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A. Pinzul – University of Alabama

A. Stern – University of Alabama

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Dual Instantons

A. Pinzul and A. Stern
Department of Physics, University of Alabama, Tuscaloosa, Alabama 35487
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We show how to map the Belavin-Polyakov instantons of the $O(3)$-nonlinear σ model to a dual theory where they then appear as nontopological solitons. They are stationary points of the Euclidean action in the dual theory, and moreover, the dual action and the $O(3)$-nonlinear σ model action agree on shell.

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Although many techniques have now been devised for finding dual descriptions of field theories, important questions and limitations remain. (For reviews see Ref. [1].) One limitation is that most of the techniques are applicable only in two space-time dimensions. Within the realm of non-Abelian T-duality, there are issues concerning the global aspects of the theory. T-dual theories are equivalent at the level of the classical dynamics, and also to several orders in perturbation theory. Moreover, from the current algebras, the dual descriptions are known to be canonically equivalent. But canonical equivalence is insufficient for proving the equivalence of Feynman path integrals [2]. More troubling is the result that the canonical equivalence between dual theories is, in general, valid only locally, as the configuration spaces of the theories can have different topological properties. We can have that certain solutions are "topological" in one theory, but not in its dual, as we shall demonstrate here. Could this lead to a breakdown in the quantum equivalence of the two theories?

The example we shall look at is that of the $O(3)$-nonlinear σ model, having a target space of S^2. From the condition of finite action in two-dimensional Euclidean space-time, one gets that the configuration space is a union of disjoint pieces, and well known topological solitons appear, namely the Belavin-Polyakov instantons [3]. The Bogomol'nyi bound ensures that these solutions are the minima in every topological sector of the theory [4]. Recently, a Lagrangian and Hamiltonian field theory was found which is *locally* canonically equivalent to the $O(3)$ nonlinear σ model, and is generalizable to nonlinear G/H models for any Lie groups G and $H \subset G$ [5]. (It is also generalizable to dynamics consistent with Poisson-Lie T duality [5–7].) However, the target space for this dual theory is topologically trivial, and finite action restrictions do not lead to any disconnected regions of the configuration space. (Of course, at the classical level the dual action can be determined only up to divergence terms since one demands equivalence only of equations of motion. But there are no divergence terms that can be added to the dual Euclidean action for the purpose of obtaining a nontrivial topology, and moreover the dual Euclidean action cannot even be made to be bounded from below.) Thus the Belavin-Polyakov instantons must appear as nontopological classical solutions in the dual theory, where their stability is not automatically assured. On the other hand, here we show that our dual action agrees on shell with the action of the $O(3)$-nonlinear σ model. It then follows that, on shell, the dual Euclidean action is bounded from below, and that classical solutions of the dual theory satisfy the Bogomol'nyi bound. For answering the question in the first paragraph, a semiclassical path integral can be computed in the dual theory and compared with analogous calculations for the $O(3)$-nonlinear σ model. We plan to report on these calculations in a forthcoming article.

In this Letter, we give an explicit construction of the instantons of the dual theory. (For dual instantons in gravity see Ref. [8].) The construction involves gluing Belavin-Polyakov instantons together with corresponding anti-instantons of opposite winding number. The dual instantons are seen to have one zero mode which is not present for Belavin-Polyakov instantons.

We first review the $O(3)$-nonlinear σ model, which we shall refer to as the primary theory, and its dual formulation [5]. The target space for the $O(3)$-nonlinear σ model is S^2, which is spanned by the fields $\psi^i(x)$, $i = 1, 2, 3$, and $\psi^i(x)\psi^i(x) = 1$. We shall specialize in two-dimensional Euclidean space-time. The standard Lagrangian density is $L = \frac{\kappa}{2}\partial_\mu\psi^i\partial_\mu\psi^i$, where κ is the coupling constant. This system can also be reexpressed in terms of SU(2)-valued fields $g(x)$ [9,10]. We take g to be in the defining representation and write $\psi^i(x)\sigma^i = g(x)\sigma^3 g(x)^{-1}$, σ_i being the Pauli matrices. This introduces an additional U(1) gauge degree of freedom, associated with $g(x) \to g(x)\exp\{-\frac{i}{2}\lambda(x)\sigma^3\}$. The corresponding U(1) connection is

$$A_\mu = i\,\mathrm{Tr}\,\sigma_3(g^{-1}\partial_\mu g)\,. \tag{1}$$

In addition, one can introduce a complex current

$$\Pi_\mu = i\epsilon_{\mu\nu}\,\mathrm{Tr}\,\sigma^+(g^{-1}\partial_\nu g), \qquad \sigma^+ = \sigma^1 + i\sigma^2\,, \tag{2}$$

which rotates in the complex plane under a gauge transformation. The gauge invariant Lagrangian may be reexpressed in terms of these currents as

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$$L = \frac{\kappa}{2}\,|\Pi_\mu|^2\,. \tag{3}$$

The equations of motion resulting from variations of g, $\delta g = -\frac{i}{2} g\sigma^i\epsilon^i$, ϵ^i being infinitesimal, state that the covariant curl of Π is zero,

$$\epsilon_{\mu\nu} D_\mu \Pi_\nu = 0\,, \tag{4}$$

where the covariant derivative is defined by $D_\mu\Pi_\nu = \partial_\mu\Pi_\nu + iA_\mu\Pi_\nu$. Along with the equations of motion (4), we have three identities. These are just the Maurer-Cartan equations, which in terms of A_μ and Π_μ, are

$$D_\mu\Pi_\mu = 0\,, \tag{5}$$

$$F = -\frac{i}{2}\,\epsilon_{\mu\nu}\Pi_\mu\Pi_\nu^*\,, \tag{6}$$

F being the U(1) curvature, $F = \epsilon_{\mu\nu}\partial_\mu A_\nu$. Finite action requires that we identify the points at infinity and compactify the Euclidean space-time to S^2. The configuration space is then a union of disconnected sectors associated with $\Pi_2(S^2)$. F is proportional to the winding number density $\rho = \frac{1}{8\pi}\epsilon^{ijk}\epsilon_{\mu\nu}\psi^i\partial_\mu\psi^j\partial_\nu\psi^k$, and the total flux is

$$\int_{S^2} d^2x\,F = 4\pi n\,, \tag{7}$$

n being the winding number.

The dual Lagrangian $\tilde{L}$ in Minkowski space was specified in [5]. It was given in terms of a complex scalar field χ and the U(1) connection A_μ (now regarded as independent field variables). It was useful to also introduce an auxiliary scalar θ. One can perform a Wick rotation to obtain the corresponding Euclidean action. We specify the Wick rotation later, and for now just assume the Euclidean Lagrangian $\tilde{L}$ to be the sum of two terms $\tilde{L} = \tilde{L}_0 + L_{BF}$, where

$$\tilde{L}_0 = \frac{\alpha}{2}\,|D_\mu\chi|^2 + \frac{i\beta}{2}\,\epsilon_{\mu\nu}(D_\mu\chi)(D_\nu\chi)^*,$$
$$L_{BF} = \theta F\,. \tag{8}$$

$\alpha\delta^{ab}$ and $\beta\epsilon^{ab}$ represent the dual metric and antisymmetric tensor, respectively. The covariant derivative $D_\mu\chi$ is defined by $D_\mu\chi = \partial_\mu\chi + iA_\mu\chi$. Under gauge transformations, $\chi \to e^{-i\lambda(x)}\chi$ and $A_\mu \to A_\mu + \partial_\mu\lambda$, while θ is assumed to be gauge invariant, and thus so is L_{BF}. As in [5], we will assume that α and β are independent of χ and A, and hence $\tilde{L}_0$ is gauge invariant. On the other hand, we allow for a nontrivial dependence on θ. The expression for these functions is given below. For the dual theory to correspond to the primary theory, we should compactify the Euclidean space-time manifold to S^2. Then, in general, A_μ is not globally defined, i.e., the curvature two-form is closed but not exact.

Following [5], it is easy to recover the equations of the primary theory, i.e., (4)–(6), starting from the dual Lagrangian $\tilde{L}$, for a certain α and β. Furthermore, although $\tilde{L}$ is not positive definite, we show that $\tilde{S} = \int_{S^2} d^2x\,\tilde{L} \geq 0$ on shell, and moreover that its numerical value is identical to that of the primary Euclidean action $S = \int_{S^2} d^2x\,L$.

We first reproduce Eqs. (4)–(6). For the equations of motion resulting from variations of χ, we can ignore the BF term. From the assumption that α and β are independent of χ, we easily recover (5), once we define Π_μ according to

$$\Pi_\mu = -\alpha D_\mu\chi + i\beta\epsilon_{\mu\nu}D_\nu\chi\,. \tag{9}$$

This definition leads to the identity $\mathrm{Im}D_\mu\chi\Pi_\mu^* = 0$, and upon using the equations of motion (5), it follows that $\epsilon_{\mu\nu}\,\mathrm{Im}(\chi\Pi_\nu^*)\,dx^\mu$ is a closed one-form. From variations of A in $\tilde{L}$, it is also exact:

$$\epsilon_{\mu\nu}\partial_\nu\theta = -\mathrm{Im}\chi\Pi_\mu^*\,. \tag{10}$$

Variations of θ in $\tilde{L}$ lead to $F = -\frac{\alpha'}{2}|D_\mu\chi|^2 - \frac{i\beta'}{2}\epsilon_{\mu\nu}(D_\mu\chi)(D_\nu\chi)^*$, the prime indicating a derivative with respect to θ. This agrees with (6) provided that $\alpha' = 2\alpha\beta$ and $\beta' = (\beta^2 + \alpha^2)$. These equations are solved by $\alpha = -\kappa/(\kappa^2 - \theta^2)$, and $\beta = \theta/(\kappa^2 - \theta^2)$, up to a constant translation in θ. κ denotes the coupling constant of the dual theory. From the Hamiltonian analysis of the Minkowski formulation of this system, it is identical to the coupling constant κ of the primary theory. It remains to obtain (4). For this we need another identity, which is obtained by inverting (9), using the solutions for α and β, to solve for $D_\mu\chi$: $D_\mu\chi = \kappa\Pi_\mu - i\theta\epsilon_{\mu\nu}\Pi_\nu$. Now take the covariant curl to get $-i\kappa\epsilon_{\mu\nu}D_\mu\Pi_\nu = F\chi - \partial_\mu\theta\Pi_\mu - \theta D_\mu\Pi_\mu$. The right-hand side vanishes upon imposing the equations of motion (5), (6), and (10), and hence we recover the equation of motion of the primary formulation (4). By comparing (8) with the Lagrangian in [5] [up to a total divergence, and where the metric tensor is diag$(1,-1)$], we see that the Wick rotation from Minkowski to Euclidean space-time affects scalar fields [$\theta \to i\theta, \chi \to i\chi, \chi^* \to i\chi^*$], as well as vectors [$\partial_0 \to i\partial_0, A_0 \to iA_0$].

The dual Lagrangian (8) can be reexpressed in several curious ways. One way is to substitute the definition of Π_μ in (9) back into $\tilde{L}_0$ and integrate by parts to get

$$\tilde{L}_0 = \chi^* D_\mu\Pi_\mu - \partial_\mu(\chi^*\Pi_\mu)\,. \tag{11}$$

The second term gives no contribution to the action for the domain S^2. (For this note that $\chi^*\Pi_\mu$ is globally defined.) Moreover, the first term, and hence the action $\tilde{S}_0 = \int_{S^2} d^2x\,\tilde{L}_0$, vanishes when evaluated on the space of classical solutions, which we denote by $\tilde{S}_0|_{\mathrm{cl}} = 0$. Alternatively, we can write $\tilde{L}_0$ quadratically in terms of the currents

$$\tilde{L}_0 = -\frac{\kappa}{2}|\Pi_\mu|^2 - \frac{i\theta}{2}\,\epsilon^{\mu\nu}\Pi_\mu\Pi_\nu^*\,. \tag{12}$$

The first term is minus the primary Lagrangian (3), while the second term is equivalent to the BF term after using (6). Other possible forms for $\tilde{L}_0$ are obtained by taking linear combinations of (11) and (12). Taking twice (11) minus (12) gives

$$\tilde{L} = \frac{\kappa}{2}|\Pi_\mu|^2 + \theta\left(F + \frac{i}{2}\epsilon^{\mu\nu}\Pi_\mu\Pi_\nu^*\right) + 2\chi^* D_\mu\Pi_\mu - 2\partial_\mu(\chi^*\Pi_\mu), \tag{13}$$

where we added the BF term. This implies that the primary and dual actions coincide on shell, $\tilde{S}|_{\rm cl} = \int_{S^2} d^2x\, \tilde{L}|_{\rm cl} = \frac{\kappa}{2}\int_{S^2} d^2x\, |\Pi_\mu|^2|_{\rm cl} = S|_{\rm cl}$, and thus the dual action evaluated on the space of classical solutions is positive definite (with the vacuum solution corresponding to vanishing currents Π_μ). The result that a dual action can be found which agrees on shell with the primary action can be generalized to G/H models for any Lie groups G and $H \subset G$ [11].

Although the space of field configurations in the dual theory is topologically trivial, the subspace of all classical solutions with finite Euclidean action is a union disconnected region. The latter is classified by the total flux, which we know from the primary theory is quantized according to (7). We can therefore say that the quantization condition is dynamically generated. It does not appear to result from any kinematic considerations of the dual theory, as, classically, all values of the flux are allowed. (In this regard, note that if the value of α at spatial infinity is restricted to being finite, a bounded Euclidean action does not necessarily imply that A must go to a pure gauge at spatial infinity.) On the other hand, a semiclassical argument based on Wilson loops $W(C) = \exp i\int_C A$ gives flux quantization, but it differs from (7). Demanding that the expectation value of $W(C)$ is independent of the coordinate patch chosen on S^2 for any closed path C gives $\int_{S^2} d^2x\, F = 2\pi \times$ integer. With this quantization condition, which is identical to the Dirac quantization of magnetic charge, we can allow for, say, merons [12]. However, such solutions are known to have infinite Euclidean action.

The instantons and anti-instantons of Belavin and Polyakov [3] are self-dual and anti-self-dual solutions, respectively, and they correspond to the minima of the Euclidean action of the primary theory in every topological sector. They are therefore "topologically" stable. For this one can write L in (3) according to

$$L = \frac{\kappa}{4}|\Pi_\mu \pm i\epsilon_{\mu\nu}\Pi_\nu|^2 \pm \frac{i\kappa}{2}\epsilon_{\mu\nu}\Pi_\mu\Pi_\nu^*. \tag{14}$$

The Bogomol'nyi bound [4] for the Euclidean action of the primary theory then follows from (6): $S = \int_{S^2} d^2x\, L \geq 4\pi\kappa|n|$, with the lower bound saturated by self-dual (instanton) configurations, i.e., $\Pi_\mu - i\epsilon_{\mu\nu}\Pi_\nu = 0$ when $n > 0$, and anti-self-dual (anti-instanton) configurations, i.e., $\Pi_\mu + i\epsilon_{\mu\nu}\Pi_\nu = 0$ when $n < 0$. Of course, the instantons (and anti-instantons) are also solutions of the dual theory, where they have the same value for the action as in the primary theory, i.e., $\tilde{S}|_{\rm cl} = 4\pi\kappa|n|$. However, n cannot represent a topological index in the dual theory, as the target space topology is trivial, and now stability cannot be assured from topology.

Below we construct the general form of the instanton solutions in the dual theory.

We first review the construction of the most general instanton solutions in the primary theory [3]. For this it was found convenient to perform a stereographic projection, and write the scalar fields ψ^i in terms of a complex function $W(x)$; $\psi^1 + i\psi^2 = 2W/(1 + |W|^2)$, $\psi^3 = (|W|^2 - 1)/(|W|^2 + 1)$. Instantons (anti-instantons) correspond to $L = 4\pi\kappa\rho$ ($L = -4\pi\kappa\rho$), ρ again being the winding number density. This is possible only for $\partial_{z^*}W = 0$ ($\partial_z W = 0$), where $z = x_0 + ix_1$, and therefore W is an analytic function of z (z^*). For the choice of boundary conditions $W \rightarrow 1$, as $|x| \rightarrow \infty$, the general instanton solution with winding number n has the form [3] $W(z) = \prod_{i=1}^n (z - a_i)/\prod_{j=1}^n (z - b_i)$, where a_i and b_i are complex constants.

To write down the currents Π_μ and connection one form A associated with the instanton, we must fix a gauge $g(W)$ for the SU(2)-valued field g. In general, this can be done only locally. A gauge choice which is everywhere valid away from the poles of $W(z)$ is

$$g_S(W) = \frac{1}{\sqrt{1 + |W|^2}}\begin{pmatrix} W^* & -1 \\ 1 & W \end{pmatrix}. \tag{15}$$

Alternatively, one that is valid away from the zeros of $W(z)$ is

$$g_N(W) = \frac{|W|}{\sqrt{1 + |W|^2}}\begin{pmatrix} 1 & -W^{-1} \\ W^{*-1} & 1 \end{pmatrix}. \tag{16}$$

Since the general solution for $W(z)$ contains poles as well as zeros, we will need to cover S^2 with at least two open regions. Say R_S^2 contains all the zeros and R_N^2 contains all the poles. We can then make the gauge choice (15) for R_S^2, and (16) for R_N^2. From the corresponding left invariant one forms, the currents (2) and U(1) connection (1) are identified. The nonvanishing z components on R_S^2 are

$$\tilde{\Pi}_z^{(S)} = \frac{1}{2}(\Pi_0^* - i\Pi_1^*) = \frac{-2\partial_z W}{1 + |W|^2}, \quad A_z^{(S)} = \frac{1}{2}(A_0 - iA_1) = -i\partial_z \ln(1 + |W|^2), \tag{17}$$

while those on R_N^2 are obtained by simply replacing W by $-W^{-1}$:

$$\tilde{\Pi}_z^{(N)} = \frac{1}{2}(\Pi_0^* - i\Pi_1^*) = \frac{2\partial_z W^{-1}}{1 + |W|^{-2}}, \quad A_z^{(N)} = \frac{1}{2}(A_0 - iA_1) = -i\partial_z \ln(1 + |W|^{-2}). \tag{18}$$

We used $\partial_{z^*}W = 0$ in both (17) and (18), which is consistent with the self-duality condition $\Pi_z = \frac{1}{2}(\Pi_0 - i\Pi_1) = 0$. The transition function $\lambda^{(NS)}$ between the two gauges in the overlapping region $R_S^2 \cap R_N^2$ is $\lambda^{(NS)} = i \ln W(z)/W(z)^*$. (The above analysis can easily be repeated for anti-instantons, corresponding to $\tilde{\Pi}_z = 0$.)

Before writing down the instanton solution in the dual theory, we first look at the implications of self-duality and anti-self-duality. The scalar fields θ and χ satisfy

$$\begin{aligned} \partial_z \theta &= \frac{1}{2}(\chi \tilde{\Pi}_z - \chi^* \Pi_z), \\ D_z \chi &= \partial_z \chi + iA_z \chi = (\kappa + \theta)\Pi_z, \\ D_z \chi^* &= \partial_z \chi^* - iA_z \chi^* = (\kappa - \theta)\tilde{\Pi}_z, \end{aligned} \tag{19}$$

$\Pi_z = 0$ for instantons and hence $D_z \chi = 0$, while $\tilde{\Pi}_z = 0$ for anti-instantons and hence $D_z \chi^* = 0$. In either case, we can then solve for the U(1) connection in terms of scalar fields. Furthermore, from (19), it follows that $(\theta - \kappa)^2 + |\chi|^2$ is a constant for $\Pi_z = 0$, while $(\theta + \kappa)^2 + |\chi|^2$ is a constant for $\tilde{\Pi}_z = 0$. Therefore, when the currents are restricted to being self-dual or anti-self-dual, the scalar fields define a two-sphere, in analogy with the scalar fields of the primary theory. [One major difference with the primary theory, though, is that while ψ^i are gauge invariant, χ is not. χ contains only one gauge invariant degree of freedom, and hence for the case of self-duality or anti-self-duality the gauge invariant observables span S^1, and not S^2. As $\Pi_2(S^1) = 0$, the topology of this space is trivial.] We can then parametrize the scalar fields in terms of a complex function, say, $V(x)$, via a stereographic projection, as was done for the primary theory. In the case of instantons, we write $\chi = 2RV^*/(1 + |V|^2)$, $\theta = R(1 - |V|^2)/(1 + |V|^2) + \kappa$, where R is the radius of the two-sphere. This expression is valid in any open subset of S^2. By comparing $D_z \chi = 0$ with the equations of motion $D_z(\tilde{\Pi}_z)^* = 0$, we get that $(\tilde{\Pi}_z)^* = G(z)^* \chi$. G is an analytic function of z, and from the last equation in (19), it is equal to $-\frac{1}{R}\partial_z \ln |V|^2$. Then up to a phase (which can be gauged away) V is either analytic or antianalytic in z. In general, both cases are needed for the global description of solutions with nonzero total flux. The global description is obtained by matching solutions in the overlapping regions. An easy way to proceed is to use our result that finite action solutions of the dual theory correspond to finite action solutions of the primary theory. By identifying the currents and connections of the primary theory (17) and (18) with those derived from the dual theory we get that $V(z) = W(z)$ in R_S^2, and $V(z^*) = -1/W(z)^*$ in R_N^2, and the transition function in $R_S^2 \cap R_N^2$ is once again $\lambda^{(NS)} = i \ln W(z)/W(z)^*$. The integration constant R drops out of the expression for the currents and connections, and hence represents a degeneracy in the space of solutions of the dual theory. This implies that the dual instantons have a zero mode which is not present for the instantons of the primary theory. In matching the solutions for χ and θ in R_S^2 and R_N^2, we must ensure that θ is gauge invariant. Then if we set $V(z) = W(z)$ in R_S^2, and $V(z^*) = -1/W(z)^*$ in R_N^2, the sign of R must switch in the two regions. The resulting dual instanton solution is

$$\begin{aligned} \chi(z) &= \begin{cases} -2RW(z)^*/(1 + |W(z)|^2) & \text{in } R_S^2, \\ -2RW(z)/(1 + |W(z)|^2) & \text{in } R_N^2, \end{cases} \\ \theta(z) &= R\,\frac{|W(z)|^2 - 1}{|W(z)|^2 + 1} + \kappa. \end{aligned} \tag{20}$$

If $(\psi^1_{(n)}, \psi^2_{(n)}, \psi^3_{(n)})$ corresponds to the n-instanton solution of the primary theory expressed in terms of the fields ψ^i, then the dual n-instanton solution can be written

$$\left(\frac{\chi^1}{R}, \frac{\chi^2}{R}, \frac{\theta - \kappa}{R}\right) = \begin{cases} (-\psi^1_{(n)}, \psi^2_{(n)}, \psi^3_{(n)}) & \text{in } R_S^2, \\ (-\psi^1_{(n)}, -\psi^2_{(n)}, \psi^3_{(n)}) & \text{in } R_N^2, \end{cases} \tag{21}$$

where $\chi = \chi^1 + i\chi^2$. Thus instantons in the dual theory are obtained by gluing instantons of the primary theory together with anti-instantons of opposite winding number, the latter being obtained by switching the orientation of one of the components. An analogous result can be found for the anti-instantons of the dual theory. In that case, the right-hand side of (21) gets replaced by $(\frac{\chi^1}{R}, \frac{\chi^2}{R}, \frac{\theta+\kappa}{R})$.

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