A Hybrid Model for Baryons

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ABSTRACT

We review the Skyrme model and discuss a method for incorporating quark degrees of freedom into the model.

The correct description of baryons is believed to come from QCD. Since QCD is difficult to work with phenomenological models have been utilized. Two currently popular models are the bag model[1] and Skyrme's topological soliton model.[2] In the bag model the fundamental particles are quarks and gluons and their dynamics is governed by QCD. From asymptotic freedom, perturbative QCD can be applied at high energies or short distances. Confinement is supposed to occur at low energies or large distances and it is incorporated into the bag model by the insertion of the bag. The internal pressure from the quarks is balanced by an ad hoc bag pressure and a classical determination of the energy of the system can be obtained by minimizing $E_{bag} \sim \frac{\alpha}{R} + \beta R^3$ with respect to the bag radius R. [The first term is the quark kinetic energy while the second is the bag energy.]

In Skyrme's model the fundamental particles are mesons, whose dynamics are governed by the non-linear chiral model. The non-linear chiral model is presently believed to be an "effective theory" for QCD which is valid at low energies or large distances. This has been established by Witten in the large N_C limit.[3] Short distances can be corrected for by adding to the Lagrangian higher order derivative terms which can be interpreted as counter terms to the non-linear chiral model. In principle, an infinite number of counter terms should be added since we are dealing with a non-renormalizable theory. Skyrme found long ago that when such higher order terms are present, there exist stable soliton solutions. These solitons (or "skyrmions") were interpreted by Skyrme as baryons. A classical determination of the energy was obtained by minimizing $E_{Skyrme} \sim AR + \frac{C}{R}$ with respect to the soliton "size" R. [The first term is due to the usual non-linear model while the second represents the short distance correction.]

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*Paper presented by A. Stern at Conference on the intersections between Particle and Nuclear Physics, May 23-30, 1984.

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A number of people[4, 5] have considered the possibility of putting these two phenomenological models together. The motivation is 1) the models have complementary regions of validity and 2) the ad hoc terms which were added to the two models and which were necessary for stability may no longer be needed; i.e., the pressure from the quarks inside a bag can be balanced by the skyrmion pressure outside the bag. Here the energy of the system can be determined classically by minimizing $E_{hybrid} \sim \frac{\alpha}{R} + AR$ with respect to R.

Next we review the Skyrme model. The Lagrangian for the SU(2) × SU(2) non-linear chiral model is

$$\mathcal{L} = \frac{F_\pi^2}{16} \mathrm{Tr}(v_\mu)^2 \qquad v_\mu = u^\dagger \partial_\mu u \tag{1}$$

where $u = u(x) \in SU(2)$ and F_π is the pion decay constant (F_π = 186 MeV). The 3 degrees of freedom in u are associated with π mesons. It is well known that there exist no static soliton solutions to (1). Skyrme was able to find a solution after adding the term $\frac{1}{32e^2} \mathrm{Tr}[v_\mu, v_\nu]^2$ to (1), where e is a dimensionless constant. The static Hamiltonian is

$$H = -\frac{F_\pi^2}{16} \mathrm{Tr}(v_i)^2 - \frac{1}{32e^2} \mathrm{Tr}[v_i, v_j]^2 \tag{2}$$

Upon requiring finite energy, $u \to$ const (which we can rotate to 1) as $x \to \infty$. As far as the u field is concerned all points at infinite are identical and spatial R^3 gets compactified to S^3. The field configurations are then classified by $\pi_3(SU(2)) = Z$. The corresponding topological index is

$$t = \frac{1}{24\pi^2} \int d^3x \, \varepsilon_{ijk} \, \mathrm{Tr}(v_i v_j v_k) \tag{3}$$

which is both conserved and an integer. Skyrme identified this integer with baryon number. He constructed a solution of the form

$$u_c(\vec{x}) = \cos\theta(r) + i\tau\cdot\hat{x}\,\sin\theta(r) \tag{4}$$

$$\theta(\infty) = 0 \qquad\qquad \theta(0) = \pi$$

which has t = 1 and was identified as an ordinary baryon.

Thus according to Skyrme the baryon is a condensed state of pions. But pions have spin 0, are bosons and have B = 0, while the composite object is supposed to have spin 1/2, 3/2, fermionic properties and B = 1. The puzzle of spin-statistics was addressed by Finkelstein and Rubenstein[6] and Williams.[7] Their arguments were based on topology and an analogy with the SO(3) rotation group. The puzzle of baryon number was answered by Balachandran, Nair, Rajeev and Stern.[8] In Ref. 8 it was shown that upon introducing fermionic fields and computing the baryon number in the background of a Skyrmion one gets $B \sim t$, where the constant of proportionality depends on the fermionic fields introduced. [It is 1 in the case of 3 quarks.]

The extension to the SU(3) × SU(3) non-linear chiral model was given by Witten.[9] The procedure was highly non-trivial due to addition of a term (called the Wess-Zumino term) to the action which eliminated certain unwanted discrete symmetries and when gauged mimicked QCD triangle anamolies.

Static properties of the Skyrmion were examined by Adkins, Nappi and Witten.[10] Among other things, they introduce a collective variable $A(t)$, $u(\vec{x},t) = A(t)u_C(\vec{x})A^{\dagger}(t)$. $A(t)$ rotates the soliton in both spin and isospin space. Upon substituting into the Lagrangian and quantizing with respect to A, they found a strong coupling-like spectrum $I = J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \cdots$, with the first two states identified as N and Δ. Using m_N and m_Δ as input parameters they computed the coupling constants e and F_π, the latter comes out about 30% off. Other numbers are also in this error range.

One can extend the above analysis to SU(3) × SU(3). The numbers do not improve but one gets an extra effect due to the Wess-Zumino term. This term is similar to the interaction of a charged particle with a magnetic monopole. In analogy to the Dirac quantization condition, one finds that quantization of the Skyrmion can be carried out only if B = integer.

Now let us add the bag model. We will follow the approach of Ref. 5. The quark fields have support in a finite volume V while the pions have support in the complementary region. The interaction between the fields occurs at the boundary ∂V. For the action we take

$$S = \int_V d^4x \; i\bar{q}\partial\!\!\!/ q + \int_{R^4-V} d^4x \, \frac{F_\pi^2}{16} \, \mathrm{Tr}(v_\mu)^2 + \int_{\partial V} d^3s^\mu \, \lambda_\mu(\bar{q}_L u q_R + \bar{q}_R u^\dagger q_L) \tag{5}$$

λ_μ is a Lagrange multiplier.

This model has a couple of difficulties from the start. First, any skyrmion in the outer region is unstable. This is due to the fact that we have cut out a hole in the spatial S^3, so the former topological classification doesn't apply. We can examine a $t = 1$ solution given by (4) with $\theta(\infty) = 0$ and $\theta(R) = \pi$, R being the radius of a spherical bag. If $u_C(\vec{x})$ is dynamical at the surface, $\theta_S \equiv \theta(R)$ can change smoothly from π to 0 and, consequently, the skyrmion decays.

It therefore appears necessary to freeze the value $u_C(\vec{x})$ at the surface ∂V. Actually, we must take $u_C(\vec{x})\big|_{\partial V}$ = const. Hence all points at the surface are identified and we have patched up the hole in S^3. The topological classification is thus the same as the Skyrme model, i.e., $t = B_{Skyrmion}$ = integer. This condition was found necessary when quantizing with the Wess-Zumino term.

The second problem is overcounting. Our picture of the baryon has 3 quarks in a bag with a skyrmion outside the bag. The total baryon number appears to be two. The solution to this problem is that the bag vacuum carries baryon number.[4, 5] The vacuum can have a non-zero baryon number if there is an asymmetry in the quark spectrum. Roughly,

$$B_{vac} = \frac{1}{3} \cdot \frac{1}{2} (N_- - N_+) \tag{6}$$

where $N_-(N_+)$ are the number of negative (positive) energy quark levels. We can regulate this quantity by insisting on $B_{vac} = 0$ when $\theta_S = 0$. The spectrum of quark states when $\theta_S = 0$ is precisely that of the MIT bag model. One obtains the same spectrum for $\theta_S = \pi$, except for a parity flip. Upon varying θ_S continuously from 0 to π one finds that 3 (for 3 colors) negative energy levels from $\theta_S = 0$ cross $E = 0$ (at $\theta_S = \frac{\pi}{2}$) and become the lowest positive energy levels at $\theta_S = \pi$. Substituting into (6), $B_{vac} = -1$ when $\theta_S = \pi$. In general, the spectrum is cyclic in π intervals of θ (except for parity), so $B_{vac} + B_{skyrmion} = 0$. The total baryon number $B = B_q + B_{vac} + B_{skyrmion}$ is thus solely determined from the quark contribution.

At this point there is no correspondence between the baryon number of the quarks and the Skyrmion. This can be remedied by requiring that there are no filled quark levels when $\theta_S = 0$. This is necessary since no skyrmion is present to confine quarks. Further we require that quarks are generated only by increasing θ_S and having filled quark levels pop out of the Dirac sea. Then $B_{quarks} = B_{skyrmion}$.

Finally we come to the numerical predictions of the model. Like Adkins, Nappi and Witten[10] we can introduce collective coordinates $A(t)$ for the skyrmion with $t = 1$. $A(t)$ has no effect on $u_C(\vec{x})$ at the boundary so the quark fields can be ignored. Quantizing the system with respect to A again leads to a strong coupling spectrum. If one tries to compute absolute masses at the simplest level (ignoring Casimir energies of the cavity, center of mass corrections, etc.) one gets the terrible result, $M \cong 4.4$ GeV. Let us instead work with relative baryon masses. Inputing $m_\Delta - m_N$ and F_π we get $R = .351$ fm. We can determine $g_{\pi NN}$ from the long behavior of the skyrmion. The result is $g_{\pi NN} = 11.3$ which agrees reasonably well with the experimental value 13.4. It should in principle be possible to calculate other properties of the model, however for most properties a more detailed knowledge of the bag vacuum is required.

To conclude, despite the result for M we believe the hybrid model is intuitively appealing and deserves serious attention. It is possible that the Wess-Zumino term may have to be modified in the presence of a spherical cavity which could potentially improve the result for M.[11] A better understanding of the interior region seems necessary for analyzing the properties of this model. Questions concerning stability and radial excitation modes are currently being investigated. Finally, it should be possible to derive this model from a more fundamental theory where quark and meson fields are defined over all space.

REFERENCES

1. A. Chodos, R. L. Jaffe, K. Johnson and C. B. Thorn, Phys. Rev. 10, 2599 (1974); T. DeGrand, R. L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. 12, 2060 (1975).
2. T. H. R. Skyrme, Proc. Roy. Soc. London, Ser. A 260, 127 (1961); Nucl. Phys. 31, 556 (1962).
3. E. Witten, Nucl. Phys. B160, 57 (1979).
4. M. Rho, A. S. Goldhaber and G. E. Brown, Phys. Rev. Lett. 51, 747 (1983); J. Goldstone and R. L. Jaffe, Phys. Rev. Lett. 51, 1518 (1983).
5. L. C. Biedenharn, Y. Dothan and A. Stern, University of Texas preprint.
6. D. Finkelstein and J. Rubinstein, J. Math. Phys. 9, 1762 (1968).
7. J. G. Williams, J. Math. Phys. 11, 2511 (1970).
8. A. P. Balachandran, V. P. Nair, S. Rajeev and A. Stern, Phys. Rev. Lett. 49, 1124 (1982); Phys. Rev. D27, 1153 (1983).
9. E. Witten, Nucl. Phys. B223, 422, 433 (1983).
10. G. S. Adkins, C. R. Nappi and E. Witten, Nucl. Phys. B228, 552 (1983).
11. A. S. wishes to thank A. Goldhaber for this remark.