and Rigelman (2017) stated that EMSs need to be able to demonstrate the ability to translate content knowledge into pedagogical practices that will likely result in meaningful mathematical learning for all students (p. 25). Just as EMSs are expected to have deep and conceptual understanding of mathematics content and pedagogy, teachers also need to develop effective teaching strategies and practices, because simply having content knowledge is not enough (Graham & Fennell, 2001).

**Types of Knowledge Needed for Teachers**

If EMSs are the ones tasked to support teachers in their content and pedagogical knowledge for teaching mathematics, research on teachers’ knowledge is also important to note. Shulman (1986) developed a framework describing types of knowledge and divided it into three dimensions: content knowledge, pedagogical content knowledge, and curricular knowledge (Table 3). Table 3 is a broad view of the types of knowledge in general, what Shulman referred to as general content knowledge (GCK) not exclusive to any specific content area in particular but encompassing all disciplines. For this study, the term EMS can be interchangeable with the term teacher that Shulman uses.

In 1986, Shulman researched the types of knowledge that one should have to be most effective in teaching mathematics to students. He brought the idea of pedagogical content knowledge (PCK) to the forefront of literature in education. PCK is the blending of content and pedagogy into an understanding of how particular subjects and content knowledge are taught and adapted to various types of learners and learning styles (Shulman, 1986; Walker, 2007).
<table>
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<tr>
<th>Dimensions of Teacher’s Knowledge</th>
<th>Definitions</th>
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<tbody>
<tr>
<td>Content Knowledge</td>
<td>The amount and organization of knowledge of the teacher. (p. 9)</td>
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<tr>
<td>Pedagogical Content Knowledge</td>
<td>Goes beyond knowledge of subject matter knowledge for teaching; it is the teachability of the content. PCK also includes the reasoning as to why specific topics are either easy or difficult based on things such as prior knowledge and conceptual understandings.</td>
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<tr>
<td>Curricular Knowledge</td>
<td>The curriculum is represented by a full range of programs designed for teaching mathematics. The teacher must know the curriculum and the tools to present the information to the students.</td>
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Shulman’s study (1986) focused on teachers, but his findings are useful when considering why EMSs also need to be properly equipped with PCK to be able to effectively facilitate the learning of the teachers or students directly or indirectly. PCK included "most useful forms of representation of these ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations in a word, the ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p. 9; Walker, 2007). Shulman found that if PCK was limited, it might hinder identifying student misconceptions, thus not being able to provide students with alternate conceptual means for exploring their misconceptions. Pedagogical
knowledge can be shared and discussed among others, but when “shaped through experiences with children,” it is more effective and powerful (Graham & Fennell, 2001, p. 116).

Shulman (1987) described pedagogical knowledge as knowledge that could be described as knowing what methods and instructional strategies were most effective for helping students learn specific content or skills, while also keeping in mind how to adapt the skills, concepts, and strategies for different learning styles. However, AMTE (2013) described pedagogical knowledge for teaching mathematics as the knowledge of learners, learning, teaching, curriculum, and assessment. This was important in the current study because EMSs should assist teachers in developing the type of knowledge that Shulman (1987) described.

Ball et al. (2008, p. 377) introduced the Mathematical Knowledge for Teaching (MKT) Framework delineating different types of knowledge (see Figure 3). MKT depicted important relationships between subject matter knowledge and pedagogical content knowledge. In these models, the teachers’ knowledge included mathematical knowledge for teaching and pedagogical content knowledge. The interchange between these two types of knowledge allowed one to make decisions about the scope and sequence of which concepts and skills were taught and at which point in the curriculum. MKT clearly distinguished a divide between content knowledge and pedagogical content knowledge. Subject matter knowledge was partitioned into three different types of knowledge: (a) common content knowledge, (b) specialized content knowledge, and (c) knowledge of the mathematical horizon, whereas pedagogical content knowledge was partitioned into the subgroups of (a) knowledge of content and students, (b) knowledge of content and teaching, and (c) knowledge of curricula.
A study by Bostic and Matney (2013) examined 469 elementary and middle school teachers’ connections between their perceived mathematical content areas of weakness from the CCSSM (NGA, 2010) as compared to the actual statewide assessment data. In the study, the participants were 21 third-grade teachers, 22 fourth-grade teachers, and 19 fifth-grade teachers. The teachers’ perspectives on mathematics content and pedagogical needs were collected through two anonymous surveys. The participants were encouraged to complete the surveys during a 2-week window; nearly one third of the Grade 6 through 9 middle school cohort (n = 148) volunteered to respond to the survey. District-level data consisted of third- through eighth-grade students’ statewide high-stakes mathematics test performance. These data allowed researchers to examine the degree to which teachers’ perceived needs matched students’ performance on statewide mathematics assessments from the prior academic year. After the
survey data were analyzed, researchers shared the results with curriculum coordinators to gather their perceptions about the research findings.

Results based on the survey data collected from the teachers indicate that the two most critical areas of need for developing deeper content knowledge were in the domains of *Operations & Algebraic Thinking* and *Numbers & Operations: Fractions*. This apparent lack of content knowledge on the part of teachers directly correlated with students’ statewide test performance and demonstrated that students across the state of Ohio struggled with these areas, based on their low achievement scores (Bostic & Matney, 2013). Nearly 18% of third-grade students in these Ohio districts did not meet the proficiency level required by the state (Bostic & Matney, 2013). However, the fourth- and fifth-grade failure rates in these domains were much higher, reporting a 24% and 42% failure rate, respectively (Bostic & Matney, 2013). Results also show that teachers preferred professional development that focused on different aspects of instructional practices, that “encouraged students’ reasoning and sense making and; improved their facility with instructional strategies that support students’ conceptual development of ideas found in the CCSSM” (Bostic & Matney, 2013, p. 15). The data revealed that teachers were pressed to make sense of every aspect of the CCSSM (e.g., student reasoning, SMPs) due to the importance of conceptual understanding in mathematics and mathematical reasoning (NGA, 2010).

Galant (2013) conducted an analysis to obtain what subject matter knowledge, pedagogical content, and curriculum knowledge were used to make and prioritize the decision-making for teaching. Galant’s study used data based on interviews from “46 third grade teachers who were presented with two mathematical tasks taken from the 2010 National Department of Basic Education (NDBE) Grade 2 and Grade 3 Numeracy Workbooks” (p. 34). The participants
in the study were required to justify the selection and sequencing of the two mathematical tasks for teaching multiplication. The interview responses highlighted weaknesses in their subject matter knowledge, pedagogical content knowledge, and curricular knowledge (Galant, 2013). In addition to the selection and implementation of a mathematical task, it appeared that the participants lacked the experience of multiple cognitive and pedagogical resources and strategies to teach mathematical concepts or skills such as multiplication (Galant, 2013). As observed in the interview responses and the data from the task selection and implementation, the lack of experience with multiple cognitive concepts and skills such as multiplication was exposed as both a weakness in pedagogical content knowledge and curricular knowledge. The data from the study by Gallant (2013) supports the data compiled during this current study pertaining to pedagogical and content knowledge for teaching mathematics, task selection, and implementation.

**Elementary Mathematics Specialists and Teacher Beliefs about Teaching Mathematics**

Nespor (1987) defined belief systems as (a) “loosely-bound systems with highly variable and uncertain linkages to events, situations, and knowledge systems” (p. 321); (b) “important influences on the ways individuals conceptualize tasks and learn from experiences” (p. 317); and (c) “play a major role in defining teaching tasks and organizing the knowledge and information relevant to those tasks” (p. 324). Thompson (1992) specified that belief systems are a person’s organized beliefs and categorized beliefs into two different types: central beliefs, which are difficult to change, and peripheral beliefs, which are more susceptible to change. Raymond (1997) defined beliefs as a person’s personal judgments about mathematics formulated from experiences in mathematics, including beliefs about the nature of mathematics, learning mathematics, and teaching mathematics. Belief systems, on the other hand, are important
influences on the ways individuals conceptualize tasks, learn from experiences, and play a role in defining teaching tasks and organizing the knowledge and information relevant to those tasks (Nespor, 1987). Similarly, Rivera (2012) described beliefs systems as the connections between a learner’s beliefs and their outcome behaviors.

Although this current study was about mathematics, it was important to note a general definition of belief and a belief system so the definitions can be connected to the EMS’s beliefs about teaching mathematics or beliefs about mathematics in a more general sense. Understanding beliefs can be challenging because beliefs “cannot be inferred directly from teacher behavior, because teachers can follow similar practices for very different reasons” (Kagan, 1992, p. 66). Kagan also indicated that teachers’ knowledge and beliefs about their area of expertise often are implicit. “Teachers are often unaware of their own beliefs, they do not always possess language with which to describe and label their beliefs, and they may be reluctant to espouse them publicly” (p. 66). Often one’s beliefs about how to teach mathematics and how one should learn mathematics are based on the personal experiences that educators had from their years of schooling beginning in early elementary school (Kagan, 1992). Despite some of the studies using the term teachers, the definitions are also applicable for EMSs for this research study.

In a case study based on interactions between a mathematics coach and an elementary teacher, Neuberger (2012) found that coaching effectiveness was a way to positively influence teachers’ beliefs and instructional practices. A public elementary school in New York City had a coaching initiative in place that required all new teachers in the district to work with EMSs, whereas all other teachers had the option to choose whether to work with an EMS or not. Evidence from teacher/coach planning sessions, classroom lessons, interview and follow up
debriefing sessions with the teachers showed that the teachers’ beliefs about mathematics and the teaching of mathematics shifted from a procedural view toward a belief that mathematics teaching is about reasoning and learning for conceptual understanding. In addition to the shifts in beliefs in mathematics, the teachers also reported having increased content and pedagogical knowledge in addition to the positive changes reported about students’ interactions and mathematical discourse during instructional time.

Hart (2002) conducted a study with preservice candidates and concluded that teacher education programs must have a protocol for assessing their effectiveness of how philosophies of teaching and learning aligned with teacher candidates’ beliefs about mathematical content and pedagogical knowledge and their practice. Her study focused on preservice candidates seeking an alternative certification for teaching in an urban setting. In a three-semester time frame, the students were required to take 6 hours of mathematics content courses as well as courses in mathematics education. The candidates took a Mathematics Beliefs Instrument (MBI) to collect data about the program’s success in changing teacher beliefs by using descriptive statistics and tracing trends across the group ($n = 14$). Data was collected in a qualitative form using weekly teaching logs. Teacher beliefs, in this context, described what preservice teacher candidates believed to be true based on their past experiences. The results suggest that the program was successful in changing teacher beliefs about mathematics. The study by Hart (2002) was important to report and relate to this current study because the shift in beliefs in mathematics took place in a setting where the candidates were supported in their learning, similarly to EMSs charged with supporting teacher and student conceptual understanding.

Wilkins (2002) claimed that among the variables of content knowledge, beliefs, and attitudes, teacher beliefs had the strongest correlation with elementary teachers’ practices.
Wilkins (2002) found that when positive beliefs and attitudes toward mathematics were evident, the classroom instruction tended to be more student-centered and with conceptual understanding as a foundation. Conversely, when participants displayed negative beliefs, the instruction was more dependent on traditional methods of instruction and less focus on procedural fluency (Wilkins, 2002).

Research indicates that teachers’ confidence in their personal mathematics abilities has a measurable influence on children’s mathematics attitudes (Stipek, Givvin, Salmon, & MacGyvers, 2001). Effective professional learning opportunities provide educators of all levels the chance to change their beliefs and methods of instruction (Loucks-Horsley et al., 2009; Stein et al., 2009). For this current research study, it was appropriate to connect the effective professional learning opportunities for EMSs and teachers alike. Some of the data collected in this study analyzed the relationship between the EMSs’ leadership knowledge and skills, their beliefs and practices about teaching and coaching mathematics with conceptual understanding, and their instructional practices pertaining to mathematics.

Although there are some limitations to collecting qualitative data such as interviews and observations about beliefs, using these qualitative methods still tends to be one of the most reliable ways to collect this type of information (Wong, 2016). In Wong’s study, \( n = 21 \) middle school science and mathematics participants’ beliefs were analyzed based on a one-year graduate online program that emphasized inquiry-based instruction and student-centered lens and frame of mind. Generally, most participants veered toward more student-centered beliefs. However, contrary to the mathematics educators, science teacher beliefs had significant changes in their beliefs regardless of their years of experience. Wong (2016) stated that, “it becomes clear that beliefs formed through years of personal experiences in the classroom may be difficult to
change” (p. 23). An inaccurate understanding and negative beliefs can be detrimental to teaching and learning and therefore must be addressed for positive learning and growing to take place. The findings support that formal education and knowledge positively impact beliefs in education.

**Limitations in Mathematics Coaching by a Literacy or Reading Specialist**

Experienced and veteran teachers seen as successful teachers are often recruited to become mathematics specialists. As experienced and accomplished teachers can be prospective specialists, they have many skills and a great wealth of knowledge about teaching (Chval et al., 2010). The assumption that successful teachers are effective specialists, and that these expert and experienced teachers need little support or guidance as they transition into their new role as mathematics specialists is simply untrue (Chval et al., 2010). “Transitioning from teaching mathematics to instructional coaching requires more than just acquiring additional competencies” (e.g., the ability to work with adult learners, facilitate grade-level meetings, or provide feedback to other teachers about their practice; Chval et al., 2010, p. 192).

Knowledgeable, highly-qualified, and successful expert teachers are given the chance to be promoted to a mathematics specialist position based on longevity at the same school, while also being considered a veteran teacher leader (Chval et al., 2010). Because these teachers have a great deal of classroom experience, they are viewed as not needing any help or support when making the transition to a new role. A change in their position, based on prior success is unfortunately considered a valid reason for a classroom teacher or specifically a reading specialist to assume a new, more general role as an instructional or literacy specialist.

In the 1930s, educators skilled in reading acted as instructional support for teachers to encourage best practices in literacy. A definition of a literacy specialist today is tied to an emphasis on the instruction of struggling readers, with less emphasis on support, resources, and
pedagogy to teach all students (Bean, Cassidy, Grumet, Shelton, & Wallis, 2002, p. 736).

According to Hall (2004), the history of literacy coaching was traced to the work of Bean and Wilson (1981), who described the emergence of the reading specialist as a support person in schools. Toll (2014) defined literacy specialists as educators who “need to be well versed in the research, theory, and practices of literacy instruction” (Toll, 2014, p. 10). Literacy specialists need to be strong planners and organized; they need knowledge of teacher professional learning and must exhibit strong interpersonal skills (Toll, 2014). When specialists make the transition from a reading specialist to an instructional or literacy specialist, which may include teaching mathematics as well, the lack of in-depth content knowledge for teaching mathematics can often hinder a specialist’s ability or perception of how effective he or she can be in supporting teachers in specific mathematical concepts. Literacy encompasses all content areas, whereas reading revolves around skills such as fluency, phonemic awareness, and comprehension. Literacy in mathematics is described as reading and writing shapes, equations, symbols, graphs, lines, formulas, proofs, and so much more.

Horton (2015) listed several key points in a blog about being a literacy specialist and having to work with teachers with mathematical content:

I always worry that, as a literacy coach, I make math teachers nervous. And honestly, it’s probably reasonable—too often, coaches like me (I should note that I’m guilty of this too), move into conversations with facilitators without a real understanding of math as a content area. In addition, we sometimes coach math teachers to do things that don’t support students’ development of math knowledge and skills. (para. 1)

Many times, specialists who are already placed in public schools come from a reading background, yet they are asked to be mathematics specialists despite their level of comfort and limited knowledge of mathematics to support teachers in mathematics (Fennell, 2011). Sweeney (2003) used the terms specialist and instructional specialist interchangeably to describe someone
who modeled new instructional strategies and provided reflective feedback when a classroom teacher put the strategies to use during an instructional lesson. The specialists observed teachers’ instruction, provided feedback, demonstrated lessons, provided support during lesson planning, incorporated co-teaching opportunities, and offered help that was focused on specific mathematical content. Many of the elements about effective literacy or reading coaching are parallel to that of effective EMSs. The differences lie in the expertise of the actual content.

Mudzimiri et al. (2014) stated the following:

> Because implementing coaching involves considerable cost and logistical effort for schools, research that examines mathematics classroom coaching is timely and relevant, with immediate policy implications for districts who employ classroom coaches and teacher educators who prepare and support coaches through professional development efforts. (p. 3)

**Summary**

This study collected information about current EMSs, regardless of title, who have been charged with coaching elementary classroom teachers in their mathematics instruction, but who may not have been properly equipped with the mathematical content and pedagogical knowledge for teaching mathematics. The literature supports the argument for EMSs to be adequately prepared in all three areas that are highlighted by AMTE (2013) as qualifications for an EMS (i.e., content knowledge, pedagogical knowledge, and leadership knowledge and skills; Fennell, 2011; Gerretson et al., 2008; Hill et al., 2005. Gaining insight into what the EMSs’ roles and responsibilities are in their schools and how their content knowledge and beliefs about coaching and teaching mathematics affect instruction are important steps toward improving mathematics education in the state. This study analyzed the relationship between the EMSs’ ability to select a high-quality mathematical task and their implementation of a mathematical task. It also examined the relationship between EMSs’ ability to select a high-quality mathematical task and
their content knowledge for teaching elementary mathematics, and examined the relationships between the EMSs’ leadership knowledge and skills, their beliefs and practices about teaching and coaching mathematics with conceptual understanding, and their instructional practices pertaining to mathematics.
CHAPTER 3
RESEARCH METHODOLOGY

Introduction

This chapter consists of a description of the research questions, the design of this study, the cluster sampling for the population, and the purposive sampling within each regional in-service cluster for classroom observations and follow-up semiformal interviews. It also describes the instruments used in the study, the data collection and analysis procedures, the limitations of the study, and an explanation of the validity and reliability of the collected and analyzed data.

Purpose of the Study

This research study examined current EMSs’ content and pedagogical knowledge for teaching mathematics, their leadership skills, and their ability to select and implement two high-quality mathematical tasks to two different classrooms of elementary students in their school. This study took place in one selected southeastern state in The United States. The push for content-specific instructional specialists is necessary for teachers’ instruction to prepare students more effectively for better performances on standardized testing, while ultimately transitioning their knowledge to the real world (Gerretson et al., 2008). Many elementary specialists are coaching outside of their own subject area of expertise of English language arts or literacy. Without a specific mathematics certification, these specialists may not be qualified to support elementary teachers in content and pedagogical knowledge for teaching mathematics (Greenberg & Walsh, 2008).
Research Questions

This research study was grounded in the AMTE (2013) qualifications necessary for a certified EMS. Thus, the following are the research questions:

Research Question 1. What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and their implementation of a mathematical task?

Research Question 2. What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and their content knowledge for teaching elementary mathematics?

Research Question 3. What are the relationships between the Elementary Mathematics Specialists’ leadership knowledge and skills, their beliefs and practices about teaching and coaching mathematics with conceptual understanding, and their instructional practices pertaining to mathematics?

Research Design

The study was initially designed to be an explanatory mixed-methods research design that used two phases of data collection. Phase 1 was intended to provide a comprehensive description of EMSs’ content and pedagogical knowledge of mathematics, beliefs about coaching and mathematics and leadership skills. Phase 2 was intended to provide an in-depth look at a subsample of coaches’ ability to plan and implement a high-quality task. These initial plans were changed due to the difficulty of obtaining completed surveys in Phase 1. While there is sufficient data from Phase 1 surveys to provide background descriptions of EMSs in the state (see Chapter 4 for a description of this data), a smaller number of participants who completed all
initial surveys and agreed to participate in Phase 2 were chosen as a convenience sample, rather than randomly, for Phase 2 of the study.

The data from the Phase 1 population were inconsistent in that not all participants completed all initial surveys; in addition, some participants did not complete all items in each survey. The convenience sample included 12 participants who completed all surveys, providing a complete profile through qualitative and quantitative data collection. They provide the in-depth data for this study (Johnson & Christensen, 2008), while a general description of the entire population from Phase 1 was reported, but not elaborated upon.

A mixed methods approach was practical to identify and analyze the data by combining inductive and deductive reasoning and employing skills such as observing EMSs through both a quantitative and qualitative lens (Creswell & Plano Clark, 2010). The explanatory sequential research design consisted of quantitative and qualitative data that were collected and analyzed during consecutive phases. The collection of supportive qualitative data provided a deeper explanation of the quantitative findings. One data source may not have been adequate to thoroughly answer all parts of the research questions outlined in this study. The mixed methods design increased the validity and reliability of the findings as different methods of gathering data can supplement one data source with another: “The limitations of one method can be offset by the strengths of another method, and the combination of quantitative and qualitative data provide a more complete understanding of the research problem than either approach by itself” (Creswell & Plano Clark, 2010, p. 8).

This research study triangulated data so the convergence and corroboration of the data could increase the strength of the findings and the internal validity (Creswell & Plano Clark, 2010; Johnson & Christensen, 2008). The trustworthiness of the qualitative data, in addition to
triangulation, multiple observations, and informal, semistructured interviews were ways to strengthen internal validity. The researcher combined and analyzed both data sets to get more precise and in-depth information about the participants.

Patton (1980) suggested “methodological mixes” for several different types of experimental and naturalistic study designs (Creswell & Plano Clark, 2010). The pragmatist view of mixed methods focuses on the merging of the two methods and analyzing the merged data. When the researcher moves from the quantitative phase to the predominately qualitative phase of data collection and analysis, there is a shift in thinking that moves from a postpositivist to a constructivist lens. A postpositivist lens assumes that context-free, experimental designs are insufficient for the information needed. In this study, it meant that observational data might be fallible and have errors. A constructivist lens allows for multiple meanings and interpretations because the researcher uses and relies on both qualitative and quantitative data collection and analysis for one to compliment the other, and for the two methods to support one another. Thus, while the research began with the quantitative data collection and analysis for Phase 1, followed by the qualitative data collection and analysis for the Phase 2 subgroup, and concluded with the interpretation and findings of the data, the data was analyzed and reported from Phase 1 and analyzed, reported and discussed from Phase 2 at different levels of depth per each instrument.

**Quantitative Data**

For EMSs to be effective in performing their roles and responsibilities, they must have the content and pedagogical knowledge needed to effectively teach mathematics. The researcher selected several surveys to discern the extent of participants’ knowledge. The criterion for selecting an initial survey was that it contains questions about the EMSs’ background and demographics, knowledge about coaching in mathematics, and beliefs in mathematics. The
following sections explain the process of selecting instruments, rationale and purpose for using each instrument, reliability and validity of the instruments, and item specifications for each instrument.

There were five instruments used in this study. The Coaching Knowledge Survey (CKS, see Appendix C) explored the EMS’ beliefs and practices about mathematics content and pedagogy regarding mathematics coaching. The Coaching Skills Inventory (CSI, see Appendix D) provided data about the EMSs’ beliefs in teaching and coaching mathematics. The Learning Mathematics for Teaching (LMT, see Appendix E) assessment quantified EMSs’ mathematical content knowledge of various skills and concepts in K-6 elementary mathematics. The Task Analysis Guide (TAG, see Appendix F) helped the researcher gauge the level of rigor of the tasks selected by the EMSs. The researcher used the Instructional Quality Assessment (IQA, see Appendix G) instrument during observations of coaches’ implementations of two, high-quality mathematics tasks. To also evaluate EMSs’ implementation of a high-quality mathematical task, the researcher used the IQA instrument.

Reliability refers to the degree to which an instrument or tool used in the study produces consistent results time and time again. Carmines and Zeller (1979) argued that reliabilities should not be below .80 for widely used scales. A higher reliability coefficient is hindered by costs in time and money (Carmines & Zeller, 1979). An alpha rating of .80 is excellent because it indicates that at least 64% of the total variance in the scores on the measure is a true score variance, but standardized tests typically use .60 or above for an alpha rating; although, the lowest acceptable Cronbach alpha for a survey is .70 (Nunnaly, 1978; Rothbard & Edwards, 2003). Similarly, for internal reliabilities to be sufficient, Schilling (2002) stated that much
lower Cronbach alphas at or above .70 were also acceptable. For this study the researcher hoped to see Cronbach alphas of .80.

**Coaching knowledge survey.** After the EMSs gave their consent to participate in this study, EMSs received the Coaching Knowledge Survey (CKS) from the Examining Mathematics Coaching project (EMC; Yopp, Burroughs, & Sutton, 2010), through an email link from the researcher. The CKS explored their beliefs about mathematics content and pedagogy regarding mathematics coaching. The CKS (Yopp et al., 2010) is a 40-item survey instrument that includes multiple choice, short answer, and open-ended questions. The CKS contains items about a coach’s beliefs, but also knowledge in two primary domains: coaching knowledge and mathematics content knowledge. The researcher chose this instrument because it contains specific items that are most closely aligned to the daily or weekly routines of EMSs, in relation to the AMTE (2013) qualifications, while simultaneously providing information about the EMSs’ beliefs about mathematics. The CKS has an internal reliability rate and Cronbach’s α scale between .822 and .935 when it was analyzed during the 5 years of the project (2009–2014). The data set from the EMC study included 51 school-based coaches and 180 coached teachers from 28 districts across eight states (Yopp, Burroughs, Sutton, Greenwood, 2014).

The CKS has a 7-point Likert scale ranging from 1 (*strongly disagree*) to 7 (*strongly agree*) on most items, or from 1 (*not at all descriptive*) to 7 (*very descriptive*) on others. In an evaluation report by Jesse, Sutton, and Linick (2014), the CKS 7-point Likert scoring scale provided the type of specific information needed to establish validity of the study and the instrument itself. The CKS also includes seven scenarios presented in a multiple-choice format. The CKS concludes with two items that ask the EMS to choose from the multiple-choice selections pertaining to how they feel about (a) teacher learning and teachers and (b) professional
development without a coaching component. The three types of validity noted in the CKS methodological explanation in the evaluation report were predictive, convergent, and concurrent (Jesse et al., 2014). The type that was most relevant to this study was the predictive validity. Predictive validity means, in theory, that the CKS should be a “predictor of coaching effectiveness” (Jesse et al., 2014, p. 5). The evaluation reported that it was because of “high correlations between the CKS and other later measures of coaching effectiveness that the evidence would have predictive validity” (Jesse et al., 2014, p. 5).

The data from the CKS partially answered Research Question 3: What are the relationships between the Elementary Mathematics Specialists’ leadership knowledge and skills, their beliefs and practices about teaching and coaching mathematics with conceptual understanding, and their instructional practices pertaining to mathematics?

Coaching skills inventory. The researcher used the Coaching Skills Inventory (CSI)—also developed by Yopp et al. (2010)—to gather data about the EMSs’ beliefs about teaching and coaching mathematics. This instrument contains 24 multiple-choice and short-answer items, along with 20 items pertaining to the participants’ demographic and background information. The first 24 items on the CSI required responses on a 5-point Likert scale, ranging from 1 (not at all effective or confident) to 5 (very effective or confident), prompting the participants to choose how they felt about the subsequent coaching items (Yopp et al., 2010). The first 24 questions of the CSI have the following section headings: coach/teacher relationships, coaching skills, mathematics content, mathematics-specific pedagogy, and general pedagogy. The remaining 20 items asked about the EMSs’ background knowledge and practices as an educator, requiring that the respondents state, for example, the highest degree or certification held or earned, the number of mathematics content courses taken in college, the highest-level mathematics course taken, and
the degree and field of study obtained while in college. The CSI also asked about the number of
years-to-date that the EMS taught, the number of years they taught mathematics only, the
number of years they have been coaching teachers, and whether the EMS had a specific
mathematics certificate to coach teachers in mathematics. The CSI further examined the EMSs’
current roles and daily activities, whereby the respondents were required to indicate how often
they led coaching sessions, the number of mathematics professional development workshops
they attended within the last year, and the grade levels in which they were certified to teach. The
internal reliability rate and Cronbach’s α scale for the CSI was .822-.935 during the 5 years of
the project (2009–2014). The data set from the EMC study included 51 school-based coaches
and 180 coached teachers from 28 districts across eight states (Yopp et al., 2014).

Carmines and Zeller (1979) argued that, for widely used scales, reliability should not be
below .80; however, attaining a higher reliability coefficient might be prohibitively expensive in
terms of both financial investment and time. Due to the small number of EMSs in this study, the
reliability of .80 may not be sufficient to make conclusions that are generalizable to the bigger
population of this state. In this study, the participants’ responses to the CSI items were analyzed
to ascertain the EMSs’ beliefs about teaching and coaching mathematics. The adoption of this
instrument was important because the results obtained by analyzing the CSI data contributed to
answering the Research Question 3: *What are the relationships between the Elementary
Mathematics Specialists’ leadership knowledge and skills, their beliefs and practices about
teaching and coaching mathematics with conceptual understanding, and their instructional
practices pertaining to mathematics?*

**Learning mathematics for teaching.** The Learning Mathematics for Teaching (LMT;
Ball et al., 2004) instrument focuses on the EMSs’ content knowledge for teaching mathematics.
Based on the criteria and items on the LMT, it was the best fit to assess the EMSs’ content knowledge for teaching mathematics. The LMT was designed to study the following: (a) structure of teacher knowledge (Hill, Schilling, & Ball, 2004), (b) how teachers learn mathematical knowledge for teaching (Hill & Ball, 2004), and (c) how teacher knowledge relates to gains in student mathematical achievement (Hill, Rowan, & Ball, in press). The 2008 version of the LMT has mathematics-released items that are referred to as the Mathematical Knowledge for Teaching (MKT) measures. The LMT’s internal reliability Cronbach’s α of .75-.80 is consistently accurate for assessing content knowledge for teaching mathematics at many levels including the grade band of K-6.

The LMT (Hill et al., 2004) has items that can measure mathematical content knowledge for teaching in the following three strands: (a) number and operations (K-6, 6-8), (b) patterns, functions, and Algebra (K-6, 6-8), and (c) Geometry (3-8). The LMT also covers some topic specific areas that include Grades 4-8 in the following areas, but these were not used for this study: (a) rational numbers, (b) proportional reasoning, (c) Geometry, and (d) data, probability, and statistics (Hill et al., 2004). Each item in the strand was piloted with over 600 elementary teachers (Hill et al., 2004). For this research study, the number and operations strand in K-6 was the only strand used to gather data and, as previously mentioned, it has a high Cronbach’s α of .75-.80. For this study, the researcher felt that the number and operations strand was the best to use for this study because of the grade levels in the study being only K-6. The LMT partially answered Research Question 2: What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and their content knowledge for teaching elementary mathematics?
**Task analysis guide.** The Task Analysis Guide (TAG; Stein, Smith, Henningsen, & Silver, 2009) helped the researcher rate and categorize the level of rigor of the tasks selected by the EMSs. Stein et al. (2000) developed the TAG based on four levels of the cognitive demand of a task (Table 1).

The authors classified *Memorization* tasks and *Procedures without Connections* tasks at a lower cognitive level, as they require students to apply facts, rules, definitions, or previously learned formulae that were simply memorized. On the other hand, they posed that *Procedures with Connections* tasks and *Doing Mathematics* tasks were at a higher cognitive level, as these tasks demand that students explore a variety of situations and gain a deeper understanding of mathematical concepts and relationships (Stein et al., 2000).

For example, although students must use prior knowledge for solving any type of task, memorization tasks “involve exact reproduction of previously seen material,” whereas mathematical tasks “require students to access relevant knowledge and experiences and make appropriate use of them” (p. 16).

In Stein, Grover, and Henningson’s (1996) study, intercoder/interrater reliability (IRR) for task analysis using the TAG ranged from 53% to 100%, with an average of 79% during the initial analysis. Intercoder reliability describes the rate of consensus between independent coders’ evaluations. In this study, the data yielded by the TAG were used to partly answer Research Questions 1 and 2: (1) *What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and their implementation of a mathematical task,* and (2) *What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and their content knowledge for teaching elementary mathematics?*
**Instructional quality assessment.** The Instructional Quality Assessment Classroom Observation Tool (IQA; Boston, 2012) was the observation instrument deemed suitable for meeting the needs of this study. The IQA was first developed to offer formative information to researchers and district leaders about the quality of instructional practices. Primarily, its purpose was to assess the design and grading of student tasks, academic rigor, expectations, self-management of learning, and accountable talk (Boston, 2012). The researcher chose the IQA for this study because it targeted the phenomenon of interest—classroom observations of the EMSs’ ability to categorize and rate the implementation of a mathematics task (Boston, 2012). For this study, the instrument was used to measure the implementation of the mathematics task at a high level of rigor, Academic Rigor 2 (AR 2). The data provided were used for the summative evaluation purposes of good instructional practices for teachers’ performances in the classroom. To increase reliability of the data collected from the IQA, a second researcher also observed and rated the EMSs’ implementation of high-quality tasks. The second observer and researcher, also a qualified and trained IQA rater and a doctoral candidate in Instructional Design and Development, had three years of experience in quantitative data analysis, earned an undergraduate degree in Elementary Education, a Master’s degree in Instructional Design and had 7 years of combined teaching experience in grades 3-12 in Mathematics, Reading, leadership, and served as a technology coordinator for a school system for 2 years. This researcher, in addition to the main researcher, observed two mathematics lessons with a selected sample comprised of a projected 12 participants across the state. Prior to the observations, both researchers calibrated the instruments to improve interrater reliability. They watched several lessons and scored the lessons based on the IQA provided rubric. When comparing individual
scores, they were found to be the same on all lesson, thus they were ready to begin the observations and scoring for the study.

The IQA is composed of 20 rubrics organized into three clusters. These clusters focus on the quality of classroom talk, the academic rigor of lessons, and the clarity and rigor of teachers’ expectations for student performance (Boston, 2012). The items on the IQA consist of the selection of a task in regard to the following questions: (a) Did the task have the potential to engage students in rigorous thinking about challenging content?; (b) Did the task have academic rigor; (c) At what level did the coach guide the students to engage with the task in implementation? (Boston, 2012). For this study, the IQA was solely used to rate the implementation of the two mathematics tasks. The data from the IQA answered Research Question 1: *What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and their implementation of a mathematical task?*

**Qualitative Data**

This study employed three main qualitative methods of data collection. The rationale and purpose for using each are explained below. The researchers scribed running records for both observations, collected lesson plans for the observed lessons, conducted informal semi-structured interviews, and collected four reflective journaling prompt entries from the convenience sample of EMSs.

**Running records of observations and lesson plans.** The researchers observed two lessons for each EMS who was in the selected subgroup. A hard copy of the lesson plans (see Appendix I) was collected before the observations or online through Qualtrics before the day of the observations. The researchers scribed the activities and conversations that took place during the classroom observations (Marshall & Rossman, 2011) on the IQA recording sheet, AR 2. Full
field notes are described as running notes taken during an observation for any duration of time. According to Glesne (2011), the researcher must allow several hours of time to read through the notes taken during the observation to clarify and expand on them and add reflective thoughts (p. 72). The researcher’s descriptive notes (Glesne, 2011) noted dialogue that occurred and details of the setting. The researcher used many senses, including “eyes, ears, and hands when observing the lesson” (Glesne, 2011, p. 72; Marshall & Rossman 2011). The researchers described the progress of the lesson, noting what was said, how the EMS responded, and the students’ or teachers’ interactions with the EMS. This qualitative data partly answered Research Question 1: *What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and their implementation of a mathematical task?*

**Follow-up informal interviews.** Informal, follow-up interviews (see Appendix M) were conducted with the selected participants from each region who were observed. The researcher audio recorded the interviews, which took place after the second observation. The interviews lasted between 7 and 20 minutes and were conducted in a private space in the school. The interview protocol and the terms of agreement were outlined in the consent form that EMSs had to sign to be participants in the study. The researcher asked participants all interview questions (see Appendix M) without any formal follow-up questioning or feedback for any responses given by the EMS. The audio recordings were transcribed immediately following the interview session (Creswell & Plano Clark, 2010, p. 178). The interviews were administered to partially address Research Questions 1 and 3: *What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and their implementation of a mathematical task,* and *What is the relationship between the Elementary Mathematics Specialists’ leadership knowledge and skills, their beliefs and practices about teaching and*
coaching mathematics with conceptual understanding, and their instructional practices pertaining to mathematics?

Reflective journal entries. Marshall and Rossman (2011) stated that “traditional qualitative research assumes that (a) knowledge is not objective Truth, but is produced intersubjectively, and (b) the researcher learns from participants to understand the meaning of their lives but should maintain a certain stance of neutrality” (p. 21). Therefore, the researcher chose to collect qualitative data that allowed each EMS to have personal input about their practice, in addition to the rationale of their choices for the mathematics instruction in their school.

The EMSs responded to four reflective journal prompts that asked the EMSs about their daily activities, lesson planning (for the two selected tasks), and their teaching, coaching, and beliefs about mathematics (see Appendix H). The first prompt asked participants to detail the lesson plans they submitted to the researcher for the two, high-quality tasks taught during the two observations. The next three entries were based on specific prompts. The first prompt asked participants to list their daily activities for one week during various time slots throughout the day (see Appendix J). The remaining two prompts were one-time responses, providing information about the EMS’s rationale for selecting and implementing one of the two high-quality mathematics tasks (see Appendix K), as well as their beliefs about teaching mathematics (see Appendix L). The research questions answered in part were Research Questions 1 and 3: What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and their implementation of a mathematical task, and What is the relationship between the Elementary Mathematics Specialists’ leadership knowledge and skills,
their beliefs and practices about teaching and coaching mathematics with conceptual understanding, and their instructional practices pertaining to mathematics?

Data Collection

The following is a description of each instrument from the appendix. Also, following the descriptions, Table 4 shows the number of participants in each in-service region in which the researcher had communication.

A = number of EMSs sent an invitation to participate
B = number of EMSs agreeing to participate and were sent three initial instruments in Phase 1
C = number of EMSs that completed and submitted CKS (Appendix C)
D = number of EMSs that completed and submitted CSI (Appendix D)
E = number of EMSs that completed and submitted LMT (Appendix E)
F = number of EMSs emailed to participate in Phase 2
G = number of EMSs participated in Phase 2 and observed with IQA (Appendix G)
H = number of EMSs that completed and submitted lesson plans for both tasks (Appendix I)
I = number of EMSs that completed and submitted the weekly reflective journal (Appendix J)
J = number of EMSs that completed and submitted the rationale for selecting and implementing a high-quality mathematics task journal entry (Appendix K)
K = number of EMSs that completed and submitted the beliefs about teaching mathematics journal entry (Appendix L)
L = number of EMSs that completed the interview (Appendix M)

Table 4

<table>
<thead>
<tr>
<th>Regions</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50</td>
<td>70</td>
<td>30</td>
<td>35</td>
<td>35</td>
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<td>15</td>
<td>35</td>
<td>15</td>
<td>35</td>
<td>365</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>10</td>
<td>0</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>44</td>
</tr>
</tbody>
</table>
Setting and Participants

The data collection took place in one state located in the southeastern region of the United States. This southern state has 11 regions designated as in-service regions by the state Department of Education. Each in-service region included several districts containing varied demographics including Title I schools, low and high socioeconomic status populations, as well as a diversity of racial majority and minority populations. Region 4, the region in which the researcher works, was excluded from this study. This exception eliminated any biases that may have arisen due to possible personal interactions with some of the participants through a professional lens as opposed to a researcher role. There was one large sample in Phase 1 that completed the CKS \((n = 40)\), CSI \((n = 39)\) and the LMT \((n = 27)\). A subset \((n = 12)\) of the larger
sample from Phase 1 was selected for observations, journals entries, and a semi-structured interview; selection was based on their completion of all Phase 1 instruments.

The state Department of Education has a list of all schools in the state organized in an Excel document. This document can be downloaded from the state Department of Education website, which has public access. The researcher filtered and sorted the schools listed in the document into 10 lists, one for each of the targeted in-service regions, selecting all schools that house kindergarten through fifth grades in the school or schools that split those grades. All alternative and special needs schools were excluded from the sample, because in most cases they did not have a mathematics coach in their school. The EMSs in the study comprised of 97% female, 82% reported being White and Non-Hispanic, 14% reported that they were of African American ethnicity, whereas the remaining 2% of the participants chose to not respond to the question. At the current writing, the population of teachers in the state is comprised of only 19% African American teachers while nearly 79% are white (Crain, 2017).

The researcher began with a population of approximately 365 EMSs from 10 of the 11 regional in-service areas of the state in which the superintendents gave prior approval to conduct research in their district. The researcher emailed a short description of the study and asked the superintendent or head of curriculum of each system or district in the state to give permission to EMSs to participate in the research study. Once the Institutional Review Board (IRB) gave approval to conduct the research and each superintendent gave his/her approval, the researcher began contacting the participants through school email addresses that were obtained from school and district websites.

If one of the participants in the study was no longer in the position of an EMS by the end of the current school year, or the end of the study’s initial data collection phases that were carried
out through the fall of 2017-2018 school year, the researcher continued to recruit EMSs in the region to replace the one(s) who had to withdraw from the study or were unable to finish the study. Of the anticipated 6-10 EMS selected from each in-service cluster, the researcher planned to select at least 1-3 EMSs through another sampling procedure for the next phase of the study, observing coaching lessons. If each region still had at least one participant in the observations’ sample, anyone who withdrew from the study did not have to be replaced. The convenience sample of 12 EMSs completed all surveys and agreed to participate in two observations in their school and a follow-up, semi-structured, informal interview session immediately following the second observation. Due to the slow response of viable participants, the recruitment process continued until the beginning of September of 2017 so that data could be collected and then finally analyzed by the projected data collection deadline in the Fall of the 2017 school year.

Study Procedures

The researcher began the study by sending out an initial recruitment email letter and consent-to-participate letter to every EMS from each cluster. All responses from the EMSs were kept strictly confidential. The researcher assigned each EMS an ID code once the initial names and schools were determined for the study. After the selections were made, the ID codes determined which participants returned their instruments, how many participants were in the study, and the demographic information from each participant; finally, they were used to align, analyze, and correlate the data pieces from each EMS. The researcher respected and protected the confidentiality of each participant. All instruments went to the EMSs’ work email addresses through Qualtrics or an email link to the LMT assessment. All addresses came from public online sources, such as the school’s website or from contacting the school and asking for the specific information. Once each participant submitted the required information from the first
phase of quantitative survey instruments, including the completion of the LMT, each EMS received a monetary incentive of a $15 gift card sent via The United States Postal Service to the school or home address.

**Data collection chart.** Table 5 displays the research questions and instruments used for the data collection and analysis. The research design is outlined below in Figure 4. After the initial recruitment email and consent forms were sent and then submitted through Qualtrics, the researcher created a link from the consent form to the CKS for all participants who answered yes to giving consent to participate in the study as laid out in Figure 4.

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Type of Data</th>
<th>Instrument</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and their implementation of a mathematical task?</td>
<td>Quantitative</td>
<td>Task Analysis Guide</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IQA</td>
</tr>
<tr>
<td></td>
<td>Qualitative</td>
<td>Reflective Journal Entries</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Interview Questions</td>
</tr>
<tr>
<td>What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and their content knowledge for teaching elementary mathematics?</td>
<td>Quantitative</td>
<td>TAG</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LMT</td>
</tr>
<tr>
<td>What are the relationships between the Elementary Mathematics Specialists’ leadership knowledge and skills, their beliefs and practices about teaching and coaching mathematics with conceptual understanding, and their instructional practices pertaining to mathematics?</td>
<td>Quantitative</td>
<td>CKS</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CSI</td>
</tr>
<tr>
<td></td>
<td>Qualitative</td>
<td>Reflective Journal Entries</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Interview Questions</td>
</tr>
</tbody>
</table>
Figure 4. Study outline.

**Timeline of the study.** This research study began in March 2017 and continued through the fall semester of the 2017-2018 school year. The following table was the research study timeline.

<table>
<thead>
<tr>
<th>Task</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling from the anticipated 10 in-service region clusters (excluding Region four).</td>
<td>(n = 56 participating EMSs across the state for the initial data collection)</td>
</tr>
<tr>
<td>CKS survey sent through Qualtrics, hyperlinked from consent form, to all participating EMSs</td>
<td>(n = 40 completed)</td>
</tr>
<tr>
<td>LMT link sent to all EMSs</td>
<td>27 completed and 5 incomplete</td>
</tr>
<tr>
<td>IQA observations (2) completed with selected EMS</td>
<td>(n = 12)</td>
</tr>
<tr>
<td>One to three EMS per region (n = 12 across the state) informal interview questions (4) and sub-questions after the last of the two observations</td>
<td></td>
</tr>
<tr>
<td>Reflective Journal Entries and Prompts (5) for the nested sample only</td>
<td>(n = 12)</td>
</tr>
<tr>
<td>CSI instrument hyperlinked after the completion of the CKS</td>
<td>(n = 39 completed)</td>
</tr>
<tr>
<td>Reflective Journal Use Only-- TAG for Task Selection</td>
<td></td>
</tr>
</tbody>
</table>

CKS survey sent through Qualtrics, hyperlinked from consent form, to all participating EMSs (n = 40 completed).
<table>
<thead>
<tr>
<th>Date</th>
<th>Data Collection/Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>March 2017</td>
<td>Submit IRB and obtain approval.</td>
</tr>
<tr>
<td>March - September 2017</td>
<td>Sampling Process</td>
</tr>
<tr>
<td></td>
<td>Recruitment email consent form, CKS, and CSI sent through Qualtrics. (projected 60-100 participants across the state)</td>
</tr>
<tr>
<td></td>
<td>After the consent form was completed and confirmed, Qualtrics automatically linked to the CKS.</td>
</tr>
<tr>
<td></td>
<td>After the completion of the CKS, automatic hyperlink to the CSI (projected 60-100 participants in 10 participating in-service region clusters)</td>
</tr>
<tr>
<td></td>
<td>After the completion of the CKS and CSI, LMT link and information sent to all EMS (projected 60-100 across the state)</td>
</tr>
<tr>
<td>April 2017-October 2017</td>
<td>Selection of participants for observations (2) (IQA &amp; running records), (projected 10-30 from the 60-100; 1-3 from each cluster).</td>
</tr>
<tr>
<td></td>
<td>Sampling of the 1-3 EMS in the smaller sample from each region for informal interview questions. (projected 10 from the whole state, 1-3 from each region).</td>
</tr>
<tr>
<td>October 2017-December 2018</td>
<td>Reflective journal entry responses (5) through Qualtrics to the projected 10-30 EMSs.</td>
</tr>
<tr>
<td></td>
<td>Complete observations (2) for convenience sample.</td>
</tr>
<tr>
<td></td>
<td>Informal interview questions with sampling group of 1-3 EMSs from each region. (projected 10-30 EMSs from the whole state, at least 1 from each in-service region).</td>
</tr>
<tr>
<td></td>
<td>Analyze and correlate all data when they are submitted.</td>
</tr>
<tr>
<td></td>
<td>Prepare data for reporting purposes.</td>
</tr>
</tbody>
</table>
**Quantitative Data**

The Coaching Knowledge Survey (CKS), the first instrument sent to all participants in the study was sent electronically through a link in Qualtrics. The researcher emailed the link to complete the consent form. If the participant answered *yes* and gave approval to be in the study, the form was then hyperlinked to begin the CKS (see Appendix C). Thirty-nine participants completed the CKS and received the prompt for the second instrument.

The Coaching Skills Inventory (CSI) was the second instrument sent in an email link via Qualtrics (see Appendix D). Once the CSI was completed, they received more information and a link to the next instrument, the LMT. Thirty-five participants completed the CSI and received the prompt for the third instrument.

After the participants submitted the completed CSI, they received a link through their email that provided a brief explanation of the Learning Mathematics for Teaching (LMT), a program code, a personal ID code, and instructions about what to do when they clicked on the link that took them directly to the LMT (see Appendix E). Twenty-seven participants completed the LMT.

Every EMS was assigned a specific ID code for them to use on any items submitted after that point to ensure their privacy to another degree. Once they completed the LMT, if they were selected as part of the observational sample, they received another email informing them of the next phase of the study accompanied by the links to all the qualitative journal entries they needed to submit. They were also asked to schedule dates with the researcher to come and conduct the two observations, which were followed by the informal, semi-structured interview.

Only the two researchers (see Appendix F) used the Task Analysis Guide (TAG). The TAG rubric helped the researchers determine the level of the mathematics tasks selected. If the
EMSs emailed the lesson plans for the observed lessons ahead of time (as requested), the researchers were able to categorize the task selection before observing the implementation of the high-quality mathematics tasks. One EMS submitted her lesson plans at the time of the observation rather than submitting it prior to arrival at the school, so that the researchers had to score her tasks after the lesson observations. Any bias created by observing the lessons prior to scoring the potential of the task was minimized because of the objective nature of the descriptors in the TAG and specific language relative to the items that were either present or not present in the lesson plans. Data about the selection of the high-quality mathematics task also came from one of the journal entries (see Appendix K).

The researchers used the Instructional Quality Assessment (IQA, see Appendix G) during the observations, to rate and score the level of rigor of the implementation of the mathematics tasks taught by the EMSs. The instrument included a code of N/A, which indicated that students did not participate in a mathematics task and that there was no mathematics task to implement. If the observation showed that no mathematical instruction was performed, the data were still collected and used in the analysis. The creators of the IQA instrument (Boston, 2012) also used the N/A code.

**Qualitative Data**

The researchers took running records of observations, also known as full field notes (Lofland & Lofland, 1995) during the observations as the EMSs implemented the high-quality mathematics tasks. Notes came from what the researchers heard from the EMSs, the students, and the cooperating teachers, if he or she played a role in the implementation of the high-quality mathematics task through possibly a co-teaching model or if the EMS was the only one
implementing the selected task. This information was noted on the IQA and used in conjunction 
with the IQA rubric.

Researchers conducted informal, semi-structured follow-up interviews (see Appendix M) 
with the subgroup sampling of selected participants from each region that were being observed 
($n = 12$). The participants were also asked to submit the other qualitative journal requirements 
that are highlighted below. Audio recorded, informal interviews took place after the last of the 
two observations and were conducted at the participants’ schools. The researcher asked the 
predetermined interview questions in a private room in the school with only the two researchers 
and the EMS present. These interviews ranged from 7 to 20 minutes. The researcher explained 
the interview protocol at the beginning of the study in the terms of agreement as well as at the 
start of the interview. The audio recordings allowed the researcher to transcribe the data shortly 
after the interview (Creswell & Plano Clark, 2010, p. 178). The researcher asked all interview 
questions without adding any additional follow-up questions or providing feedback for any 
responses provided by the EMSs.

Most participants submitted reflective journal entries through Qualtrics. The participants 
kept a daily journal for one week. Each entry took approximately 20 minutes to complete. The 
EMSs submitted electronically or gave the researcher a hard copy of the two mathematics tasks 
they taught or implemented in two different elementary classrooms (see Appendix I). The EMSs 
explained their rationale for selecting the tasks as being high quality and reflected on how the 
tasks were implemented in the elementary mathematics classrooms (see Appendix K). The 
EMSs also submitted another journal entry explaining their beliefs about teaching mathematics 
and their instructional pedagogy in mathematics. The researcher used the data to look for the
relationships between the EMSs’ instructional pedagogy in mathematics and the EMSs’ beliefs about teaching mathematics (see Appendix L).

**Data Analysis**

This study used descriptive statistics to analyze the data from the CKS, CSI, LMT, TAG, and IQA. A Spearman ($r_s$) correlational analysis evaluated the relationship between the TAG and the LMT. Qualitative methods of analysis explored the data from the observational running records, informal interview questions answered by the EMSs, and the journal entries submitted by the EMSs. The data collection and analysis concluded when the researcher analyzed the correlations of the quantitative data from the TAG and LMT, and coded the qualitative data such that it was comprehensible for the reader and reporting purposes.

The main variables that addressed the three research questions were the following: (a) the EMSs’ ability to select a high-quality mathematics task and their ability to implement a high-quality mathematics task, (b) the EMSs’ ability to select a high-quality mathematical task and their content knowledge of mathematics, and (c) the instructional practices and the EMSs’ beliefs about teaching and coaching mathematics. The validity of the findings was supported with statistical, descriptive data in conjunction with descriptive qualitative data. All measures were taken to secure the privacy of all participants to ensure confidentiality and hopefully allow the participants to be honest in their responses for all parts of the study in which they participated.

**Quantitative Data**

The researcher analyzed the statistical data for this research project using a Spearman rank-order analysis and determined the following descriptive statistics (a) mean, median, mode, and range; (b) outliers in the data; (c) frequency tables; and (d) the standard deviation. A Spearman correlation is a nonparametric correlation used when two ordinal, interval, or ratio
variables are compared. The Spearman rank-order correlation coefficient, $r_s$, measured the statistical dependence, strength, or direction between the two ranked variables (Fields, 2013). The Spearman correlation allowed the researcher to assess the relationship between the rank values of the variables in each of the compared instruments, the Learning Mathematics for Teaching (LMT) and the Task Analysis Guide (TAG) assessments. The research question variables for the study were based on the EMSs’ ability to select a high-quality mathematics task and their content knowledge for teaching elementary mathematics (LMT & TAG). The researcher analyzed statistical data to investigate the relationship between the EMSs’ ability to select a high-quality mathematics task (TAG) and their ability to implement a high-quality mathematics task (IQA), between the EMSs’ ability to select a high-quality mathematical task (TAG) and their content knowledge of mathematics (LMT), and lastly, between the EMSs’ beliefs about teaching and coaching mathematics and their instructional practices pertaining to mathematics (CKS & CSI).

The researcher used the StatsToDo website Power analysis calculator to find the estimated minimum number for the participant size for each instrument to maintain high reliability. The CKS 40-item survey needed a minimum of eight participants for an expected internal Cronbach’s $\alpha$ of .80 or higher. The CKS has an internal reliability and Cronbach’s $\alpha$ ranging between .822 and .935, with the probability of achieving a Type I error of .05 (the probability of rejecting a true null hypothesis; Statstodo, 2016). The CKS had a chance of a Type II error, which meant rejecting a false null hypothesis, with a calculated effect of .8 (Power [1-ß]; Statstodo, 2016). The CKS (Yopp et al., 2010) surveyed the EMSs’ beliefs and practices about mathematics coaching.
There were three types of validity in the methodology explanation of the CKS in the evaluation report: predictive, convergent, and concurrent (Jesse et al., 2014). Predictive validity was the only type used in this study. Predictive validity means that, “in theory, the CKS should be a predictor of coaching effectiveness. There were high correlations between the CKS and other later measures of coaching effectiveness that the evidence would have predictive validity” (Jesse et al., 2014, p. 5). The reason the other two validity features were not valid for this study is because they were relevant to the 5-year longitudinal study and the many other types of specific data collection used in the Examining Mathematics Coaching study. During the data analysis phase, the initial intent was for the CKS data to be adjusted to use only Items 3, 4, and 5. The rationale for using only Items 3, 4, and 5 was that those items were the only ones in the high-level coaching subscale and the most aligned and precise information for this study (Jesse et al., Evaluation Report, 2014, p. 15). However, the researcher analyzed and reported all items of the CKS.

Analysis of the ordinal and interval quantitative data included descriptive statistics, and the comparative data for the specific research questions were qualitative in nature and therefore coded separately. When analyzing the ordinal data from the CKS, the researcher purposely divided the test items into two main constructs that also aligned to the research question about beliefs about teaching and coaching mathematics as well as instructional practices in mathematics. The researcher collected CKS data from 39 participants.

Analysis of the CSI included descriptive statistics, as previously described with the CKS. The internal reliability rate and Cronbach’s α for the CSI is .822 to .935. With the probability of Type I error (α) of .05, the Power (1-β) of .8, 24 items on the survey, the expected Cronbach’s Alpha of .935, and the hypothesis in Cronbach’s Alpha as a 0 for the null hypothesis; in order to
keep the reliability rate high, the generated sample size estimated for the CSI was only approximately five participants. The current study collected data from the CSI from 35 participants.

The researcher used the LMT assessment to assess the EMSs’ mathematical content knowledge for teaching. The LMT had an internal reliability rate and Cronbach’s α of .75 to .80 and was the best fit for this study to assess the EMSs’ mathematical content knowledge. With the probability of Type I error (α) of .05, the Power (1-β) of .8, 25 items on the LMT, the expected Cronbach’s Alpha of .80, and the hypothesis in Cronbach’s Alpha as a 0 for the null hypothesis, it was determined that in order to keep the reliability rate high, the generated sample size for the LMT was estimated at approximately nine participants. The current study collected completed LMT data from only 27 participants; five participants from the 32 that started the assessment did not complete it. The data from the CKS, CSI, and LMT for the general population of the study was analyzed and reported, and an in-depth analysis and following discussion was performed for the smaller sample of 12 EMS who completed all the surveys and agreed to participate in Phase 2.

The researcher analyzed the TAG quantitatively using descriptive statistics. Also, the researcher analyzed the TAG using a Spearman rank-order correlation with the data from the LMT as soon as both instruments’ data were collected. The EMSs selected two mathematics tasks that they thought were of high-quality, then the researchers categorized the tasks based on the characteristics of the potential of the tasks: identifying if they were in quadrant I, II, III, or IV based on the TAG. The TAG and IQA were coded similarly (see the next section).

The researcher and other doctoral student researcher observed and scored the observations using the IQA. The TAG and the IQA were both scored and analyzed using an
intercoder/interrater agreement or interpretive convergence (Tinsley & Weiss, 2000). Intercoder reliability measures only “the extent to which the different judges tend to assign exactly the same rating to each object” (Tinsley & Weiss, 2000, p. 98). Should there have been a discrepancy in the ratings between the two raters, the focus was on getting to a standard coefficient that was statistically necessary in quantitative research (Kuckartz, 2014). Stemler (2004) cautioned raters that interrater reliability was more a function of the situation as opposed to the assessment tool itself. There are three main different types of interrater reliability: consensus estimates, consistency estimates, and measurement estimates (Stemler, 2004). Consistency estimates are based on the assumption that it is not necessary for the raters to agree on the exact score as long as they are both consistent in their scoring (Stemler, 2004). Cohen (1968) provided an alternative weighted kappa that allowed researchers to correct disagreements based on the extent of the discrepancy. It was mainly used for categorical data with an ordinal structure, such as in a rating system that categorizes high, medium, or low with a specific characteristic. In this case, if a participant was rated high by one coder and low by another, the score resulted in a lower interrater reliability estimate than when a subject was rated as high by one coder and medium by another. When there was a discrepancy, the coders reviewed the notes taken during the observations. Based on the key words recorded, the score was once again rated against the instrument’s rubric in order for both coders to come to an agreement.

All observation instruments were included in the analysis, even if there was no mathematics performed, because the request was for two, high-quality mathematics tasks to be selected and implemented. If students did not engage in a mathematics task, N/A was used as outlined in the scoring expectations section (Boston, 2012). The only way the data was coded
N/A was if there was an activity connected to the lesson that had nothing to do with mathematics or any part of the lesson.

**Qualitative Data**

The qualitative data encompassed running records, lesson plans, informal semi-structured interviews, and journal entries. The researcher began by transcribing the informal interviews before coding and organizing them and the notes from the running records (Glesne, 2011). The researcher read through the data multiple times to gain a sense of patterns revealed and tried to make sense of the data in order to make the coding a little easier. When the researcher coded the qualitative data, there were certain key words or phrases to keep in mind; these were for the most part based on the research questions.

According to Gibbs (2007), “Coding is how you define what the data you are analyzing are about” (p. 38). Coding can also be referred to as indexing (Glesne, 2011). It is the process of organizing and sorting qualitative data. Codes serve to label, compile, and organize your data using words, phrases, or symbols to tag specific things visible in the qualitative data collection. Coding allowed the researcher to summarize and synthesize what was happening in the data. In linking the data collected and interpreting it, coding became the basis for developing the analysis.

The researcher coded the data as a whole and also line by line (Glesne, 2011). When coding line by line, the researcher immersed herself in the data to reveal any concepts that may not have been visible when looking at the data as a whole. Data was coded using descriptive methods and uncovered patterns or relationships. The researcher used a codebook in the NVivo (11) for Mac software to organize the words or phrases and then group them into common themes and subgroups. The online codebook was used at the beginning of data collection so that
it would reflect the earliest major codes and evolved after new subcodes were assigned; at that point, the subcodes were explained (Glesne, 2011). Different colors were added each time new codes were assigned; this helped to identify the codes because they stood out more clearly. Once all different colored codes were established, the researcher created themes based on the codes. As the researcher codified new themes, she added them to the online codebook. Thick descriptions (Geertz, 1973) captured the findings of this study, validated the purpose, and ensured the rigor of the methodology. Lincoln and Guba (1985) described thick descriptions as a way for the researcher to achieve external validity by elaborating on something in great detail to obtain generalizable conclusions that may be applicable for other people, situations, studies, the location of a study, or even while the study is being conducted.

Observations were used to help provide context for the quantitative data attained through thick descriptions. The thick descriptions came from the detailed notes from the running records taken by the researcher and other observer during the observations. The powerful, thick descriptions gave the researcher an idea of “the voices, feelings, actions, and meanings of interacting individuals” (Denzin, 1989, p. 83). The researchers described the progress of the lesson, noting what was said, how the EMS responded, and the students’ or teachers’ interactions with the EMS. The researcher reread the data and highlighted multiple times, color-coding based on the newest codes that may have changed, been combined, or were deleted from the initial coding phase (Creswell & Plano Clark, 2010). Once all codes were set, and the researcher created phrases from the common codes that came up in the data, she then created a memo to make sense of the phrases. After the second of the two observations, the EMS and the two researchers went to a private area to conduct the semi-structured informal interview. As previously mentioned, the IQA data came from 12 participants, each providing two lessons for
the observations. During the two observations for each EMS, the researchers recorded the running record notes from all 12 participants on the IQA recording sheets.

The follow-up interviews (see Appendix M) took place with the selected EMSs from each region that were also observed (n = 12). The audio recording during the informal interviews took place in a private area in the school after the last of the two observations. The researcher asked all interview questions without adding any follow-up questions or feedback for any responses provided by the EMS. The audio recordings allowed the researcher to go back and transcribe the data, preferably soon after the interview; the recordings also served as a backup system for recording the information (Creswell & Plano Clark, 2010, p. 178). The researcher transcribed the audio recordings and then wrote them in a memo format to make sense of the responses. The coded data revealed patterns in the responses. Once the data were coded and some patterns and themes created, the researcher related the findings to the research questions: (a) the EMS’s knowledge about what their roles should be, (b) the qualifications of an EMS, (c) the relationship between content and pedagogical knowledge, (d) what the EMS believe good mathematics instruction looks like, and (e) how to select and implement a high-quality mathematics task at a high level of rigor. The researcher collected, transcribed, and coded the interview data from all 12 participants.

The EMSs electronically submitted or gave the researcher a hard copy of the two mathematics tasks they implemented in an elementary classroom (see Appendix I). The EMS also explained their rationale for selecting the tasks as high quality, and reflected on how they thought the tasks were implemented in the elementary mathematics classroom (see Appendix K). The EMSs submitted a reflective prompt explaining how their beliefs for teaching mathematics related to their instructional pedagogy in mathematics (see Appendix L).
Participants submitted the reflective journals through Qualtrics, so the researcher could analyze and code the entries as soon as possible after the submission date. At the end of the qualitative data collection phase, all data were merged so the researcher could analyze it, compare it to corresponding instruments if need be, and then formally write it up for reporting purposes. The intentional and systematic coding strategies and phases are located at the beginning of the qualitative data analysis section (e.g., initial coding methods, using a codebook, creating charts of the frequently used words and phrases, emerging patterns and themes, thick descriptions) and were used to code, sort, and interpret the data. The researcher collected and coded the qualitative data from the journal entries of all 12 participants, but not all 12 participants completed all five of the journal entries.

**Triangulation and Merging of Quantitative and Qualitative Analyses**

The researcher combined all qualitative and quantitative data to make connections and comparisons from each data instrument based on the research questions for the 12 EMSs. As the patterns appeared and revealed the common phrases, the researcher aligned them with the indicators on the individual instruments and the criteria from the AMTE (2013) qualifications to be an effective EMS and combined them into a quantitative and qualitative table (Creswell & Plano Clark, 2010). This table is in the following chapter, which provides the data analysis and interpretation.

This research study triangulated data so that the convergence and substantiation of the data could increase the strength of the findings and solidify the internal validity (Creswell & Plano Clark, 2010; Johnson & Christensen, 2008). Triangulation captured different perspectives of the same data to cross-validate the data and see them from both a qualitative and quantitative standpoint.
Once the researcher collected all data from the CKS, CSI, LMT, TAG, and IQA, she coded the journaling entries and interview questions and then made comparisons with the quantitative data based on their common themes. She then merged the quantitative and qualitative data sets from selected participants to determine whether the findings from the qualitative data supported or did not support the quantitative results from the selected smaller sampling group. By taking all the qualitative data pieces and finding the corresponding quantitative instruments from the subsample participant group, the researcher determined which, if any, correlations could be made when looking at the individuals and then viewing the group as a whole. In using a sequential approach, the intent was to explain the results of the quantitative data by also supporting it with the qualitative data set.

When looking at Research Question 1, “What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and their implementation of a mathematical task,” the researcher looked for words and actions from the EMS and the students from the TAG instrument to determine the level of rigor in the selection of the mathematics task. For example, the TAG and IQA were compared to the journal entry about the EMS’s implementation and selection of the mathematics task (see Appendix K), lesson plans provided to the researcher (see Appendix I), and the interview questions (see Appendix M). The researcher also compared the data from the CSI, the journal entry, and the interview questions about the EMS’s beliefs about teaching and coaching mathematics pertaining to their instructional practices (see Appendix L). Lastly, the researcher compared the data from the CKS with the journal entry, and the interview questions about their beliefs about teaching and coaching mathematics pertaining to their instructional practices (see Appendix L). The comparisons were made as the researcher looked for specific words that came up in the journal
entries that applied to the research question and what common words or phrases came up when examining the interview questions in looking for similarities.

**Limitations**

Every effort was made to decrease the number of limitations in this research study. One limitation was the researcher’s primary job as an EMS for a specific in-service region in the state. The limiting factor was that the researcher had conducted 10 years of work in the state and was personally acquainted with many other Alabama Mathematics, Science, and Technology Initiative (AMSTI) specialists, and she had also trained many teachers and EMSs, specifically in Region 4, and potentially in many other regions in the state. The limitation was the possible influence the researcher would have on the persons and affiliations, specifically in Region 4. Thus, eliminating Region 4 from the study’s population ensured that this limitation would be minimized. The researcher also realized that previous relationships with participants from other regions could have been a threat or limitation, but excluding the biggest hindrance minimized this concern. A potential risk was that the data would be contaminated if the EMSs intentionally answered how they thought it needed to be answered instead of what the reality was at the time. Triangulation of several data sources minimized this risk.

Some limitations of a correlational study are that when there is a relationship between any two variables, it may not necessarily mean that it is a causal relationship. This means that a correlation does not show that one variable causes the other to change, simply that they both exist as described. To strengthen the study, the researcher added the journaling, observations, and interviews to assist in describing this relationship.

When looking at the data from the LMT it is important to note that the correlation data changes when the highest scores and lowest scores are omitted. The TAG also has a ceiling
effect. When the 2 EMSs who scored the highest on the TAG were taken out of the data set, the
Spearmen rho correlation was close to .50. When the two EMSs were put back into the data set,
the correlation was negative. Changing the variables of the highest and lowest scores and
interpreting the changing data is important to keep in mind. If the sample in the data set were
bigger, the correlation would not move as much as when the sample is small such as only 12
EMSs.

Validity Threats

Every study has threats to validity. Of the many types of validity (e.g., construct,
content, internal, external, statistical; Creswell & Plano Clark, 2010), Campbell and Staley
(1963) addressed two types in terms of the experimental design used in this particular study:
internal and external. Internal validity referred to whether there was adequate evidence to
support the claim of a treatment or a specific condition. External validity referred to one being
able to generalize the outcomes of a treatment or a condition. An observation time threat in this
study was taken into consideration to ensure that as many observations were done as close
together as possible to limit the factor of maturation. Maturation could be a factor in this study if
the observations were too far apart between participants, particularly in cases where some of
them had training over the data collection period to improve their content and pedagogical
knowledge for teaching mathematics or in the coaching realm in general. There was no true
threat for the observations being too close together. The researcher and cooperating EMSs
collaborated when setting the dates. All observations for each EMS took place on the same day,
but at different times and with different classes.

Other internal validity threats in this study may have been that The Hawthorne Effect
may have been evident if the participants changed their behaviors and way of teaching because
they knew they were being observed for research purposes. This is like the John Henry effect in
that the participants may have also done something extra and made it a competition if they knew
they were being compared with others in a sample.

In research studies, compensation can cause external validity issues (Johnson &
Christensen, 2008). In this study, to reduce the chances of another validity issue, a general
compensation (i.e., $15 gift card) was provided to all participants who completed the first three
requirements of the study after agreeing on the consent form (i.e., CKS, CSI, and LMT).

Reliability

Reliability refers to the degree to which an instrument or tool used in the study produces
consistent results time and time again. To increase the reliability of this study, the methods, data
collection, and data analysis were intentional, detailed, and organized; examples included the
implementation and analysis using the specific instruments and thorough coding procedures.
The researcher documented all procedures to increase the ability to replicate this study in another
location.

The qualitative data in this study used thick descriptions to capture the findings of this
study to validate the purpose and ensure rigor of the methods. There were also two
rater/observers, including the researcher, using the TAG and IQA instruments, which helped
increase the reliability of the quantitative results. The instruments in this study had high
reliability rates that ensured the consistency of the results of the surveys.

Summary

This study occurred in one selected southeastern state in The United States. The purpose
of this study was to evaluate current elementary mathematics specialists’ content and
pedagogical knowledge for teaching mathematics. It also examined the specialists’ leadership
skills and their ability to select and teach high-quality mathematics tasks to students. This mixed methods study used an explanatory sequential design where the researcher collected both quantitative and qualitative data, beginning with the collection and analysis of the quantitative data, followed by the collection and analysis of the qualitative data to prepare both sets for reporting purposes (Creswell & Plano Clark, 2010).

This study will add to the existing research studies regarding mathematics coaching, mathematics specialists’ content and pedagogical knowledge for teaching mathematics, the ability to select high-quality mathematics tasks paired with high-quality instruction, and the implementation of a mathematics task to a high level of rigor. The study will also add to the existing research on literacy and instructional coaches who have been charged with helping teachers with mathematics instruction in an EMS role.
CHAPTER 4

ANALYSIS OF PHASE 1 DATA

This chapter presents the results of the analysis of data collected and examined current Elementary Mathematics Specialists’ (EMSs) content and pedagogical knowledge for teaching mathematics and the EMSs’ leadership knowledge and skills. All participants came from public elementary schools in the same state and had the responsibility of coaching classroom teachers in mathematics, in some capacity. Although this state has state funded EMSs, some school systems and districts still chose to have school-based or system-based EMSs. This is important to note because of the low number of EMSs that agreed to participate in Phase 2 of the study \( n = 12 \) and whose data are discussed in the following chapter. It is also important to note that this state also does not clearly define the qualifications for an EMS, thus the EMSs may not be true EMSs according to AMTE (2013) standards. In Phase 1, the researcher collected the data from all participants through online surveys and questionnaires that are reported in this chapter. It is important to note that the data collected from the EMSs in Phase 1 of the study as well as the EMSs from Phase 2, is reported in separate chapters to clearly distinguish between the two groups of EMSs. The data from Phase 1 EMSs provides a general understanding of EMSs in the state in terms of their knowledge and beliefs as explored through the data collection. A fuller discussion in Chapter 5 of the 12 EMSs’ subsample that includes both Phase 1 and Phase 2 data provides the data for in-depth answers to the research questions.
Participants

This study included 44 participants from one southeastern state in the United States.

Table 8 shows the number of participants in each in-service region who responded to the researcher’s request for participation. The numbers below differ depending on the participant’s interest in the study. There were 365 EMSs contacted to be part of the study. Of the 365 who were contacted on multiple occasions within a one-year time frame, 44 agreed to take part in the study. After 44 EMSs gave consent to be in the study, once the study began with the first instrument, there were only 39 EMSs that were part of the study because 4 EMSs decided to not move forward after their consent was given.

Table 7

Participant Information Per Each In-Service Region

<table>
<thead>
<tr>
<th>Regions</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Invitations sent to EMS</td>
<td>50</td>
<td>70</td>
<td>30</td>
<td>35</td>
<td>35</td>
<td>45</td>
<td>15</td>
<td>35</td>
<td>15</td>
<td>35</td>
<td>365</td>
</tr>
<tr>
<td>EMS agreeing to participate</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>10</td>
<td>0</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>44</td>
</tr>
</tbody>
</table>

Region 8 has seven districts, six of which were demographically described as low-performing, low-income, high-poverty, and mainly African-American populated. In an article published by the state department of education (Yawn, 2017) reported that 27 schools in one of Region 8’s districts were under scrutiny by the state department of education for being low-performing (Yawn, 2017). The schools from Region 7 were diverse, representing multiple demographic populations, and all had separate funding specifically for EMSs. This is important to note because in the state as a whole, there would generally be someone in the school labeled as an instructional coach, curriculum coach, or literacy coach who would be responsible for the
mathematics content coaching in addition to the other curricular areas. However, Region 7 specifically chose to apply the funding for teacher units to specifically hire EMSs. Region 7 had the largest representation in the study, perhaps because of the presence of an EMS in each elementary school. Region 1 also had a large representation of EMSs \((n = 5)\) in the study; this region also had specific funding for EMSs in some of their elementary schools. From the 44 original participants who agreed to be in the study, only 39 total participants submitted the completed CKS, while only 35 EMSs successfully submitted the CSI. Something important to note is the decrease in the number of participants that gave consent to be in the study \((n = 44)\), to the number of EMSs that completed the CKS \((n = 39)\), to the number of EMSs that completed the CSI, \((n = 35)\). Of the 35 participants who submitted the CSI, 34 reported identifying as female, whereas only 1 identified as male. The ethnic diversity of the group tended toward those of Caucasian descent \((n = 29)\), with five African-Americans, and one who chose not to respond to the question. Although the majority of elementary educators in this state are female and only 19% are of African American descent, the data is still not meant to be generalizable with such a small sampling.

**Phase 1: Quantitative Data Results**

The quantitative data from the instruments included data from the total number of participants who began and completed the instrument; some of the items within each instrument were omitted by some of the participants. The quantitative calculations were based on the total number of responses per item, indicated as \((n)\) within each instrument. The data are reported in the order in which EMSs completed the instruments.
Coaching Knowledge Survey

The CKS (Appendix C) explored the EMSs’ beliefs and practices about mathematics coaching. Table 8 below shows the EMSs by region who completed the CKS (n = 39).

Table 8

*Participant Information for the CKS Per Each In-Service Region*

<table>
<thead>
<tr>
<th>Regions</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participants who completed CKS</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>9</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>39</td>
</tr>
</tbody>
</table>

There were 12 items in three sections of the CKS. The first five questions were broken down into sub-items, for a total of 49 items encompassing Sections 1 and 2. Responses to the items in these first two sections used a Likert scale of 1-7. Five scenarios of instructional coaching situations plus a final question about teacher learning comprised the third section; this section employed multiple-choice responses. Each section provided a different perspective on coaches’ beliefs and practices related to instructional coaching. The first section focused on how a coach’s beliefs and practices generally related to instructional coaching. The second section focused on how coaches practiced instructional coaching in the context of their school settings, which included building relationships with teacher and administrators. The third section explored coaching decisions based on the situations. One of the purposes of the CKS was to distinguish high-level coaching from low-level coaching (Yopp, Burroughs, & Sutton, 2010). The researchers also developed the content coaching scale that includes measures about working with principals, although it does not have as strong reliability and validity evidence as the high- and low-level coaching scales (Yopp et al., 2010).
Sections 1 and 2 comprised 49 items about beliefs and practices related to instructional coaching in general and in specific coaching settings. The data showed mixed responses in that respondents’ answers varied unpredictably between high-level and low-level coaching categories. EMSs (n = 39) showed an overall tendency for high-level coaching, but there was a good variety of high numbers in low-level coaching items as well. Responses to specific CKS items placed respondents into the categories of high level coaching (n = 18), low-level coaching (n = 10), and context coaching (n = 8).

The following responses are some examples that indicate that the participants were thinking like a high-level coach. When asked how they would respond when a teacher says something that the EMS found confusing, the majority (n = 36) of the 39 EMSs who responded to this question said they responded by paraphrasing what they heard back to the teacher. Thirty-two of the 36 EMSs who responded to this question encouraged the teachers to reflect on similarities and differences among mathematics topics in the curriculum. The majority of the 37 EMSs who answered this question (n = 33) encouraged teachers to set personal improvement goals for mathematics instruction. When EMSs were asked about how closely their coaching practices aligned with the statement, I help teachers reflect on discrepancies between espoused beliefs and actual practices, 33 of the 37 EMSs agreed with this statement. Having a collaborative relationship with the teachers and being able to have difficult conversations with them are two qualities of EMS leadership knowledge and skills according to AMTE (2013). Based on the data mentioned, the EMSs felt comfortable with the teachers they worked with and had positive relationships with them.

Some examples of responses indicating the participants thought like low-level coaches are as follows: Of the 39 EMSs who responded to this question, 25 EMSs disagreed that
beginning teachers needed more coaching than 25-year veterans. Research supports that all teachers need access to coaching whether they have been teaching for one year or 20 years because mathematics intelligence can be altered after new and effective professional learning (Fennell, 2011). All but one of the 39 EMSs who responded to this question did not agree that a teacher could learn new mathematics; that is, a teacher’s basic mathematical intelligence cannot be changed. Because veteran teachers have a great deal of classroom experience, other school professionals often view them as not needing any help or support when making the transition to a new role (Chval et al., 2010). Unfortunately, this is not true when they transition to EMSs.

The diversity of EMS responses to the following statement also showed low-level coaching thinking: “As a mathematics coach, I support mathematics teachers by tutoring their struggling students.” Twenty EMSs disagreed with this statement, whereas 14 agreed, and five EMSs did neither. The distribution of the responses to this item was surprising in that according to AMTE (2013) an EMS’s role is not to tutor struggling students. The role of an EMS is to support the teacher. As a whole, the population of EMSs’ answers was considered to be on a high level of coaching. For a complete listing of items and responses, see Appendix N.

For Section 3 of the CKS, only 37 of the 39 EMSs read and responded to five scenarios that explored situations that might occur between coaches and teachers during coaching instruction in the classroom setting. Scenario 1 is a situation in which the coach feels the teacher grasps only part of a concept, but the teacher is not taking responsibility for further study. The coach determines what the difficulty is by asking the teacher to explain the concept. Table 9 shows conflicting data in that some of the participants (n = 23) chose a response that would encourage further conversation, a positive result. The other EMSs (n = 14) chose a response that would likely shut down further conversation, a negative result. EMSs must have a multitude of
practices and educational experiences to support teachers with formative assessment and differentiating instruction for all learners (NCSM, 2008). When EMSs have leadership knowledge and skills, paired with content and pedagogical knowledge, empirical evidence supports the positive impact of a mathematics specialist on teachers’ instructional practices (Becker, 2001; Campbell, 1996; & McGatha, 2008).

Table 9

Coaching Knowledge Survey Scenario #1

<table>
<thead>
<tr>
<th>Answer Choices</th>
<th>“You’re almost right!” followed by a clear explanation of exchanges.</th>
<th>“Let me paraphrase your explanation,” followed by a clear restatement of the teacher’s approach.</th>
<th>“It’s clear you struggle with some aspects of exchanges. Let’s go through this again together.”</th>
<th>“You confused me during your explanation about the exchanges. Can you provide a better explanation that helps me clear up that confusion?”</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Responses</td>
<td>10 (27%)</td>
<td>13 (35%)</td>
<td>4 (11%)</td>
<td>10 (27%)</td>
<td>37</td>
</tr>
</tbody>
</table>

Scenario 2 places the coach in a classroom, observing a lesson. The teacher does not realize that some students do not understand the concept. The teacher has only lectured about it, not eliciting student comments or examining student thinking. The responses from which the participants had to choose framed possible ways to approach the teacher and express concern. The participants were divided in their responses. Seventeen EMSs chose to address the misconception immediately, which showed that their positive relationship with the teacher allowed them to interject at any point during the instruction. Ten EMSs either chose the response, lack of discussion or lack of formative assessment during the instruction. The last two
responses might indicate that the EMSs were not confident enough in their relationship with the teacher to enable them to interrupt the lesson.

Table 10

*Coaching Knowledge Survey Scenario 2*

<table>
<thead>
<tr>
<th>Answer Choices</th>
<th>Making sure that this student misconception is addressed so that students don’t leave this class with wrong thinking.</th>
<th>Making sure that the teacher engages with the coach in a conversation about student thinking and learning.</th>
<th>Making sure that the teacher uses formative assessment strategies in the next class taught.</th>
<th>Making sure that the lesson is retaught.</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Responses</td>
<td>17 (46%)</td>
<td>10 (27%)</td>
<td>10 (27%)</td>
<td>0</td>
<td>37</td>
</tr>
</tbody>
</table>

Scenario 3 focuses on teacher resistance when a coach is not wanted or presumably, needed by a veteran sixth-grade teacher. The teacher states that when a problem occurs with a student, she prefers to continue teaching as she has taught for the last 25 years. The participants had to choose from descriptions that identified how the teacher would likely solve the problem. An overwhelming majority of EMSs ($n = 32$) selected the response indicating that the teacher depends on a process in which her beliefs about problem resolutions are determined internally. The data from Scenario 3 suggests that EMSs realized that sometimes a teacher’s beliefs could not be changed at a certain time due to factors such as personal experiences.
### Coaching Knowledge Survey Scenario 3

<table>
<thead>
<tr>
<th>Answer Choices</th>
<th>Number of Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>A process in which the teacher acts according to what the teacher believes peers will approve of.</td>
<td>1 (3%)</td>
</tr>
<tr>
<td>A process in which the teacher steps outside of the situation to reflect on the problem objectively.</td>
<td>1 (3%)</td>
</tr>
<tr>
<td>A process in which the teacher’s beliefs are determined internally.</td>
<td>32 (89%)</td>
</tr>
<tr>
<td>A process in which the teacher’s beliefs are substantiated by reflecting on what works.</td>
<td>2 (6%)</td>
</tr>
<tr>
<td>Total</td>
<td>36</td>
</tr>
</tbody>
</table>

Scenario 4 places the EMS in the classroom with a teacher during her whole group mathematics instruction after meeting with the teacher the previous day. While observing the lesson, the coach notices the students’ and teacher’s weak conceptual understanding of place value in subtraction. The teacher placed blame on the previous year’s teacher after observing that many students did not understand what they were doing during the lesson. Most EMSs ($n = 23$) responded that the coach should try to teach the next day’s lesson to the same class, modeling through the use of base-10 blocks for subtraction to build conceptual understanding. Part of the EMSs’ role as specified by AMTE (2013) is to strengthen teachers’ procedural fluency from conceptually understanding.
Table 12

Coaching Knowledge Survey Scenario # 4

<table>
<thead>
<tr>
<th>Answer Choices</th>
<th>The coach should postpone the discussion of the teacher’s conceptual understanding because bringing it up this early in their relationship would undermine the trust and rapport in their relationship.</th>
<th>The coach should try to teach the next day’s lesson to the same class, modeling the lesson using base-10 blocks for subtraction.</th>
<th>The coach should explain to the teacher that the students are weak at the subtraction procedure because the teacher didn’t address the conceptual basis of the algorithm in class, and the coach should recommend resources for the teacher to use in class.</th>
<th>The coach should ask if the teacher is more concerned about establishing students’ proficiency with the subtraction algorithm or establishing their conceptual understanding.</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Responses</td>
<td>2 (6%)</td>
<td>23 (64%)</td>
<td>3 (8%)</td>
<td>8 (22%)</td>
<td>36</td>
</tr>
</tbody>
</table>

Scenario 5 is a coaching scenario with an EMS and a teacher conversing about a specific teaching strategy, and then asks the coach to decide what should happen next. The coach feels that the teacher knows enough about the strategy to implement it. Also, the teacher has developed a plan for implementation. Item responses were closely distributed between two choices. One suggests the coach should provide continued support for the teacher in planning and implementation \((n = 17)\), whereas the other option allows the teacher to implement her plans and then ask for additional help as needed \((n = 18)\). Developing a strategic plan with the teacher for continuing support and modeling, if necessary, is the best answer choice because that is one of the major roles of an EMS (AMTE, 2013).
Table 13

Coaching Knowledge Survey Scenario 5

<table>
<thead>
<tr>
<th>Answer Choices</th>
<th>Develop a plan with the teacher for continued coaching support on the strategy and the possible modeling of the strategy.</th>
<th>Leave the teacher alone to try it out a few times so the teacher can grow comfortable with the strategy and gain ownership of it.</th>
<th>Check on the teacher occasionally to make sure the teacher is using the strategy.</th>
<th>Wait for the teacher to ask for further support to avoid appearing &quot;pushy.&quot;</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Responses</td>
<td>17 (46%)</td>
<td>18 (49%)</td>
<td>2 (5%)</td>
<td>0</td>
<td>37</td>
</tr>
</tbody>
</table>

As a response to the question, which of the following is true about teacher learning, most of the EMSs \(n = 28\) responded that coaches could influence teacher traits such as intelligence, which contradicts earlier responses in the survey to the question of being able to improve mathematics intelligence. A smaller group of EMSs \(n = 8\) agreed that coaches should differentiate their coaching, based on teacher intelligent quotient assessments. Although the idea of a fixed intelligence is well supported, effective coaching can harness that intelligence to engender teacher learning.

The data from the CKS showed mixed responses in that respondents’ answers varied between high-level and low-level coaching categories. The EMSs \(n = 39\) showed an overall tendency for high-level coaching.
Table 14

Coaching Knowledge Survey Final Item

<table>
<thead>
<tr>
<th>Answer Choices</th>
<th>Number of Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teachers come to us with fixed intelligence.</td>
<td>1 (3%)</td>
</tr>
<tr>
<td>Teacher traits such as intelligence can be influenced by coaches.</td>
<td>28 (76%)</td>
</tr>
<tr>
<td>Teachers are born with traits such as intelligence that cannot be changed.</td>
<td>0</td>
</tr>
<tr>
<td>Coaches should differentiate coaching based on teacher intelligence quotient assessments.</td>
<td>8 (22%)</td>
</tr>
<tr>
<td>Total</td>
<td>37</td>
</tr>
</tbody>
</table>

Coaching Skills Inventory

The CSI (Appendix D) explored EMS beliefs about teaching and coaching mathematics while measuring the EMSs’ perspective on his or her own level of effectiveness and confidence with assorted coaching responsibilities. Table 16 shows the EMSs by region who completed the CSI (n = 35).

Table 15

Participant Information for the CSI Per Each In-Service Region

<table>
<thead>
<tr>
<th>Regions</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participants who completed CSI</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>9</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>39</td>
<td></td>
</tr>
</tbody>
</table>

The inventory’s first 24 items used a 5-point Likert scale to measure participants’ (n = 35) perception of their coaching effectiveness. This section on coaching effectiveness was presented in five categories: (1) coach/teacher relationships, (2) coaching skills, (3) mathematics
content, (4) mathematics-specific pedagogy, and (5) general pedagogy. The remaining 20 items in the CSI asked about EMSs’ background and practices as an educator.

**Perceptions of coaching effectiveness.** The mode for the responses to items in Sections 1, 2, 4, and 5 was either a 4 or 5, with only one item rated at a 3. These responses expressed high confidence in the EMSs’ perceived effectiveness. In Section 3, mathematics content, modes of responses were spread more widely, with one mode of 3, one of 4, and one item with three modes (3, 4, 5), reflecting less confidence in the EMSs’ mathematical content knowledge. More specifically, the areas of weakness reported by the EMSs included their confidence in coaching mathematics for the particular grade levels to which they were assigned. The EMSs also did not feel confident helping teachers apply and connect mathematics, nor did they feel confident coaching teachers on number sense and relevant computations. For a complete listing of items and responses, see Appendix O.

**EMSs’ background and practices.** Coaching overall should be a relationship-based collaboration used to move teachers in the right direction of implementing new instructional strategies and making instructional shifts while at the same time encouraging professional growth (Showers, 1985). The following are the purposes, suggested by Showers (1985), that coaching encompasses: (a) building communities of teachers who continuously engage in the study of their craft, (b) developing the shared language and set of common understandings necessary for the collegial study of new knowledge and skills, and (c) providing a structure for the follow-up to training that is essential for acquiring new teaching skills and strategies (p. 19). Feger, Woleck, and Hickman (2004) described that most types of coaching fell under two headings: coaching knowledge and skills. EMSs are expected to show preparedness to teach 21st century mathematics skills. Paired with leadership knowledge and skills, EMSs must also
be equipped with content and pedagogical knowledge for teaching mathematics (AMTE, 2013). EMSs must also have rich, quality professional development opportunities to grow professionally in mathematics content areas.

**Certifications, degrees held, and content courses.** Some EMSs held multiple certifications for teaching. Seventy-seven percent ($n=34$) reported being certified to teach elementary school, 18% ($n = 8$) held middle school certification, whereas 5% ($n = 2$) were certified to teach at the secondary level. Some of the EMSs indicated they had more than one certification. None of the 35 EMSs however had a special certification for teaching mathematics. Only three EMSs from the whole population had a specific certification or endorsement for coaching teachers. EMSs’ ($n = 35$) highest degrees held ranged from a bachelor’s degree to multiple master’s degrees to National Board Certification. More specifically, their areas of major focus for 32 of the 35 participants were not in a mathematics field. Four who did have a mathematics degree stated they had a minor in another field, but the other field was not elaborated upon. Twenty-seven participants held a bachelor’s degree in elementary education, one in secondary education, and eight stated they had a major in an “other” category that was also not elaborated upon. Thirty-two held master’s degrees, one in secondary education, 19 in elementary education, and 12 in an “other” category that was not further explained.
Figure 5. Highest degree/certification held by the elementary mathematics specialist

The EMSs as a whole \((n = 34)\) were not prepared by a specific degree to hold their mathematics specialist position. Most did not have sufficient mathematics content courses in college or thereafter. Over half \((55\%)\) of the EMSs in the big population as a while did not take any mathematics content courses in college; only one EMS took four mathematics content courses while in college. A few EMSs \((n = 4)\) reported taking statistics, while only two reported taking a calculus course. Because most of the EMSs \((n = 18)\) did not take mathematics content courses in college, this data seems to support the lack of confidence in the EMSs’ ability to help teachers with specific mathematics content and concepts based on conceptual understanding.
Roles and responsibilities. Several EMSs ($n = 23$) reported they were hired to specifically work with teachers as an instructional coach, and one EMS reported simultaneously holding a position as a classroom teacher (see Figure 7). Only nine EMSs reported they were hired to have multiple responsibilities including coaching other teachers, but not working as a classroom teacher, whereas the remaining EMS ($n = 2$) reported their situation was none of the options listed. If the EMSs are hired to help coach teachers with mathematics content and pedagogy, and they are given many other responsibilities on top of coaching mathematics, the situation begs the question of their effectiveness as coaches.
Years of teaching and coaching experience. The EMSs’ \( n = 35 \) teaching experience ranged from 2 years to 30 years in K-8 schools, whereas the number of years of coaching experience in K-8 schools ranged from 0 to 10 years. The numbers of years the EMSs have been in their current position ranged from 0 to 14 years of experience. The wide distribution of years of experience allowed the data to be seen from a wide range of age levels in addition to the variety of years of teaching and coaching experiences.

Coaching by the numbers. The EMSs \( n = 36 \) reported that they served anywhere from 0 teachers to as many as 80 classroom teachers. One EMS reported serving 38 elementary teachers in addition to also serving 15 middle school teachers. The single EMS to state that she served elementary and middle school teachers was only certified in elementary education. This raises questions about what grade the middle school students were in. Most elementary certifications serve from Kindergarten to Grade 6. If this is the case, the EMS was certified to coach the middle school teachers as long as they were only sixth-grade teachers. If they were not only sixth-grade teachers, she was not technically qualified to teach or coach them based on her
level of certification. This may figure into the EMS’s lack of mathematical content knowledge at the secondary level. And, most of the time if someone is certified to teach in the middle grades, you generally have to choose a specific content area in which to focus on (i.e. mathematics, reading and language arts, science, etc.).

When the EMSs ($n = 33$) were asked to report how many coaching cycles they were assigned to do each school year, the responses were disturbing. Two EMSs stated that they were not sure, whereas another coach reported having to do over 190 coaching cycles in a school year. The high number is an outlier, but this particular EMS reported that she currently coached 39 teachers. If each EMS and teacher had 5 coaching cycles together, that is equivalent to approximately 190 coaching cycles in one school year.

**Professional development.** EMSs reported receiving from zero to over 100 hours of specific mathematics professional development hours within the last 12 months. The EMSs ($n = 3$) who reported having at least or over 100 hours of mathematics professional development were noteworthy as they were from systems that have separate funding for EMSs and their daily responsibilities revolved around mathematics content, pedagogy, and instructional coaching. Also, the three EMSs that reported having over 100+ hours of specific mathematics professional development within the last 12 months were also part of Phase 2 of the study. This is important to note because it seems as if the EMSs that were part of Phase 2 of the study seemed to be some of the top EMSs compared to the group as a whole.
Table 16

*Hours of Mathematics Professional Development in the Last 12 Months*

<table>
<thead>
<tr>
<th>Number of Hours</th>
<th>n</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>No math hours (0)</td>
<td>10</td>
<td>29.0</td>
</tr>
<tr>
<td>1-20 hours</td>
<td>11</td>
<td>32.0</td>
</tr>
<tr>
<td>21-50 hours</td>
<td>4</td>
<td>12.0</td>
</tr>
<tr>
<td>50-99 hours</td>
<td>6</td>
<td>18.0</td>
</tr>
<tr>
<td>100+ hours</td>
<td>3</td>
<td>9.0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>34</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Although the bulk of the EMSs lacked college mathematics content courses, it appears that most of them did not pursue developing their content or pedagogical knowledge through in-teacher professional development. These facts, combined with their lack of confidence in coaching teachers with mathematics content, support the contention that lack of preparation is a hindrance to the EMS’s ability to perform his or her responsibilities to the level that is expected by AMTE (2013) qualifications for an EMS.

**Learning Mathematics for Teaching**

The Learning Mathematics for Teaching (LMT, see Appendix E) assessment was used to identify EMS mathematical content knowledge based on various skills and concepts in elementary mathematics. The LMT instrument was sent to EMSs \( n = 39 \) who completed or were in the midst of completing the first two instruments during Phase 1 of the study. Thirty-two EMSs began the LMT assessment; five did not finish or stopped after logging into the assessment. Table 18 shows the EMSs by region who completed the LMT \( n = 27 \).
The items on the LMT varied in levels of difficulty and assessed EMSs’ knowledge and skills in various areas of elementary mathematics that an average teacher should know. The items included questions about multiplication, division, place value, integers, fractions, decimals, ratios, and proportions; see Appendix E for sample items. Upon login, participants were randomly assigned to complete Form A or Form B. Half of the EMSs ($n = 14$) were randomly assigned Form A, whereas the other EMSs ($n = 13$) were randomly assigned Form B. Form A had 28 items; Form B had 29 items, which were not arranged in the same order as on Form A.

To report the overall scores from both exams consistently, scores were scaled. The distribution of the scaled scores ranged from -2.4861 to 2.2748, which indicated that most EMSs were in the bell curve for a normal distribution. The minimum scaled score for the whole populations was -2.4861, the maximum was 2.2748, the mean was 0.5354, and the standard deviation was calculated as 1.11. Items including fractional reasoning, the divisibility rules, distribution property, prime and composite numbers, and a comparison-meaning problem including subtraction were missed most often by the majority of the EMSs; most of these standards are seen in the higher elementary grades (CCSSM, 2010). See Table 22 for the EMSs’ scaled scores on the LMT. The EMSs ($n = 3$) who had the majority of the highest scores on the LMT were part of Phase 2 of the study, which could indicate that their interest in the study was, to some extent, predicated on their interest in mathematics. The lowest scaled scores from the
EMSs \((n = 3)\) were not part of Phase 2 of the study, due to their discretion. They chose not to move forward with the study after completing the first three items and ending with the LMT.

**Summary**

The data reported in Chapter 4 provides an understanding of the content knowledge, pedagogical knowledge, beliefs about mathematics and coaching, and the leadership skills and knowledge of the participants in Phase 1 of this study. Chapter 5 reports data from Phase 1 and Phase 2 for 12 of the participants who completed all surveys and agreed to Phase 2 observations of coaching lessons.
CHAPTER 5
ANALYSIS OF PHASE 2 DATA

Quantitative Data Results for Convenience Sample

This chapter presents the analysis of the data collected from a subset group of EMSs (n = 12) and examined their content and pedagogical knowledge for teaching mathematics; the EMSs’ leadership knowledge and skills; and their ability to select and teach a grade level appropriate, high-quality mathematical task to two different classrooms of elementary students in their school. All participants came from public elementary schools in the same state and had the responsibility of coaching classroom teachers in mathematics, in some capacity. Although this state has state-funded EMSs, some school systems and districts still chose to have school-based or system-based EMSs. This distinction may have bearing on the qualifications and mathematics knowledge of some coaches. In Phase 1, the researcher collected the initial data from all participants through online surveys and questionnaires. After submission of the quantitative items during Phase 1, data were collected during Phase 2 through lesson plans, classroom observations, participant interviews, and journal entries from the subset of participants (n = 12). A second, trained researcher assisted with data collection for the two classroom observations in Phase 2 of the study. The qualitative phase was conducted to support the quantitative results and strengthen any possible relationships between the data sources.
Research Questions

This research study was grounded in the AMTE (2013) qualifications necessary for a certified EMS. Thus, the following are the research questions that were answered by EMSs in Phase 2 of the study:

Research Question 1. What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and their implementation of a mathematical task?

Research Question 2. What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and their content knowledge for teaching elementary mathematics?

Research Question 3. What are the relationships between the Elementary Mathematics Specialists’ leadership knowledge and skills, their beliefs and practices about teaching and coaching mathematics with conceptual understanding, and their instructional practices pertaining to mathematics?

Participants

Phase 2 was comprised of a convenience sampling of 12 participants, including 11 Caucasian females and one African-American female. Their participation in Phase 2 was dependent upon submitting all items for Phase 1 and confirming with the researcher to be part of Phase 2. In addition to the Phase 1 data collected, they participated through two observations and a semi-formal structured interview and submitted multiple journal entries.

Years of teaching and coaching. The EMSs in the subset had a wide range of years of teaching experience. They ranged from 6 years to 25 years in K-8 schools, whereas the number of years of coaching experience in K-8 schools ranged from 0 to 10 years. The numbers of years
the EMSs had been in their current position ranged from 0 to 14. The wide distribution of years of experience allowed the data to be seen from an expansive range of age levels in addition to the variety of years of teaching and coaching experiences. Table 18 shows the number of years of teaching experience of each EMS in Phase 2, as well as the number of years in a coaching position. Years in coaching are far less than years of teaching, perhaps because the EMSs were not able to stay in a specialist position for an extended period of time, perhaps because funding allotted for coaches may not have been stable. The years of coaching are not necessarily consecutive years due to the nature of the coaching position being used as a teacher unit or the lack of specific funding.

Table 18

<table>
<thead>
<tr>
<th>EMS</th>
<th>Years of Teaching</th>
<th>Years of Coaching</th>
<th>Years in Current Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anna</td>
<td>18</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Brooke</td>
<td>8</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Connie</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Denise</td>
<td>16</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Ellen</td>
<td>24</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Fran</td>
<td>8</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Gwen</td>
<td>13</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Helen</td>
<td>13</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>*Ingrid</td>
<td>11</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Jennifer</td>
<td>25</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Katie</td>
<td>12</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
Lilly 10 5 2

Total 164 39 27

Note. *Coach and classroom teacher

Coaching Knowledge Survey

The Coaching Knowledge Survey (CKS, see Appendix C) explored EMSs beliefs and practices about mathematics coaching. Table 20 below shows the EMSs by region who completed the CKS ($n = 12$). For Phase 2 only, the number of participants by region is the same for the CKS, CSI, and LMT instruments.

Table 19

Participant Information for the CKS Per Each In-Service Region

<table>
<thead>
<tr>
<th>Regions</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participants who completed CKS</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>

The 49 total items encompassing three sections provided a different perspective on coaches’ beliefs and practices related to instructional coaching. The first section focused on how a coach’s beliefs and practices generally relate to instructional coaching; the second focused on how coaches practice instructional coaching in the context of their school settings, which included building relationships with teacher and administrators; and the third section explored coaching decisions based on situations. Distinguishing high-level coaching from low-level coaching was one of the purposes of the instrument (Yopp et al., 2010).

The following responses are some examples indicating the participants’ thinking at a high-level in the context of coaching. When asked how they would respond when a teacher says
something the EMS found confusing, the majority of EMS \((n = 10)\) said they responded by paraphrasing what they heard back to the teacher. Most EMSs \((n = 8)\) encouraged the teachers to reflect on similarities and differences among mathematics topics in the curriculum. Most EMSs \((n = 9)\) encouraged teachers to set personal improvement goals for mathematics instruction.

And, when EMS were asked about how closely their coaching practices align with the statement, “I help teachers reflect on discrepancies between espoused beliefs and actual practices,” many EMSs \((n = 9)\) agreed with this statement. Having a collaborative relationship with the teachers and being able to have difficult conversations with them are two qualities of EMS leadership knowledge and skills according to AMTE (2013). Based on the data mentioned, the EMSs seemed to feel comfortable with the teachers they worked with and seemed to have positive relationships with them. Brooke and Gwen did not agree to three of the four items reported above about paraphrasing feedback to the teacher, encouraging the teachers to reflect on similarities and differences among mathematics topics in the curriculum, encouraging teachers to set personal improvement goals for mathematics instruction, and helping teachers reflect on discrepancies between espoused beliefs and actual practices; whereas the other 10 EMSs only disagreed with one or two of the four total items reported. Because these items were in a high-level coaching category, the fact that Brooke and Gwen did not agree with three of the four items suggested they have less confidence in their coaching decisions compared to what is considered for high-level coaches.

Examples of responses indicating that the participants think like a low-level coach are as follows: EMSs \((n = 9)\) disagreed when prompted that beginning teachers need more coaching than 25-year veterans. When EMSs were given the prompt that a teacher can learn new mathematics, but the teacher’s basic mathematical intelligence cannot be changed, all 12 of the
EMSs disagreed. One item that the EMSs did not have an overall tendency to agree or disagree with was when given the statement, “as a mathematics coach, I support mathematics teachers by tutoring their struggling students.” Six EMSs disagreed with this statement, five agreed, and one EMS did not agree or disagree. The distribution of the responses to this item was surprising in that according to AMTE (2013) an EMS’s role is not to tutor the struggling students. The role of an EMS is to support the teacher. Connie and Jennifer responded opposite to the rest of the group and agreed to two of the three items reported in the previous low-level coaching context. They indicated that they did not support mathematics teachers by tutoring their struggling students, nor did they believe that beginning teachers need more coaching than 25-year veteran teachers. Because these items were considered to be in a low-level coaching category, the fact that Connie and Jennifer did not agree may indicate contradictions in their thinking. For a complete listing of items and responses, see Appendix N.

Coaching Skills Inventory

After submitting the CKS, the EMSs took the CSI. The CSI’s first 24 items used a 5-point Likert scale to measure participants’ \((n = 12)\) perception of their coaching effectiveness. This section on coaching effectiveness was presented in five categories: (1) coach/teacher relationships, (2) coaching skills, (3) mathematics content, (4) mathematics-specific pedagogy, and (5) general pedagogy. The remaining 20 items in the CSI asked about EMSs’ background and practices as an educator.

Perceptions of coaching effectiveness. The mode for the responses to items in sections one (coach/teacher relationships), two (coaching skills), four (mathematics-specific pedagogy) and five (general pedagogy) was either a 4 or 5, with only one item rated a 3. These responses expressed high confidence in the EMSs’ perceived effectiveness. In section three, mathematics
content, modes of responses were spread more widely, with one mode of 3, one of 4, and one item with three modes (3, 4, 5), reflecting less confidence in the EMSs’ mathematical content knowledge. More specifically, the areas of weakness reported by the EMSs included their confidence in coaching mathematics for the particular grade levels to which they were assigned. The EMSs also did not feel confident helping teachers apply and connect mathematics, nor did they feel confident coaching teachers on number sense and relevant computations. For a complete listing of items and responses, see Appendix O.

**Certifications, degrees held, and content courses.** The convenience group of EMSs held a variety of degrees and certifications: Masters in Arts \((n = 2)\), Bachelor in Science \((n = 1)\), Masters in Business Administration \((n = 1)\), Masters in Elementary Education \((n = 1)\), Masters in Education \((n = 3)\), Masters and National Board Certified \((n = 1)\), Masters in Gifted Education \((n = 1)\), and multiple masters degrees \((n = 1)\) that were not further elaborated upon. One specialist chose to not respond to this item. All of the EMSs had a degree in Elementary Education except for two, Ellen and Helen. They stated that their major for their bachelor’s degree was something other than education, but did not state what the other degree was. All EMSs had a degree in elementary education, whereas three EMSs also had degrees in secondary education. The secondary education degree was not elaborated on in terms of a specific content area. Overall, the subset group of EMSs did not hold a specific degree to support their mathematics coaching position. Most of them reported not having a sufficient amount of mathematics content course in college or thereafter. Over half (55%) of the EMSs did not take any mathematics content courses in college; only 1 EMS took four mathematics content courses while in college. A few EMSs \((n = 4)\) reported taking statistics, whereas only 2 reported taking a calculus course in college, but did not go into any further explanations. The EMSs reported having taken one to
four mathematics content courses in college, outside of the mathematics methods course. The courses varied from algebra, pre-calculus, calculus, trigonometry, statistics, and a specialized honors mathematics course. Those EMSs who did not take more than two mathematics content courses in college or if their highest course taken was algebra, noted that they lacked confidence in their ability to help the teachers with specific mathematics content. Some EMSs chose to not state how many mathematics courses they took in college and simply stated the highest course they took. It is unclear if algebra was a basic algebra class or an advanced algebra course.

Roles and responsibilities. Several EMSs (n = 6) reported they were hired specifically to work with teachers as an instructional coach (Helen, Katie, Lilly, Brooke, Gwen, and Anna). Only 1 EMS, Ingrid, reported simultaneously holding a position as a classroom teacher. Five EMSs reported they were hired to have multiple responsibilities including coaching other teachers, but not working as a classroom teacher (Jennifer, Connie, Denise, Fran, and Ellen). If the EMSs were hired to help coach teachers with mathematics content and pedagogy, and they were given many other responsibilities on top of coaching mathematics, the situation begs the question of their effectiveness as coaches. This point will also be addressed in the qualitative journal entries data that were collected in Phase 2 of the study.

Coaching by the numbers. Currently, the EMSs (n = 12) reported that they served anywhere from 0 to as many as 80 classroom teachers. One EMS reported serving 38 elementary teachers in addition to also serving 15 middle school teachers. Although the EMS stated that she served elementary and middle school teachers, she was only certified in elementary education. Most elementary certifications and degrees serve from kindergarten to Grade 6. If this was the case, the EMS was certified to coach the middle school teachers as long as they were only sixth-grade teachers. When the EMSs (n = 12) were asked to report how many
coaching cycles they were assigned to do each school year with each teacher they coached, the responses were disturbing. One EMS stated that she did not have a coaching cycle with each teacher she was assigned to coach. The item on the CSI did not provide them the opportunity to state what kept the EMS from more mathematics coaching opportunities. The other EMSs reported having anywhere from zero to 12 coaching cycles (planning, co-teaching/modeling, debriefing) per teacher they were responsible for coaching. The EMSs’ reports of not having coaching cycles with teachers raises questions about what the EMSs are doing with their time each day if they are not coaching teachers in mathematics.

**Professional development.** EMSs reported receiving from zero to over 100 hours of specific mathematics professional development hours within the last 12 months. The EMSs ($n = 3$) who reported having at least or over 100 hours of mathematics professional development are noteworthy as they were from systems that have separate funding for EMSs, and their daily responsibilities revolve around mathematics content, pedagogy, and instructional coaching. This sample has a similar distribution compared to the total EMSs in the study, but the level of mathematics content courses taken in college and teacher professional development was slightly increased with the EMSs in Phase 2. These specific data mentioned, combined with their lack of confidence in coaching teachers with mathematics content, support the contention that lack of preparation is a hindrance to the EMSs’ ability to perform their responsibilities to the level that is expected by AMTE (2013) qualifications for an EMS.
Learning Mathematics for Teaching

Upon the completion of the CKS and CSI, the EMSs in the subset sample (n = 12) completed and submitted the final instrument in Phase 1 of the study: the Learning Mathematics for Teaching assessment (LMT, see Appendix E). The LMT assessment identified the mathematical content knowledge of the EMS based on various skills and concepts in elementary mathematics. The LMT score was used to determine if there was a relationship between the EMSs’ content knowledge for teaching mathematics and their ability to select a high-quality task. The items on the LMT varied in level of difficulty and assessed the EMSs’ knowledge and skills in various areas of elementary mathematics that an average teacher would know. The items included multiplication, division, place value, and scenarios of how to solve problems (see Appendix E for sample items). Upon login, participants were randomly assigned to complete Form A or Form B. Half of the EMSs (n = 6) were randomly assigned Form A, while the other EMSs (n = 6) were randomly assigned Form B. Form A had 28 items; Form B had 29 items and items were not arranged in the same order as on Form A. To report the overall scores from both exams consistently, scores were scaled. Table 22 shows the EMSs’ scaled scores on the LMT.
The EMSs (n = 3) who had the highest scores on the LMT, which assessed their mathematics content knowledge, also reported the greatest amount of mathematics professional development within the last 12 months.

An item including fractional reasoning was missed by only 3 of the EMSs in the subset sample; but was missed by 20 of the 32 EMSs in the whole population. Most fractional type problems that were seen on the LMT are seen in the higher elementary grades (CCSSM, 2010). The CSI did not specifically ask what grade(s) the EMSs previously taught, but based on the grade level tasks selected, the data may allow the inference that the three EMSs who missed the fractional reasoning items spent the majority of their classroom teaching experience in a lower elementary grade. The mean for the 12 EMSs that completed Phase 2 was .88. It is also important to note that even though the EMSs’ scores in Phase 2 were among the top 20% of the entire elementary school teacher population, the scores reveal a lack of mathematics content knowledge for purported mathematics experts in the state schools or districts. Table 21 shows the EMSs’ scaled scores from the LMT; Figure 8 shows the frequency of their scores relative to their content knowledge in mathematics. Note that 9 of the EMSs’ scored better than the average teacher.

Table 21

<table>
<thead>
<tr>
<th>EMS</th>
<th>Scaled Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fran</td>
<td>2.27475</td>
</tr>
<tr>
<td>Anna</td>
<td>2.17958</td>
</tr>
<tr>
<td>Lilly</td>
<td>2.04946</td>
</tr>
<tr>
<td>Jennifer</td>
<td>1.75572</td>
</tr>
</tbody>
</table>
Quantitative Data Results for the Convenience Sample

I now turn to the Task Analysis Guide (TAG), and the Instructional Quality Assessment (IQA) only used with Phase 2 participants. The quantitative calculations were based on the total number of responses per item, indicated as \((n)\) within each instrument. The focus of the data

<table>
<thead>
<tr>
<th>Name</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ellen</td>
<td>1.55831</td>
</tr>
<tr>
<td>Connie</td>
<td>1.3683</td>
</tr>
<tr>
<td>Denise</td>
<td>0.658824</td>
</tr>
<tr>
<td>Brooke</td>
<td>0.489644</td>
</tr>
<tr>
<td>Helen</td>
<td>0.154447</td>
</tr>
<tr>
<td>Katie</td>
<td>-0.02175</td>
</tr>
<tr>
<td>Gwen</td>
<td>-0.74025</td>
</tr>
<tr>
<td>Ingrid</td>
<td>-1.12035</td>
</tr>
</tbody>
</table>
sources in Phase 2 revolved around the high-quality tasks selected and implemented by the 12 EMSs.

Two instruments were used together to complete the scoring of the coaching lesson plans that included high-quality tasks and the implementation of the tasks from the plans. The researchers used the TAG to determine the level of potential of the task based on the lesson plans submitted and then recorded the implementation of the tasks’ score using the IQA (Academic Rigor, AR 2) rubric.

**Task Analysis Guide**

The TAG (see Appendix F) instrument was used by the researchers to help gauge the level of rigor (scale of 1-4) of the EMS’ selection of two mathematics tasks, as presented in their lesson plans. The TAG consists of characteristics of tasks in each level of cognitive demand (Memorization, Procedures without connections to concepts or meaning, Procedures with connections to concepts and meaning, and Doing mathematics). Classifying a mathematical task at a high level means students are engaged in high-leveling thinking. High-quality tasks were described as

the potential to engage students in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships, such as: doing mathematics, using complex thinking or things such as solving problems with procedures with connections, applying a broad general procedure that remains closely connected to mathematical concepts. (TAG, see Appendix F).

The total overall scores from the EMSs \( n = 12 \) ranged from a 1 to 4 for the potential of the task. A higher score indicated the mention of a high level of best practices and high rigor of the two tasks, whereas the lower scores did not meet certain criteria such as students’ performing procedures with connections to concepts and meaning and doing the mathematics. The table below shows the scores given by each researcher for each EMS’s rating of the potential of the
task. Of the 48 total scores given for the EMSs \((n = 12)\), the majority of the scores for the potential of the task were scored as a 3 or 4. The EMSs strengths indicated on the TAG were that students were (a) engaged in deeper levels of understanding of mathematics concepts and ideas, (b) making connections among multiple representations, (c) self-monitoring or self-regulating their own cognitive processes, and (d) actively examining the task constraints that may limit possible solution strategies and solutions (Stein et al., 2009). There were scores of 2 given \((n = 8)\) and scores of 1 \((n = 2)\) also given for the potential of the task. These EMSs did not meet the criteria of students doing the mathematics and making connections to mathematical concepts.

Table 22

EMSs’ Scores on Task Analysis Guide

<table>
<thead>
<tr>
<th>EMS</th>
<th>Potential of the Task</th>
<th>Potential of the Task</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Researcher 1</td>
<td>Researcher 2</td>
</tr>
<tr>
<td>Anna</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Task 1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Task 2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Brooke</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Task 1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Task 2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Connie</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Task 1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Task 2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Denise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Task 1</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Task 2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Ellen</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Task 1</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Task 2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Fran</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Task 1</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Task 2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>*Gwen</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Task 1</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Task 2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Helen</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Task 1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Task 2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
If the potential of the task was scored a 1 or 2 on the TAG, the chance of the task being implemented with a high level of rigor was very slim. The importance of recognizing that task selection is the first step in effective student discourse, collaboration, and problem-solving opportunities while doing and learning the mathematics concepts and skills is critical.

**Instructional Quality Assessment**

The Instructional Quality Assessment (IQA, see Appendix G) instrument was used by the researchers to help gauge the level of rigor (scale of 1 to 4) of the EMSs’ implementation of two mathematics tasks. Table 24 shows the scores given by each researcher for each EMS’s implementation of the task. Although the majority (n = 25) of the potential of the task scores were coded as a 4, the score dropped for many of the EMSs when implementing the same task (see Table 24). The mean for Tasks 1 and 2 for Researcher 1 was 3.296. The mean for Tasks 1 and 2 for Researcher 2 was 3.25. As seen in Tables 22 and 24, there were only two EMSs, Gwen and Ingrid, who received a perfect score of 4, for both the potential of the task and the implementation of the task. The implementation of their tasks had characteristics that may require the students to do some of the following:

<table>
<thead>
<tr>
<th></th>
<th>Task 1</th>
<th>Task 2</th>
<th>Task 1</th>
<th>Task 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ingrid</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Jennifer</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Katie</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Lilly</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

* EMS that submitted lesson plan at the time of the observation.
Develop an explanation for why procedures work; identify patterns, form and justify generalizations based on the patterns; make conjectures and support conclusions with mathematical evidence; make explicit connections between representations, strategies, or mathematical concepts and procedures; or follow a prescribed procedure in order to explain/illustrate a mathematical concept, process, or relationship. (IQA, see Appendix G)

Table 23

EMSs’ Scores on Instructional Quality Assessment (IQA, AR 2)

<table>
<thead>
<tr>
<th>EMS</th>
<th>Task 1 AR 2 Implementation of the Task</th>
<th>Task 2 AR 2 Implementation of the Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anna</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Brooke</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Connie</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Denise</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Ellen</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Fran</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Gwen</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Helen</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Ingrid</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Jennifer</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>
Table 24

Task Analysis Guide and Instructional Quality Assessment Frequency Table for Task Potential and Implementation

<table>
<thead>
<tr>
<th>Rubric Score</th>
<th>Potential Frequency</th>
<th>Implementation Frequency</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>14</td>
<td>29</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>18</td>
<td>38</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td>12</td>
<td>25</td>
</tr>
<tr>
<td>Total</td>
<td>48</td>
<td>48</td>
<td>100</td>
</tr>
</tbody>
</table>

Summary of Task Potential and Implementation

Task set up is the task the teacher gives to the students to complete (Smith et al., 1996). The task level can range from a low-level task students complete through verbal cues from the teacher to a high-level task requiring students to apply prior knowledge and new knowledge to solve the task (Smith et al., 1996). Task implementation is how the student works on the task, based on how the teacher proposes the task to be completed per the teacher’s instruction (Smith et al., 1996). The data from the TAG and IQA from the EMSs in Phase 2 of the study (n = 12) showed that when an EMS selected a high-quality task, more times than not, they implemented it at the same level of the potential of the task or one level below. For example, Anna, Denise, Ellen, Fran, Gwen, Ingrid, Jennifer, Katie, and Lilly had at least one of their four tasks they selected scored as a level 4. When they implemented their task, it was implemented at a level 3
There was one instance when an EMS, Connie, selected a level 3 task, according to the criteria on the TAG rubric, but her implementation of the task was scored as a level 1 on the IQA by one of the researchers. This was a rarity and the only time that a task was initially scored a higher score for the potential of the task and then was scored more than 2 points lower for the task implementation. There were only two instances out of the 24 observations where an EMS selected a task scored as a level 2 task, and yet it was implemented at a higher level, a level 3. This did not happen often, but due to Anna and Lilly’s instructional decisions to allow students more problem-solving opportunities, more mathematical reasoning and discourse to occur, or more opportunities for conceptual understanding to take place, the implementation score was higher than the potential score.

If the potential of the task was scored a 1 or 2 on the TAG, there was a very slight chance the implementation of the task would be higher than a 1 or 2. Task selection is critical when thinking about the level of rigor to increase students’ mathematical discourse and provide multiple opportunities for problem solving and mathematical reasoning. Although 18 of the 48 lessons were implemented at a level 1 or 2, which is still only 37% of the total lessons reviewed. Half of the EMSs earned low-level scores for implementation of the tasks, which brings a question about the EMSs’ ability to implement a high-quality task with a high level of rigor.

**Statistical Analysis of TAG and LMT Using a Spearman Rank Correlation Coefficient**

The statistical package SPSS version 25.0 was used to run a Spearman Correlation between the LMT and TAG to determine if there was a relationship in the data, and the significance. The Spearman Rank Correlation Coefficient $R_s$ value measures the strength or relationship between the two data sets. The statistical significance was calculated on the probability ($p$) values. A small $p$ value is typically less than or equal to 0.05 and indicates strong evidence against the null hypothesis, so the null hypothesis is rejected. A $p$ value greater than
0.05 indicated a weak correlation against the null hypothesis, therefore you fail to reject the null hypothesis. The purpose of the analysis was to discover how the EMSs’ content knowledge for teaching mathematics correlated with their ability to select a high-quality task. The Spearman correlation determined that there was not a statistically significant correlation in the EMSs’ content knowledge for teaching mathematics and their ability to select a high-quality task because \( r_s \) is -.023 and \( p = .944 \). The EMSs had a mean score of .88 on the LMT, whereas the mean score for the TAG was 3.167. The TAG is rated on a maximum scale of 4.00, thus a 3.167 is a relatively high score. A high score on the TAG suggested the EMSs did a good job of selecting the mathematics tasks with a high level of rigor. The standard deviation for the TAG is .937 whereas the standard deviation for the LMT is 1.15.

Table 25

**SPSS Data**

<table>
<thead>
<tr>
<th></th>
<th>TAG</th>
<th>LMT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Mean</td>
<td>3.27</td>
<td>.88</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>.80</td>
<td>1.15</td>
</tr>
<tr>
<td>Range</td>
<td>2.50</td>
<td>3.395</td>
</tr>
<tr>
<td>Minimum</td>
<td>1.50</td>
<td>-1.12</td>
</tr>
<tr>
<td>Maximum</td>
<td>4.00</td>
<td>2.27</td>
</tr>
</tbody>
</table>

The correlation analysis of these instruments was not at a high enough level of statistical significance to claim that there was a strong correlation between the data from the TAG and LMT.

**Qualitative Data**

The qualitative data collected in Phase 2 of the study included journal entries written by the 12 participants in the subset sample, which included the written lesson plans that were
evaluated and scored by the two trained researchers using the TAG, the researchers’ running
records notes collected from two classroom observations, the scores of the task implementation
using the IQA rubric, and interviews conducted after the observations. What follows are
thematic descriptions gleaned from reading and coding the data (Gibbs, 2007; Glesne, 2011;
Saldana, 2013).

Journal Entries

The journal prompts for this study were: (1) Send 2 lesson plans (high-quality tasks) that
will be implemented in a classroom (see Appendix I); (2) Keep a 1-week daily journal with all
your activities (see Appendix J); (3) Select one of the tasks or lessons you chose to submit and
explain your rationale for selecting the task as a high-quality task and reflect on how you feel the
task was implemented (see Appendix K); and (4) What does it look like when a teacher is
successfully teaching mathematics (see Appendix L)?

Journal prompt 1: lesson plans. Twenty-four lessons were coded for this report. Table
28 shows the number of lessons from each grade level and the content domains addressed in the
lesson plans. Most of the lessons were in the upper elementary grades \( n = 16 \); there were only
8 in the early elementary grades. Katie, Lilly, Ingrid, Connie, Jennifer, Anna, Fran, Gwen, and
Denise all selected at least one high-quality task to implement in the higher grades (3 - 5). Five
of the EMSs (Helen, Katie, Ingrid, Gwen, and Denise) who elected to implement a task in the
higher elementary grades also had lower scores on the LMT mathematics content assessment.
The same five EMSs who scored the lowest on the LMT also felt very confident about their level
of comfort with coaching and teaching mathematics content to teachers as data from the CSI
showed.
Notice that most lessons taught content from the domains Numbers in Base-Ten \((n = 10)\) and Operations in Algebraic Thinking \((n = 7)\). The question raised from this data is whether teachers believed they understood the content they were teaching. If they were unaware of their own misconceptions about the content knowledge as they planned and implemented the tasks, this fact could explain the subsequent problems teachers and students have in mathematics learning. There are a total of 26 times a specific domain was mentioned in the lesson plans, but only 24 lessons that were implemented. The reason there are more domains than lessons is because some of the EMSs selected a lesson that may have covered more than one content domain.

Table 26

*Grade Levels and Domains Addressed in the Lesson Plans*

<table>
<thead>
<tr>
<th>Domains</th>
<th>Kindergarten</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
<th>Fourth</th>
<th>Fifth</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operations in Algebraic Thinking</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Numbers &amp; Operations in Base-Ten</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Geometry</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Numbers &amp; Operations-Fractions</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td></td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement &amp; Data</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A high-quality task includes such criteria as having students represent their thinking in multiples ways and providing problem-solving opportunities. Helen, Lilly, Ellen, Jennifer,
Anna, and Fran planned lessons \( (n = 6) \) that would employ concrete manipulatives, whereas Ingrid was the only one who planned lessons \( (n = 1) \) that included visual and pictorial models. Research states that a high-quality task should focus students’ attention on the use of procedures with connections where students can represent their thinking in “multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations) and are able to make connections among multiple representations” (Task Analysis Guide, Stein & Smith, 1998, p. 16). This thinking and reasoning “helps students develop meaning” (Task Analysis Guide, Stein & Smith, 1998, p. 16). Helen was the only EMS who stated that she used technology during the lessons. Some observations of EMSs’ lessons gave the EMSs high implementation scores due to unplanned activities the students were engaged in. Therefore, although the EMSs implemented problem-solving tasks, they scored low on the potential for a high-quality task, as the activities were not in their plans. A teacher can tell students about mathematics concepts, but research shows that lecturing alone does not develop deep conceptual understanding (Hiebert & Carpenter, 1992; Skemp, 1976, 1987, 1999). Rather, conceptual understanding is best developed when new concepts and procedures are introduced through the process of solving problems in which the new ideas and procedures are embedded (Lester & Charles, 2003). As only half of the 24 lesson plans analyzed mentioned intentionally embedding problem-solving opportunities for the students, the lessons did not align with descriptors of a high level of rigor from the TAG rubric nor did they align with best practices in mathematics education when teaching for procedural fluency from conceptual understanding is the goal.

Eight EMSs mentioned an exit ticket, but a specific instructional step planned for after the lesson was only mentioned twice. Research states that EMSs could be asked to design effective professional development for teachers, conduct classroom observations, assist teachers
in analyzing data and determining instructional next steps, plan and provide intervention for struggling teachers, and organize effective and research-based resources for teachers (Chval, Arbaugh, Lannin, van Garderen, Cummings, Estapa, & Huey, 2010; Campbell & Malkus, 2011). EMSs must know a multitude of practices and educational experiences to support their work with teachers with formative assessment and differentiated instruction for all learners (NCSM, 2008).

**Journal prompt 2: EMSs’ explanations for task selection.** Table 27 shows the common themes that emerged when analyzing and coding the journal entry, which asked for EMSs to discuss why they believed their tasks, were high quality ($n = 9$). Three EMS chose not to submit this journal entry.

Table 27

*Themes in High-Quality Task Selection Explanations*

<table>
<thead>
<tr>
<th>Nodes</th>
<th>Number of times Referenced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Engagement/Experiences in Small Groups/Discourse</td>
<td>49</td>
</tr>
<tr>
<td>Differentiated Instruction/Strategies/Task Flexibility</td>
<td>23</td>
</tr>
<tr>
<td>Productive Struggle/Teacher Involved in Student Learning/Questioning Students to Explain Their Thinking</td>
<td>19</td>
</tr>
<tr>
<td>Multiple Entry Points/Problem-Solving/Inquiry</td>
<td>16</td>
</tr>
<tr>
<td>Standards for Mathematical Practice</td>
<td>11</td>
</tr>
<tr>
<td>Common Core State Standards for Mathematics</td>
<td>10</td>
</tr>
<tr>
<td>Formative Assessment and Next Steps</td>
<td>8</td>
</tr>
<tr>
<td>Real World Connection/Research-Based</td>
<td>5</td>
</tr>
</tbody>
</table>
The most mentioned phrases or words were about student engagement, student discourse, or small group experiences ($n = 49$), followed by task differentiation and flexibility and using multiple strategies ($n = 23$). Most of the responses were limited in what actually constituted a high-quality task per the TAG and IQA rubrics. The explanations did not include phrases that related to students developing a deeper understanding of mathematical concepts and ideas, making connections among multiple representations to help develop meaning, and requiring the students to explore and understand mathematical relationships. Some examples of high-scoring responses are as follows.

Gwen stated she selected the task as high-quality because she “felt it was accessible to all students; there was a low floor and high ceiling which I felt was critical for this collaborative activity.” She also selected it because “it addressed the current standards outlined in the current curriculum unit.” Furthermore, Gwen stated that the high-quality task involved various levels of mathematics and included the opportunity for self-differentiation as students could decide which strategy they felt most comfortable using to solve the problem. Furthermore, the collaborative part was encouraging for some reluctant learners as they were able to learn from others within their group, yet share their understanding from their level.

She considered that the task was a high-quality and rich task because it was “standards based” and gave the students “opportunity to apply different problem-solving strategies.” It was “accessible to all students” and gave them opportunities for collaboration. It also included three Standards for Mathematical Practice.
Lilly’s responses were similar to those of Gwen in that she stated that she chose the task because she always starts by seeing if the task is “developmentally appropriate and if it fits well into the goals of the overall unit.” Furthermore, she made sure the task was “hands-on and engaging for the students.” Lilly tried to ensure that the task allowed students to “engage in as many of the math practice standards as possible” and that it had “an element of discovery.” She closed by saying that she “tries to create a task that will enable learning to occur rather than teaching.”

When Ingrid was asked her reason for choosing the task as high-quality she affirmed that (a) it aligned with the content standard for the specific grade, (b) explored fractional parts of a whole fraction notation and comparison of fractions with unlike denominators, (c) had a problem-solving context, (d) students had to make sense of the question that is to be answered, and (e) they had to design a model based on the information. She further explained that the task was “open to a variety of strategies in order to solve it” while also anticipating that the students would engage in mathematical discourse while solving the task collaboratively. She also added that the task offered multiple mathematical concepts while solving it. The task lent itself to being a high-quality mathematical task because it allowed the students to apply the Eight Standards for Mathematical Practice for problem solving. They had to persevere to solve the problem while also drawing appropriate models and diagrams that connected to the problem. In their mathematical discourse, the students had to construct viable arguments while also attending to precision in the justification of their reasoning.

Jennifer said she selected her high-quality task “after a discussion with the classroom teacher.” They both felt that the “use of manipulatives would be beneficial to students understanding of the concept of multiplication.” Furthermore, the task was high quality because
it allowed students to use manipulatives to work on multiplication and they were “able to gain a better understanding that multiplication is a relationship between the number of groups and the number of objects in each group.” Jennifer also expressed that the task was considered high quality because it “allows students to create their own methods of grouping and explaining their thinking.” It also allowed students to “explain their reasoning using drawings and numbers…and allowed students to write a word problem to correspond to their equation to give them further opportunities to explain their reasoning.”

Some responses did not elicit high scores on the TAG or IQA rubrics because they did not make connections to mathematics relationships, have students explore the mathematics through problem solving and mathematical reasoning, or give students opportunities to learn procedural fluency from conceptual understanding. Denise stated, “I tried to make it align with what teachers were currently teaching. I wanted a real world connection and considered areas of weakness from students’ test scores. My gut told me it had the right criteria to be high-quality.” Brooke expressed that she selected the task as high quality because it was “a research-based strategy to help improve number sense.” Anna’s responses were similar to Brooke’s in that she expressed that she selected the task as high quality because the activity was an “important way for us to help our students develop number sense.” She considered the task to be a high-quality task because it “helps students deepen their conceptual understanding of mathematics and helps them develop some of the Standards for Mathematical Practice.” Katie stated that she selected the task because it was “from a curriculum that was research-based, aligned to the CCSSM, and it was adapted to meet the needs of her small group of students.” Additional criteria that she considered made it high-quality was that it was “based on mathematical practices, it supports the standards, and relates directly to enhance the needs of the students based on data and
observation.” Although these reasons were all valid, they were insufficient because they did not align to what a high-quality task should entail (Task Analysis Guide, Stein & Smith, 1998, p. 16).

**Journal prompt 3: EMSs’ description of successful mathematics teaching.** EMSs were asked to explain what it looks like when a teacher is successfully teaching mathematics and what the classroom would look and sound like. Although they were able to describe their expectations of a successful lesson, their own lessons that were observed by the researchers did not display the practices they described in the journals. Denise indicated that she would describe a teacher who is successfully teaching mathematics by saying, “I want the students to become comfortable in their knowledge and use of the math.” Lilly described a successful mathematics teacher as “someone in the role of a facilitator of learning. She has collaborative groups and students are sharing multiple strategies. The emphasis is on the why, not the how of mathematics.” She also stated that the Standards for Mathematical Practice would also be evident in the mathematics lesson. The classroom culture would have the “teacher questioning and scaffolding the students and continually using formative assessment and adjusting the lesson based on the assessment.” She also expressed that the students would be “using hands-on manipulatives to solve a challenging task while collaborating with one another and sharing ideas.” These practices were not observed in their own lessons, although other important practices were. Lilly opted not to answer the question about what it would look like if she taught mathematics today.

The following are examples from EMSs whose responses aligned the most with key words and phrases described in the mathematics with the connections and doing mathematics sections of the TAG when describing how they teach mathematics if they were in the classroom.
Fran described a successful mathematics teacher as someone who uses a variety of formative assessment on a regular basis. She emphasized the need to teach the standards and to understand the progression of their learning. She also thought it was important to use math tasks and ask students a lot of questions to push their thinking; “there is little telling and lots of learning in a direction through questioning.” Fran’s data from the observations showed that she did ask her students many questions during the lesson, which supported her responses about the importance of questioning. An effective mathematics lesson was described by Fran as beginning with a number talk, a mini-lesson, or some sort of a warm up to get the students to start thinking mathematically and then work on math for about 30-40 minutes. The lesson would close with the teacher bringing everything back together and discussing the mathematics the students did and how they met their math goal for the day. As the researchers also observed, Fran expressed that she believed that the teacher should be walking around monitoring the students’ learning while being involved in math conversation by asking questions and clearing up any misconceptions during the task time. When Fran described her mathematics classroom she stated that she was fairly enthusiastic and energetic while she’s teaching. She asked open-ended questions that led students to discoveries. This was also observed when she was asking the students questions about the text she was reading to them, in which they were doing counting activities. She hoped that her enthusiasm wore off on her students and they enjoyed learning math as much as she did.

When Katie described a successful mathematics teacher she stated that knowledge of the progression of the standards was important. She said the teacher would plan intentional lessons and be aware of the mathematical practices with purposeful questioning. The lessons would lead the students to discovery. She also stressed the importance of the teacher using formative
assessments to drive their instruction and scaffold learners in small group instruction while also using summative assessments to determine next instructional steps.

Katie and Fran’s journal entries offered well-reasoned and perceptive responses to the prompts and aligned very closely with AMTE (2013) qualities of a certified elementary mathematics specialist; their responses on the CKS, CSI, and their interview questions also aligned closely to their multiple journal entries.

**Interviews**

The semistructured interviews between the EMSs and the researchers were divided into six main questions: (1) What are the necessary knowledge and skills for an EMS (i.e. content, pedagogy, and leadership)? (2) What is the relationship between content knowledge and pedagogy? (3) What was your preparation to be an EMS? (4) How has your preparation specifically supported you in assisting your teachers? (5) How was your time utilized in terms of professional development with your whole school, at grade level, and with individual coaching with a teacher? and (6) What do you believe is good mathematics instruction and how do you specifically support this expectation?

The EMSs used several similar words and phrases more than 20 times throughout the EMSs’ interviews, including coaching experiences ($n = 25$), leadership knowledge and skills ($n = 28$), networking with others ($n = 25$), professional development opportunities for professional growth ($n = 26$), the importance of relationships with other educators or the students ($n = 29$), supporting other teachers ($n = 23$), and the importance of having teaching experiences ($n = 27$). Most of the nodes listed were centered on classroom experiences and the importance of personal relationship building.
When the EMSs mentioned coaching experiences, Katie referred to “continually spending time receiving professional development to increase my abilities and learn new methods to reach all learners.” Fran explained some of her coaching experiences as providing planning sessions with a grade-level team or individual meetings to discuss student work. Coaching experiences were mentioned 25 times throughout the 12 interviews.

Coaching knowledge was mentioned 18 times when the EMSs participated in the interview portion of Phase 2 of the study. Brooke stated that she “feels that her instruction and coaching have drastically improved due to workshops, training and direct coaching.” Fran said that a big part of being a leader was making sure you had a relationship with the “boots on the ground so they are open to having me in their classrooms and learning from me at the same time.” When the EMSs mentioned anything that was coded under leadership knowledge (n = 28) and skills, they stated things such as having good, professional relationships with the teachers, the administrators, and all other school faculty and staff. Other practices the EMSs named under the leadership knowledge theme were about the EMSs receiving professional development about coaching and mathematics instruction. Although coaching knowledge is important, equally as important to the EMSs is having pedagogical knowledge, which was coded 12 different times during the EMS’ interviews. “Knowing how to teach the math is as equally important as knowing the content standards,” mentioned 7 times, or practice standards, student experiences, and students showing mathematical reasoning (n = 14). Students collaborating (n = 8), having mathematical discourse (n = 1), using manipulatives (n = 7), and having opportunities for conceptual understanding (n = 7) are all key components in students having procedural fluency (n = 1) from conceptual understanding learning. The EMSs recognized that they should be differentiating their instruction (n = 8) to meet the needs of all the learners in the classroom. The
EMS and teacher should be the facilitators of the learning \((n = 4)\) and have flexibility \((n = 8)\) in their teaching and pedagogy. Like teachers, EMSs include as important that they should focus on formative assessment and next instructional steps \((n = 4)\). When students were working on high-quality tasks \((n = 3)\), students, teachers, and EMSs should be involved in the learning \((n = 1)\), focusing on problem solving \((n = 2)\), and there should be a level of productive struggle \((n = 5)\). EMSs stated that students should be asked a variety of questions \((n = 1)\) while working on mathematics tasks with real-world connections \((n = 6)\).

When thinking about themselves and classroom teachers, only four EMSs stated it was important to plan lessons with teachers, be organized and prepared for the lessons while focusing on the relationship between the EMSs and the teachers and between the EMSs and the rest of the school personnel. The importance of relationships was one most often mentioned by the EMSs \((n = 29)\). Connecting with the theme of relationships, networking with others was mentioned 25 times, and supporting teachers was mentioned 23 times. When the EMSs spoke about professional development opportunities for themselves as well as the teachers, it was mentioned 26 times while EMSs also thought that teaching experience was \textit{fairly important} \((n = 27)\). When relationships are in place with the students and the teachers, a few EMSs hoped that there would be a positive shift in student and teacher learning and thinking; it was mentioned six times. EMSs only mentioned research-based texts or resources four times when they were speaking about how they assisted teachers when they asked for help or when they had been assigned to help specific teachers. Only two EMSs mentioned having a special education background that helped them be more prepared for their current position.

Fran expressed that some of the necessary knowledge and skills for an EMS were a basic knowledge of the standards progressions and being able to connect with people in a leadership
role. She also felt it was important to have a connection with people and be their cheerleader. She further described the importance of the relationship between content knowledge and pedagogy by stating the “importance of Math Ed research” and the pedagogy of how kids acquired math knowledge presented by the teachers. When considering Fran’s preparation in becoming an EMS and how her experiences prepared her in supporting teachers with mathematics content and their instructional practice, she explained that she had been a mathematics facilitator with multiple grades. She accrued a lot of hours working with teachers across the district and was able to see the big picture of what was happening with mathematics instruction in the whole school. She also stated that her undergraduate degree in special education and graduate degree in gifted education also gave her credibility with teachers due to her experiences and education with a wide spectrum of learners. This knowledge and education supported her to “lead the teachers in a direction that spoke to the needs of each group of kids.”

In her first 3 years of coaching, she did a lot of work with Gifted Education and met with the enrichment teacher from the school every other Friday for about an hour. Her collaboration with the enrichment teacher opened the door for teachers to join their collaboration. Fran’s previous experiences as a special education teacher also opened the doors for a relationship with the special education teachers. She was invited to assist by looking at Individualized Education Plan (IEP) goals, how the goals would be obtained, and the scaffolds and manipulatives that would be used to meet the goals. These opportunities gave Fran the foundation for building stronger relationships with the teachers, although building relationships was an area of weakness for her based on the data from the CSI and CKS.

Fran expressed the importance of good mathematics instruction, and she stressed the importance of “pre-assessing students, teaching the standards, using games and tasks,” to teach
mathematics. Fran thought formative assessment was a good way to “get a pulse of what the students know” and to see their growth. Supported by her actions during the observations, Fran liked students to work independently as well as with partners or small groups to have opportunities for discourse.

When examining and analyzing the qualitative data in order to support the quantitative findings, there were many connections made that showed relationships between the data sets. Below, in the mixed methods data findings section, the connections made between the data from several quantitative and qualitative instruments to support the findings are elaborated.

Mixed Methods Data Findings

The EMSs’ leadership knowledge and skills were assessed using the Coaching Skill Inventory, and their practices about teaching and coaching mathematics were assessed using the data from both the CSI and CKS. Their instructional practices were assessed using quantitative instruments and observations, journal entries, and their answers to specific interview questions. What follows is a description of six of the participants’ stories of their work as EMSs.

Fran. In the 8 years Fran taught, she coached for 3 years. She had a Master’s degree in gifted education and was certified to teach at the elementary and middle school levels. Fran took 2 college mathematics courses during her undergraduate program, the highest level being college statistics, but did not take any mathematics courses during her graduate degree. Fran participated in over 100 hours of mathematics professional development in the 12 months prior to the study, and at the end of the school year, she completed a coaching cycle with 32 teachers in her school.

Fran’s initial responses from the CKS data indicate strong coaching practices based on what the teachers’ needs were, used student evidence to lead her discussion with the classroom
teachers, while she also focused on mathematics content knowledge and having hard conversations about clearing up possible misconceptions. Fran’s responses on the CKS showed that she seeks to help teachers identify their needs for mathematics content and instruction, coupled with helping them understand her leadership role. When she discussed these points in her journal entries and interview, she noted that these practices were critical in relationship building. She stated that using state assessment data and student work samples in her coaching conversations with teachers was useful for planning the next steps for the classroom instruction. Based on her responses, she knew that her job was to coach teachers in mathematics content and pedagogy. She also believed that although she did model or co-teach a lesson with a classroom teacher, her role was not to tutor the students. She also recognized that just as students have different learning styles, teachers did as well. She presumed the teachers’ learning styles and worked with them and coached them in the most effective ways for the teachers to understand and adjust their practices for the better.

Fran’s responses on the CSI showed that she felt most effective when she modeled instruction for teachers and was able to coach them by deepening their understanding of mathematical content. Fran felt confident in her mathematical knowledge and reasoning in the grades she was responsible for teaching and coaching. She also felt effective when coaching teachers about how to incorporate problem solving into their lessons or incorporating mathematics conceptually, through inquiry or discovery-based learning and allowing students multiple opportunities to be engaged in mathematics sense making. Fran understood that both teachers and students needed to have multiple opportunities for exploration and mathematics reasoning when they attacked new concepts in mathematics content or pedagogy. Fran employed effective questioning strategies during her lesson implementations and encouraged the
students through positive reinforcements. While walking around the room and monitoring the students as they were working independently, and then when they worked with a partner, Fran asked them assessing and advancing questions that gauged their understanding and mathematical reasoning. She confirmed the students’ reasoning with verbal affirmations and words of positive reinforcement. It was obvious that she believed in creating safe learning environments because the students were not afraid to answer questions when being called upon. Fran’s classroom management was very good in that she was able to allow the students to have mathematical discourse with one another while still having the class noise and off-task behaviors under control. Fran’s responses aligned with the AMTE (2013) characteristics of an effective EMS, specifically in pedagogical knowledge for teaching mathematics as well as leadership knowledge and skills that are needed to be an effective EMS. Fran’s experiences and education contributed to her success as a coach.

Fran scored a 4 and 3 on the observations of her lessons on the IQA, respectively, and a scaled score of 2.27475 on the LMT, which was one of the highest scores overall. Fran’s high scores on the LMT complimented her data from the mathematics content coaching heading on the CSI. The combination of Fran’s high number of specific mathematics professional development hours in the last 12 months, paired with her high content knowledge exhibited in the LMT and the CSI content coaching section supported her contention that she was effective in mathematics content coaching. The researcher’s observations supported these findings by seeing her range of in-depth content knowledge, the pedagogical knowledge she exhibited when implementing the tasks, and the specific instructions she gave the students to allow them opportunities to reason mathematically while providing them with concrete manipulatives as needed. Fran surprisingly did not score well on the section of the CSI regarding helping teachers
create an environment of open discussion and being able to give constructive criticism to the teachers. Before, during, and after the lessons, it was evident that Fran did not have the best communication or working relationship with the classroom teachers. She came in the rooms, implemented great lessons with the students, but did not say anything to the teacher during the lessons, nor did she speak with them in depth after the lessons were over. The researchers did not hear any questions, comments, or ideas for any next steps between Fran and the classroom teachers. Her relationship with the teachers lacked substance. Her leadership style in the school resulted in a lot of telling the teachers what to do and how to do it and did not foster building relationships with the teachers. This observation supported and explained her poor scores relative to her confidence in helping teachers with personal feedback and giving them constructive criticism. Although, she did recognize that an EMS “has to have the skills to connect with people and to lead.” She stated that she liked to begin with telling a story to “get people on board with your leadership,” not in terms of “tricking them” but more as a form of engagement. However, her teaching showed she had created an effective working relationship with the students; the discussions and mathematical discourse were lively, but on point. All students were engaged. Students worked independently, then worked with their groups. They had assigned roles in their groups, and completed their tasks while Fran walked around, monitored the work and gave students positive feedback. Overall, Fran’s exceptional content and pedagogical knowledge enabled her to use active teaching in the classroom for effective coaching, although her relationship with teachers outside the classroom environment was a more difficult area for her to excel at that time.

Katie. In the 12 years that Katie taught, she coached for 2 years. She had a master’s degree in elementary education and was also National Board Certified in Early Childhood
Generalist. Katie was certified to teach at the elementary level only. Katie took one college mathematics course during her the graduate program, but did not indicate which course. She participated in approximately 8 hours of mathematics professional development in the last 12 months, previous to the study, and coached 34 teachers at the time she completed the survey.

Like Fran, Katie also had very high scores on all of the Phase 1 instruments. She selected two; level three, high-quality tasks and implemented them effectively. Katie believed EMSs should use state mathematics assessment data to help develop a coaching plan with teachers while also using students’ formative assessment data to guide coaching conversations and determine next instructional steps with the teacher. Katie believed coaching conversations with her teachers should be grounded in mathematics content while also providing teachers with feedback. In her interview, Katie spoke about how relationship building was very important in helping the teachers trust her enough to allow her to come into their rooms and not feel like she was there to tell them what to do. A positive conversation with the teachers about how the students were doing and how to adjust instruction was how she liked to approach things, as opposed to telling teachers what to do differently. Part of her role as an EMS, she provided the teachers with an understanding of how the mathematics they taught supported learning beyond the grade level they taught. She also stated that part of her role as an EMS was to help teachers plan their lessons and make them feel comfortable enough to ask for help with things they were not as confident in teaching. Katie also thought it was important to encourage teachers to set personal improvement goals for mathematics instruction, while keeping in mind the school’s ultimate mathematics goal. She took precautions to ensure that the lessons that she modeled did not inadvertently send a message that she was the expert and the teacher was not. Many things mentioned in her interview reflected the importance of relationship building with her teachers.
During the observations of her lessons, Katie showed confidence in working with teachers; she noted she was a mathematics trainer for a state initiative and was used to teaching adult learners. She also taught multiple grades while she was in the classroom and earned respect from many of the teachers from working with them on the same grade level or teaching the same content at one time. Katie’s depth of content knowledge resulting from her teaching experiences in multiple grades, her professional development hours in mathematics, as well as her leadership opportunities and training mathematics teachers on behalf of the state department, were some of the factors that allowed her to be more effective as an EMS than someone without those experiences. She elaborated on her past experiences in her interviews as well as reporting that same information on the CSI and in the journal entries.

Katie shared that her days were filled with many duties other than mathematics coaching that were assigned to her by her administrator. She followed up by stating that any extra time she had was dedicated to coaching individual teachers in their classrooms or having informal conversations in the hallways about questions they had for her at the spur of the moment regarding content and pedagogy. Although the researchers did not observe Katie interact with teachers in a coaching situation, the fact that she had frequent informal conversations with teachers suggested that she had a positive relationship with many of the teachers. Katie felt confident that she helped teachers incorporate inquiry-or discovery-based learning into their teaching while still focusing on student engagement and sense making. She herself had the ability to do this, as observed in her lessons. She reported that she felt confident coaching teachers on how to establish a safe classroom environment so the students could have mathematical discourse with one another and learn cooperatively.
Also like Fran, Katie had both undergraduate and master’s degrees in Elementary Education, is National Board Certified, and taught for 12 consecutive years. She worked in two different schools and had coached for only 2 years. Katie was hired specifically as an instructional coach to work with teachers. Despite being hired specifically to coach classroom teachers, in one of her journal entries Katie mentioned that much of her time was given to assisting with computer-based testing in the computer lab, conducting Dyslexia screenings, creating performance assessments for teachers, helping with reading screening tests, and meeting with different teams to discuss some shifts in classroom instruction. She conducted two coaching sessions per teacher. Katie reported coaching 24 teachers at her school over the current school year. On top of all the other responsibilities, she still managed to coach multiple teachers throughout the school year, but her efforts and time for them was limited.

In her interview, Katie stated that she was trained by the Alabama Math, Science, and Technology Initiative (AMSTI) in mathematics and science in multiple grades and was also a trainer for the state initiative. She felt as if the opportunities helped prepare her for the EMS position. Katie had also attended training for the Mobile Mathematics Initiative. In the last 12 months, Katie attended 8 hours of mathematics specific professional development, yet in her interview, she mentioned that she did not spend a lot of time delivering professional development to the staff, but rather spent any extra time in the classrooms with teachers.

Jennifer. In the 25 years that Jennifer taught, she only taught mathematics for 8 years. Jennifer was a relatively new coach, having only been coaching for 2 years. She had a master’s in Elementary Education and was certified to teach at the elementary and middle school levels. While studying for her master’s degree, Jennifer had 1 college mathematics course, which was
Jennifer participated in only 6 hours of specific mathematics professional development in the last 12 months and only coached 11 teachers in her school.

Jennifer’s data from the CKS was comparable to Fran and Katie’s, but Jennifer was not as confident in some of the things the others were comfortable doing in their EMS position. Jennifer’s responses indicate that she thought an effective mathematics coach should provide teachers with an understanding of how the mathematics they taught supported learning beyond the grade level they taught. Jennifer believed that an effective EMS coached teachers on teacher-identified needs as well as those noted by the coach or administrators. She also believed that teachers could not learn new mathematics and that powerful and effective coaching could not change teachers’ basic thematic intelligence. Jennifer contradicted herself, however, when she stated that an EMS could influence teacher traits such as intelligence, and their intelligence could indeed be altered. Jennifer also reported that she did not think that once a teacher knew about a research-based strategy for improving student learning, he or she would begin using the strategy. Some of the responses from Jennifer suggest that she seemed to be set in her ways of teaching and coaching and did not think that teachers had flexibility in learning new things. She also reported that she did not encourage teachers to include summaries of what students learned in each of the lessons they taught. Researchers did not observe Jennifer interact with any classroom teachers. She had her own classroom as an EMS and the students came to her for the small group lessons. The researchers noted in both of her observed lessons that she did not wrap up at the end of the lesson to improve students’ sense making; perhaps because she did not see the value in it when she worked with teachers or students in her pullout classroom.

Jennifer did not feel very effective in the coach/teacher relationship nor in mathematics content. She did however feel extremely effective when coaching teachers in general pedagogy
and incorporating investigative inquiry-based or discovery-based mathematics learning into their lessons. On six of the eight item prompts about coaching teachers in general and specific mathematics pedagogy, she reported having a lot of confidence in her ability to help teachers. Again, her rapport with teachers was not observed, as she did not have any interactions with the classroom teacher other than picking up the students and then sending them back to their room. Jennifer felt confident in coaching teachers about successful classroom management, which is complimentary to pedagogical practices. Jennifer had wonderful classroom management during her small group lessons and was able to keep the students engaged and on-task during the entire lesson.

Jennifer’s education ranged from having received undergraduate and graduate degrees in elementary education, and she was certified to also teach middle-level education. The CSI did not ask the EMSs to elaborate on the middle school degrees or certifications, so it is unclear if Jennifer’s middle-level education certification was in a specific content area. She had many years of teaching experience, 25 to be exact. She taught mathematics for 8 of those years and had been a coach for 2 years, yet she still felt uncomfortable with certain responsibilities in her current role as an EMS. Jennifer stated that her current assignment had multiple responsibilities that included coaching classroom teachers, but not working as a classroom teacher. In her 1-week at a glance journal entry that she submitted (see Appendix J), Jennifer stated that in that week she proctored science, reading, and mathematics assessments, went on a fieldtrip, prepared for the Robotics competition because she was also the team’s sponsor, and did some side-by-side science coaching with some teachers. She typically only had 2 coaching cycles per teacher, per school year and only coached 11 teachers during the current school year. In comparison to the numbers from Fran and Katie, this caseload seemed very low. Jennifer participated in only 6
hours of mathematics professional development in the last 12 months compared to Fran’s 100+ and Katie’s 8 hours, but all of her extra experiences as a coach for other initiatives such as AMSTI also gave her an advantage. Because she was a teacher trainer for a state initiative, she had prior experience in working with adult learners and knew what to expect when she became a coach; in addition to her high level of specific content and pedagogical knowledge, that was a prerequisite for her to be considered to be a trainer. Despite Jennifer’s struggles with many responsibilities in her current role, her score on the LMT was one of the highest of all EMSs, with a scaled score of 1.75572. The high score on the LMT suggested that she understood the mathematics content but possibly had difficulty with changes in specific mathematics pedagogy with the new ways and strategies of teaching mathematics. The lessons observed did not suggest that she had trouble with the specific strategy she showed the students, but perhaps her showing the students the strategy in the form of a procedure supported the fact that she was not able to allow the strategy to be seen as a strategy and have flexibility in her thinking to not teach it as a procedure. Jennifer also was nervous and lacked confidence during the observations and in her responses during the interview. She struggled with understanding the questions asked as well as understanding the specific education vocabulary, such as content and pedagogical knowledge, used in asking the question.

Jennifer communicated that the necessary knowledge and skills for an EMS began with being aware of the progression of standards and beginning where the students are and taking them where they need to go. During her interview, Jennifer discussed the relationship between content knowledge and pedagogy as, “My mind doesn't work quickly. I mean I have to really… I'm more of a deep thinker. I have to think about things. Let’s go to the next one.” When she spoke about her preparation before becoming an EMS and how she feels it has assisted her in
supporting teachers with mathematics content and their instructional practices, Jennifer affirmed that

it's getting worse. I've not been prepared like I want to be prepared, to my knowledge. My preparation is on the job preparation; I guess you could say. I'm learning from AMSTI state specialists as we go. I do quite a bit of research on the Internet. I study the standards; I go to as much content development as I can. My development is pretty much on the job training. Also interacting with teachers, gleaning from them as much as I can and researching and getting the most out of what I can to help them.

Jennifer did not have a lot of confidence in teaching specific mathematics content despite her high, scaled score on the LMT. She said, “I don't have a strong math content, but I'm learning to love it and to really want to help to develop these children.” Despite her will to learn more each day, she still seemed to be unprepared for her position as an EMS.

**Helen.** In the 13 years that Helen taught, she coached for 4 years. Helen had two undergraduate degrees one in Elementary Education and another in a different field, which was not indicated. She also received a master’s degree in elementary education. During her undergraduate degree, Helen took one calculus course, but she did not take any mathematics courses during her graduate degree coursework. Helen participated in 40 hours of mathematics professional development in the last 12 months and coached 24 teachers in her school at the time she completed the survey at the end of the school year.

Helen received low scores on her lessons, 1s and 2s, for the TAG and IQA. In her interview, Helen defined good mathematics instruction as moving away from a “sit and get and rote memorization.” Helen described good mathematics instruction by mentioning the use of manipulatives and moving from the concrete to the abstract and facilitating the learning process. She also pointed out the importance of productive struggle and how it helps the teacher determine what the students understand about the mathematics. Her response in the interview,
however, contradicts the notes taken by the researchers during her observations. The observations showed that there was more telling of what the students needed to do and how to do it as opposed to student exploration and mathematical reasoning taking place. Although technology was used to get the lesson started, and the students created a foldable to help with the learning of the geometric shapes and classifications, there was little mathematical content learned in the lesson. There were no embedded problem-solving opportunities for the students to grapple with during their learning. Despite stating that students needed to have productive struggle and less time to sit and get, the notes from the researchers contradicted what she stated needed to happen during mathematics instruction.

Helen explained the relationship between mathematics content knowledge and pedagogy by saying that pedagogy is the way we teach and the theory behind it. She also thought, “it comes naturally to natural teachers.” She scored low scores on the observations, but during her interview, she gave specific characteristics of good teaching that were evident in the IQA and TAG rubrics and she defined what she thought were pedagogy and content knowledge for teaching mathematics. Instead of answering the interview question and discussing the relationship between content knowledge and pedagogy, Helen tried to define both and what she thought they looked like and meant. She was not able to connect the two or speak about their relationship. Furthermore, Helen did explain what she thought were the necessary knowledge and skills of an EMS. Helen explained that an EMS needs:

a basic knowledge of [pauses] those core components of math. What it means to teach a kid to add, subtract, multiply and divide, fractions, those, those kinds of things. Not just the [pauses] basic knowledge of it. Because I can have knowledge in mathematics, but [pauses] not have the knowledge of how to teach it. I think there has to be a wide range of knowledge of strategies umm... on top of just the basic knowledge. You've got to have some strategies and you have to have
practiced those as a mathematics specialist. Umm…you can't go in not knowing what to expect when kids give you an answer. So, I think umm… and you definitely have to be flexible and willing to, that when things don't go well, be able to redirect teachers and students.

As seen in her response, Helen had many moments of hesitation during her reply. When Helen finished responding, the researcher stated, “If you feel as if you already answered these, then you can just say so. What type of knowledge do you need? What specific instructional strategies do you need? What specific leadership skills do you need?” This was asked because the researcher did not feel as if the question had been fully answered and wanted to ensure that Helen had the opportunity to answer each part of the question. Helen then continued with the following response (with many pauses):

Umm… I guess I would add to that familiarity with our state standards, whatever they are at the current time. Umm… Just a familiarity with what each grade level is responsible for so that we know where content will be taught. Umm… It's very difficult to be the math specialist for the building and not know scope and sequence of, of mathematics. Umm… As far as strategies, [sigh] they are wide and varying. I think that the more practice and the more that, that coach can, can learn new ideas, workshops, or from watching other teachers, just from talking to colleagues. Umm…you never know what strategy's going to be the one that you pull out for that one set of kids that's really going to get them where they need to be. Umm…and leadership skills, you've definitely got to be organized. I don't know if I necessarily fit that, but you've got to be organized and you've got to umm…be willing to manage that time and give the time necessary.

Although there were many key words and phrases Helen stated that aligned with the research from AMTE (2013) for the preparation of an EMS, they were not seen in her lesson plans or in the observations when she had to select a high-quality task and implement it with a high level of rigor. In her interview, Helen hesitated on multiple occasions, repeated words, and was not confident in her responses.

On the CSI, Helen denoted that she did not have confidence in her organizational skills but felt effective in all areas of coaching including content knowledge, general pedagogy,
leadership knowledge and skills, coaching knowledge, coach and teacher relationships, and mathematics specific pedagogy. During her observations, Helen did not portray high-quality pedagogical instruction. She taught by doing a lot of telling and the students simply recorded the information told to them by the teacher or another student that answered aloud. Helen’s data from the CKS showed that she was both knowledgeable and skilled in mathematics content and pedagogy, although her actual practice was very far from what her knowledge base showed on all the instruments. Helen had some of the lowest scores on the TAG and IQA (all 1’s and 2’s) for her lesson plans recorded by the TAG and observations recorded by the IQA rubric. Although her data on the CKS and CSI suggested that she was confident in her coaching knowledge and pedagogy, and the LMT scaled score of 0.154447 showed her confidence in her mathematics content knowledge, she was not able to answer some of the interview questions without grappling over the questions for an extended period of time with many hesitations. The cumulative data suggest that for her, there was not a strong correlation between mathematics content knowledge and her ability to articulate her pedagogy.

**Connie.** In the 6 years that Connie has taught, the current school year was her first year to serve as a coach. During her undergraduate degree in elementary education, Connie took three college mathematics courses, the highest level being algebra. Connie did not take any mathematics content courses during her graduate degree coursework. Connie did not participate in any mathematics professional development in the last 12 months and did not coach any teachers during the school year.

Connie scored a 3 and a 1 on the first IQA for task implementation and a 3 and 2, for the second observation of implementing a high-quality task. She stated that she chose the task as a high-quality task because it was “selected from a reputable website for high-
quality mathematical tasks that are organized by specific grade levels.” During the implementation portion of the task, the researchers gave her a lower score than they did for the potential of the task. This decline in the score was due to the task not being implemented with a high level of rigor. Based on characteristics from the IQA rubric, “There was little ambiguity about what needed to be done and how to do it”; the students did not “make connections or attach meaning to what they were solving”; and the focus of the selected task when implemented was on “producing correct answers rather than developing mathematical understanding” (IQA, Boston, 2012).

During Connie’s interview, she described good mathematics instruction as providing students with differentiated instruction and a variety of strategies and letting them choose the strategy that worked best for them. She also stated that there should be instruction in the form of a whole group, small groups, partners, and independent work. Her statements did not align with what was observed by the researchers during the implementation of the tasks. For one observation, a small group of students were asked to use two numbers from the worksheet to add together to get the sum of 1,000. Some students did not try and simply sat there disrupting the others’ learning. One student did not use the numbers provided on the sheet and came up with two addends that were much easier to work with than the ones on the worksheet. During the small group lesson, two of the four students were arguing with one another, distracting the others, and did not want to do anything but sit at the round table. Connie attempted to help some of the students with a strategy, but they were not able to follow along and were confused. The standard chosen to focus on was for the grade level below the grade in which the students were in that school year. Before the start of the lesson, Connie explained to the researchers that this
group was a very low intervention group that she was going to be pulling into her room for a small group lesson. While debriefing the small group lesson with the students, Connie asked the students a few questions about the numbers provided on the sheet and then proceeded to solve the problems using the U.S. Standard Algorithm as a memorized procedure to solve the problem and get a quick answer. Connie was not able to implement a Level-3 task as a high level due to her observed lack of conceptual understanding of the mathematics content and the lack of being able to use more concrete or visual models to assist the students in concrete and conceptual understanding. The task was not on the students’ level. After implementation and observation of both lessons, Connie was asked during the interview to explain what she thought was the relationship between mathematics content and pedagogy. She said the following; “I’m going to have to think about that for a second. Can you ask it again? I think content knowledge; I guess is more of the right or the wrong. This is factual, whereas pedagogy is more of how you teach it and your beliefs about how you teach math.” Instead of discussing the relationship between the content knowledge and pedagogy, Connie tried to define both terms and what she thought they looked like in the classroom. Furthermore when asked to explain the necessary knowledge and skills of an EMS, Connie stated, “Knowing the content, knowing the standards [pauses], umm… understanding the standards and umm… I guess having a lot of strategies on how to teach that.” She continued to explain the necessary knowledge and skills of an EMS:

I guess, convey that to teachers and being umm… I don't know, I guess like when I think of an effective coach, I think of somebody who is [pauses] umm… I guess, creates a safe environment for teachers, not intimidating, [pauses] umm… welcoming, like you can just go in their room and ask for help when you need it. [pauses] umm.. If they don't know the answer, they'll get the answer type of thing. [pauses]
As with Helen’s responses, Connie also had many pauses and moments of uncertainty when she responded to the question asked about the knowledge and skills for an EMS. What Connie expressed in her interview about what she thought good mathematics instruction was, the data gathered from the instruments versus the data from the interview questions did not match with what the researchers’ noted during the observations. In fact, the researchers noted that she did exhibit knowledge of good mathematics instruction when she was in the classroom setting, although she struggled with answering many of the interview questions and seemed very nervous. She had multiple pauses in her responses, which in turn suggested that she perhaps did not understand the specific mathematics content or coaching vocabulary used in the interview questions.

**Ingrid.** In Ingrid’s 11 years of teaching, she taught mathematics for 8 years and coached for 3 years. Ingrid took two college mathematics courses, the highest level being pre-calculus, while she earned her Bachelor’s in Elementary Education. Ingrid participated in 18 hours of specific mathematics professional development in the last 12 months and only coached four teachers in her school.

Ingrid scored all 4s for the potential and the implementation of both tasks. She began the first lesson by telling the students that they had been working on multiplication and division and that they were going to do an activity. Ingrid prompted the students to think about what they needed to do when they solved a word problem. They responded with, “We need to show a model, read for understanding, use a strategy, and show our work,” to name a few. After reading the word problem together, she asked what the important information was that they read. After some student responded, she had the students turn and talk to a partner about a good way to represent this problem and how they could show their work to reflect their thinking. She
consistently included mathematical discourse in the problem-solving lesson. During the students’ conversations with their partners, Ingrid walked around and monitored what the students were saying, and she asked questions to have them justify their thinking. After speaking and checking in with all groups, Ingrid told the students to begin working together in their own math journals. She monitored the students during their partner work, asked advancing and assessing questions to gauge their understanding, asked questions about their visual models, and monitored to ensure that the students were working collaboratively with one another. Ingrid’s actions were all listed in the Level 4 sections of the TAG and IQA rubrics. After the students had time to think through the task, Ingrid had a few of the partners present their work to the class. Ingrid’s task was scored a 4 because the students were “doing mathematics.” The students explained their thinking and justified their rationale for how they chose to solve it based on the multiple strategies they were previously taught. They made explicit connections between representations and concepts or procedures. The students also supported their conjectures with evidence, and the task challenged them to use their mathematical reasoning and understanding to support their illustrations. The observations in both lessons coincided with the interview question about what good mathematics instruction should look like. Ingrid stated that effective questioning and thorough responses showed that someone knew and understood what they had to verbally explain. She had the students discuss the mathematics in detail to check for their level of understanding. To create a safe classroom environment, she felt as if it was also important to have the students discuss their mistakes to show them the learning that took place.

Ingrid also thoroughly explained her understanding of the relationship between mathematics content and pedagogy. She began by saying, “You can’t teach it if you don’t understand it yourself. You have to be able to have the content knowledge and be able to teach it,
and teach it well.” This statement directly connected to and is reflected in what she tried to instill in the students during the lessons. She pushed the students with their conceptual understanding in order for them be able to articulate their thinking and reasoning verbally during their explanations. Ingrid furthermore explained what she thought was adequate preparation needed to be an EMS. She explained that an EMS must be knowledgeable of the content that they had to teach. She also specified the importance of how knowledge builds and progresses. She furthermore explained the leadership aspect of being an EMS. She affirmed that an EMS had to be teachable to learn from others and gain more knowledge to convey it back to the teachers. Everything Ingrid verbalized aligned to what she tried to do when growing professionally as well instilling this mindset in the students’ thinking. She pushed the students in their conceptual understanding and expected them to justify their thinking when conveying it to others.

Ingrid’s data from the CKS and CSI revealed her passion for building relationships with her teachers. She felt very confident employing practices that build teacher and coach relationships, such as assisting with creating environments for open discussions about instructional practices and helping teachers set goals and objectives to support improving their instruction. She did not feel as comfortable or effective in providing feedback to the teachers or having open discussions about constructive criticisms with the teachers. Because Ingrid was a classroom teacher for part of the day and an EMS for part of the day, she found it to be a struggle to give feedback to teachers, as she felt she was not seen as a mathematics coach or leader by some of the teachers. She expressed this concern in her interview, in addition to informal conversations upon the researchers’ arrival to the school, and after the observations. She enjoyed modeling lessons and instruction, coaching mathematical content, and helping with mathematics specific and
general pedagogy. However, Ingrid did not feel as confident or effective with the mathematics content for all the grade levels she coached. This finding corroborated her low score of 1.12035 on the LMT, which measured her content knowledge in teaching mathematics. She also mentioned not feeling as effective in coaching teachers in number sense and computation topics, which also confirmed the low content knowledge on her LMT. When she was asked to gauge her level of confidence on content-specific pedagogy, Ingrid felt moderately effective in coaching teachers on using applications and connections in their mathematics classes and coaching teachers on engaging students in mathematical abstraction and sense making.

Ingrid had doubts about her own coaching abilities. She felt somewhat effective or confident in coaching teachers on intellectual rigor, creating environments where students listened to one another, coaching teachers on the use of cooperative learning, and effective classroom management. Although her observations supported strong classroom management skills and her students appeared as if she had created a safe learning environment because they collaborated with one another, she still felt that her ability to coach teachers on these topics was not as effective.

Summary

This chapter presented the statistical analysis and qualitative data collected and examined from 12 current Elementary Mathematics Specialists’ (EMS) content and pedagogical knowledge for teaching mathematics, the EMSs’ leadership knowledge and skills, and their ability to select and teach a grade level appropriate, high-quality mathematical task to two different classrooms of elementary students in their school. Chapter 6 provides the interpreted data findings from Chapter 5 and concludes with implications for future research.
CHAPTER 6

INTERPRETATION OF RESULTS

Chapter 6 includes an overview of the research study and the discussion of the data. This chapter concludes with the implications for preparation of an EMS and recommendations for future research. This mixed methods study analyzed the content knowledge, pedagogical knowledge, and leadership knowledge and skills (AMTE, 2013) of elementary mathematics specialists (EMSs). In addition, the study examined EMSs’ ability to select and implement a high-quality mathematical task. The study was carried out over 2 consecutive school years, in multiple elementary schools, in one southeastern state in the United States. Quantitative methods included a Spearman Rank Correlation Coefficient and descriptive statistics. The traditional qualitative methods of observation, interview, and item analysis and interpretation from written journal entries were also compiled during data gathering.

The study began with the premise that EMSs should have the qualifications of a certified EMS proposed by AMTE (2013). Standards from AMTE (2013) were used as the conceptual framework for describing the knowledge and abilities of EMSs. The three parts of the AMTE (2013) standards are shown in Figure 1 in Chapter 1. The following discussion is organized to answer each research question in turn.

Research Question 1

What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and their implementation of a mathematical task?
The data from the TAG and IQA from the EMSs in Phase 2 of the study ($n = 12$) showed that when an EMS selected a high-quality mathematical task, more times than not, they implemented it at the same level of the potential of the task or one level below. Anna, Denise, Ellen, Fran, Gwen, Ingrid, Jennifer, Katie, and Lilly all had at least one of their tasks scored as a Level 4 potential based on the TAG rubric. When the EMSs implemented their task, it was implemented at a level 3 or 4 based on the IQA rubric. Three different relationships between task potential and implementation were identified in the data.

**High-level task potential and low-level task implementation.** Connie selected a Level 3 task according to the criteria on the TAG rubric, but her implementation of the task was scored as a Level 1 on the IQA by one of the researchers. This was a rarity and the only time that a task initially earned a higher score for the potential of the task and then was scored more than 2 points lower for the task implementation based on the IQA. This was a concern because Connie was not able to take a high-quality task and implement it at a high level of cognitive demand. When she implemented the task with the students, she lowered the level of rigor and cognitive demand through her instructional choices. The drop in scores from the TAG to the IQA was a major concern because Cai, Moyer, Nie, and Wang, (2006) stated that high-quality mathematical tasks are important vehicles for developing students’ capacity to think deeply and critically and develop true understanding when solving basic mathematics problems. If the students are not engaged in high-quality tasks, we do not know if the students are learning mathematics conceptually, through problem solving opportunities provided to them that also allow all of the students multiple entry and exit points while having mathematical discourse with their peers. In the 6 years that Connie had been in education, she was only in her first year of her new coaching
role. During her undergraduate degree in elementary education, Connie took three college mathematics courses, with the highest level only being algebra. It was not stated which type of algebra course was taken. Unfortunately, Connie did not take any mathematics content courses during her graduate degree coursework, nor did she participate in any mathematics professional development in the prior 12 months. The relationship between Connie’s high-level potential for her tasks and low-level implementation appears to be a result of her lack of strong mathematics pedagogical content knowledge needed to implement a high-quality task with a high level of rigor and cognitive demand.

**Low-level task potential and high-level task implementation.** The first step in students solving a task with high rigor and allowing opportunities for the students to have purposeful mathematics discourse, collaboration with one another, and problem solving opportunities while doing and learning the mathematics, is that the EMS must be able to distinguish the differences between a high-quality task and a lower level task. Some tasks “have the potential to engage students in complex forms of thinking and reasoning, whereas others focus on the memorization or use of rules or procedures” (Stein et al., 2009, p. 4). There were only two instances in the 24 observations where two EMSs (Anna and Lilly) selected a task that was scored as a Level 2, yet they were both able to implement the task at a Level 3. To be considered effective, an EMS must make sure teachers are also experts in mathematics content and pedagogy to teach students the standards to attain procedural fluency and conceptual understanding (Campbell, Ellington, Haver, & Inge, 2013). Just as EMSs are expected to have deep and conceptual understanding of mathematics content and pedagogy, teachers also need to develop effective teaching strategies and practices, because simply having content knowledge is not enough (Graham & Fennell, 2001). Lester and Charles (2003) stated that conceptual learning
happens when relevant mathematics concepts and skills are integrated into the process of teaching through solving problems. It is vital to understand mathematics content and pedagogy at levels that enable the EMSs to represent mathematics in multiple ways and use various instructional strategies so that students have a variety of ways to understand the concepts taught (Ball, 1990). Because Anna and Lilly’s instructional decisions to allow students more problem-solving opportunities, more mathematical reasoning and discourse to occur, or more opportunities for conceptual understanding to take place than was foreseen in the task alone, the initial scoring increased after it was implemented. Anna and Lilly showed that they knew how to implement a high-quality task including providing the students with opportunities for mathematical discourse and asking them purposeful questions to assess and advance their thinking, yet they struggled with the selecting a task with high rigor and a high cognitive demand. Shulman (1986) found that if PCK was limited, it might hinder identifying student misconceptions, thus not being able to provide students with alternate conceptual means for exploring their misconceptions. So, if the EMSs have limited PCK, they may not be fully equipped to fulfill the role of a qualified EMS. The relationship between Anna’s and Lilly’s low-level potential for their tasks and high-level implementation appears to be a result of having strong pedagogical knowledge from almost 30 years of combined teaching experience, but perhaps a lack in creating or selecting a high cognitive demand task due to not having enough experiences with characteristics or qualities of higher level tasks and what they should entail. This specific scenario was rare, but with a strong foundation in teaching and instructional strategies Anna and Lilly were able to overcome the lack of selecting a high-quality task and still provide beneficial, conceptual learning opportunities for the students despite the low-level potential of the tasks. The task Anna and Lilly chose to implement may not have required a deep
level of mathematics content knowledge in order to implement it with a high level of cognitive demand. The Teaching and Learning Principle from *Principles to Actions* calls for teachers to be equipped with pedagogical knowledge, knowledge needed for teaching mathematics effectively to all students (NCTM, 2014). The National Council of Supervisors of Mathematics (2008) stated that EMSs must have a multitude of instructional practices and educational experiences to support teachers with formative assessment and differentiating instruction for all learners. In the case of Anna and Lilly, strong pedagogical knowledge still provided an excellent learning opportunity for the students.

**High-level task potential and high-level task implementation.** Gwen and Ingrid both received a perfect score of 4 for both the potential and the implementation of the tasks. High-level tasks generally have multiple entry and exit points that allow students to approach the task in different ways based on their prior knowledge of problem solving and mathematical reasoning, as well as have various ways to express the solution (Van de Walle et al., 2015). High-level tasks can feature multiple representations and provide opportunities for mathematical discourse (Stein et al., 1996). Gwen stated in one of her journal entries that she selected the task as high quality because she “felt it was accessible to all students; there was a low floor and high ceiling which I felt was critical for this collaborative activity.” She also selected it because “it addressed the current standards outlined in the current curriculum unit.” Furthermore, Gwen stated that the high-quality task involved various levels of mathematics and included the opportunity for self-differentiation as students could decide which strategy they felt most comfortable using to solve the problem. Furthermore, the collaborative part was encouraging for some reluctant learners as they were able to learn from others within their group, yet share their understanding from their level.
She considered that the task was a high-quality and rich task because it was based on the content standards, was accessible to all students, and allowed students to apply different problem-solving strategies as they solved the task.

Ingrid explained that she choose the task as high quality because (a) it aligned with the content standard for the specific grade, (b) explored fractional parts of a whole fraction notation, and comparison of fractions with unlike denominators, (c) had a problem-solving context, (d) students had to make sense of the question that is to be answered, and (e) they had to design a model based on the information. She further explained that the task was “open to a variety of strategies in order to solve it” while also anticipating that the students would engage in mathematical discourse while solving the task collaboratively. She also added that the task offered multiple mathematical concepts while solving it and lent itself to being a high-quality mathematical task because it allowed the students to apply the eight Standards for Mathematical Practice. The task also supported the productive struggle from the students while they continued to persevere to solve the problem, while drawing appropriate models and diagrams that connected to the problem. And finally, Ingrid stated that the task supported mathematical discourse because the students had to construct viable arguments while attending to the meticulousness and accuracy in their justification of their mathematical reasoning. In Ingrid’s 11 years of teaching, she taught mathematics for 8 years and coached for 3 years.

Although Ingrid was one of the youngest EMSs in the study and has only earned her bachelor’s degree as of yet, her 18+ hours of specific mathematics professional development in the last 12 months has given her the confidence in knowing what a high-quality task should entail and how to implement it with a high level of cognitive demand. The relationship between Gwen and Ingrid’s high-level potential for their tasks and high-level implementation appears to
be a result of their educational experiences, preparation for the EMS position, mathematics-
specific professional development opportunities, and strong pedagogical knowledge and
understanding from almost 25 years of combined teaching experience between the two. These
are contributing factors as to why the relationship between selecting high-quality tasks and
implementing them with a high-level of rigor are supported for the best learning situation for the
students to learn with conceptual understanding.

**Factors affecting task potential and implementation.** In their lesson plans, EMSs most
often planned for student engagement, student discourse, or small group experiences \(n = 49\),
followed by task differentiation and flexibility and using multiple strategies \(n = 23\). However,
the plans most often did not include phrases that related to students developing a deeper
understanding of mathematics concepts and ideas, making connections among multiple
representations to help develop meaning, and requiring the students to explore and understand
mathematical relationships. Formative assessment and next instructional steps were only
specifically mentioned a couple of times out of the 24 lessons that were observed. The findings
indicate that EMSs were not aware of the importance of determining next instructional steps
when formatively assessing the students. If EMSs are to effectively help teachers develop
students’ conceptual understanding in mathematics, then EMSs must be able to identify and
select high-quality mathematics tasks and implement them effectively with high cognitive
demand. The relationship between selecting a high-level task and implementing it at a high level
may be affected by the EMSs’ knowledge of selecting a rigorous task to begin with. As shown
from the data from Anna and Lilly, perhaps it is better for an EMS to have a strong foundation in
pedagogical knowledge to still be able to take a lower level task and implement it at a higher
level as opposed to not being able to make in-the-moment instructional adjustments to allow the students more conceptual understanding opportunities.

**Research Question 2**

*What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and their content knowledge for teaching elementary mathematics?*

**Task Analysis Guide and Learning Mathematics for Teaching**

The TAG and LMT were used to collect quantitative data for Research Question 2.

McGatha and Rigelman (2017) expressed the breadth of knowledge needed by EMSs in terms of knowledge of content and teaching: EMSs need to be able to (a) select and implement a mathematical task to students (Jackson, Shahan, Gibbons, & Cobb, 2012), (b) facilitate mathematical discourse (Smith & Stein, 2011), (c) prompt and promote student collaborative mathematical thinking (Franke et al., 2009; Webb et al., 2014), and (d) provide suitable scaffolding for students’ critical problem solving (Baxter & Williams, 2010). To facilitate mathematical discourse, promote students’ thinking, and scaffold problem-solving with mathematics, EMSs must have in-depth content knowledge, paired with the knowledge to select a high-quality mathematics task to make the conceptual understanding and learning meaningful for the students and teachers they coach.

**High-level task potential and high content knowledge.** The LMT assessment was used to identify the mathematical content knowledge of the EMS based on various skills and concepts in elementary mathematics. The subset group of EMSs had a mean score of .88 on the LMT. However, when the EMSs’ LMT scores were analyzed for a correlation with the TAG scores, the results indicate a weak correlation. The statistically weak correlation is counterintuitive, but
those with high scores on the TAG and LMT were observed as highly competent in the classroom, coaching teachers. Fran scored all 4s on the TAG for the potential of the task and a 4 and three 3s on the observations of her lessons on the IQA, respectively. She also had a scaled score of 2.27475 on the LMT, which was one of the highest overall scores. Fran’s high scores on the LMT complimented her data from the mathematics content coaching heading on the CSI. She also reported receiving 100+ hours of specific mathematics professional development in the prior 12 months. The combination of her high number of professional development hours, paired with her high content knowledge exhibited in the LMT and the CSI content coaching section supported her contention that she was effective in mathematics content coaching.

Jennifer was in a similar situation as Fran. Jennifer’s score on the LMT was one of the highest of all EMS, 1.75572. The high score on the LMT suggested that she understood the mathematics content. Jennifer seemed to know what a high-quality task looks like based on her scores of 3s and a single 4 for her task selection. She said, “I don't have a strong math content, but I'm learning to love it and to really want to help to develop these children.” Notwithstanding her will to learn more each day, she still seemed to think she was unprepared for her position as an EMS, which was contrary to her ability to select a high-quality task and her high mathematics content knowledge based on the data from the LMT. Jennifer may seem to be a timid person when she knows someone is observing her and recording data on her ability to select a high-quality task and evaluating her content knowledge for teaching mathematics. In 2000, the National Survey of Science and Mathematics Education found that only 60% of the participating teachers in elementary schools felt that they were qualified to teach mathematics (Banilower, Smith, & Weiss, 2002). Mathematics tends to be an area in which many elementary teachers feel unprepared to teach in depth, in addition to not feeling comfortable talking about mathematics in
general (Banilower et al., 2002). Despite Jennifer’s high scores in her task selection and her content knowledge for teaching mathematics, she still felt unprepared.

**High-level task potential and low content knowledge.** Ingrid did not feel as confident or effective with the mathematics content for all the grade levels she coached. This finding validated her low score of -1.12035 on the LMT. Ingrid did not feel effective in coaching teachers in number sense and computation topics as well as coaching teachers on strategies for engaging students in mathematical abstraction and sense making. Ingrid spoke about her many doubts and insecurities about her own mathematics content knowledge and coaching abilities. She scored all 4s from both researchers on both of her observations, one in fifth-grade in the Numbers and Fractions domain solving a task with fractional parts of a pizza, and in fourth-grade Operations and Algebraic Thinking using multiplication and division to solve word problems. Her lack of specific mathematics content knowledge may hinder her as she moves into other mathematics concepts and skills. When an EMS is equipped with all three suggested areas of expertise (content knowledge, pedagogical knowledge, and leadership knowledge and skills), according to AMTE (2013) the EMS is considered a well-rounded, qualified EMS. Many of the 2010 mathematics standards require a deeper understanding of mathematical content knowledge by the EMS, the teacher, and ultimately the students (Rowland, 2015). EMSs are expected to show preparedness to teach 21st century mathematics skills.

Gwen earned perfect scores on her task selection and implementation. However, her mathematics content knowledge was one of the lowest scaled scores on the LMT, -0.74025. She missed or did not attempt items in multiple content areas of the LMT, including fractions and division, which are seen in the higher elementary grades. Gwen taught one lesson in second grade focusing on addition and subtraction within 100, which is in the Operations and Algebraic
Thinking domain. Gwen’s other lesson was in third grade, focusing on the only addition and subtraction standard in third grade in the Number and Base Ten domain. Perhaps Gwen selected lessons for the study in grades that did not have a higher level of mathematical knowledge that would focus on higher fractional reasoning, multistep division word problems, or decimals and percentages, her areas of weakness. Gwen may have earned a high score on the TAG and IQA, but her low score on the LMT may be due to her selecting and implementing high-quality mathematics tasks that did not allow for a deep understanding of mathematics. Gwen may have chosen to implement tasks in the lower elementary grades because she was comfortable with the content in the lower grades. Because the LMT provides items that are appropriate for elementary grades through Grade 6, perhaps she performed so low on the LMT because many of the items were in the higher grades such as grades 4-6. She began the (third grade) lesson with a Number Talk to get the students to start thinking about numbers and focused on using correct vocabulary usage. She wrote the expression 49 + 49 on the board and asked the students to solve it mentally. Once they solved the expression, she asked some to share their strategies as she recorded them on the board. After the warm-up activity she went into her “rich task” as she referred to it. She explained that she created the task on her own to push on the students’ higher learning while also assessing the students on what they already knew. She explained the task after students read it independently and explained any vocabulary they may have not known in the problem. The students worked in their table groups. Gwen walked around asking questions to push the students’ thinking and then brought them together at the end of the lesson to recap the lesson. Although Gwen showed strong pedagogical knowledge in how she implemented both high-quality tasks with a high level of cognitive demand, her content knowledge was not a visible struggle for her in the lessons she taught due to the lower level of mathematics that she
chose to focus on. She reported a fairly high amount of specific mathematics professional development hours in the prior 12 months, 25+ hours. Most of the EMSs who had the highest scores on the LMT (Anna, Fran, and Lilly) also reported some of the greatest amount of mathematics professional development within the last 12 months, reporting over 100+ hours. Like Connie’s experience, Gwen’s lack of mathematical content knowledge was compensated for by her pedagogical content knowledge during the observations of early-grade instruction. It was also important to note that the mathematical task may not have required a deep understanding of specific mathematics content in order to select or implement the mathematics task at a high level.

EMSs need to be able to demonstrate the ability to translate content knowledge into pedagogical practices that will likely result in meaningful mathematical learning for all students (McGatha & Rigelman, 2017, p. 25). EMSs must be equipped with the mathematical content knowledge of the standards to effectively facilitate the learning of others. Shulman (1986) found that if PCK was limited, it might hinder identifying student misconceptions, thus not being able to provide students with alternate conceptual means for exploring their misconceptions. Having EMSs who are well prepared in both content and pedagogical content knowledge is crucial to being a qualified EMS.

**Research Question 3**

*What are the relationships between the Elementary Mathematics Specialists’ leadership knowledge and skills, their beliefs and practices about teaching and coaching mathematics with conceptual understanding, and their instructional practices pertaining to mathematics?*
Coaching Knowledge Survey and Coaching Skills Inventory

The CKS and CSI were used to collect quantitative data for Research Question 3. Supporting the quantitative data, the EMSs responded to the written journal prompt describing what successful mathematics teaching looks like and responded to semiformal interview questions about necessary knowledge and skills for an EMS; that is, content, pedagogy, and leadership, the relationship between content knowledge and pedagogy, their preparation to be an EMS and how it has supported them in assisting their teachers, and finally what they believe good mathematics instruction looks like.

To support the data findings from the EMSs, McGatha and Rigelman (2017) expressed the breadth of knowledge needed by EMSs in terms of knowledge of content and teaching. EMSs need to be able to (a) select and implement a mathematical task to students (Jackson, Shahan, Gibbons, & Cobb, 2012), (b) facilitate mathematical discourse (Smith & Stein, 2011), (c) prompt and promote student collaborative mathematical thinking (Franke, Webb, Chan, Ing, Freund, & Battery, D, 2009; Webb, Franke, Ing, Wong, Fernandez, Shin, & Turrão, 2014), and (d) provide suitable scaffolding for students’ critical problem solving (Baxter & Williams, 2010). Just as EMSs are expected to have deep and conceptual understanding of mathematics content and pedagogy, teachers also need to develop effective teaching strategies and practices because simply having content knowledge is not enough (Graham & Fennell, 2001).

Strong understanding of relationship between content and pedagogy and beliefs about good mathematics instruction. Graham & Fennell (2001) made the case for both strong content knowledge and strong pedagogical content knowledge. Fran’s story is representative of the effectiveness of having both in place. She represents one of the three EMSs of the convenience sample whose performance during observed lessons and confidence in her
explanations of her choices during interviews provided living examples of the effectiveness of Graham and Fennell’s suppositions. Fran clearly communicated her knowledge and understanding about the many relationships between content and pedagogy. She expressed the content knowledge needed to understand the mathematics content conceptually as well as the standards for mathematical practice that the students utilize. Fran expressed the importance of how content is connected to pedagogy, specifically when implementing high-quality tasks. Fran stressed the importance of research in mathematics education and further discussed the importance in formative assessment. She communicated and displayed the significance of strong coaching practices by helping teachers identify their needs for mathematics content and instruction. Fran understood that both teachers and students alike must have multiple opportunities for exploration and mathematics reasoning when they attacked new concepts in mathematics content or pedagogy. Shulman (1986) explained PCK as the blending of content and pedagogy into an understanding of how particular subjects and content knowledge are taught and adapted to various types of learners and learning styles (Shulman, 1986; Walker, 2007). Fran believed that if she did not have deep conceptual knowledge, it would be difficult to coach teachers in mathematics content and pedagogical instructional practices, hence her 100+ hours of specific mathematics professional development within the prior 12 months. With her near perfect scores for the task selection and implementation, deep mathematics content knowledge, and her ability to articulate the necessary knowledge and skills to be an effective and adequately prepared EMS, Fran is a well-rounded example of a prepared EMS who recognizes the importance of continuing to grow professionally.

**Limited relationship understanding and low content knowledge.** Katie’s position as an EMS is similar to Fran’s in some aspects, but she displayed a weaker level of mathematics
content knowledge and some responses to specific questions about the tasks she selected to implement were not as strong as Fran’s. Katie explained that she selected one of the tasks “because it was from a curriculum that was research-based, aligned to the CCSSM, and it was adapted to meet the needs of her small group of students.” Although these reasons were valid, they were insufficient because they did not align with what a high-quality task should entail (Task Analysis Guide, Stein & Smith, 1998). Like Fran, Katie also emphasized the importance of using formative assessments to drive the instruction and determine next instructional steps. Although Katie had the third lowest score for her mathematics content knowledge, it seemed to be somewhat stronger in the lessons she chose to implement. This unforeseen finding could have been because of her teaching experiences in multiple grades, in addition to her leadership opportunities and training mathematics teachers at one point in her career. These experiences were possible factors that allowed her to be a more effective EMS in certain aspects than someone without the specific experiences and previous coaching opportunities. Katie valued her professional growth by stating, “I continually spend time receiving professional development to increase my abilities and learn new methods to reach all learners.” Compared to Fran’s 100+ hours of professional development, Katie only reported 8+ hours of professional development in mathematics in the prior 12 months. The lack of more professional development opportunities may have contributed to her inability to select Level 4 tasks and implement them with the highest level of cognitive demand for the students.

Helen was similar to Katie in some data analyses, but Helen displayed much lower levels of understanding in general in addition to her low level of content knowledge. Helen was not able to clearly explain the relationship between mathematics content knowledge and pedagogy. She simply said that content is what you teach and pedagogy is
the way we teach and the theory behind it. She also stated that pedagogy “comes naturally to natural teachers.” Instead of discussing the relationship between content knowledge and pedagogy, Helen tried to define what they meant and what she thought they looked like. She was not able to connect the two or speak about their relationship. Helen defined good mathematics instruction as moving away from a “sit and get and rote memorization” and by mentioning the use of manipulatives and moving from the concrete to the abstract and facilitating the learning process. She also pointed out the importance of productive struggle and how it helped the teacher determine what the students understood about the mathematics. Her responses aligned with many things that the Mathematics Teaching Practices and Principles to Actions (NCTM, 2014) state about good mathematics instructional practices; however, her responses contradicted the notes taken by the researchers during her observations. During the lessons, she frequently told the students what to do and how to do it as opposed to encouraging student exploration and mathematical reasoning. A teacher can tell students about mathematics concepts, but research shows that lecturing alone does not develop deep conceptual understanding (Hiebert & Carpenter, 1992; Skemp, 1976, 1987, 1999). Contrary to Helen’s expressed, but misplaced confidence in her mathematics content knowledge, Helen had the fourth lowest scaled scores on the LMT, 0.154447. In addition to low content knowledge, Helen received the lowest scores on her task selection and implementation within the subset group. Helen grappled with articulating the necessary knowledge and skills of an EMS and had many moments of hesitation during her interview. Although she stated many key words and phrases that aligned with the research from AMTE (2013) for the preparation
of an EMS, they were not observed in her lesson plans or in the observations when she had to select a high-quality task and implement it with a high level of rigor.

Although Helen completed 40 hours of specific mathematics professional development within the prior 12 months, it was not reflected in her content or pedagogical knowledge. After repeated reminders from the researcher, Helen failed to submit three journal items including the rationale for selecting a task as high quality, her beliefs about teaching mathematics, and her weekly journal reflection reporting her daily activities for one week. Helen’s cumulative data proposed that she did not have a strong foundation in mathematics content knowledge, she was not able to successfully articulate the relationship between content and pedagogy, nor was she able to successfully select and implement a high-quality task with students.

If an EMS showed strengths in their leadership knowledge and skills with teachers, and if their beliefs and practices about teaching and coaching mathematics with conceptual understanding aligned with their instructional practices and pedagogy pertaining to mathematics, they displayed the most effective characteristics of an adequately prepared and operational EMS. However, if an EMS showed limited ability in their leadership knowledge and skills with teachers, had difficulty in sustaining positive and professional relationships with their teachers, and their beliefs and practices about teaching and coaching mathematics with true conceptual understanding did not completely align with their instructional practices in a mathematics classroom, they would not be considered adequately prepared to be the most effective EMS. EMSs without a mathematics-specific EMS certification may not have the required expertise in the three areas previously listed to be most effective in a school or for a district. As a result, EMSs without the specific
certification may not be adequately qualified to support elementary teachers with improving their mathematics instruction (Greenberg & Walsh, 2008; Reys & Fennell, 2003), which in turn negatively impacts student success in real-world application of mathematical reasoning.

Lack of Preparation in Deep Content and Pedagogical Knowledge

AMTE (2013) described content knowledge for teaching mathematics as a deep understanding of the mathematics content developed in Grades K-8, whereas pedagogical knowledge for teaching mathematics was defined as knowledge of learners, learning, teaching, curriculum, and assessment. The National Council of Supervisors of Mathematics (NCSM, 2008) stated that pedagogical knowledge and mathematical content knowledge are both central and vital components to successful mathematics teaching and coaching. Ball and colleagues (2008) affirmed that an “unpacking” of mathematical ideas that is needed to make “features of particular content visible to and learnable by students” (p. 400) is essential for teaching mathematics. Many of the most recently adopted mathematics standards require a deeper understanding of mathematical content knowledge by the EMS, the teacher, and ultimately the students (Rowland, 2015).

Helen and Brooke exhibited low scaled scores regarding taking the LMT, which gauged their elementary mathematics content knowledge. Their low scores from the task selection and implementation implied that they visibly struggled with selecting and implementing a high-quality task. They both received the lowest scores from the whole group of EMSs. The struggle may have been due to their lack of in-depth content knowledge paired with the limited knowledge of the most effective pedagogical practices in mathematics instruction. Several studies in elementary mathematics literature have supported positive changes in teachers’
practices because of interactions with a qualified and effective EMS, including actively engaging students, emphasizing reasoning and problem-solving over skills-based lessons, using students’ work to inform instruction, and effectively planning lessons (AMTE, 2013). It was challenging for the EMSs to exhibit procedural fluency when they had limited content knowledge grounded in conceptual understanding. A second implication from this study is that EMSs must be adequately prepared in all three areas that are highlighted by AMTE (2013) as qualifications for an EMS (Fennell, 2011; Gerretson et al., 2008; Hill et al., 2005).

Leadership Knowledge and Skills

Leadership knowledge and skills were described by AMTE (2013) as (a) having a broad view of many aspects and resources to support and facilitate effective instruction and professional growth in mathematics; (b) the ability to take on nonevaluative leadership roles; and (c) being able to plan, develop, implement, and evaluate professional development programs. Bay-Williams, McGatha, Kobett, and Wray (2013) described five coaching skills needed for supporting teachers’ growth and professional development: building trust, establishing rapport, listening, paraphrasing, and posing questions (p. 1).

These five coaching skills align with what AMTE (2013) described as aspects of leadership knowledge and skills (Bay-Williams et al., 2013). McGatha and Rigelman (2017) stated that EMSs should focus on developing trust and establishing confidentiality with the teachers they work with and support, but also keep their relationships with administrators at the forefront. They also suggested that EMSs may be asked to (a) attend professional development sessions or conferences, (b) enroll in classes, or (c) take on learning opportunities by possibly joining professional organizations (p. 29).
Brooke and Gwen did not seem to have strong relationships with teachers in the schools, based on their data specifically about coaching relationships. They did not agree that paraphrasing feedback to the teacher was the best way to address a misconception the teacher had when incorrectly teaching a concept or skill. They also did not believe that encouraging the teachers to reflect on similarities and differences among mathematics topics in the curriculum was important for them to do to clear up misconceptions about mathematics. Brooke and Gwen did not feel that encouraging teachers to set personal improvement goals for mathematics instruction was beneficial to the teachers, nor did they feel that it was their role to help teachers reflect on discrepancies between espoused beliefs and actual practices. Many of the things listed seemed to be due to the lack of trust or a true professional relationship between the coach and the teachers. Many times, if there are misconceptions that need to be addressed or other difficult things that need to be discussed among the EMS and the teacher, and if there is a positive relationship between the two, the concerns are not seen to be as harsh or taken personally. The findings suggest that Gwen and Brooke are less confident in their level of coaching and decision-making when they are placed in difficult situations. When EMSs have leadership knowledge and skills, paired with content and pedagogical knowledge, empirical evidence supports the positive impact of a mathematics specialist on teachers’ instructional practices and student learning (Becker, 2001; Campbell, 1996; & McGatha, 2008). A third implication of this research study is that coaches’ abilities to develop personal relationships with their teachers are key to their effectiveness in improving instruction. Coaches need professional development in these areas, as well as content and pedagogical content knowledge.
Future Research

This study informed and supported state qualification guidelines for a certified EMS while the data revealed the deficits in current EMSs’ mathematical content and pedagogical knowledge and their leadership skills. Future research should focus on the possible relationships of EMSs’ beliefs about high-quality mathematical task selection, the implementation of high-quality mathematical tasks, and their beliefs and practices related to instructional coaching, all grounded in conceptual understanding in mathematics. The Task Analysis Guide and the Instructional Quality Assessment for the task selection and potential seemed to collect the same type of data. There was inefficiency in collecting the data when attempting to utilize both instruments when analyzing the tasks. Developing more effective rubrics for similar studies would provide clearer links between preparation for the EMSs’ position and their effectiveness in improving instruction.

Research states that EMSs could be asked to design effective professional development for teachers, conduct classroom observations, assist teachers in analyzing data and determining instructional next steps, plan and provide intervention for struggling teachers, and organize effective and research-based resources for teachers (Campbell & Malkus, 2011; Chval, Arbaugh, Lannin, van Garderen, Cummings, Estapa, & Huey, 2010). I recommend conducting a study examining coaching effectiveness, coupled with changes in teaching practices, and concluding with examining student learning outcomes based on the EMSs’ preparation, professional development treatment, and coaching effectiveness between the EMS and cooperating teacher(s). This study contributed to the existing literature regarding mathematics coaching and the limited preparation of EMSs, specifically in this southeastern state of the United States. However, a study of effective professional development treatments to strengthen EMSs’ content and
pedagogical knowledge for teaching and coaching mathematics, along with a focus on effective mathematics coaching instructional strategies, would help solidify the relationship between EMS and teacher preparation in relation to student outcomes. If EMSs are tasked to support teachers in their content and pedagogical knowledge for teaching mathematics, extending research on teachers’ knowledge would also be useful.

**Discussion and Implications**

According to the Association of Mathematics Teacher Educators (AMTE, 2013) and the National Council of Teachers of Mathematics (NCTM, 1981, 1984), there should be certified EMSs in schools or districts to appropriately support teachers in mathematics and ultimately promote gains in students’ mathematical understanding and achievement (Dossey, 1984; Fennell, 2006; NMAP, 2008; Reys & Fennell, 2003). The literature supports the argument for EMSs to be adequately prepared in all three areas that are highlighted by AMTE (2013) as qualifications for an EMS (i.e., content knowledge, pedagogical knowledge, and leadership knowledge and skills; (Fennell, 2011; Gerretson et al., 2008; Hill et al., 2005).

The experiences and preparation of the 12 EMSs support the contention that often, EMSs are coaching outside of their own subject area of expertise; they are often education generalists (AMTE, 2013; NCTM, 2000; NMAP, 2008; NRC, 1989) or former reading or literacy coaches in elementary schools (McGatha & Rigelman, 2017). Katie, along with some of the other EMSs in the subset group, was hired specifically to coach classroom teachers. Katie reported that much of her time was given to assisting with computer-based testing in the computer lab, conducting Dyslexia screenings, creating performance assessments for teachers, helping with reading screening tests, and meeting with different teams to discuss some shifts in classroom instruction. Although Katie showed deep content knowledge when implementing her specific tasks and
displayed strengths in her coaching knowledge and skills through her positive rapport with her teachers, Katie was qualified to coach, but was not given time to do so effectively. The inconsistency of job assignments, and many times unexplained responsibilities assigned to mathematics coaches prevent the mathematics coaches from doing what they were hired to do: use their leadership knowledge and skills paired with their pedagogical and content knowledge to improve mathematics instruction. Gaining insight into what the EMSs’ roles and responsibilities are in their schools and how their content knowledge and beliefs about coaching and teaching mathematics affect instruction are important steps toward improving mathematics education in the state.

Neufeld and Roper (2003) defined the EMS’s primary roles as engaging teachers in collaborative practices, while at the same time leading them to use instructional practices out of their normal comfort zones. EMSs are then able to go deeper in their practices with the teachers and see shifts in thinking while engaging students in meaningful learning. Neufeld and Roper (2003) stated that in one of their promising districts, employing full-time or part-time coaches in schools who could provide ongoing, sustained support to principals and teachers would positively impact classroom instruction and student achievement (p. iii). An implication of this study is that as long as qualified EMSs are required to take on many responsibilities outside of mathematics coaching and fulfill multiple roles that do not directly affect improvements in mathematics instructional practices, students will continue to be exposed to only surface knowledge about mathematics procedures versus students having procedural fluency from conceptual understanding.

A third implication of the study is that coaches may not have sufficient mathematical knowledge to coach elementary school teachers in their classroom instruction of mathematics
content. The National Council of Supervisors of Mathematics (NCSM, 2008) stated that pedagogical knowledge and mathematical content knowledge are both central and vital components to successful mathematics teaching and coaching. EMSs must have a deep and conceptual understanding of the elementary mathematics progression of topics across all domains and grade levels through the middle school level and utilize their knowledge while working with teachers (ems&tl, 2009, 2011; NCSM, 2008). Ball and colleagues (2008) stated that mathematical knowledge that is needed for everyday life is important, but an “unpacking” of mathematical ideas that is needed to make “features of particular content visible to and learnable by students” (p. 400), is essential for teaching mathematics. This type of knowledge for teaching is described by Ball and others (2008) as things such as, but not limited to, understanding how certain words have different meanings in a mathematics context, knowing and understanding the difference between mathematical justifications versus explanations, and determining whether particular problem-solving strategies work in general. EMSs must be properly equipped with the types of in-depth knowledge needed to be effective and agents of positive change in mathematics instruction.
REFERENCES


Fennell, F. (2011, Summer/Fall). We need elementary mathematics specialists now, more than ever: A historical perspective and call to action. *NCSM Journal, 53-60.*


APPENDIX A
THE UNIVERSITY OF ALABAMA
HUMAN RESEARCH PROTECTIONS PROGRAM

Nicolette Nalu, Principal Investigator from the University of Alabama, is conducting a study called Elementary Mathematics Instructional Coaches: Prerequisites of a Qualified Mathematics Specialist. She is under the direction of Drs. Diane Sekeres and Dr. Cynthia Sunal. She wishes to find out: (a) What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and the implementation of a mathematical task? (b) What is the relationship between the Elementary Mathematics Specialists’ ability to select a high-quality mathematical task and an Elementary Mathematics Specialists’ content knowledge for teaching elementary mathematics? (c) What are the relationships between the Elementary Mathematics Specialists’ leadership knowledge and skills, their beliefs and practices about teaching and coaching mathematics with conceptual understanding, and their instructional practices pertaining to mathematics?

Taking part in this study involves Elementary Mathematics Specialists completing two online surveys that will take approximately 10-25 minutes each. The overall purpose for these surveys is to build a profile of the Elementary Mathematics Specialists (EMSs) serving schools in Alabama. These surveys contain questions about content and pedagogical knowledge for teaching mathematics, leadership skills, beliefs about teaching and coaching mathematics, the nature of high-quality mathematical tasks for developing conceptual understanding, and ways to implement a high-quality mathematics task. Also, a selected subgroup of 1-3 EMSs in each region will be asked to teach two lessons (implement 2 high-quality mathematics tasks) that develop conceptual understanding, and submit reflective journal entries. Each lesson will be observed and some coaches will be asked to participate in an informal, semistructured follow-up interview.

The researcher will protect your confidentiality. Your name will never be used or appear anywhere in the reports for this work. All data will be secured at the researcher’s home. Only Nicolette Nalu will have access to all of the data pieces except for the other approved and certified observation researcher who will have access to only the observational data that they will assist in collecting. The data will be coded with ID codes assigned by Nicolette Nalu. Only summarized data will be presented at meetings or in publications. There will be no direct benefits to you other than a gift card for $15 after all of the surveys and necessary documents have been submitted by the predetermined due dates set by the researcher. The findings for research in elementary mathematics coaching will be useful to the state of Alabama and the rest of the country. There are no foreseeable risks although there may be unforeseen risks. The chief risk is that some of the questions and or activities may make you uncomfortable. You may skip any questions you do not want to answer.

If you have questions about this study, please contact Nicolette Nalu at 251-510-2580 or by email at nnalu@ua.edu or Diane Sekeres at desekere@ua.edu. If you have questions about your rights as a research participant, contact Ms. Tanta Myles (the University Compliance Officer) at (205) 348-8461 or toll-free at 1-877-820-3066. If you have complaints or concerns about this study, file them through the UA IRB outreach website at http://osp.ua.edu/site/PRCO_Welcome.html. Also, if you participate, you are encouraged to complete the short Survey for Research Participants online at this website. This helps UA improve its protection of human research participants.

YOUR PARTICIPATION IS COMPLETELY VOLUNTARY. You are free not to participate or stop participating any time before you submit your answers.

If you understand the statements above, are at least 19 years old, and freely consent to be in this study, click on the _____ (CONTINUE or I AGREE) button to begin.
Dear Specialist/Coach,

My name is Nicolette Nalu and I am a Ph.D. candidate at The University of Alabama. As part of my degree requirements I must complete a dissertation research project and I have chosen to investigate the role of the elementary school mathematics coach. This topic is of significant interest to me because I am currently employed by the Alabama Mathematics, Science, and Technology Initiative, (AMSTI) as a mathematics specialist and I work directly with teachers and students on a daily basis. I feel strongly that the findings from this study will be beneficial to the field of mathematics education and, specifically provide important information to elementary schools, districts, principals, and mathematics specialists.

As an elementary specialist your insight is extremely valuable not only to me, but for the purpose of this research study. Therefore, I am writing to request your participation in this study. The study itself will take place in March-October of 2017, and will include a couple of surveys, a mathematics content assessment, and for a randomly selected group of coaches from each region, five reflective journal prompts or entries, two in-class observations and one, audio recorded, informal, semistructured interview with 10-30 of the randomly selected Elementary Mathematics Specialists across the state that are selected from the observation subgroup in each region.

Included in this email, you will find a link to the first survey. Please respond to all items on the survey according to the instructions provided, your honest responses will be sincerely appreciated. Please do your BEST to complete this first survey as soon as possible and then you will be directed to the next survey instrument used for data collection.

Your responses will be kept strictly confidential and I will be the only one who will assign each participant an ID code once the initial names and schools have been determined. You have been selected to have the opportunity to participate in this study through a random sampling from your area. After the selections have been made, the ID code assigned to you, by the researcher will be used only to determine which surveys are returned, how many participants are in the study, and so the data can be analyzed and correlated to the other pieces of your own data. Be assured that your anonymity will be respected and your cooperation will be greatly appreciated.

If you have any questions regarding the study or your participation, please feel free to contact me at 251-510-2580 or you may email me at nnalu@ua.edu. The data analysis of the results will be available upon your request. Additionally, if you have questions regarding the IRB or confidentiality process, that information is on the initial consent form.

Thank you in advance for considering helping me with my research study,

Nicolette I. Nalu
## Coaching Knowledge Survey

The following survey is designed to capture information about your beliefs and practices related to instructional coaching.

Enter your name or ID code: [ ]

1. Please indicate your level of agreement with each of the following statements, from 1 (strongly disagree) to 7 (strongly agree).

<table>
<thead>
<tr>
<th>Statement</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. An effective mathematics coach coaches only on teacher-stated needs.</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>b. Beginning teachers need more coaching than 25-year veterans.</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>c. When a teacher says that she or he doesn’t want any coaching, an effective mathematics coach respectfully does not try to persuade the teacher to accept coaching.</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>d. Sometimes an effective mathematics coach has to oppose school or teacher actions that are not good for students’ mathematics learning.</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>e. Teachers will adapt to whatever method of coaching is used.</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>f. An effective mathematics coach gets input from a school’s principal on which teachers need to improve their mathematics</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
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</table>
g. Number sense is a prerequisite for algebraic thinking. ○ ○ ○ ○ ○ ○ ○ ○

h. A coach should put no pressure on teachers to improve their practices. ○ ○ ○ ○ ○ ○ ○ ○

i. In general, teachers need coaches to model a lesson with a particular strategy before they will incorporate it with fidelity. ○ ○ ○ ○ ○ ○ ○ ○

j. A teacher can learn new mathematics, but the teacher’s basic mathematical intelligence cannot be changed. ○ ○ ○ ○ ○ ○ ○ ○

2. Please indicate your level of agreement with each of the following statements, from 1 (strongly disagree) to 7 (strongly agree).

<table>
<thead>
<tr>
<th></th>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>More Disagree than Agree</th>
<th>Neither Disagree nor Agree</th>
<th>More Agree than Disagree</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>Once a teacher knows about a research-based strategy for improving student learning, the teacher will begin using the strategy.</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>An effective mathematics coach provides teachers with an understanding of how the mathematics they teach supports learning beyond the grade level they teach.</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td></td>
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<tr>
<td>c.</td>
<td>An effective mathematics coach uses state mathematics assessment data when developing a coaching plan with teachers.</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td>An effective mathematics coach asks the principal what she or he believes the teachers’ needs are.</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
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</tr>
<tr>
<td>e.</td>
<td>A student’s intelligence can be changed through excellent teaching.</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td></td>
</tr>
<tr>
<td>f.</td>
<td>Teachers generally have similar teaching styles.</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td>○ ○ ○ ○</td>
<td></td>
</tr>
</tbody>
</table>
g. When a teacher says something that isn’t quite mathematically correct, an effective mathematics coach says, “You are almost right,” and then gives the teacher a clear explanation of the correct mathematics.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>More Disagree than Agree</th>
<th>Neither Disagree nor Agree</th>
<th>More Agree than Disagree</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
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</table>

h. An effective coach sticks to the coaching objectives established with a teacher at the beginning of the school year.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>More Disagree than Agree</th>
<th>Neither Disagree nor Agree</th>
<th>More Agree than Disagree</th>
<th>Agree</th>
<th>Strongly Agree</th>
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<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

i. Teachers can influence students’ learning styles.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>More Disagree than Agree</th>
<th>Neither Disagree nor Agree</th>
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j. An effective mathematics coach gives feedback to the principal about teachers who are struggling in the classroom.

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<th>Strongly Disagree</th>
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<th>More Disagree than Agree</th>
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3. Please indicate the degree to which each of the following statements is descriptive of your coaching practices, from 1 (not at all descriptive) to 7 (very descriptive).

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<tr>
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<th>Strongly Disagree</th>
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<th>Agree</th>
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<tr>
<td>a. When a teacher says something I find confusing, I paraphrase what I heard and say it back to her or him.</td>
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<td>b. I collect students’ mathematics work from a teacher’s classroom to guide our coaching conversations.</td>
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<tr>
<td>c. When decisions about mathematics instruction are being made, I ensure that the decision-makers interpret research literature accurately.</td>
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<td>d. I coach teachers on needs that I observe in the teacher, even when the teacher is unaware of these needs.</td>
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<tr>
<td>e. As a mathematics coach, I support mathematics teachers by tutoring their struggling students.</td>
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<td>f. I have difficult conversations with teachers, when necessary, about mathematics misconceptions they hold.</td>
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<tr>
<td>g. I always make sure that coaching conversations</td>
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with mathematics teachers are grounded in the mathematics content.

h. I meet with the principal to discuss the school’s vision for mathematics instruction.

i. I encourage teachers to include, in each lesson they teach, summaries of what students learned.

j. I provide feedback to teachers about whether or not the school is meeting its vision for mathematics instruction.

4. Please indicate the degree to which each of the following statements is descriptive of your coaching practices, from 1 (not at all descriptive) to 7 (very descriptive).

<table>
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<tr>
<th></th>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>More Disagree than Agree</th>
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<th>More Agree than Disagree</th>
<th>Agree</th>
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<tr>
<td>a. I try to provide the teachers I coach with an understanding of how the mathematics they teach supports learning beyond the grade level they teach.</td>
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<td>b. I ask the principal what he or she believes the mathematics teachers’ needs are.</td>
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<td>c. I encourage the teachers I coach to reflect on similarities and differences among mathematics topics in the curriculum.</td>
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<td>d. I help teachers plan their lessons.</td>
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<td>e. I ask the teachers I coach what aspects of mathematics teaching they need help with.</td>
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<td>f. I try to help teachers understand my role as mathematics coach.</td>
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<td>g. I encourage teachers to include algebraic thinking in the lessons on number sense and operations.</td>
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<td>h. I do not alter the coaching plan developed with the teacher at the beginning of the school year.</td>
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</table>
i. I help teachers identify consistencies and inconsistencies between their own practices and the practices recommended by the National Council of Teachers of Mathematics.

j. I work with principals or other administrators to form a clear message to teachers about effective mathematics instruction.

5. Please indicate the degree to which each of the following statements is descriptive of your coaching practices, from 1 (not at all descriptive) to 7 (very descriptive).

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<th>i.</th>
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6. Given the following scenario, please select the response that you feel most closely answers the question.

A coach has been working with a teacher on using base-10 pieces for subtraction with regrouping. After several discussions, the coach feels that the teacher still doesn’t understand this model and suspects that the teacher needs to take more personal responsibility for conceptual understanding of this topic. In their next meeting, the coach asks the teacher for a verbal explanation of the model. In the explanation, the teacher clearly understands how to remove base-10 pieces as they correspond to subtraction, but the teacher is confused about how to exchange larger place values for smaller ones. Which is the most powerful response to help the teacher take ownership of developing a personal knowledge base?

- a. “You’re almost right!” followed by a clear explanation of exchanges.
- b. “Let me paraphrase your explanation,” followed by a clear restatement of the teacher’s approach.
- c. “It’s clear you struggle with some aspects of exchanges. Let’s go through this again together.”
- d. “You confused me during your explanation about the exchanges. Can you provide a better explanation that helps me clear up that confusion?”

7. Given the following scenario, please select the response that you feel most closely answers the question.

A coach has watched a teacher teach a lesson on ordering fractions. During the short seatwork time, the coach noticed that many of the students were ordering the fractions based on the size of the numerators without considering the denominators. The teacher lectured for most of the class period and did not elicit student comments or examine student thinking. From a coaching perspective, what is the most important consideration?

- a. Making sure that this student misconception is addressed so that students don’t leave this class with wrong thinking.
- b. Making sure that the teacher engages with the coach in a conversation about student thinking and learning.
- c. Making sure that the teacher uses formative assessment strategies in the next class taught.
- d. Making sure that the lesson is retaught.

8. Given the following scenario, please select the response that you feel most closely answers the question.

A veteran sixth-grade teacher tells an instructional coach that mathematics-coaching services are not wanted and definitely not needed. This middle-school teacher of 25 years says retirement is soon approaching and, therefore, changes in instruction “will not occur.” The teacher continues by saying, “I already use the tried-and-true method, which I am more than happy with.” The teacher describes this method as: first, answering homework questions; second, introducing a new topic; third, giving the students some seatwork on the new topic; and fourth, walking around to check the students’ seatwork. The teacher says, “If I notice students struggling with the new topic, I’ll go over it again.” The teacher declares this method to be the “best approach anyone has ever come up with” and feels no need to change anything. “Furthermore,” the teacher says, “I will not attend any more useless professional development sessions.” Based on the information in this scenario, which of the following most likely describes how the teacher affirms beliefs about problem students?
a. A process in which the teacher acts according to what the teacher believes peers will approve of.

b. A process in which the teacher steps outside of the situation to reflect on the problem objectively.

c. A process in which the teacher’s beliefs are determined internally.

d. A process in which the teacher’s beliefs are substantiated by reflecting on what works.

9. Given the following scenario, please select the response that you feel most closely answers the question.

After meeting for the first time a day earlier, a coach and teacher mutually agreed that the coach can come to the teacher’s classroom to observe a subtraction lesson. While observing the lesson, the coach notices that the students’ understanding of the importance of place value notions in subtraction is weak. The coach also notices that the teacher’s conceptual understanding of the subtraction with regrouping algorithm appears weak. After the lesson, the teacher says to the coach, “Only about half of the students could correctly perform the subtraction procedure. I think the teacher before me didn’t teach subtraction very well.” What should the coach do next?

a. The coach should postpone the discussion of the teacher’s conceptual understanding because bringing it up this early in their relationship would undermine the trust and rapport in their relationship.

b. The coach should try to teach the next day’s lesson to the same class, modeling the lesson using base-10 blocks for subtraction.

c. The coach should explain to the teacher that the students are weak at the subtraction procedure because the teacher didn’t address the conceptual basis of the algorithm in class, and the coach should recommend resources for the teacher to use in class.

d. The coach should ask if the teacher is more concerned about establishing students’ proficiency with the subtraction algorithm or establishing their conceptual understanding.

10. Given the following scenario, please select the response that you feel most closely answers the question.

A coach and teacher have discussed a teaching strategy in detail. The coach feels that the teacher knows enough about the strategy to implement it, and the teacher has developed a plan to implement it. At this point, the coach should:

a. Develop a plan with the teacher for continued coaching support on the strategy and the possible modeling of the strategy.

b. Leave the teacher alone to try it out a few times so the teacher can grow comfortable with the strategy and gain ownership of it.

c. Check on the teacher occasionally to make sure the teacher is using the strategy.

d. Wait for the teacher to ask for further support to avoid appearing “pushy.”
11. Please select the response that you feel most closely answers the question. Which of the following is true about teacher learning?
   ○ a. Teachers come to us with fixed intelligence.
   ○ b. Teacher traits such as intelligence can be influenced by coaches.
   ○ c. Teachers are born with traits such as intelligence that cannot be changed.
   ○ d. Coaches should differentiate coaching based on teacher intelligence quotient assessments.

12. Please select the response that you feel most closely answers the question. Which of the following is true about teachers and professional development without a coaching component?
   ○ a. About 10 percent of teachers remember what is presented in professional development, and 5 percent implement it.
   ○ b. Most teachers remember what is presented in professional development, but only about 5 percent implement it.
   ○ c. About 50 percent of teachers remember what is presented in professional development, and about 50 percent implement it.
   ○ d. Most teachers will try the new teaching strategy once.

You have reached the end of the Coaching Knowledge Survey. If you are finished, please submit your responses according to your instructions.

APPENDIX D

COACHING SKILLS INVENTORY (CSI)

The following survey is intended to measure your perspective on your own level of effectiveness or confidence with various coaching responsibilities.

Enter your name or ID code:

For each of the following 24 questions, please rate the items on a scale from 1 to 5 based on how effective (or confident) you are with the various coaching functions, with 1 meaning not at all effective (or confident) and 5 meaning very effective (or confident).

I. Coach/Teacher Relationships

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<tbody>
<tr>
<td>1</td>
<td>How effective do you feel observing lessons and giving teachers feedback?</td>
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<td>2</td>
<td>How effective do you feel creating environments where teachers reflect openly on their instructional practices?</td>
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<td>3</td>
<td>How effective do you feel helping teachers set goals and objectives aimed at improving their instruction?</td>
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<td>4</td>
<td>How effective do you feel creating an environment of open discussion and constructive criticism with teachers?</td>
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II. Coaching Skills

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<tr>
<td>5</td>
<td>How effective do you feel modeling instruction for teachers?</td>
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<td>6</td>
<td>How effective do you feel coaching teachers on mathematical content?</td>
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<td>7</td>
<td>How effective do you feel coaching teachers on general (not necessarily mathematics-specific) pedagogy? (Examples of general pedagogy include but are not limited to engaging students, use of questioning)</td>
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strategies, use of cooperative learning, and classroom management.)

8. How effective do you feel coaching teachers on mathematics-specific pedagogy? (Examples of mathematics-specific pedagogy include but are not limited to incorporating inquiry, discovery or investigative mathematics into lessons, and incorporating problem-solving and conceptual understanding into lessons.)

III. Mathematics Content

9. How confident are you with the mathematics taught at the grade levels that you coach?

10. How confident are you with the mathematical reasoning behind mathematics taught at the grade levels that you coach, meaning the understanding of why we teach it, how it relates to other mathematics topics, and why it is valid?

11. How effective do you feel coaching teachers on number sense and computation topics relevant to their classrooms?

IV. Mathematics-Specific Pedagogy

12. How effective do you feel coaching teachers on creating and using mathematical applications and connections for/in their mathematics classes?

13. How effective do you feel coaching teachers on incorporating mathematics conceptual understanding into their lessons?

14. How effective do you feel coaching teachers on incorporating genuine mathematical problem solving into their lessons?

15. How effective do you feel coaching teachers on incorporating investigative, inquiry-based or discovery-based mathematics learning into their lessons?

16. How effective do you feel coaching teachers on engaging students in mathematical abstraction or sense making?

V. General Pedagogy

17. How effective do you feel coaching teachers on “reading” or detecting students’ levels of understanding?

18. How effective do you feel coaching teachers on using questioning strategies such as higher-order questioning, open questions or wait time?

19. How effective do you feel coaching teachers on encouraging intellectual rigor, constructive criticism or challenging of ideas?

20. How effective do you feel coaching teachers on encouraging student participation?

21. How effective do you feel coaching teachers on using strategies to
increase student collaboration or dialogue among students?

22. How effective do you feel coaching teachers on creating an environment where students listen to one another?

23. How effective do you feel coaching teachers on the use of cooperative learning?

24. How effective do you feel coaching teachers on classroom management?

The following 20 questions are about your background and practices as an educator.

25. What is the highest degree that you hold? (Choose one)
   - [ ] BA or BS
   - [ ] MA, MS, or MEd
   - [ ] Multiple MA, MS, or MEd
   - [ ] PhD or EdD
   - [ ] Other (describe)

26. Please indicate the number of mathematics content courses (not including methods courses) that you completed as part of your collegiate study for the bachelor’s degree, and please identify the highest-level math course you took. (Number of courses (your best estimate, in numeral form, such as “0” or “3”):)

   Highest-level math course taken (For example, “college algebra.” Enter “none” if that applies):

27. If you have earned a graduate degree, please indicate the number of mathematics content courses (not including methods courses) that you completed as part of your collegiate study for that degree, and please identify the highest-level math course you took. (Skip this question if you do not have a graduate degree.) (Number of courses (your best estimate, in numeral form, such as “0” or “2”):)

   Highest-level math course taken (For example, “calculus.” Enter “none” if that applies):

28. Which of the following are true about your field(s) of study for the bachelor’s degree? (Choose all that apply)
   - [ ] I have a major in mathematics
   - [ ] I have a major in a mathematics-intensive field (e.g., engineering, statistics, physics)
   - [ ] I have a major in another field
   - [ ] I have a minor in mathematics
   - [ ] I have a minor in a mathematics-intensive field (e.g., engineering, statistics, physics)
   - [ ] I have a minor in another field
29. What was your major field of study for the bachelor's degree?
   o Elementary education
   o Secondary education: mathematics
   o Secondary education: mathematics-intensive field (e.g., physics, chemistry)
   o Secondary education: other field
   o Mathematics
   o Other mathematics-intensive field (e.g., engineering, statistics, physics)
   o Other (please specify)

30. If you have a graduate degree as well, what was your major field of study for that degree?
   o I don't have a graduate degree
   o Elementary education
   o Secondary education: mathematics
   o Secondary education: mathematics-intensive field (e.g., physics, chemistry)
   o Secondary education: other field
   o Mathematics
   o Other mathematics-intensive field (e.g., engineering, statistics, physics)
   o Other (please specify)

31. Including this school year, and rounding up to a whole numeral ...
   a. How many school years have you taught (not coached) on a full-time basis at any grade level within grades K-12? (numeral, 0 or above)
   b. How many of those school years included teaching at any grade level within grades K-8? (numeral, 0 or above)
   c. How many of those school years included teaching mathematics at any grade level within grades K-8? (numeral, 0 or above)

32. Which of the following best describes your current assignment?
   o a. I was hired specifically as an instructional coach to work with teachers.
   o b. I am a classroom teacher who also coaches other classroom teachers.
   o c. I have multiple responsibilities that include coaching other classroom teachers but not working as a classroom teacher.
   o d. None of the above.
33. Including this school year to date, and rounding up to a whole numeral ...
   a. How many school years have you served as a coach in one or more schools within grades K-12? (numeral, 0 or above) [ ]
   b. How many of those school years included coaching teachers at any grade level within grades K-8? (numeral, 0 or above) [ ]
   c. How many of those school years included coaching teachers of mathematics at any grade level within grades K-8? (numeral, 0 or above) [ ]

34. Including this school year to date and rounding up to a whole numeral, how many years have you worked in your present position as a coach in your present school district? (numeral, 0 or above) [ ]

35. At what levels are you certified to teach? (Choose all that apply) [ ]
   o Not certified at this time
   o Elementary education
   o Middle-level education
   o Secondary education [ ]

36. Do you hold a specific certificate or endorsement for teaching mathematics?
   o Yes
   o No

37. Do you hold a specific certificate or endorsement for coaching teachers?
   o Yes
   o No

38. Please indicate the number of teachers that you currently coach. (numeral, 0 or above) [ ]

39. For many, a typical coaching session has three primary components: 1) the pre-lesson conference; 2) the lesson observation; and 3) the post-lesson conference. In the spaces provided, please indicate the length (in minutes) for each component of a typical coaching session that you conduct with a teacher, and briefly describe each component.
   Component: [ ]
   Number of Minutes (numeral): [ ]
   Component: [ ]
   Number of Minutes (numeral): [ ]
40. During a complete school year, how many coaching sessions (entered as a numeral) do you typically conduct with each teacher assigned to you? 

41. Please indicate the number and names of mathematics courses that you have completed as an enrolled student in the past 12 months.

   Number of courses (numeral, 0 or above): 

   Name(s) of courses (if none, leave blank): 

42. Please indicate (in numeral form) the number of hours of mathematics professional development that you have participated in over the past 12 months (not including mathematics instructional coaching). If none, enter “0.” 

43. Your gender: 
   - I prefer not to answer
   - Female
   - Male

44. Which of the following best describes your race/ethnicity? (Choose one)
   - I prefer not to answer
   - African-American (not of Hispanic origin)
   - American Indian or Alaskan Native
   - Asian or Pacific Islander
   - Hispanic
   - White (not of Hispanic origin)
   - Other (please specify)
You have reached the end of the Coaching Skills Inventory. If you are finished, please submit your responses according to your instructions.

APPENDIX E

Learning Mathematics for Teaching (LMT)

Content Knowledge for Teaching Mathematics Measures (MKT measures) Released Items, 2008

Sample

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sisters select as being true?

(Mark YES, NO, or I’M NOT SURE for each item below.)

a) 0 is an even number.

b) 0 is not really a number. It is a placeholder in writing big numbers.

c) The number 8 can be written as 008.

4. Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

a) Four is an even number, and odd numbers are not divisible by even numbers.

b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).

c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.

d) It only works when the sum of the last two digits is an even number.
5. Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)

a) 5/4
b) 5/3
c) 5/8
d) 1/4

KNOWLEDGE OF CONTENT AND STUDENTS ITEMS

11. Students in Mr. Hayes’ class have been working on putting decimals in order. Three students — Andy, Clara, and Keisha — presented 1.1, 12, 48, 102, 31.3, .676 as decimals ordered from least to greatest. What error are these students making? (Mark ONE answer.)

a) They are ignoring place value.
b) They are ignoring the decimal point.
c) They are guessing.
d) They have forgotten their numbers between 0 and 1.
e) They are making all of the above errors.

12. You are working individually with Bonny, and you ask her to count out 23 checkers, which she does successfully. You then ask her to show you how many checkers are represented by the 3 in 23, and she counts out 3 checkers. Then you ask her to show you how many checkers are represented by the 2 in 23, and she counts out 2 checkers. What problem is Bonny having here? (Mark ONE answer.)

a) Bonny doesn’t know how large 23 is.
b) Bonny thinks that 2 and 20 are the same.
c) Bonny doesn’t understand the meaning of the places in the numeral 23.
d) All of the above.
22. Ms. Brockton assigned the following problem to her students:
How many 4s are there in 3?
When her students struggled to find a solution, she decided to use a sequence of examples to help them understand how to solve this problem. Which of the following sequences of examples would be best to use to help her students understand how to solve the original problem? (Circle ONE answer.)

a) How many: 4s in 6? 4s in 5? 4s in 4? 4s in 3?

b) How many: 4s in 8? 4s in 6? 4s in 1? 4s in 3?

c) How many: 4s in 1? 4s in 2? 4s in 4? 4s in 3?

d) How many: 4s in 12? 4s in 8? 4s in 4? 4s in 3?

23. Ms. Williams plans to give the following problem to her class: Baker Joe is making apple tarts. If he uses of an apple for each tart, how many tarts can he make with 15 apples?
Because it has been a while since the class has worked with fractions, she decides to prepare her students by first giving them a simpler version of this same type of problem. Which of the following would be most useful for preparing the class to work on this problem? (Circle ONE answer.)

I. Baker Ted is making pumpkin pies. He has 8 pumpkins in his basket. If he uses of his pumpkins per pie, how many pumpkins does he use in each pie?

II. Baker Ted is making pumpkin pies. If he uses of a pumpkin for each 4 pie, how many pies can he make with 9 pumpkins?

III. Baker Ted is making pumpkin pies. If he uses of a pumpkin for each 4 pie, how many pies can he make with 10 pumpkins?

a) I only

b) II only

c) III only

d) II and III only

e) I, II, and III
### APPENDIX F

**TASK ANALYSIS GUIDE (TAG; STEIN, SMITH, HENNINGSEN, & SILVER, 2009)**

<table>
<thead>
<tr>
<th>Lower-Level Demands</th>
<th>Higher-Level Demands</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Memorization Tasks</strong></td>
<td><strong>Procedures With Connections Tasks</strong></td>
</tr>
<tr>
<td>Involves either producing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory. Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. Have no connection to the concepts or meaning that underlies the facts, rules, formulae, or definitions being learned or reproduced.</td>
<td>Focus students’ attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. Suggest pathways to follow (explicitly or implicitly) that are broad, general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.</td>
</tr>
</tbody>
</table>

| **Procedures Without Connections** | **Doing Mathematics** |
| **(II)** | **(IV)** |
| Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. Have no connection to the concepts or meaning that underlies the procedure being used. Are focused on producing correct answers rather than developing mathematical understanding. Require no explanations, or explanations that focus solely on describing the procedure that was used. | Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships. Demands self-monitoring or self-regulation of one’s own cognitive processes. Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task. Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required. |
APPENDIX G

INSTRUCTIONAL QUALITY ASSESSMENT CLASSROOM OBSERVATION TOOL

COVER PAGE – COMPLETE FOR EACH LESSON AND ATTACH TO FIELD NOTES, COPY OF INSTRUCTIONAL TASK, AND SCORE SHEET

Background Information
Date of observation: _____________________ Start Time: ____________________________ District: ____________________________
Grade: ____________________________

Classroom Context
Observer: ____________________ End Time: ___________________
School: ______________________ Day 1 or Day 2 ________________
Total number of students in the classroom: ____________________________________ Boys _____ Girls ______
Sketch of seating arrangement(s):

Mathematical Topic of the Lesson:

Field Notes (attach).
IQA Mathematics Lesson Observation Rubrics and Checklists, Melissa Boston ©2012. For permission to use, contact Melissa Boston, bostonm@duq.edu, 412-396-6109
### RUBRIC 1: Potential of the Task

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
</table>
| 4     | The task has the potential to engage students in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships, such as:  
- Doing mathematics: using complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example); OR  
- Procedures with connections: applying a broad general procedure that remains closely connected to mathematical concepts.  
  The task must explicitly prompt for evidence of students’ reasoning and understanding.  
  For example, the task **MAY** require students to:  
  - solve a genuine, challenging problem for which students’ reasoning is evident in their work on the task;  
  - develop an explanation for why formulas or procedures work;  
  - identify patterns; form and justify generalizations based on these patterns;  
  - make conjectures and support conclusions with mathematical evidence;  
  - make explicit connections between representations, strategies, or mathematical concepts and procedures.  
  - follow a prescribed procedure in order to explain/illustrate a mathematical concept, process, or relationship. |
| 3     | The task has the potential to engage students in complex thinking or in creating meaning for mathematical concepts, procedures, and/or relationships. However, the task does not warrant a “4” because:  
- the task does not explicitly prompt for evidence of students’ reasoning and understanding.  
- students may be asked to engage in doing mathematics or procedures with connections, but the underlying mathematics in the task is not appropriate for the specific group of students (i.e., too easy or too hard to promote engagement with high-level cognitive demands);  
- students may need to identify patterns but are not pressed for generalizations or justifications;  
- students may be asked to use multiple strategies or representations but the task does not explicitly prompt students to develop connections between them;  
- students may be asked to make conjectures but are not asked to provide mathematical evidence or explanations to support conclusions. |
| 2     | The potential of the task is limited to engaging students in using a procedure that is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task.  
- **There is little ambiguity about what needs to be done and how to do it.**  
- The task does not require students to make connections to the concepts or meaning underlying the procedure being used.  
- **Focus of the task appears to be on producing correct answers rather than developing mathematical understanding (e.g., applying a specific problem solving strategy, practicing a computational algorithm).**  
OR There is evidence that the mathematical content of the task is at least 2 grade-levels below the grade of the students in the class. |
| 1     | The potential of the task is limited to engaging students in memorizing or reproducing facts, rules, formulae, or definitions. The task does not require students to make connections to the concepts or meaning that underlie the facts, rules, formulae, or definitions being memorized or reproduced. |
| 0     | The task requires no mathematical activity. |
| N/A   | Students did not engage in a task. |
### Common Characteristics of Tasks at each Score Level

<table>
<thead>
<tr>
<th>High-Level Cognitive Demands</th>
<th>Low-Level Cognitive Demands</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4</strong> Making Explicit Connections</td>
<td><strong>1</strong> Memorization</td>
</tr>
<tr>
<td>Doing Mathematics or Procedures with Connections (Stein &amp; Smith, 1998) that provide opportunities to reflect on the mathematical work that has been done. Require connections and reasoning to be explicit, visible.</td>
<td>Procedures without connections (Stein &amp; Smith, 1998).</td>
</tr>
<tr>
<td>Problem-solving tasks where students must develop their own solution strategy and explain their thinking.</td>
<td>Focus on correctness of steps in the procedure and on the correctness of the result, but not on making connections or understanding why the procedure works or makes sense to use.</td>
</tr>
<tr>
<td>Tasks that involve multiple strategies or representations, strategy choice, AND require an explanation, justification, comparison, etc.</td>
<td>Algorithmic, rote procedures.</td>
</tr>
<tr>
<td>If a method or procedure is prescribed, the purpose is to develop an understanding of a larger mathematical concept, idea, or procedure, AND the task has explicit requirement to explain “why”, justify, compare, describe the mathematical connections or underlying ideas, etc.</td>
<td>Most tasks that have a single, prescribed strategy or pathway to follow, with no request for explanations of students’ thinking, no opportunities for making sense of the procedure.</td>
</tr>
<tr>
<td>Tasks that provide opportunities to use and discuss more than one strategy or representation simultaneously – i.e., the student switched back and forth between two or more representations in solving and explaining the problem.</td>
<td>Might request explain “how” or “show your steps.”</td>
</tr>
<tr>
<td>Tasks that involve complex, multi-step problems for which students must illustrate their reasoning about mathematics as they solve the task.</td>
<td>Tasks that require students to solve several of the same type of problem – too many problems to spend time thinking about or creating meaning for mathematical ideas or processes.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>3</strong> Making Implicit Connections</th>
<th><strong>2</strong> Procedures without Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doing Mathematics or Procedures with Connections (Stein &amp; Smith, 1998) but do not require the connections and reasoning to be visible.</td>
<td>Problems without connections (Stein &amp; Smith, 1998).</td>
</tr>
<tr>
<td>Problem-solving tasks where students must develop their own solution strategy, but are not asked to explain their thinking.</td>
<td>Focus on correctness of steps in the procedure and on the correctness of the result, but not on making connections or understanding why the procedure works or makes sense to use.</td>
</tr>
<tr>
<td>Tasks that involve more than one strategy and/or representation, BUT do not require an explanation.</td>
<td>Algorithmic, rote procedures.</td>
</tr>
<tr>
<td>If a method or procedure is prescribed, the purpose is to develop an understanding of a larger mathematical concept, idea, or procedure, BUT without explicit requirement to explain “why”, justify, compare, etc.</td>
<td>Most tasks that have a single, prescribed strategy or pathway to follow, with no request for explanations of students’ thinking, no opportunities for making sense of the procedure.</td>
</tr>
<tr>
<td>Tasks that allow students to develop an understanding of a mathematical concept, idea, or procedure, but do not explicitly require students to explain “why”, justify, compare, etc.</td>
<td>Might request explain “how” or “show your steps.”</td>
</tr>
<tr>
<td>Tasks that provide opportunities for problem solving, but either the problem itself or the mathematics in the problem is not complex enough to warrant deep reasoning or reflection on mathematical ideas.</td>
<td>Tasks that require students to solve several of the same type of problem – too many problems to spend time thinking about or creating meaning for mathematical ideas or processes.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>2</strong> Procedures without Connections</th>
<th><strong>1</strong> Memorization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focus on correctness of steps in the procedure and on the correctness of the result, but not on making connections or understanding why the procedure works or makes sense to use.</td>
<td>Memorization of vocabulary, math facts (timed tests).</td>
</tr>
<tr>
<td>Algorithmic, rote procedures.</td>
<td>Most fill-in-the-blank, one-word or one-number questions.</td>
</tr>
</tbody>
</table>

Note-taking.
RUBRIC 2: Implementation of the Task

At what level did the teacher guide students to engage with the task in implementation?

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
</table>
| 4     | Students engaged in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships, such as:  
• Doing mathematics: using complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example); OR  
• Procedures with connections: applying a broad general procedure that remains closely connected to mathematical concepts.  
There is explicit evidence of students’ reasoning and understanding.  
For example, students may have:  
• solved a genuine, challenging problem for which students’ reasoning is evident in their work on the task;  
• developed an explanation for why formulas or procedures work;  
• identified patterns, formed and justified generalizations based on these patterns;  
• made conjectures and supported conclusions with mathematical evidence;  
• made explicit connections between representations, strategies, or mathematical concepts and procedures.  
• followed a prescribed procedure in order to explain/illustrate a mathematical concept, process, or relationship. |
| 3     | Students engaged in complex thinking or in creating meaning for mathematical concepts, procedures, and/or relationships. However, the implementation does not warrant a “4” because:  
• there is no explicit evidence of students’ reasoning and understanding.  
• students engaged in doing mathematics or procedures with connections, but the underlying mathematics in the task was not appropriate for the specific group of students (i.e., too easy or too hard to sustain engagement with high-level cognitive demands);  
• students identified patterns but did not form or justify generalizations;  
• students used multiple strategies or representations but connections between different strategies/representations were not explicitly evident;  
• students made conjectures but did not provide mathematical evidence or explanations to support conclusions. |
| 2     | Students engaged in using a procedure that was either specifically called for or its use was evident based on prior instruction, experience, or placement of the task.  
• There was little ambiguity about what needed to be done and how to do it.  
• Students did not make connections to the concepts or meaning underlying the procedure being used.  
• Focus of the implementation appears to be on producing correct answers rather than developing mathematical understanding (e.g., applying a specific problem solving strategy, practicing a computational algorithm).  
OR There is evidence that the mathematical content of the task is at least 2 grade-levels below the grade of the students in the class. |
| 1     | Students engage in memorizing or reproducing facts, rules, formulae, or definitions. Students do not make connections to the concepts or meaning that underlie the facts, rules, formulae, or definitions being memorized or reproduced. |
| 0     | Students did not engage in mathematical activity. |
| N/A   | The students did not engage in a task. |
### Mathematics Lesson Checklist

Check each box that applies:

<table>
<thead>
<tr>
<th>A</th>
<th>The Lesson <strong>provided</strong> opportunities for students to engage with high-level cognitive demands:</th>
</tr>
</thead>
</table>
| ✔️ Students | o engaged with the task in a way that addressed the teacher’s goals for high-level thinking and reasoning.  
  o communicated mathematically with peers.  
  o had appropriate prior knowledge to engage with the task.  
  o had opportunities to serve as mathematical authority in classroom  
  o had access to resources that supported their engagement with the task. |
| ✔️ Teacher | o supported students to engage with the high-level demands of the task while maintaining the challenge of the task  
  o provided sufficient time to grapple with the demanding aspects of the task and for expanded thinking and reasoning.  
  o held students accountable for high-level products and processes.  
  o provided consistent presses for explanation and meaning.  
  o provided students with sufficient modeling of high-level performance on the task.  
  o provided encouragement for students to make conceptual connections. |
| B | The lesson **did not provide** opportunities for students to engage with high-level cognitive demands: |
| * The task | o expectations were not clear enough to promote students’ engagement with the high-level demands of the task.  
  o was not rigorous enough to support or sustain student engagement in high-level thinking.  
  o was too complex to sustain student engagement in high-level thinking (i.e., students did not have the prior knowledge necessary to engage with the task at a high level).  |
| * The teacher | o Allowed classroom management problems to interfere with students’ opportunities to engage in high-level thinking.  
  o provided a set procedure for solving the task  
  o shifted the focus to procedural aspects of the task or on correctness of the answer rather than on meaning and understanding.  
  o Gave feedback, modeling, or examples that were too directive or did not leave any complex thinking for the student.  
  o Did not press students or hold them accountable for high-level products and processes or for explanations and meaning.  
  o Did not give students enough time to deeply engage with the task or to complete the task to the extent that was expected.  
  o Did not provide students access to resources necessary to engage with the task at a high level. |

<table>
<thead>
<tr>
<th>C</th>
<th>The <strong>Discussion</strong> provides opportunities for students to engage with the high-level demands of the task.</th>
</tr>
</thead>
</table>
| Students: | o Use multiple strategies and make explicit connections or comparisons between these strategies, or explain why they choose one strategy over another.  
  o use or discuss multiple representations and make connections between different representations or between the representation and their strategy, underlying mathematical ideas, and/or the context of the problem  
  o identify patterns or make conjectures, predictions, or estimates that are well grounded in underlying mathematical concepts or evidence.  
  o generate evidence to test their conjectures. Students use this evidence to generalize mathematical relationships, properties, formulas, or procedures.  
  o (rather than the teacher) determine the validity of answers, strategies or ideas. |
**Part 3: Scoring Sheet**

COMPLETE A SCORING SHEET FOR EACH OBSERVATION. CONNECT TO COVER SHEET

Observer: ______________________ Lesson Code: ______________________

<table>
<thead>
<tr>
<th>Academic Rigor</th>
<th>Rater 1</th>
<th>Rater 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rubric 1: Potential of the Task</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rubric 2: Implementation of the Task</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rubric 3: Student Discussion Following the Task</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Rubric AR-Q: Questioning</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Rubric AR-X: Mathematical Residue</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Accountable Talk</th>
<th>Rater 1</th>
<th>Rater 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rubric AT1: Participation</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Rubric AT2: Teacher’s Linking</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Rubric AT3: Students’ Linking</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Rubric AT4: Asking (Teacher Press)</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Rubric AT5: Providing (Student Responses)</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

IQA Mathematics Lesson Observation Rubrics and Checklists, Melissa Boston ©2012. For permission to use, contact Melissa Boston, bostonm@duq.edu, 412-396-6109
APPENDIX H
REFLECTIVE JOURNAL ENTRIES COMPILATION

1. Send me the 2 lessons plans (high-quality tasks) that will be implemented in a classroom. If this is not part of your role, please indicate so and explain.

2. Keep a 1-week, Monday through Friday daily log of what your day-to-day activities entail. (e.g. data meeting, meeting off campus, administrative position, coaching cycles, classroom assistance, modeling etc.) Please indicate which subject you assisted with if necessary and give as much information as possible that you would like to share.

3. The EMS will select one of the tasks or lessons they chose to submit and explain their rationale for selecting the task as a high-quality task as well as reflect on how the task was implemented in the elementary mathematics classroom.

4. The EMS will submit a reflection of how their mathematical beliefs aligned or did not align with the implementation of the lesson plan.

5. The EMS will submit a reflection of how their beliefs about teaching mathematics may or may not have any relation to the EMS’s instructional pedagogy in mathematics.
APPENDIX I

LESSONS PLAN TEMPLATE

Lesson/Task 1 Title:

Objective:

Standard:

Before/Opening of the lesson:
Lesson/Task 2:

Objective:

Standard:

Before/Opening of the lesson:

During the lesson:
After/Closing the lesson:

Role:

Subject Area:

Grade Level:
APPENDIX J

WEEKLY REFLECTIVE JOURNAL ENTRY TEMPLATE

<table>
<thead>
<tr>
<th>Times</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>7:00-8:00</td>
<td></td>
<td></td>
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<tr>
<td>8:00-9:00</td>
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<td>9:00-10:00</td>
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<td>10:00-11:00</td>
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<tr>
<td>11:00-12:00</td>
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<tr>
<td>12:00-1:00</td>
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<tr>
<td>1:00-2:00</td>
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<td>2:00-3:00</td>
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<tr>
<td>3:00-4:00</td>
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<tr>
<td>4:00-5:00</td>
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</tr>
</tbody>
</table>

Some of the things that you can elaborate on are the following:

- If you spent time in a classroom coaching a teacher in any subject area
- If you spent time in a classroom co-teaching any subject
- If you spent any time in a classroom modeling something for the teacher
- If you helped with assessments in the school
- If you helped the administrator with job
- If you went to a meeting and what type of meeting did you go to?
- If you had data or team meetings at your school
- If you are shared by more than one school and had to go from one to the other
- If you have to facilitate professional development for anyone (to whom and what was it about?)

(Chval et al., 2010; Fullan & Pomfet, 1977; Mudzimiri et al., 2014; Neufeld & Roper, 2003; Staub et al., 2003)
APPENDIX K

RATIONALE FOR SELECTING AND IMPLEMENTING A HIGH-QUALITY MATHEMATICS TASK

1. Reflect on how you selected the mathematics task. How did you choose it?

2. What helped you decide it was a high-quality mathematical task?
3. What criteria made it a high-quality mathematical task?

4. How did you implement the task?
5. How did you feel the task was implemented?
APPENDIX L

BELIEFS ABOUT TEACHING MATHEMATICS

1. How would you describe a teacher who is successfully teaching mathematics?

2. What would the classroom look and sound like during a mathematics lesson? (What are you as the teacher doing? What are the students doing?)
3. Describe yourself teaching mathematics today.

4. How do you support a teacher who is not teaching mathematics successfully?
5. What resources do you use to support a teacher who is struggling?

6. What instructional support do you provide to teachers who struggle with teaching mathematics?
APPENDIX M

FOLLOW-UP INFORMAL INTERVIEW QUESTIONS (AUDIOTAPED)

What are the necessary knowledge and skills of an elementary mathematics specialist?
   What type of knowledge do you need?
   What specific instructional strategies do you need?
   What specific leadership skills do you need?

What is the relationship between mathematics content knowledge and pedagogy?

   Sub-questions:
   What was your preparation for your position as an elementary mathematics specialist?

   How has your preparation assisted you in supporting teachers with mathematics content
   and their instructional practice?

   If so, please provide some examples to support this. If not, please provide some rationale
   for supporting this.

During the 2016-2017 school year, estimate the amount of professional development hours you
have spent with the teachers at your school in the following settings: (round to the nearest .5
hour)

   ______ whole school faculty
   ______ grade level or content specific group
   ______ individualized assistance

What do you believe is good mathematics instruction? How do you support this expectation?
Please provide specific examples.
### APPENDIX N

**COACHING KNOWLEDGE SKILLS DESCRIPTIVE SURVEY ITEMS 1-49**

<table>
<thead>
<tr>
<th>Question</th>
<th>Mean</th>
<th>Mode</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a. An effective mathematics coach coaches only on teacher-stated needs.</td>
<td>5.666</td>
<td>6</td>
<td>0.9271</td>
</tr>
<tr>
<td>b. Beginning teachers need more coaching than 25-year veterans.</td>
<td>4.871</td>
<td>6</td>
<td>1.7648</td>
</tr>
<tr>
<td>c. When a teacher says that she or he doesn’t want any coaching, an</td>
<td>4.564</td>
<td>5</td>
<td>1.5354</td>
</tr>
<tr>
<td>effective mathematics coach respectfully does not try to persuade the teacher to accept coaching.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. Sometimes an effective mathematics coach has to oppose school or</td>
<td>2.487</td>
<td>2</td>
<td>1.1441</td>
</tr>
<tr>
<td>teacher actions that are not good for students’ mathematics learning.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e. Teachers will adapt to whatever method of coaching is used.</td>
<td>4.717</td>
<td>5</td>
<td>1.2762</td>
</tr>
<tr>
<td>f. An effective mathematics coach gets input from a school’s principal on which teachers need to improve their mathematics instruction.</td>
<td>2.410</td>
<td>3</td>
<td>1.1172</td>
</tr>
</tbody>
</table>
g. Number sense is a prerequisite for algebraic thinking.

h. A coach should put no pressure on teachers to improve their practices.

i. In general, teachers need coaches to model a lesson with a particular strategy before they will incorporate it with fidelity.

j. A teacher can learn new mathematics, but the teacher’s basic mathematical intelligence cannot be changed.

2a. Once a teacher knows about a research-based strategy for improving student learning, the teacher will begin using the strategy.

b. An effective mathematics coach provides teachers with an understanding of how the mathematics they teach supports learning beyond the grade level they teach.

c. An effective mathematics coach uses state mathematics assessment data when developing a coaching plan with
teachers.

d. An effective mathematics coach asks the principal what she or he believes the teachers’ needs are.
ed. A student’s intelligence can be changed through excellent teaching.
f. Teachers generally have similar teaching styles.
g. When a teacher says something that isn’t quite mathematically correct, an effective mathematics coach says, “You are almost right,” and then gives the teacher a clear explanation of the correct mathematics.
h. An effective coach sticks to the coaching objectives established with a teacher at the beginning of the school year.
i. Teachers can influence students’ learning styles.
j. An effective mathematics coach gives feedback to the principal about teachers who are struggling in the classroom.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Mean Dislike</th>
<th>Mean Like</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>d. An effective mathematics coach asks the principal what she or he believes the teachers’ needs are.</td>
<td>2.461</td>
<td>2</td>
<td>1.3148</td>
</tr>
<tr>
<td>e. A student’s intelligence can be changed through excellent teaching.</td>
<td>2.157</td>
<td>2</td>
<td>1.1746</td>
</tr>
<tr>
<td>f. Teachers generally have similar teaching styles.</td>
<td>5.512</td>
<td>6</td>
<td>1.3351</td>
</tr>
<tr>
<td>g. When a teacher says something that isn’t quite mathematically correct, an effective mathematics coach says, “You are almost right,” and then gives the teacher a clear explanation of the correct mathematics.</td>
<td>3.435</td>
<td>2</td>
<td>1.6188</td>
</tr>
<tr>
<td>h. An effective coach sticks to the coaching objectives established with a teacher at the beginning of the school year.</td>
<td>4.102</td>
<td>3</td>
<td>1.4831</td>
</tr>
<tr>
<td>i. Teachers can influence students’ learning styles.</td>
<td>2.897</td>
<td>3</td>
<td>1.3532</td>
</tr>
<tr>
<td>j. An effective mathematics coach gives feedback to the principal about teachers who are struggling in the classroom.</td>
<td>3.128</td>
<td>3</td>
<td>1.3412</td>
</tr>
</tbody>
</table>

1= Strongly Agree, 2=Agree, 3=More Agree Than Disagree, 4=Neither Agree Nor Disagree, 5=More Disagree Than Agree, 6=Disagree, 7=Strongly Disagree
<table>
<thead>
<tr>
<th>Items</th>
<th>Mean</th>
<th>Mode</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3a. When a teacher says something I find confusing, I paraphrase what I heard and say it back to her or him.</td>
<td>1.974</td>
<td>2</td>
<td>0.8106</td>
</tr>
<tr>
<td>b. I collect students’ mathematics work from a teacher’s classroom to guide our coaching conversations.</td>
<td>1.948</td>
<td>2</td>
<td>0.9444</td>
</tr>
<tr>
<td>c. When decisions about mathematics instruction are being made, I ensure that the decision-makers interpret research literature accurately.</td>
<td>2.205</td>
<td>2</td>
<td>0.8638</td>
</tr>
<tr>
<td>d. I coach teachers on needs that I observe in the teacher, even when the teacher is unaware of these needs.</td>
<td>2.256</td>
<td>2</td>
<td>0.9925</td>
</tr>
<tr>
<td>e. As a mathematics coach, I support mathematics teachers by tutoring their struggling students.</td>
<td>4.307</td>
<td>2</td>
<td>1.7343</td>
</tr>
<tr>
<td>f. I have difficult conversations with teachers, when necessary, about mathematics misconceptions they hold.</td>
<td>2.078</td>
<td>2</td>
<td>0.9967</td>
</tr>
<tr>
<td>g. I always make sure that coaching conversations with mathematics teachers are grounded in the mathematics content.</td>
<td>2.157</td>
<td>2</td>
<td>1.2417</td>
</tr>
<tr>
<td>h. I meet with the principal to discuss the school’s vision for mathematics instruction.</td>
<td>1.743</td>
<td>2</td>
<td>0.7151</td>
</tr>
<tr>
<td>i. I encourage teachers to include, in each lesson they teach, summaries of what students learned</td>
<td>2.794</td>
<td>2</td>
<td>1.3607</td>
</tr>
<tr>
<td>j. I provide feedback to teachers about whether or not the school is meeting its vision for mathematics instruction.</td>
<td>2.552</td>
<td>2</td>
<td>1.3496</td>
</tr>
<tr>
<td>4a. I try to provide the teachers I coach with an understanding of how the mathematics they teach supports learning beyond the grade level they teach.</td>
<td>1.578</td>
<td>1, 2</td>
<td>0.5987</td>
</tr>
<tr>
<td>b. I ask the principal what he or she believes the mathematics teachers’ needs are.</td>
<td>2.459</td>
<td>2</td>
<td>1.0433</td>
</tr>
<tr>
<td>c. I encourage the teachers I coach to reflect on similarities and differences among mathematics topics in the curriculum.</td>
<td>2.305</td>
<td>2</td>
<td>1.0090</td>
</tr>
<tr>
<td>d. I help teachers plan their lessons.</td>
<td>2.189</td>
<td>2</td>
<td>1.0230</td>
</tr>
<tr>
<td>e. I ask the teachers I coach what aspects of mathematics teaching they need help with.</td>
<td>1.837</td>
<td>1, 2</td>
<td>0.8979</td>
</tr>
<tr>
<td>f. I try to help teachers understand my role as mathematics coach.</td>
<td>1.7837</td>
<td>2</td>
<td>0.7124</td>
</tr>
<tr>
<td>g. I encourage teachers to include algebraic thinking in the lessons on number sense and operations.</td>
<td>2.243</td>
<td>2</td>
<td>1.1156</td>
</tr>
<tr>
<td>h. I do not alter the coaching plan developed with the teacher at the beginning of the school year.</td>
<td>5.5135</td>
<td>5, 6</td>
<td>0.9315</td>
</tr>
<tr>
<td>i. I help teachers identify consistencies and inconsistencies between their own practices and the practices recommended by the National Council of Teachers of Mathematics.</td>
<td>2.8108</td>
<td>2</td>
<td>1.0498</td>
</tr>
</tbody>
</table>
j. I work with principals or other administrators to form a clear message to teachers about effective mathematics instruction.  
5a. When a teacher says something I find confusing, I say, “That confused me,” and ask the teacher to rethink it.  
b. I help teachers reflect on discrepancies between espoused beliefs and actual practices.  
c. I take precautions to ensure that my demonstration lessons do not inadvertently send a message that I am the expert and the teacher is not.  
d. I reflect on state assessment data to identify curriculum areas that need to be strengthened.  
e. I use student work when coaching mathematics teachers.  
f. I provide feedback to the principal about whether or not the school is meeting its vision for mathematics instruction.  
g. I encourage teachers to set personal improvement goals for mathematics instruction.  
h. When a teacher complains about the school’s vision for mathematics, I ask the teacher about her or his vision for mathematics.  
i. I coach my newer teachers more than experienced ones.

<table>
<thead>
<tr>
<th>Item</th>
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<th>Code</th>
<th>Standard Error</th>
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<tr>
<td>j</td>
<td>2.000</td>
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<td>5a</td>
<td>4.702</td>
<td>5, 6</td>
<td>1.5962</td>
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<td>b</td>
<td>2.4324</td>
<td>2</td>
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<tr>
<td>c</td>
<td>1.702</td>
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<td>0.6610</td>
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<tr>
<td>d</td>
<td>1.621</td>
<td>1, 2</td>
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<td>e</td>
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<td>f</td>
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<td>g</td>
<td>2.108</td>
<td>2</td>
<td>0.9656</td>
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<td>h</td>
<td>2.783</td>
<td>2</td>
<td>1.2278</td>
</tr>
<tr>
<td>i</td>
<td>3.729</td>
<td>3</td>
<td>1.6098</td>
</tr>
</tbody>
</table>

1=Very Descriptive, 2=Descriptive 3=More Descriptive Than Not Descriptive, 4=Neither Descriptive Nor Not Descriptive, 5=More Not Descriptive Than Descriptive, 6=Not Descriptive, 7=Not Descriptive At All
APPENDIX O

COACHING SKILLS INVENTORY SURVEY

<table>
<thead>
<tr>
<th>Question</th>
<th>Mean</th>
<th>Mode</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How effective do you feel observing lessons and giving teachers feedback?</td>
<td>4.057</td>
<td>4</td>
<td>0.7647</td>
</tr>
<tr>
<td>2. How effective do you feel creating environments where teachers reflect openly on their instructional practices?</td>
<td>4.057</td>
<td>4</td>
<td>0.6835</td>
</tr>
<tr>
<td>3. How effective do you feel helping teachers set goals and objectives aimed at improving their instruction?</td>
<td>3.857</td>
<td>4</td>
<td>0.7724</td>
</tr>
<tr>
<td>4. How effective do you feel creating an environment of open discussion and constructive criticism with teachers?</td>
<td>3.714</td>
<td>4</td>
<td>1.0166</td>
</tr>
<tr>
<td>5. How effective do you feel modeling instruction for teachers?</td>
<td>4.114</td>
<td>5</td>
<td>0.8667</td>
</tr>
<tr>
<td>Question</td>
<td>Score</td>
<td>Scale</td>
<td>Effect Size</td>
</tr>
<tr>
<td>--------------------------------------------------------------------------</td>
<td>-------</td>
<td>-------</td>
<td>-------------</td>
</tr>
<tr>
<td>6. How effective do you feel coaching teachers on mathematical content?</td>
<td>3.942</td>
<td>4, 5</td>
<td>0.9983</td>
</tr>
<tr>
<td>7. How effective do you feel coaching teachers on general (not necessarily mathematics-specific) pedagogy?</td>
<td>4.205</td>
<td>4, 5</td>
<td>0.8082</td>
</tr>
<tr>
<td>8. How effective do you feel coaching teachers on mathematics-specific pedagogy?</td>
<td>3.914</td>
<td>4</td>
<td>0.9194</td>
</tr>
<tr>
<td>9. How confident are you with the mathematics taught at the grade levels that you coach?</td>
<td>3.8</td>
<td>3</td>
<td>0.9330</td>
</tr>
<tr>
<td>10. How confident are you with the mathematical reasoning behind mathematics taught at the grade levels that you coach, meaning the understanding of <em>why</em> we teach it, <em>how</em> it relates to other mathematics topics, and <em>why</em> it is valid?</td>
<td>3.771</td>
<td>4</td>
<td>0.9102</td>
</tr>
<tr>
<td>11. How effective do you feel coaching teachers on number sense and computation topics relevant to their classrooms?</td>
<td>3.885</td>
<td>3, 4, 5</td>
<td>0.9321</td>
</tr>
</tbody>
</table>
12. How effective do you feel coaching teachers on creating and using mathematical applications and connections for/in their mathematics classes? 3.542 3 0.9185

13. How effective do you feel coaching teachers on incorporating mathematics conceptual understanding into their lessons? 3.742 4 0.8520

14. How effective do you feel coaching teachers on incorporating genuine mathematical problem solving into their lessons? 3.771 4 0.8773

15. How effective do you feel coaching teachers on incorporating investigative, inquiry-based or discovery-based mathematics learning into their lessons? 3.571 4 1.0371

16. How effective do you feel coaching teachers on engaging students in mathematical abstraction or sense making? 3.571 4 0.9167

17. How effective do you feel coaching teachers on “reading” or detecting students’ levels of understanding? 3.571 4 0.9787
<table>
<thead>
<tr>
<th>Question</th>
<th>Rating</th>
<th>Scale</th>
<th>Effectiveness</th>
</tr>
</thead>
<tbody>
<tr>
<td>18. <strong>How effective do you feel coaching teachers on using questioning strategies such as higher-order questioning, open questions or wait time?</strong></td>
<td>4.000</td>
<td>4</td>
<td>0.9393</td>
</tr>
<tr>
<td>19. <strong>How effective do you feel coaching teachers on encouraging intellectual rigor, constructive criticism or challenging of ideas?</strong></td>
<td>3.714</td>
<td>4</td>
<td>0.5994</td>
</tr>
<tr>
<td>20. <strong>How effective do you feel coaching teachers on encouraging student participation?</strong></td>
<td>4.228</td>
<td>5</td>
<td>0.8075</td>
</tr>
<tr>
<td>21. <strong>How effective do you feel coaching teachers on using strategies to increase student collaboration or dialogue among students?</strong></td>
<td>4.028</td>
<td>4</td>
<td>0.7853</td>
</tr>
<tr>
<td>22. <strong>How effective do you feel coaching teachers on creating an environment where students listen to one another?</strong></td>
<td>4.171</td>
<td>4</td>
<td>0.7853</td>
</tr>
<tr>
<td>23. <strong>How effective do you feel coaching teachers on the use of cooperative learning?</strong></td>
<td>4.085</td>
<td>4</td>
<td>0.8530</td>
</tr>
<tr>
<td>24. <strong>How effective do you feel coaching teachers on classroom management?</strong></td>
<td>3.971</td>
<td>4</td>
<td>0.8219</td>
</tr>
</tbody>
</table>

1= Not at all Effective, 2= Somewhat Effective, 3= Effective, 4= Somewhat Effective, 5= Very Effective
February 13, 2018

Nicolette Nalu
Department of Curriculum & Instruction
College of Education
The University of Alabama
Box 870232

Re: IRB # 17-OR-091 “Elementary Mathematics Instructional Coaches: Prerequisites of a Qualified Mathematics Specialist”

Dear Ms. Nalu:

The University of Alabama Institutional Review Board has granted approval for your renewal application. You have also been granted the requested waiver of documentation of informed consent. Your renewal application has been given expedited approval according to 45 CFR part 46. Approval has been given under expedited review category 7 as outlined below:

(7) Research on individual or group characteristics or behavior (including, but not limited to, research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices, and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies.

Your application will expire on February 12, 2019. If your research will continue beyond this date, complete the relevant portions of Continuing Review and Closure Form. If you wish to modify the application, complete the Modification of an Approved Protocol Form. When the study closes, complete the appropriate portions of FORM: Continuing Review and Closure.

Should you need to submit any further correspondence regarding this proposal, please include the above application number.

Good luck with your research.

Sincerely,

[Redacted]
Director & Research Compliance Officer
Office for Research Compliance

358 Rose Administration Building | Box 870127 | Tuscaloosa, AL 35487-0127
205-348-8461 | Fax 205-348-7189 | Toll Free 1-877-820-3066
Appendix A

THE UNIVERSITY OF ALABAMA
HUMAN RESEARCH PROTECTIONS PROGRAM

Nicolete Nala, Principal Investigator from the University of Alabama, is conducting a study called Elementary Mathematics Instructional Coaches: Prerequisites of a Qualified Mathematics Specialist. She is under the direction of Drs. Diane Sekeres and Stefanie Livers. She wishes to find out: (a) what is the relationship between the Elementary Mathematics Specialists' ability to select a high quality mathematical task and the implementation of a mathematical task? (b) What is the relationship between the Elementary Mathematics Specialists' ability to select a high quality mathematical task and an Elementary Mathematics Specialists' content knowledge for teaching elementary mathematics? (c) What is the relationship between the elementary Mathematics Specialists' beliefs about teaching and coaching mathematics and their instructional practices pertaining to mathematics?

Taking part in this study involves Elementary Mathematics Specialists completing two online surveys that will take approximately 10-25 minutes each. The overall purpose for these surveys is to build a profile of the Elementary Mathematics Specialists (EMSs) serving schools in Alabama. These surveys contain questions about content and pedagogical knowledge for teaching mathematics, leadership skills, beliefs about teaching and coaching mathematics, the nature of high-quality mathematical tasks for developing conceptual understanding, and ways to implement a high-quality mathematical task. Also, a randomly selected subgroup of 2-3 EMSs in each region will be asked to teach two lessons that develop conceptual understanding and submit reflective journal entries. Each lesson will be observed and some teachers will be asked to participate in a follow-up interview.

Your confidentiality will be protected by the researcher. Your name will never be used or appear anywhere in the reports for this work. All data will be locked up in a filing cabinet at the researcher’s home. Only Nicolete Nala will have access to all of the data pieces except for the other approved and certified observation researcher who will have access to only the observational data that she will assist in collecting. The data will be coded with ID codes assigned by Nicolete Nala. Only summarized data will be presented at meetings or in publications. There will be no direct benefits to you other than a Visa gift card for $15 at the end of the study. All of the surveys and necessary documents have been submitted by the predetermined due dates set by the researcher. The findings will be useful to the state of Alabama and the rest of the country for research in elementary mathematics coaching. There are no foreseeable risks although there may be unforeseen risks. The chief risk is that some of the questions and/or activities may make you uncomfortable. You may skip any questions you do not want to answer.

If you have questions about this study, please contact Nicolete Nala at 251-510-2340 or by email at analu@ua.edu or Drs. Stefanie Livers at sfivers@ua.edu or Diane Sekeres at dsekeres@ua.edu. If you have questions about your rights as a research participant, contact Ms. Tanta Myles (the University Compliance Officer) at (205) 348-8461 or toll-free at 1-877-820-3066. If you have complaints or concerns about this study, file them through the UA IRB outreach website at http://osp.ua.edu/site/FRCO_Welcome.html. Also, if you participate, you are encouraged to complete the short Survey for Research Participants online at this website. This helps UA improve its protection of human research participants.

YOUR PARTICIPATION IS COMPLETELY VOLUNTARY. You are free to participate or stop participating any time before you submit your answers.

If you understand the statements above, are at least 19 years old, and freely consent to be in this study, click on the (CONTINUE or I AGREE) button to begin.

UNIVERSITY OF ALABAMA IRB CONSENT FORM APPROVED 2-9-17
EXPIRATION DATE 2-9-17