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INNER RING STRUCTURES IN GALAXIES AS DISTANCE INDICATORS. IV. DISTANCES TO SEVERAL GROUPS, CLUSTERS, THE HERCULES SUPERCLUSTER, AND THE VALUE OF THE HUBBLE CONSTANT

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ABSTRACT

The extragalactic distance scale derived in previous papers of this series from the diameters of inner ring structures (r) in galaxies is applied to the determination of the distances of six groups and clusters, and one supercluster, in the distance range $10 \leq \Delta \leq 112$ Mpc. The clusters are well distributed on the sky and are used to derive a value of the Hubble constant free of the perturbing effects of the Local Supercluster. Several definitions of the Hubble ratio, for bound and unbound systems, and several frames of reference are introduced and used. Radial velocities V_c corrected for solar motion in a frame of reference defined by the nearby galaxies ($2 \leq \Delta \leq 32$ Mpc) are shown to yield a Hubble ratio which is substantially independent of direction. An analysis of four bound groups and one unbound grouping in the distance interval $10 \leq \Delta \leq 16$ Mpc yields a mean Hubble ratio $H_c = 92 \pm 4$ (internal m.e.) km s^{-1} Mpc^{-1}. A similar analysis of two more distant bound systems, the IC 4329 Group at $\Delta = 45$ Mpc and the Hercules Supercluster at $\Delta = 112$ Mpc, yields $H_c = 96 \pm 5$, or 93 ± 5 when plausible cosmological corrections are considered. The constancy of the Hubble ratio over the distance range $10 \leq \Delta \leq 112$ Mpc gives no support to nonlinear concepts of the redshift-distance relation and demonstrates the absence of Malmquist bias in the $\mu_0(r)$ scale. The value of the Hubble constant derived from the rings over this range is $H_0 = 93 \pm 4$ (internal m.e.) km s^{-1} Mpc^{-1} and ± 10 (external m.e.) when the zero point error of our tertiary indicators is included.

Subject headings: cosmology — galaxies: clusters of — galaxies: structure

I. INTRODUCTION

The diameters of inner ring structures in spiral and lenticular galaxies are metric distance indicators possessing many advantages. Unlike most other tertiary and quaternary indicators, rings are relatively easy to measure, require no correction for galactic or internal absorption, are little affected by inclination, and are present in nearly 50% of all disk galaxies covering a wide range of Hubble types from lenticulars to late spirals; finally, they are sufficiently large to be detected out to distances in excess of 100 Mpc. In previous papers of this series (Buta and de Vaucouleurs 1982 = Paper II, 1983 = Paper III) the absolute calibration and typical errors of ring diameters as distance indicators were thoroughly discussed, and applications were made to ~450 galaxies generally nearer than 60 Mpc. In the present paper, we extend the application of ring diameters to the determination of the distances of five nearby groups and clusters, and of the more distant IC 4329 Group and Hercules Supercluster, to derive a value of the Hubble constant free of local perturbations. Because the ring calibration was derived from our tertiary indicators (de Vaucouleurs and Bollinger 1979; Bottinelli *et al.* 1980), we are not making an independent absolute determination of the Hubble constant H_0 with this analysis, but instead are primarily testing its uniformity over the sky and over a much larger range of distances than previously considered (de Vaucouleurs and Peters 1981; de Vaucouleurs *et al.* 1981).

The groups and clusters chosen for this study include five bound systems—viz., the Dorado Cloud, the Virgo E Cluster, the Virgo S Cloud, and the Fornax I Cluster in the distance range 10 to 16 Mpc, and the IC 4329 Group at a distance of 45 Mpc—plus one unbound (optical) grouping, the Grus Cloud extending from 6 to 25 Mpc, and the Hercules Supercluster at a mean distance of 112 Mpc. Their apparent distribution in supergalactic coordinates is shown in Figure 1 illustrating the good sky coverage (four in the northern galactic hemisphere, three in the southern hemisphere) and the lack of dependence on direction of the finally derived values of the Hubble ratio H_c as defined and discussed in §§ II and V.

II. CORRECTED VELOCITIES AND HUBBLE RATIOS

In order to derive an unbiased value of the Hubble constant from the mean ring modulus and the observed mean heliocentric velocity V of a group or cluster of galaxies, we must first correct it for the velocity component which represents merely the reflection of the peculiar solar motion. This motion depends, of course, on

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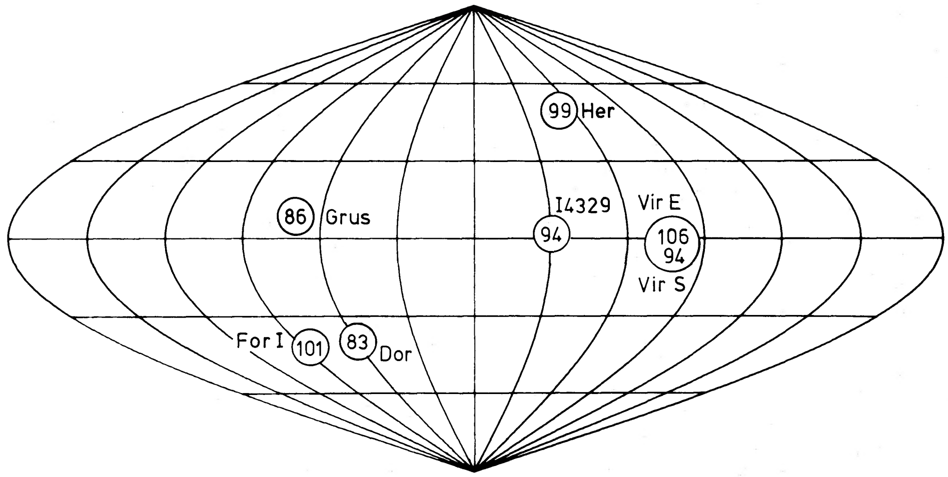

FIG. 1.—Distribution of the seven groups and clusters in supergalactic coordinates. The number in each circle is the value of the mean Hubble ratio H_c derived from ring diameters of probable members of each configuration. Note the good sky coverage and the lack of significant dependence of H_c on direction.

the adopted frame of reference. Traditionally this has been defined either by the Galaxy or by the Local Group.

In *galactic* astronomy the current convention (IAU Commission 33) is that the (stellar) local standard of rest (LSR) is moving at 250 km s^{-1} toward $l = 90°$, $b = 0°$ and that the peculiar motion of the Sun relative to the LSR is 20 km s^{-1} toward 18^h, $+30°$ (1900). In *extragalactic* astronomy the current convention (adopted by IAU Commission 28 in 1976) is that the total solar motion relative to the velocity centroid of the Local Group is $V_s = 300$ km s^{-1} toward $l_s = 90°$, $b_s = 0°$. Application of the corresponding correction to the observed heliocentric velocity of a galaxy gives the "corrected velocity," $V_0 = V + \Delta V = V + 300 \cos A$, where A = apex angle. Values of ΔV and V_0 are listed, for example, in the Second Reference Catalog (RC2) (de Vaucouleurs, de Vaucouleurs, and Corwin 1976).

This "corrected velocity," V_0, is neither the galactocentric velocity nor the velocity relative to the Local Group, but a compromise, intermediate value. The current best estimate of the total solar motion relative to the Local Group ($\Delta < 2$ Mpc) is $V_s' = 335 \pm 20$ km s^{-1} toward $l_s' = 107° \pm 5°$, $b_s' = 15° \pm 4°$, the weighted means of five recent determinations (de Vaucouleurs and Peters 1981, Table 1).

However, the Local Group itself has a peculiar motion in the Local Supercluster (de Vaucouleurs 1958, 1978*a*). The current best estimate of the total solar motion relative to a frame of reference defined by the nearby galaxies outside the Local Group ($2 < \Delta < 32$ Mpc) is $V_s'' = 320 \pm 30$ km s^{-1} toward galactic coordinates $l_s'' = 120° \pm 10°$, $b_s'' = +28° \pm 5°$ (supergalactic $L_s'' = 26° \pm 5°$, $B_s'' = +18° \pm 9°$). These are the weighted means of two recent determinations from 200 spiral galaxies having distances derived from the luminosity index (de Vaucouleurs and Peters 1981) and from 300 spiral galaxies having distances from H I line widths (de Vaucouleurs *et al.* 1981).

In the present paper we will reduce the heliocentric velocities V to (*a*) the conventional "corrected velocity" frame, V_0, (*b*) the Local Group frame, V_G, and (*c*) the nearby galaxies frame, V_c. We believe that the last is the proper one to use to derive the correct value of the Hubble constant, free of the effects of local motions. This value should be independent of direction.

A second question is the choice of a proper definition of the Hubble constant. For *field* galaxies obeying a linear isotropic velocity-distance relation, the Hubble ratio can be calculated in both the linear form,

$$\langle H \rangle = \langle V_0 \rangle / \langle \Delta \rangle = \Sigma V_0 / \Sigma \Delta \,, \tag{1}$$

and in the logarithmic form,

$$\log \bar{H} = \langle \log V_0 \rangle - 0.2(\langle \mu_0 \rangle - 25) \,. \tag{2}$$

As noted by de Vaucouleurs and Bollinger (1979, hereafter EDS VII), $\langle H \rangle$ and $\bar{H}$ are not identical in the presence of accidental errors in V_0 and μ_0. If V_0 were error-free and the scatter were due only to $\sigma(\mu_0)$ = const., independent of distance, $\bar{H}$ would be the correct value. If μ_0 were exact and errors in V_0 were dominant, $\langle H \rangle$ would be the correct value. In the real case where both variables contribute to the scatter, $H^* = (\langle H \rangle \bar{H})^{1/2}$ may be adopted as a compromise.

For *cluster* galaxies which, in principle, occupy in the (V, Δ)-diagram a small domain defined by the velocity dispersion σ_v (including measuring errors) and the internal accidental errors in the relative distance moduli, $\sigma(\delta\mu_0)$ = const., the correct value is given by

$$\log (H) = \log \langle V_0 \rangle - 0.2(\langle \mu_0 \rangle - 25) \,, \tag{3}$$

where

$$\langle V_0 \rangle = \frac{1}{N} \sum V_0 \pm \frac{\sigma_v}{(N_1)^{1/2}} \,, \tag{4}$$

and

$$\langle \mu_0 \rangle = \frac{1}{N} \sum \mu_0 \pm \left[\sigma_0{}^2 + \frac{\sigma^2(\delta\mu_0)}{N_2} \right]^{1/2} . \quad (5)$$

Here we must make a distinction between the purely accidental internal errors in the differential moduli, $\delta\mu_0$, which are reduced when the sample size N_2 increases, and the zero point error, $\sigma_0 = 0.2$ mag, of the $\mu_0(r)$ scale which cannot be so reduced. Note that the numbers N_1, N_2 of objects having known velocities or distance moduli may be different. In general $N_1(V_0) > N_2(r)$ and one might argue that in a cluster $\langle V_0 \rangle$ is better determined by the larger sample of galaxies with known redshifts than by the often small sample of those with rings. There is no guarantee, however, that the latter is an unbiased sample of the former and, if so, one might just as well prefer to derive $\langle V_0 \rangle$ from the smaller ring sample only, even though it will be less precise. In practice we will use both approaches and adopt as the most appropriate mean velocity $\langle V_0 \rangle = [\langle V_0(N_1) \rangle \langle V_0(N_2) \rangle]^{1/2}$.

In the following §§ III–IV where individual clusters are discussed we will use only V_0 and the corresponding values of H^* or (H) to permit direct comparison with earlier studies. In § V, we will use also V_G and V_C to derive the most probable weighted mean value of H_0.

III. DISTANCES TO NEARBY GROUPS AND CLUSTERS

a) *The Virgo Cluster*

Tables 1 and 2 summarize respectively the available ring moduli for 17 probable members of the Virgo S Cloud and seven probable members of the Virgo E cluster. Membership in both components has been discussed extensively (de Vaucouleurs 1961, 1982; see also de Vaucouleurs and de Vaucouleurs 1973). A few objects not specifically listed as members in these papers were assigned membership on the basis of position within a 6°.5 radius centered at $\alpha = 12^h27^m$, $\delta = +13°.5$. All but two were included in Table 4 of Paper III; the weights assigned to the distance moduli $\mu_0(r)$ are based on the *internal* mean errors calculated according to the formulae given in Appendix B of Paper III.[3]

TABLE 1

RING MODULI FOR PROBABLE MEMBERS OF THE VIRGO S CLOUD

Object	Revised Type	V_0	$\mu_0(r)$	w	Notes
N4152	SAB(rs)c	2036	32.69	1.58	
N4165	SAB(r)a?	1420	32.22	1.60	a
N4189	SAB(rs)cd?	2013	31.54	1.58	b
N4237	SAB(rs)bc	806	31.45	2.09	
N4394	(R)SB(r)b	717	31.24	3.51	
N4440	SB(rs)a	627	32.00	2.78	a
N4501	SA(rs)b	2060	30.63	1.76	
N4519	SB(rs)d	1078	30.63	2.45	c
N4548	SB(rs)b	403	30.67	3.13	
N4567	SA(rs)bc	2121	31.46	1.36	
N4568	SA(rs)bc	2168	30.56	1.06	
N4571	SA(r)d	275	30.01	1.27	
N4579	SAB(rs)b	1730	29.77	1.97	
N4647	SAB(rs)c	1286	31.36	1.46	
N4651	SA(rs)c	749	30.03	1.58	
N4654	SAB(rs)cd	970	29.85	2.80	
N4689	SA(rs)bc	1554	30.75	1.66	
Unweighted means		1295	30.99		
Internal m.e.		±157	±0.21		
σ_1			0.85		
Weighted mean			30.98		
Internal m.e.			±0.20		
σ_1			0.83		

[a] Weight adjusted to be consistent with later spirals (see text).
[b] Ring diameter $d_r = 0'.42$ is based on a PSS print.
[c] Ring diameter $d_r = 0'.63$ is based on a PSS print.

TABLE 2

RING MODULI FOR PROBABLE MEMBERS OF THE VIRGO E CLUSTER

Object	Revised Type	V_0	$\mu_0(r)$	w	Notes
N4309	LAB(r)0$^+$	998	30.86	3.51	
N4340	LB(r)0$^+$	844	30.12	6.16	
N4371	LB(r)0$^+$	898	30.05	6.16	
N4429	(R)LA(r)0$^+$	1029	31.40	1.71	
N4459	LA(r)0$^+$	1039	(32.72)	...	a
N4596	(R)LB(r)0$^+$	1882	30.33	5.83	
N4608	LB(r)0^0	1758	30.45	6.16	
Unweighted means		1109	30.54		
Internal m.e.		±272	±0.19		
σ_1			0.51		
Weighted mean			30.38		
Internal m.e.			±0.16		
σ_1			0.39		

[a] Suspect inner ring may be a nuclear ring; excluded from means.

The unweighted mean modulus for the Virgo S cloud is $\langle \mu_0(r) \rangle = 30.99 \pm 0.21$ (exclusive of zero point error), and the weighted mean is 30.98 ± 0.20. For the E cluster, the unweighted mean of six of the lenticulars in Table 2 is $\langle \mu_0(r) \rangle = 30.54 \pm 0.19$ (exclusive of zero point errors), and the weighted mean modulus is 30.38 ± 0.16. The differential modulus, $\Delta\mu_0(S-E) = 0.60 \pm 0.25$ (weighted) or 0.46 ± 0.28 (unweighted), appears to be significant relative to the mean errors (which are exclusive of the common zero point error). Its indication that the E cluster is slightly closer than the S Cloud is consistent with a variety of other indicators and the mean distances are also in good agreement with values determined from other methods (Table 3).

Figure 2*a* illustrates the correlation between log V_0 and $\mu_0(r)$ for all 24 galaxies in Tables 1 and 2. In agreement with previous findings (de Vaucouleurs and de Vaucouleurs 1973; EDS VII), the ring moduli indicate that the Hubble law is not in evidence in the Virgo Cluster which, by this criterion, appears to be a bound system (see, however, de Vaucouleurs 1982 for the velocity structure of the S Cloud).

b) *The Grus Cloud*

This well-known southern cloud of nearby galaxies (de Vaucouleurs 1956*a*, 1975) consists of about two dozen

[3] The weights for a few objects of types Sa and earlier ($T < 2$) were adjusted to lower values because we believe the errors in Paper III for these objects are probably too small relative to those for the later spirals. Internal mean errors were computed by removing the zero point error, 0.2 mag, of the $\mu(r)$ scale, since it is common to all the distance moduli.

TABLE 3

DISTANCE MODULUS OF VIRGO CLUSTER

Source		Method	Virgo E Cluster	Virgo S Cloud	Rem
de VAUCOULEURS	1977	Brightest Globular Clusters	30.4 ± 0.3		(1)
VISVANATHAN	1978	Color-Luminosity Relation of Ellipticals	(30.60 ± 0.39) 30.28 ± 0.3:		(2) (3)
HANES	1979	Luminosity Function of Globular Clusters	(30.70 ± 0.3) 30.3 ± 0.2		(4) (5)
De VAUCOULEURS	1979	Luminosity Index, B_T^O, D_O of 6 Spirals		30.53 ± 0.2:	(6)
"	1979	Luminosity Index, Metric Diameter (A_e) of 9 Spirals		30.79 ± 0.26	(7)
"	1979	Type I Supernovae in 5 Ellipticals	30.25 ± 0.25		(8)
MOULD *et al*.	1980	Infra-red T-F Relation H Mag of 23 Spirals		(30.98 ± 0.2:) 30.57 ± 0.2:	(9) (10)
BOTTINELLI *et al*.	1980	Revised T-F Relations B_T^O, D_O of 35 spirals		30.88 ± 0.15	(11)
De VAUCOULEURS AND OLSON	1981	Velocity Dispersion in 34 E,L Galaxies	30.51 ± 0.2		(12)
BUTA AND De VAUCOULEURS	1981	Inner Ring Diameters in 6 E,L and 17 Spirals	30.38 ± 0.25	30.98 ± 0.3	(13)
VISVANATHAN	1981	Visual T-F Relation V_{26} Mag of 20 Spirals		(31.15 ± 0.2): 30.59 ± 0.2	(14) (15)
		Means	30.36 ± 0.2	30.73 ± 0.2	

(1) De Vaucouleurs 1977.
(2) Visvanathan 1978, with $A_B = 0.0$ and Local Group zero point on long scale for $\mu_0(\text{Hya}) = 3.03$.
(3) Corrected for zero point with $A_B = 0.2$, $\mu_0(\text{Hya}) = 3.29$, and revised magnitudes.
(4) Hanes 1979, with $A_B = 0.0$ and $\langle M_v(\text{RR})\rangle = 0.6$.
(5) Corrected for zero point with $A_B = 0.2$ and $\langle M_v(\text{RR})\rangle = 0.8$.
(6) De Vaucouleurs 1979, Table 3.
(7) De Vaucouleurs 1979, Table 4.
(8) De Vaucouleurs 1979, Table 6.
(9) Mould, Aaronson, and Huchra 1980, with Local Group zero point (M31, M33 on long scale) corrected to $\mu_0(\text{Hya}) = 3.29$.
(10) Corrected for zero point (M31, M33 on short scale, de Vaucouleurs 1978*b*).
(11) Bottinelli *et al.* 1980.
(12) De Vaucouleurs and Olson 1982: zero point consistent with (6) and (11).
(13) This paper: zero point consistent with (6) and (11).
(14) Visvanathan 1982 with $A_v = 0.0$ and Local Group zero point (M31, M33 on long scale) corrected to $\mu_0(\text{Hya}) = 3.29$.
(15) Corrected for $A_v = 0.15$ and zero point (M31, M33 on short scale, de Vaucouleurs 1978*b*).

bright spiral galaxies between right ascensions 22^h30^m and 23^h50^m and declinations $-35°$ and $-45°$ (supergalactic coordinates $\langle L\rangle \approx 250°$, $\langle B\rangle \approx +8°$). Table 4 lists the $\mu_0(r)$ moduli which are available in Paper III for six members of the cloud. The unweighted mean modulus is $\langle\mu_0(r)\rangle = 30.59 \pm 0.45$ (exclusive of zero point error), and the weighted mean modulus is 30.82 ± 0.43. However, here log V_0 correlates well with the individual moduli (Fig. 2*b*), which strongly suggests that the Grus Cloud is an accidental optical grouping, not a physically bound system. In this case, the Hubble ratio, calculated via equations (1) and (2), $H^* = 100 \pm 14$ km s^{-1} Mpc^{-1}, is in excellent agreement with that found for the general direction of the supergalactic anticenter sector, in which the Grus cloud lies (area *E*, $H^* = 100.7$, in EDS VII, Table 2). An independent study of distances in the Grus Cloud has been made by Aaronson *et al.* (1981), who find that $\langle V_0/\Delta\rangle = 93 \pm 7$ (internal m.e.) for spirals whose distances were determined from their *H*-band version of the Tully-Fischer relation. Their

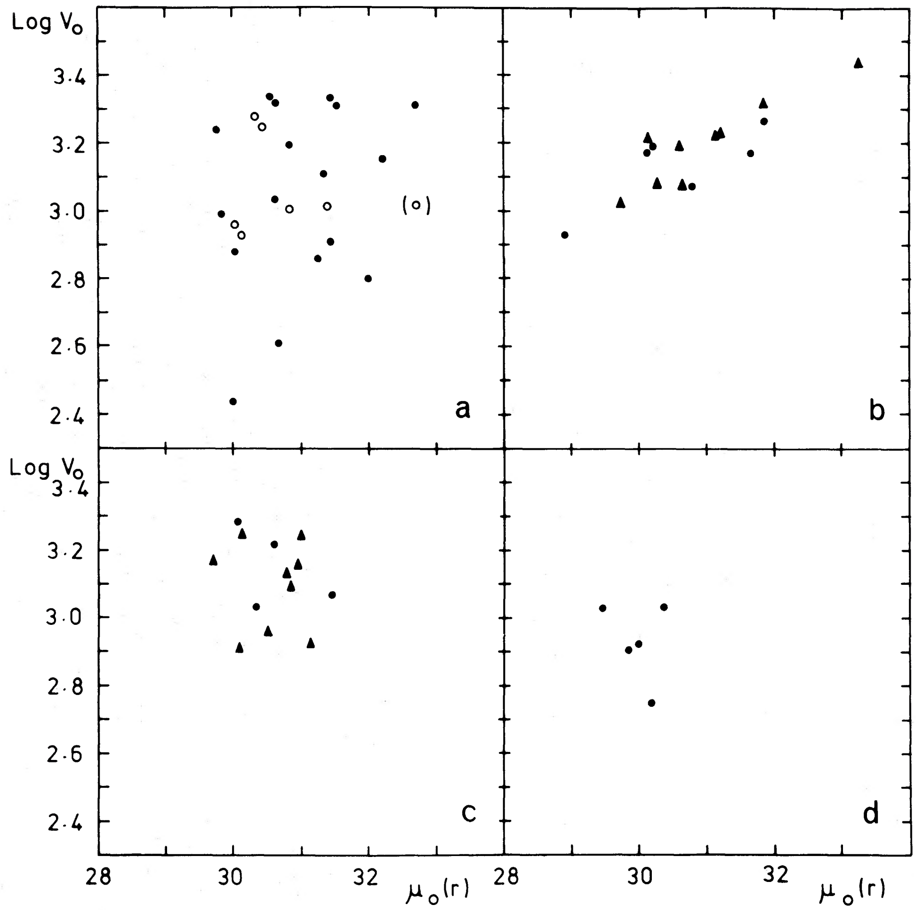

FIG. 2.—Velocity-distance correlations for the nearby ($10 \leq \Delta \leq 16$ Mpc) groups and clusters: (*a*) Virgo S Cloud (*filled circles*) and Virgo E Cluster (*open circles*); (*b*) Grus Cloud; (*c*) Fornax I Cluster; and (*d*) Dorado Cloud. In (*b*) and (*c*), the filled circles represent the objects in Tables 4 and 5 of this paper while the filled triangles represent objects from Aaronson *et al.* (1981), corrected for zero point (see text).

TABLE 4

RING MODULI FOR SPIRALS IN THE DIRECTION OF THE GRUS CLOUD

Object	Revised Type	V_0	$\mu_0(r)$	w	Notes
I5240	SB(r)a	1476	31.65	3.41	a
N7418	SAB(rs)cd	1481	30.14	2.22	
N7421	SB(rs)bc	1838	31.85	2.70	b
N7424	SAB(rs)cd	849	28.89	1.51	
I5273	SB(rs)cd:	1194	30.77	1.92	
N7531	SA(r)bc	1550	30.22	1.81	
Unweighted means		1398	30.59		
Internal m.e.		±138	±0.45		
σ_1			(1.09)		c
Weighted mean			30.82		
Internal m.e.			±0.43		
σ_1			(1.05)		c

[a] Weight adjusted to be consistent with later spirals (see text).
[b] Classification parameters $T = 4$ and $L = 4$ are from de Vaucouleurs 1979.
[c] Large value of $\sigma_1(\mu_0)$ is not significant since cloud is not a physical grouping; see text.

results confirm the strong velocity distance correlation in the Cloud (see Fig. 2*b*). However, their zero point is based on the mean distance modulus of M31 and M33 after Sandage and Tammann (1976) corrected by +0.26 mag to allow for the revision of the Hyades modulus (see de Vaucouleurs 1978*b*, Appendix B). If, instead, we define the zero point by the corresponding values in de Vaucouleurs 1978*c* (= EDS II), the Hubble ratio of Aaronson *et al.* becomes $\langle V_0/\Delta \rangle = 112 \pm 8$ (internal mean error).[4]

It will be noted, further, that Aaronson *et al.* calculate H as the simple average $\langle V_0/\Delta \rangle$ of the individual velocity-distance ratios. This is a biased estimate because the relative errors in V_0, $\delta V_0/V_0$, are roughly inversely proportional to V_0, while the errors in Δ, $\delta\Delta/\Delta$, are

[4] Aaronson, Mould, and Huchra (1980) state correctly that the zero point difference is $\langle \delta\mu_0(\text{ST VII} - \text{EDS II}) \rangle = +0.41$ mag, but give 1.18 as the corresponding distance ratio instead of the correct value dex $[0.2\delta\mu_0] = 1.21$.

independent of Δ (hence V_0) and, consequently, in the Hubble diagram, $V_0(\Delta_0)$, the mean of the slopes is not the slope of the mean relation. As discussed in § II, the correct procedure is to use equations (1) and (2) and adopt H^* as a compromise. This technique applied to the data of Aaronson *et al.* for the Grus Cloud gives $H^* = 108 \pm 8$ (corrected for zero point).

c) *The Fornax I Cluster*

The Fornax I Cluster has been the subject of many studies (de Vaucouleurs 1956*a*, 1975; Hodge 1959, 1960, 1978; Jones and Jones 1980 and references therein). Distance moduli are available in Paper III for four probable members of this cluster (Table 5). The unweighted mean modulus is $\langle\mu_0(r)\rangle = 30.61 \pm 0.30$ (exclusive of zero point error), and the weighted mean is 30.67 ± 0.30. Confirmation that the cluster is a bound system is provided by Figure 2*c*, which shows that there is no velocity-distance relation among the four probable members. This is in agreement with Aaronson *et al.* (1981), who find no velocity-distance relation among seven probable spiral members having distances from the H-band Tully-Fisher relation (these are plotted for comparison in Fig. 2*c*, after correction by -0.41 mag for zero point).

The weighted mean distance modulus of 30.67 and the radial velocity $\langle V_0\rangle = 1453 \pm 199$ km s^{-1} of the four galaxies in Table 5 give, by equations (3)–(5), $(H) = 107 \pm 21$ km s^{-1} Mpc^{-1}, necessarily with a large uncertainty because of the small sample, but in good agreement with the mean value, $H^* = 103$, for the relevant supergalactic area K (EDS VII, Table 2) and with the mean value derived by Aaronson *et al.* (1981), $\langle V_0/\Delta\rangle = 109 \pm 17$ (corrected for zero point).[5] However, here again, Aaronson *et al.* calculate H as the mean of the individual slopes V_0/Δ which is inappropriate in a bound cluster, because, as explained in § II, the error distribution is Gaussian in μ_0, but not in Δ. By application of equations (3)–(5) to the data of Aaronson *et al.* we find $(H) = 104 \pm 14$ (corrected for zero point), in good agreement with our value within their combined mean errors (exclusive of zero point error).

[5] The mean redshift of the four probable members is in good agreement with that given by de Vaucouleurs (1975) for the mean of the cluster, $\langle V_0\rangle = 1464$ km s^{-1} (from 12 galaxies), and with that given in Paper II ($\langle V_0\rangle = 1489 \pm 66$ from 18 galaxies), but is higher than that given by Jones and Jones (1980), $\langle V_0\rangle = 1395 \pm 49$ km s^{-1} (from 54 galaxies); if this latter value is adopted instead of 1453 ± 199, then $(H) = 102 \pm 15$ km s^{-1} Mpc^{-1}.

TABLE 5

Ring Moduli for Probable Members of the Fornax I Cluster

Object	Revised Type	V_0	$\mu_0(r)$	w	Notes
N1317	SAB(r)a	1918	30.05	3.98	
N1326	(R)LB(r)0$^+$	1167	31.44	5.83	
N1350	(R′)SB(r)ab	1649	30.61	6.16	
N1437	SA*B*(rs)bc	1078	30.33	4.76	a
Unweighted means		1453	30.61		
Internal m.e.		±199	±0.30		
σ_1			0.60		
Weighted mean			30.67		
Internal m.e.			±0.30		
σ_1			0.60		

[a] Weight adjusted to be consistent with earlier spirals (see text).

d) *The Dorado Cloud*

This large nearby cloud of southern galaxies (de Vaucouleurs 1975), which forms part of the "Southern Supergalaxy" (de Vaucouleurs 1953, 1956*a*), comprises three groups centered near NGC 1433, 1566, and 1672. The unweighted mean modulus of five probable members, listed in Table 6, is $\langle\mu_0(r)\rangle = 29.96 \pm 0.15$ (exclusive of zero point error), and the weighted mean modulus is $\langle\mu_0(r)\rangle = 30.00 \pm 0.13$. Some of the best and nearest examples of the SB (r) type of spiral are present in this cloud (notably NGC 1433 and 1512). The plot in Figure 2*d* shows no velocity-distance relation, indicating that the cloud may be a bound system. The mean redshift of the five galaxies in Table 6 is $\langle V_0\rangle = 867 \pm 96$ km s^{-1}, and the mean Hubble ratio calculated by equations (3)–(5) is $(H) = 87 \pm 11$ km s^{-1} Mpc^{-1}. If, instead, we use the mean velocity $\langle V_0\rangle = 896 \pm 47$ km s^{-1} of 14 probable members of the Dorado Cloud listed by de Vaucouleurs (1975), we find $(H) = 90 \pm 7$ km s^{-1} Mpc^{-1}; both values are in fair agreement with the mean value, $H^* = 103$ in supergalactic area K (EDS VII).

e) *The IC 4329 Group*

This small southern group includes about two dozen galaxies, many with inner rings. Membership has been discussed by Corwin (1967) and by Sandage (1975*a*); the nine possible members possessing inner rings are listed in Table 7A. For seven of these galaxies, the ring diameters and Hubble types were obtained from two prime-focus plates taken with the 3.9 m reflector of the Anglo-Australian Observatory. For the remaining two, these parameters were determined from either SRC-J or ESO-B southern sky survey films. The Hubble types (estimated by R. B.) are in good agreement with independent estimates by Holmberg *et al.* (1978).

TABLE 6

Ring Moduli for Probable Members of the Dorado Cloud

Object	Revised Type	V_0	$\mu_0(r)$	w	Notes
N1433	SB(r)a	802	29.84	6.16	
N1493	SB(r)cd	835	29.98	6.16	a
N1512	SB(r)a	558	30.18	6.16	
N1553	LA(r)0^0	1064	29.46	2.45	
N1672	SB(rs)b	1076	30.34	4.07	a, b
Unweighted means		867	29.96		
Internal m.e.		±96	±0.15		
σ_1			0.34		
Weighted mean			30.00		
Internal m.e.			±0.13		
σ_1			0.28		

[a] Weight adjusted to be consistent with earlier spirals (see text).
[b] Classification SB(r*s*)b based on du Pont 2.5 m reflector plate is from Sandage and Brucato 1979; ring diameter $d_r = 2'.34$ is based on an ESO B southern sky survey film.

TABLE 7A

RING MODULI FOR POSSIBLE MEMBERS OF THE IC 4329 GROUP

Object	Type	d_r	V_0	$\mu_0(r)$	Notes
N5298	SB(rs)b	24''.0:		34.15:	a, b
N5302	SAB(rs)a	35.2	3115	32.51	a
I4328	SAB(rs)ab	27.1		33.33	a
A1345.3−3017	SAB(r)a	32.1		32.71	a
A1345.4−3020	SB:(rs)a sp	23.7		34.12:	a
A1347.6−3024	LAB(r)0$^+$?	19.1		33.34:	a
A1347.7−2948	(R)LB(rs)0$^+$	24.6		33.54	a
A1348.5−3054	SB(r)bc	30.5		33.05	c
A1350.0−3028	SB(r)ab	33.6		33.61	d
Unweighted mean				33.37	
				±0.19	
σ_1				0.56	
Weighted mean				33.27	e
				±0.18	
σ_1				0.53	

[a] Classification and ring diameter based on AAO prime focus plates.
[b] Pseudo-ring feature weak; d_r is average of new measure from AAO plate (21'') and that given by de Vaucouleurs and Buta 1980 based on a Boyden 1.5 m plate.
[c] Classification and ring diameter based in ESO B southern sky survey film.
[d] Classification and ring diameter based on SRC-J southern sky survey film.
[e] Values marked with a colon (:) are given half-weight.

TABLE 7B

RADIAL VELOCITIES OF PROBABLE MEMBERS OF THE IC 4329 GROUP

Object	l	b	V_0
N5291	317.01	30.93	4151
N5292	316.92	30.40	4266
N5302	317.33	30.75	3115
N5304	317.59	30.62	3509
N5357	319.10	30.50	4806
I4329	317.45	30.94	4243
I4329A	317.51	30.92	4640
A1345−30	317.19	30.47	5060

The unweighted mean modulus for the nine possible members is $\langle\mu_0(r)\rangle = 33.37 \pm 0.19$ (exclusive of zero point error), and the weighted mean is 33.27 ± 0.18. Unfortunately, only one of these objects (NGC 5302) has a published radial velocity; however, velocities for seven other probable (nonringed) members are available from Sandage (1975*a*), and the corrected values (from RC2) are listed in Table 7B.[6] The mean radial velocity from the eight presumed members is $\langle V_0\rangle = 4224 \pm 230$ km s^{-1}. If the group is bound, then, with $\langle\mu_0(r)\rangle = 33.27 \pm 0.18$, the corresponding Hubble ratio, $(H) = 94 \pm 9$, agrees within errors with the value, $H^* = 102$, found in EDS VII for supergalactic area D.[7]

[6] Excludes NGC 5328 and 5330 since their projected separation from the center of the group is $\geq 2°$ or 1.6 Mpc at $\Delta = 45$ Mpc, which is well beyond the main body of the IC 4329 group. These galaxies may, however, be part of another small group at nearly the same redshift as the IC 4329 group.

[7] Note that NGC 5302 has both the smallest redshift and the smallest distance modulus. It could be in the foreground, in which case its Hubble ratio would be 98, a normal value for area D.

IV. THE HERCULES SUPERCLUSTER

Tarenghi *et al.* (1979) have presented an extensive collection of data on radial velocities and morphological types for 150 bright galaxies in the regions of three well-known Abell clusters in Hercules, A2147, 2151, and 2152, which are subunits of a large supercluster (Shapley 1934; de Vaucouleurs 1956*b*, 1971; Corwin 1971). The Hubble types in Tarenghi *et al.* (1979), estimated by Thompson, indicate that many spirals and lenticulars are present in the clusters, some of which have inner rings. The detection of inner rings in the Hercules Supercluster, whose mean radial velocity is close to $+11{,}000$ km s^{-1}, provides an important application of our new distance indicator and allows us to test the distance range over which it can be satisfactorily applied. At the same time ring diameters in Hercules can be used to make a reliable determination of the Hubble ratio free of any local perturbations and pertaining to a volume of space large enough and of mean density low enough to give a close approximation to the Hubble *constant* H_0.

a) *Data*

Our analysis of the Hercules Supercluster is based on film copies of the five IIIa-J plates (exposed through a GG385 filter) and one 098-04 plate (exposed through an RG610 filter) which were taken in 1975 by Thompson (see Tarenghi *et al.* 1979) with the 4 m Mayall telescope of Kitt Peak National Observatory. The scale of the plates, 18''.59 mm^{-1}, is suitable for the observation of intermediate scale features in the brighter galaxies in Hercules, whose apparent photographic diameters range from about 30'' to 1'. One of us (R. B.) searched each of the films for ringed galaxies, and a total of 31 definite or probable rings were found, ranging from about 5'' to 22'' in apparent diameter. Of these, 20 are SB galaxies, 11 are SAB galaxies, and none is an SA galaxy. Radial velocities from Tarenghi *et al.* (1979) are available for only 11 of the objects, but apparent magnitudes and diameters of most of the others indicate that they are probable members also.

Tables 8 and 9 list the designations, Hubble types, ring diameters, radial velocities (if available), reduced ring diameters (see § IV*b*[i]), and ring distance moduli for 17 of the SB and nine of the SAB (including SAB and SAB) ringed galaxies found on the 4 m films. Since many of these objects are not in the *Catalog of Galaxies and of Clusters of Galaxies* (Volume **2**, Field 108, Zwicky and Herzog 1963) or the *Uppsala General Catalog of Galaxies* (Nilson 1973), the 1950 positions were measured by means of overlays on the Palomar Sky Survey prints. All type estimates given are the means of two separate estimates (by R. B.) made several months apart. They are in good agreement with independent estimates by Corwin (private communication).[8]

The apparent ring diameters d_r are also averages of two independent measurements made several months apart. The comparison between the separate measures indicates that the internal precision is excellent, with $\sigma_1(d_r) = 0.03$ mm $\approx 0''.55$, or $\sigma(d_r)/\langle d_r \rangle \approx 4.5\%$. The conversion to angular size was made with a plate scale of $18''.59$ mm^{-1}, and the final estimates have, in addition, been corrected for pincushion distortion according to formulae given by Chiu (1976). Tables 8 and 9 show that $\langle d_r \rangle = 14''.1 \pm 0''.9$ for the 17 SB galaxies while $\langle d_r \rangle = 10''.9 \pm 0''.8$ for the nine SAB galaxies. The mean logarithmic difference, $\Delta \log \langle d_r \rangle = 0.11$, is smaller than the value, 0.15, expected from the analysis in Paper II; as shown in § III*b*, this is probably due to selection effects. There are no independent published estimates of the diameters of these rings. The only ring large and distinct enough to be recognizable on the Palomar Sky Survey is that in A1559.9+1634; its diameter measured on the POSS print is $d_r = 21''$, which is in good agreement with the more precise estimate given in Table 8.

b) *Analysis*

Because the Hercules Supercluster is much richer in galaxies than the smaller groups and clusters discussed in § III, and also is much more distant, size-of-sample effects and potential Malmquist-type biases are important to consider. To account for such effects, we analyze in this section the distribution of reduced logarithmic ring diameters for SB and SAB galaxies in Hercules separately, and compare the distributions with those found for the nearer galaxies listed in Paper III.

[8] Comparison with independent estimates of 10 objects by Corwin yielded for the types $\langle T_B - T_C \rangle = 0.2 \pm 0.3$ ($\sigma_{BC} = 0.8$) and families $\langle F_B - F_C \rangle = 0.15 \pm 0.1$ ($\sigma_{BC} = 0.3$). A similar comparison of independent estimates by Thompson of nine objects in our sample yielded $\langle T_B - T_T \rangle = -1.3 \pm 0.7$ ($\sigma_{BT} = 2.0$) and $\langle F_B - F_T \rangle = 0.0 \pm 0.2$ ($\sigma_{BT} = 0.5$). Since the comparison between the first two sources shows no systematic differences and small accidental errors, we suspect that Thompson's classifications are too late by about 1 stage interval and have larger accidental errors.

TABLE 8

RING MODULI AND DATA FOR PROBABLE SB MEMBERS OF THE HERCULES SUPERCLUSTER

Object	Type	T	d_r	V_0	x	$\mu_0^{II}(r)$	Notes
A1559.0+1627	SB(rs)ab	2	15''.8	11386	0.297	35.27	a
A1559.3+1634	SB(rs)a	1	17.5	13156	0.389	34.78	a
A1559.9+1634	SB(r)b	3	22.0	9381	0.474	34.36	a
A1601.2+1734	LB(r)0$^+$	−1	10.4	...	0.265	35.41	b
A1602.1+1801	(R′:)SB(r)0/a	0	13.8	...	0.336	35.05	b
A1603.0+1727	LB(r)0$^+$	−1	10.3	...	0.260	35.43	b
A1603.3+1749	LB(r)0$^+$	−1	12.2	...	0.332	35.06	b
A1603.4+1824	SB(r)b	3	10.8	11399	0.164	35.90	b
A1603.5+1809	SB(rs)b	3	11.2	12325	0.179	35.82	b
I1186	SB(r)b	3	16.3	11172	0.344	35.01	b
A1604.0+1726	SB(r)b	3	14.4	...	0.290	35.28	b
I1189	(R)SB(rs)0/a	0	20.7	12006	0.512	34.17	b
A1604.1+1830	SB(rs)a	1	11.2	...	0.196	35.75	b,c
A1603.1+1759	SB(rs)0/a	0	10.9	...	0.236	35.56	b,c
I1195	SB(r)bc	4	12.6	12232	0.352	34.97	b
A1604.8+1809	SB(rs)a	1	19.5	...	0.438	34.55	b
A1604.0+1627	SB(r)0/a	0	10.5	...	0.218	35.64	d
Unweighted means		1.2	14''.1		0.311	35.18	
Internal m.e.		±0.4	±0.9		±0.025	±0.12	
σ_1		1.7	3.9		0.102	0.51	
Weighted mean					0.317	35.15	
Internal m.e.					±0.025	±0.12	
σ_1					0.101	0.51	

[a] In A2147.
[b] In A2151.
[c] Weight of 0.5 assigned.
[d] In A2152.

TABLE 9

RING MODULI AND DATA FOR PROBABLE SAB MEMBERS OF THE HERCULES SUPERCLUSTER

Object	Type	T	d_r	V_0	x	$\mu_0{}^{II}(r)$	Notes
A1559.9+1604	SAB(r)ab	2	10″0	13281	0.247	35.50	a
A1601.3+1737	SAB(r)bc	4	11.3	...	0.455	34.46	b
A1601.6+1630	SAB(rs)0/a	0	11.8	11698	0.418	34.64	c
A1602.2+1747	LAB(r)0$^+$	−1	9.8	...	0.387	34.79	b
A1602.4+1729	SAB:(r)0/a	0	6.8	...	0.254	35.46:	b, d
A1603.2+1749	(R)LAB(r)0$^+$	−1	10.9	...	0.435	34.56	b
A1603.2+1803	SAB?(r)a	1	16.0	...	0.501	34.23:	b, d, e
A1604.3+1802	SAB(r)b	3	9.4	11163	0.253	35.46	b
A1604.4+1755	SAB(r)0/a	0	12.5	...	0.369	34.89	b
Unweighted means		0.9	10.9		0.369	34.89	
Internal m.e.		±0.6	±0.8		±0.032	±0.16	
σ_1		1.8	2.5		(0.096)	(0.48)	f
Weighted means					0.368	34.89	
Internal m.e.					±0.030	±0.15	
σ_1					(0.090)	(0.45)	f

[a] In A2147.
[b] In A2151.
[c] In A2152.
[d] Weight of 0.5 assigned.
[e] Resembles NGC 4274 with foreshortened bar in plane of vision; could be an SB.
[f] The small value of σ_1 is caused by incompleteness of the sample (see text).

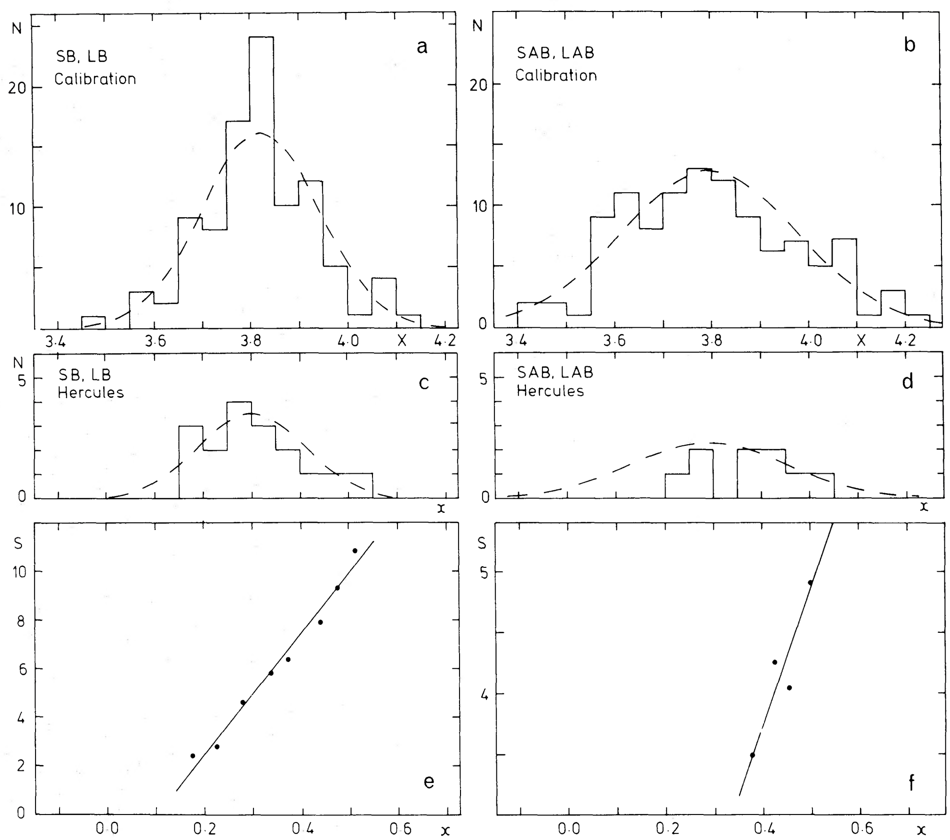

FIG. 3.—(a) Histogram of the distribution of reduced logarithmic *linear* ring diameters (in kpc) $X = \log D_r{}^c$ (0, 2.5) for 97 SB, LB galaxies in the calibrating sample of Paper II. (b) Same as (a) but for 108 SAB, LAB galaxies. (c) Histogram of the distribution of reduced logarithmic *apparent* ring diameters (in units of 0′.1) $x = \log d_r{}^c$ (0, 2.5) for 17 SB, LB galaxies in the Hercules Supercluster. (d) Same as (c) but for 11 SAB, LAB galaxies. In each of these plots the dashed curve is the Gaussian frequency function which best represents the distribution; for the Hercules galaxies, these were derived from graphs of eq. (7), which are illustrated in Fig. 3e for 17 SB, LB galaxies, and in Fig. 3f for six SAB, LAB galaxies.

i) Ringed SB Galaxies

For the purpose of this analysis, we define

$$x \equiv \log d_r^c(0, 2.5) = \log d_r - 0.15F - C_1(T - 2.5) \quad (6a)$$

to be the reduced logarithmic *apparent* ring diameter (in units of 0.'1), and

$$X \equiv \log D_r^c(0, 2.5) = \log D_r - 0.15F - C_1(T - 2.5) \quad (6b)$$

to be the reduced logarithmic *linear* ring diameter (in kpc), where F is the family index and $C_1 = 0.05$ for $-2 \le T \le 2$ or -0.12 for $3 \le T \le 7$. Figure 3*c* shows the distribution of reduced apparent diameters x for the 17 Hercules galaxies in Table 8.[9] Since all these objects are SB or LB types ($F = 1$), the principal purpose of this reduction is to remove the strong effect of type, especially for $T > 2$. Figure 4 shows that the reduced apparent diameters are essentially independent of type over the range $-1 \le T \le 4$, indicating that the correction formula determined from the nearer galaxies is also valid for the Hercules galaxies. The spread in the reduced diameters is seen to be very large; but we ask whether it is consistent with the dispersion of the reduced linear diameters X among the nearby galaxies in the calibrating sample of Paper II. The situation is analogous to that of globular cluster luminosity functions (Hanes 1977), which are Gaussian with a large enough dispersion that incompleteness could bias the mean magnitude if not properly accounted for. For the Hercules analysis, we compile in Table 10*a* and illustrate in Figure 3*a* the distribution of X values for 97 SB galaxies in Paper III having distance moduli $\mu_0^w(\Lambda_c)$ (de Vaucouleurs 1979) or $\mu_0^w(V_M)$ (Bottinelli *et al.* 1980) from optical and radio indicators. The sample consists of all the SB galaxies used in Paper II to calibrate the two-parameter formula (eq. [6b]), except that galaxies possessing (rs)-type pseudo rings were retained and given half weight since several galaxies in our Hercules sample are (rs) types. The distribution of SB ring diameters is found to be nearly Gaussian with a mean $\bar{X} = 3.819 \pm 0.012$ (with weights as in Paper II) and a dispersion $\sigma_{12} = 0.120$, as indicated by the curve in Figure 3*a*. This weighted mean is in good agreement with the adopted zero point, 3.810, derived in Paper II from solutions to a sample of 226 lenticulars and spirals of all families. The dispersion, $\sigma_{12} = 0.120$, includes the effects of error in the tertiary distance moduli. For a mean modulus $\langle\mu_0\rangle$ based on both $\mu_0^w(\Lambda_c)$ and $\mu_0^w(V_M)$, this error will be $\sigma(\langle\mu_0\rangle) \approx 0.30$–$0.35$ mag, so that errors in the distances contribute $\sigma_2 = 0.06$–0.07 to the dispersion of the reduced ring diameters. The corrected dispersion, $\sigma_1 = 0.10$, is slightly lower than the estimate $\sigma_1 = 0.112$ ($5\sigma_1 = 0.56$ mag) made in Paper II via a comparison between $\mu_0^{\mathrm{II}}(r)$ and $\mu_0^w(V_M)$ for 38 well-observed SB spirals. Because $\mu_0^w(\Lambda_c)$ depends on T, the correlation between $\mu_0^{\mathrm{II}}(r)$ and $\mu_0^w(\Lambda_c)$ will tend to cause the calculated σ_1 to be too low, but apparently the reduction is only slight. We will adopt $\sigma(X) = 0.11 \pm 0.01$ for the true dispersion (cosmic scatter plus measuring errors) in the distribution of reduced logarithmic ring diameters in LB and SB galaxies.

This analysis of SB galaxies can now be used to estimate the mean logarithmic reduced apparent ring

[9] We are excluding from this analysis the three SB systems of Table 12 whose rings and parent galaxies are so small ($d_r \le 7''$, $d_G \lesssim 20''$) that the classifications (both family and stage) are very uncertain. For reasons given in § IV*c* we suspect that most of these objects are in the background of the Hercules Supercluster. However, were they to be included in the analysis, the derived distance to Hercules would be increased by less than 10% (the modal diameter would not change). We believe that the distance is better determined from Table 8 where the classifications and ring diameters are most reliable.

TABLE 10

DISTRIBUTION OF REDUCED SB AND LB RING DIAMETERS

X or x	$\bar{X}$ or $\bar{x}$	N	$\langle V_0\rangle$
a) Nearby Calibrating Sample			
4.10–4.15	4.101	1	1701
4.05–4.10	4.069	4	2089
4.00–4.05	4.033	1	1141
3.95–4.00	3.979	5	2151
3.90–3.95	3.922	12	2156
3.85–3.90	3.880	10	1577
3.80–3.85	3.827	24	2132
3.75–3.80	3.778	17	1386
3.70–3.75	3.725	8	2204
3.65–3.70	3.677	9	1730
3.60–3.65	3.626	2	2840
3.55–3.60	3.591	3	1700
3.50–3.55	...	0	...
3.45–3.50	3.474	1	1649
b) Hercules Supercluster			
0.50–0.55	0.512	1	12006
0.45–0.50	0.474	1	9381
0.40–0.45	0.438	1	...
0.35–0.40	0.371	2	12694
0.30–0.35	0.337	3	11172
0.25–0.30	0.278	4	11386
0.20–0.25	0.224	2	...
0.15–0.20	0.176	3	11862

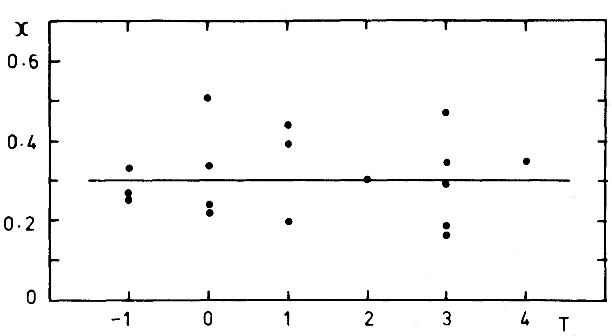

FIG. 4.—Reduced logarithmic apparent ring diameter x versus stage T for 17 SB, LB galaxies in the Hercules Supercluster. Note the lack of significant residual dependence of x on stage, indicating that eq. (6a) adequately removes the effect of Hubble type on the ring diameters.

diameter $\bar{x}$ for galaxies in Hercules, if we assume that the sample is essentially complete to a certain limit and that it can be characterized by a normal error law with a true dispersion $\sigma_1 = 0.11$. Figure 3*c* shows that the frequency distribution of apparent diameters is probably complete (over the area surveyed) down to diameters significantly smaller than the true mean of the distribution; the approximate cutoff in apparent diameters, $d_r \approx 10''$, for galaxies with reliable Hubble types indicates that incompleteness does not seriously bias the mean values in Table 8. Nevertheless, to illustrate the procedure and evaluate the size of the bias we will pursue this analysis. To calculate the true mean of the Gaussian distribution, we derive the slope of the regression line,

$$S = \ln N + x^2/2\sigma^2 = \text{const.} + (\bar{x}/\sigma^2)x\ , \qquad (7)$$

after Hanes (1977). Values of N and x are compiled in Table 10B for eight intervals of x; the correlation between x and S for $\sigma = 0.11$ is illustrated in Figure 3*e*, which shows that a linear representation (i.e., a Gaussian frequency function) is a good fit to the points. The mean regression line,

$$S = -2.527 + 24.967x\ ,$$

yields $\bar{x} = 0.302 \pm 0.027$. The weighted mean of the 17 values in Table 8, $\bar{x} = 0.317 \pm 0.025$, is only slightly larger than this, indicating that, as expected, we have detected a large fraction of the SB ringed galaxies in Hercules. The mean actually depends slightly on the adopted value of σ, ranging from $\bar{x} = 0.310$ for $\sigma = 0.10$ to 0.294 for $\sigma = 0.12$.[10] This range corresponds to less than 0.10 mag in the calculated distance modulus, and hence the small uncertainty in $\sigma(\leq 0.01)$ will not appreciably influence the derived value of the Hubble constant.

The distance modulus of the Hercules Supercluster is then estimated by simply comparing the mean reduced apparent ring diameter with the corresponding mean of the linear diameters in the near sample via

$$\mu_0(r) = 5[\bar{X} - \bar{x} + 3.536]\ , \qquad (8)$$

which, with $\bar{X} = 3.819 \pm 0.012$ and $\bar{x} = 0.302 \pm 0.027$, yields $\mu_0(r) = 35.27 \pm 0.15$ (exclusive of zero point error, or ± 0.25 inclusive). The corresponding distance is $\bar{\Delta} = 113 \pm 13$ Mpc. The mean radial velocity from eight of the 17 objects in Table 8 is $\langle V_0 \rangle = 11{,}632 \pm 394$ km s^{-1}; the mean distance modulus from these objects alone, $\langle \mu_0(r) \rangle = 35.04 \pm 0.30$, leads to $\Delta = 102 \pm 14$ Mpc and $(H) = 114 \pm 32$ km s^{-1} Mpc^{-1}, necessarily with a large error. Since the Hercules Supercluster is presumably bound, we can also use the mean radial velocity, $\langle V_0 \rangle = 11{,}080 \pm 108$ km s^{-1} (Tarenghi *et al.* 1980) of 123 probable members in the redshift range $8500 \lesssim V_0 \lesssim 14{,}000$ km s^{-1}. Then with $\bar{\Delta} = 113 \pm 13$ Mpc, the corresponding Hubble *constant* is $(H_0) = 98 \pm 12$ km s^{-1} Mpc^{-1}, which is in excellent agreement with the unbiased global estimate, $H_0 = 100 \pm 10$ km s^{-1} Mpc^{-1}, obtained in EDS VII, and with the more recent values of 96 ± 10 (de Vaucouleurs and Peters 1981) and 103 ± 10 (de Vaucouleurs *et al.* 1981) derived from combined solutions for H_0 and solar motion from field galaxies in the distance range $2 < \Delta < 32$ Mpc.

[10] Note that the effect is very much smaller than in the case of the globular clusters in the Virgo Cluster (Hanes 1977), because in the present case the counts reach well past the maximum of the frequency function.

ii) *Ringed SAB Galaxies*

Because SAB rings are on the average smaller and show a larger dispersion than SB rings, they are more prone to incompleteness and Malmquist bias at the distance of the Hercules Supercluster. That such incompleteness is probably present in our SAB sample is indicated by Table 9, where the mean reduced logarithmic ring diameter, $\bar{x} = 0.368 \pm 0.030$, for the nine SAB objects is larger than that for the set of SB galaxies, $\bar{x} = 0.317 \pm 0.025$. To further illustrate the bias, we show in Figures 3*b* and 3*d* histograms, respectively, of the distribution of X for a sample of 108 nearby SAB galaxies used in our calibrating sample of Paper II, chosen according to the same precepts as for the SB galaxies in § IV*b*(i), and of x for the SAB rings in Hercules. The data for these histograms are compiled in Table 11. The distribution of logarithmic linear diameters is seen to be approximately Gaussian, with a mean $\bar{X} = 3.793 \pm 0.018$ and dispersion $\sigma = 0.183$. This mean is in good agreement with the adopted zero point, 3.810, obtained from

TABLE 11

DISTRIBUTION OF REDUCED SAB AND LAB RING DIAMETERS

X or x	$\bar{X}$ or $\bar{x}$	N	$\langle V_0 \rangle$
a) Nearby Calibrating Sample[a]			
4.20–4.25	4.230	1	2802
4.15–4.20	4.177	3	1356
4.10–4.15	4.140	1	4258
4.05–4.10	4.071	7	2290
4.00–4.05	4.023	5	2500
3.95–4.00	3.982	7	2436
3.90–3.95	3.919	6	1665
3.85–3.90	3.864	9	1604
3.80–3.85	3.833	12	2531
3.75–3.80	3.784	13	2595
3.70–3.75	3.723	11	1708
3.65–3.70	3.684	8	2602
3.60–3.65	3.623	11	2070
3.55–3.60	3.576	9	1525
3.50–3.55	3.537	1	1237
3.45–3.50	3.480	2	1184
3.40–3.45	3.435	2	937
b) Hercules Supercluster			
0.50–0.55	0.501	1	...
0.45–0.50	0.455	1	...
0.40–0.45	0.427	2	11698
0.35–0.40	0.378	2	...
0.30–0.35	...	0	...
0.25–0.30	0.253	2	11163
0.20–0.25	0.247	1	13281

[a] Excluding three objects with $\log D_r^c < 3.3$.

solutions in Paper II. Although the sample is small, it is possible to roughly estimate the true mean of the distribution of SAB rings in Hercules via an analysis identical to that in § IV*b*(i) for the SB galaxies. The corrected dispersion of the nearby SAB sample is $\sigma \approx 0.17$, but here there is poor agreement with the analysis in Paper II, where we estimated $\sigma_1 = 0.14$ ($5\sigma_1 = 0.70$ mag) via a comparison between $\mu_0^{\rm II}(r)$ and $\mu_0^{w}(V_M)$ for 42 well-observed SAB spirals. We adopt $\sigma_1 = 0.16 \pm 0.02$ for the Hercules analysis and, assuming that the counts are complete to $x = 0.35$, the mean regression line via equation (7) is (Fig. 3*f*)

$$S = -0.834 + 11.374x ,$$

whose slope indicates that $\bar{x} = 0.291 \pm 0.053$. This is in good agreement (within the large mean error) with the value $\bar{x} = 0.302 \pm 0.027$ for the SB galaxies, but because σ is somewhat uncertain and because the counts are not complete beyond the true mean of the population, the derived mean is more sensitive to σ. Thus $\bar{x} = 0.321$ for $\sigma = 0.14$, while $\bar{x} = 0.263$ for $\sigma = 0.18$. We adopt $\bar{x} = 0.291 \pm 0.060$, which yields $\mu_0(r) = 35.19 \pm 0.31$ (exclusive of zero point error, or ± 0.37 inclusive) and a mean distance $\bar{\Delta} = 109^{+20}_{-17}$ Mpc. Only three of the galaxies in Table 9 have known redshifts. If we again assume that $\langle V_0 \rangle$ for our SAB sample is close to that for the entire Hercules complex, $\langle V_0 \rangle = 11{,}080 \pm 108$, then the Hubble constant is $(H_0) = 102 \pm 17$, again, in excellent agreement with the global estimates already discussed. The weighted mean distance modulus of the Hercules Supercluster (with weights $w = 1$ for SB and 0.25 for SAB galaxies) is $\langle \mu_0(r) \rangle = 35.25 \pm 0.14$ (internal m.e.). This is in excellent agreement with the preliminary estimate $\mu_0(\Lambda_c) = 35.1$ derived by Corwin (1977) from an analysis of magnitudes and diameters of Hercules galaxies in conjunction with the corrected luminosity index $\Lambda_c = (T + L_c)/10$.

c) *Possible Ringed Galaxies in the Background of the Hercules Supercluster*

Table 12 lists five galaxies in the Hercules region that possess very small inner rings. The ring moduli for these galaxies range from 36.3 to 37.0, and most are more than 2 σ from the mean modulus, 35.25, derived in § IV*b*. The histogram in Figure 5 shows that these objects may represent an isolated group in the background of the Hercules Supercluster. Because the gap in Figure 5 is not statistically significant and because redshifts are unavailable for any of these galaxies, this conclusion is not definite; but we note that Tarenghi *et al.* (1979) have suggested that another supercluster beyond Hercules may be present at a redshift of 19,000 km s^{-1} or more. The corresponding redshift modulus, $\mu_0(H_0) = 36.5$ (for $H_0 = 95$), is sufficiently close to the mean ring modulus, 36.6, of the five galaxies in Table 12, to be considered as supporting this suggestion.

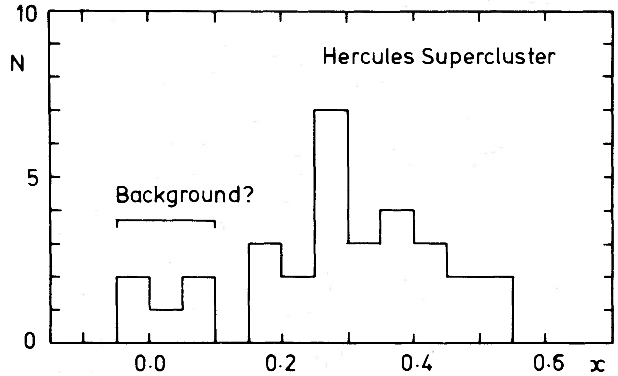

FIG. 5.—Histogram of the distribution of reduced logarithmic apparent ring diameters x for 31 ringed galaxies in the direction of the Hercules Supercluster. Note the possible concentration of the objects into two groups. Objects with $x > 0.15$ are probable members of the Hercules Supercluster, but those with $x < 0.10$ could be in the background.

However, the very small sizes of both the rings and their parent galaxies make it difficult to classify these galaxies accurately. They mark the current practical limit of the range of applicability of this distance indicator, because at distances in excess of 250 Mpc ($\mu_0 = 37$), even the largest SB rings may be 5″ in diameter or less. For distances less than 150 Mpc ($\mu_0 = 36$), ring diameters should be useful as distance indicators at the resolution of good prime-focus plates obtainable with 4–6 m class

TABLE 12

POSSIBLE RINGED GALAXIES IN THE BACKGROUND OF THE HERCULES SUPERCLUSTER

Object	Type	T	d_r	x	$\mu_0^{\rm II}(r)$	Notes
A1602.3+1648 …………	SB:(r:)a sp	1	7″.3	0.010	36.68:	[a]
A1602.3+1700 …………	SB:(r)bc	4	6.6	0.071	36.37:	[a]
A1603.1+1632 …………	SB(r)a	1	7.0	−0.008	36.77	
A1603.3+1730 …………	SAB(rs)a	1	5.4	−0.046	36.96	
A1558.6+1638 …………	SAB:(r)ab:	2:	8.3	0.091	36.28:	[a]
Unweighted mean …………………			6.9	0.024	36.61	
			±0.5	±0.025	±0.13	
σ_1 ………………………………			1.1	0.057	0.28	
Weighted mean ……………………				0.009	36.68	
				±0.024	±0.12	
σ_1 ………………………………				0.056	0.28	

[a] Assigned half-weight in weighted mean.

telescopes, because at this resolution ($\sim 1''$, giving $\sim 10^3$ pixels in the visible image) it is still possible to make fairly reliable estimates of revised morphological types.

V. THE HUBBLE CONSTANT IN DIFFERENT FRAMES OF REFERENCE

a) *Dependence on Distance and Direction*

The purpose of this section is to summarize the results of §§ III–IV regarding the derivation of the Hubble ratio, but in addition we study its dependence, if any, on the adopted frame of reference, that is, V_0, V_G, or V_c as described in § II. The detailed summary for the four nearby bound groups and clusters ($10 \leq \Delta \leq 16$ Mpc) is given in Table 13A, that for the two distant ($\Delta = 45$ and 112 Mpc) clusters is given in Table 13B, while values for the one unbound grouping (Grus) is given in Table 13C. In each case the Hubble ratio is derived according to the formulae given in § II. In Tables 13A and 13B, two cases

TABLE 13A

Hubble Ratio Determinations for Nearby Bound Groups and Clusters

Group or Cluster (1)	ℓ b L B (°) (2)	$\langle\mu_0(r)\rangle$ $\bar{\Delta}$ (3)	$\langle V_0\rangle$ $\langle V_G\rangle$ $\langle V_c\rangle$ (4)	N_2 N_1' (5)	$(H_0)'$ $(H_G)'$ $(H_c)'$ (6)	$\langle V_0\rangle$ $\langle V_G\rangle$ $\langle V_c\rangle$ (7)	N_1 (8)	$(H_0)''$ $(H_G)''$ $(H_c)''$ (9)	$(H_0)^*$ $(H_G)^*$ $(H_c)^*$ (10)	Notes (11)
Dorado	262	30.00	867	5	86.7	896	14	89.6	88.1	a
Cloud	-44	±0.13	±96	5	±10.9	±47		±7.1	±9.3	
	237	10.0	935		93.5	960		96.0	94.7	
	-40	±0.6	±90		±10.6	±46		±7.4	±9.2	
			814		81.4	844		84.4	82.9	
			±95		±10.7	±46		±6.8	±9.0	
Virgo E	284	30.38	1234	6	103.6	1000	42	84.0	93.3	b
Cluster	+75	±0.16	±188	6	±17.5	±60		±8.0	±12.8	
	103	11.9	1142		95.9	908		76.2	85.5	
	-3	±0.9	±186		±17.1	±60		±7.6	±12.4	
			1383		116.1	1150		96.5	105.9	
			±185		±17.7	±60		±8.7	±13.3	
Fornax I	237	30.67	1453	4	106.7	1395	54	102.5	104.6	c
Cluster	-55	±0.30	±199	4	±20.8	±49		±14.6	±17.4	
	263	13.6	1548		113.7	1491		109.5	111.6	
	-42	±1.9	±200		±21.5	±49		±15.5	±18.6	
			1399		102.8	1342		98.6	100.7	
			±197		±20.3	±49		±14.1	±17.4	
Virgo S	284	30.98	1295	17	82.5	1350	26	86.0	84.2	b
Cloud	+75	±0.20	±157	17	±12.6	±150		±12.4	±12.5	
	103	15.7	1222		77.8	1258		80.1	78.9	
	- 3	±1.5	±153		±12.1	±150		±12.1	±12.1	
			1445		92.0	1500		95.5	93.7	
			±157		±13.1	±150		±13.0	±13.0	

Explanation of table 13A.—Col. (1), name of group or cluster. Col. (2), mean galactic coordinates (l, b) and supergalactic coordinates (L, B) from de Vaucouleurs 1975. Col. (3), weighted mean ring modulus $\langle\mu_0(r)\rangle$ and mean antilogarithmic distance $\bar{\Delta} = \text{dex}\,[0.2(\langle\mu_0(r)\rangle - 25)]$. Col. (4), mean radial velocities corrected for IAU standard solar motion with respect to velocity centroid of Local Group (V_0), revised version of the latter after de Vaucouleurs and Peters 1981 (V_G), and motion of Local Group itself within Local Supercluster (V_c, de Vaucouleurs and Peters 1981; de Vaucouleurs *et al.* 1981). Col. (5), N_2 = number of objects with ring moduli, N_1' = number of these same objects with radial velocity available. Col. (6), Hubble constant calculated via eq. (3) using data in cols. (3) and (4). Col. (7), group mean velocities (see notes for references). Col. (8), N_1 = Number of objects in total group mean velocity. Col. (9), Hubble constant from cols. (3) and (7) via eq. (3). Col. (10), $(H)^* = [(H)^{(\prime)}(H)^{(\prime\prime)}]^{1/2}$.

TABLE 13B
HUBBLE RATIO DETERMINATIONS FOR THE DISTANT CLUSTERS

Group or Cluster	ℓ / b / L / B / (o)	$\langle\mu_o(r)\rangle$ / $\bar{\Delta}$ / $(\langle\mu_o(r)\rangle)$ / $(\bar{\Delta})$	$\langle V_o\rangle$ / $\langle V_G\rangle$ / $\langle V_c\rangle$	N_2 / N_1'	$(H_o)'$ / $(H_G)'$ / $(H_c)'$	$\langle V_o\rangle$ / $\langle V_G\rangle$ / $\langle V_c\rangle$	N_1	$(H_o)''$ / $(H_G)''$ / $(H_c)''$	$(H_o)^*$ / $(H_G)^*$ / $(H_c)^*$	Notes
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
IC 4329	318	33.27	(3115)	9	(98.0)	4224	8	93.7	(95.8)	d
Group	+31	±0.18			...	±230		±9.1		
	149	45.1	(3005)		(94.5)	4113		91.2	(92.8)	
	+4	±3.7			...	±230		±9.1		
		(32.51)	(3134)	1	(98.6)	4242		94.1	(96.3)	
		...			...	±230		±9.2		
		(31.8)								
		...								
Hercules	31	35.27	(11632)	17	(114.2)	11080	123	97.9	(105.7)	e
Supercluster	+44	±0.15	±394		±12.2	±108		±6.8		
(SB,LB)										
	109	113.2	(11517)		(113.0)	10965		96.9	(104.6)	
	+49	±7.8	±394		±12.1	±108		±6.7		
		(35.04)	(11630)	8	(114.1)	11078		97.9	(105.7)	
		±0.22	±394		±12.2	±108		±6.8		
		(101.9)								
		±10.3								
Hercules	31	35.19	(12047)	9	(109.8)	11080	123	101.6	(105.6)	e
Supercluster	+44	±0.31	±636		±15.3	±108		±14.5		
(SAB,LAB)										
	109	109.1	(11932)		(108.8)	10965		100.5	(104.6)	
	+49	±15.6	±636		±15.2	±108		±14.4		
		(35.20)	(12045)	3	(109.8)	11078		101.5	(105.6)	
		±0.28	±636		±15.3	±108		±14.5		
		(109.7)								
		±14.2								

EXPLANATIONS.—Same as in Table 13A, but since $N_1' < N_2$ in all of these cases, col. (3) gives the mean ring modulus $\langle\mu_0(r)\rangle$ and antilogarithmic distance $\bar{\Delta}$ for only the subset of galaxies which have both a ring modulus and a radial velocity available. In this case, $(H)'$ is computed via these quantities and the radial velocities in col. (4).

are considered. First, the Hubble ratio $(H)'$ is derived via equation (3) using only the mean radial velocity and ring modulus from the subset of galaxies with rings. Second, the Hubble ratio $(H)''$ is computed again via equation (3) but using the better defined mean radial velocity of the entire group. For the IC 4329 Group and the Hercules Supercluster, the former value is not well defined because radial velocities are not available for many of the galaxies with rings. In these cases, the Hubble ratio $(H)'$ computed from the mean distance and velocity of the (r) objects alone has low weight and is shown for completeness only. For all the groups in Table 13A, we will adopt the geometric mean $(H)^*$ of $(H)'$ and $(H)''$ as the best estimate of the Hubble ratio in each case.

In Table 14 we summarize mean values of the Hubble ratios from Table 13 for the five groups and clusters closer than 20 Mpc. The Hubble ratios for these groups are especially sensitive to the adopted reference frame, but the means are fairly insensitive because the groups are well distributed on the sky (Fig. 1). As described in § II, we believe that the Hubble constant is best determined with respect to the frame of reference defined by the nearby galaxies ($2 < \Delta < 32$ Mpc) because in this frame it should be, and is, substantially independent of direction. This is shown in Figure 1, where each cluster or group is labeled with its value of $(H_c)^*$ or $H_c{}^*$ if nearer than 20 Mpc, or $(H_c)''$ for the IC 4329 Group and the Hercules Supercluster. No significant trend of H with

TABLE 13C

Hubble Ratio Determination for the (Unbound) Grus Cloud

Group or Cluster (1)	ℓ b L B (o) (2)	$\langle\mu_0(r)\rangle$ $\langle\Delta\rangle$ (3)	N_2 (4)	$\langle\log V_0\rangle$ $\langle\log V_G\rangle$ $\langle\log V_c\rangle$ (5)	$\langle V_0\rangle$ $\langle V_G\rangle$ $\langle V_c\rangle$ (6)	$\langle H_0\rangle$ $\langle H_G\rangle$ $\langle H_c\rangle$ (7)	$\bar{H}_0$ $\bar{H}_G$ $\bar{H}_c$ (8)	H_0^* H_G^* H_c^* (9)
Grus Cloud	348 -65	30.59 ...	6	3.133 ±0.048	1398 ±138	96.6 ±13.4	103.7 +15.2 -13.1	100.1 ±13.8
	250 + 8	14.5 ...		3.145 ±0.046	1433 ±138	99.0 ±13.9	106.4 +15.8 -13.7	102.6 ±14.3
				3.063 ±0.057	1202 ±138	83.1 ±11.2	88.2 +12.5 -10.8	85.6 ±11.4

Explanations.—Cols. (1) and (2), same as in Table 13A. Col. (3), unweighted mean ring modulus and distance (Mpc). Cols. (4), (5), and (6), see explanations for cols. (4) and (5) in Table 13A. Col. (7), Hubble ratio computed from eq. (1). Col. (8), mean antilogarithmic Hubble ratio via eq. (2). Col. (9), $H^* = [\bar{H}\langle H\rangle]^{1/2}$.

Notes to Table 13.—(*a*) Group mean radial velocity is based on 14 of 15 objects listed by de Vaucouleurs (1975) in the subcomponents of this cloud, G16, G21, and G22, with radial velocities from RC2. (*b*) Mean cluster radial velocity is from de Vaucouleurs and de Vaucouleurs 1973. (*c*) Mean cluster radial velocity is from Jones and Jones 1980. (*d*) Mean group radial velocity is from Sandage 1975*a*, based on velocities given in RC2. (*e*) Mean supercluster radial velocity is from Tarenghi *et al.* 1980.

direction appears to be present in the map. By comparison, the corresponding maps (not shown) for the velocities reduced only to the conventional V_0 or to the centroid of the Local Group (V_G) show a pronounced minimum in the general direction of the Virgo Cluster and a maximum toward the supergalactic anticenter sector, as has been so often demonstrated for a variety of galaxy samples (see, e.g., EDS VII and references therein).

The mean Hubble ratios in Table 14 were calculated in two ways: (i) the unweighted mean, and (ii) a mean weighted by the inverse of the formal variance, σ^{-2}, of the internal mean error. The means are seen to be somewhat sensitive to the weights, but we note that internal mean errors based on small numbers of objects are not necessarily representative of the true relative errors. In any case, if, as argued in § II, we adopt the values of $\langle(H_c)^*\rangle$ in the third column of Table 14 as the best estimates, then the mean Hubble ratios from the nearby groups and clusters are 94 (unweighted) and 91 (weighted). We adopt $H = 92 \pm 4$ (internal m.e.) as representative.

The IC 4329 Group and the Hercules Supercluster are much more distant than these nearby configurations. At $\Delta = 45$ Mpc, the IC 4329 Group may already be outside the Local Supercluster; and at $\Delta \approx 110$ Mpc, the Hercules Supercluster should give a close approximation to the Hubble constant, H_0, defined as the low-density limit of the Hubble ratio. Nevertheless, the values of $(H)_c$ derived from these two configurations are not significantly different from the mean derived from the nearby groups and clusters. Table 13B gives $(H)_c'' = 94 \pm 9$ km s^{-1} Mpc^{-1} for the IC 4329 Group and

TABLE 14

Mean Hubble Ratios for the Nearby Groups and Clusters ($10 \le \Delta \le 16$ Mpc)[a]

$\langle(H_0)'\rangle_*$ $\langle(H_G)'\rangle_*$ $\langle(H_c)'\rangle_*$	$\langle(H_0)''\rangle_*$ $\langle(H_G)''\rangle_*$ $\langle(H_c)''\rangle_*$	$\langle(H_0)^*\rangle_*$ $\langle(H_G)^*\rangle_*$ $\langle(H_c)^*\rangle_*$	Notes
95.9 ±4.8 96.7 ±5.9 95.6 ±6.3	92.4 ±3.7 92.9 ±6.4 92.1 ±3.0	94.1 ±3.7 94.7 ±5.8 93.8 ±4.4	b
92.4 ±4.5 93.1 ±5.3 90.6 ±5.5	89.6 ±2.6 88.9 ±5.8 90.1 ±3.0	91.8 ±3.3 92.3 ±4.9 90.7 ±4.3	c

[a] The asterisk subscript on all of these means is to indicate that the Grus cloud has been averaged into each case.

[b] Means unweighted.

[c] Means weighted by the inverse variance of the internal mean error of each Hubble constant determination in Table 13.

$(H)_c'' = 99 \pm 5$ for the Hercules Supercluster (note, however, that the latter values are uncorrected for possible cosmological effects; see § V*b*). The constancy of the Hubble ratio over a range of distances $10 \leq \Delta \leq 112$ Mpc is an important result of this analysis and shows that in our adopted reference frame any residual effects of the Local Supercluster must be small. (This is because most of the deceleration caused by the Local Supercluster has already been compensated by reducing the velocities to V_c). The mean Hubble ratio from all the groups and clusters is $\langle H_c \rangle = 95.0$ (unweighted) or 94.7 (weighted by the inverse formal variance of the internal mean error in each of the three estimates). We adopt $H_c = 95 \pm 4$ (internal m.e.) km s^{-1} Mpc^{-1} as the best estimate of the mean value of the Hubble ratio in the distance interval $10 \leq \Delta \leq 112$ Mpc derived from the (r) indicator.

b) *Cosmological Effects*

The mean redshift of the Hercules Supercluster, $\langle Z \rangle = 0.037$, is sufficiently large to require consideration of cosmological effects and, specifically, of the effect of geometric optics on the relation between apparent metric diameter d_r and linear diameter D_r. This depends on the cosmological model which introduces an arbitrary choice. For example, in the usual case of the Robertson-Walker metric for a homogeneous isotropic model (with $\Lambda = 0$) equations (30)–(32) of Sandage (1975*b*) give

$$H_0(+1) = \frac{d_r}{D_r} \frac{cZ}{(1+Z)^2} ,$$

$$\text{or} \quad H_0(+1)/H_c = (1+Z)^{-2} , \quad \text{for} \quad q_0 = +1 ; \qquad (9a)$$

$$H_0(0) = \frac{d_r}{D_r} \frac{cZ(1+Z/2)}{(1+Z)^2} ,$$

$$\text{or} \quad H_0(0)/H_c = \frac{1+Z/2}{(1+Z)^2} , \quad \text{for} \quad q_0 = 0 ; \qquad (9b)$$

$$H_0(-1) = \frac{d_r}{D_r} \frac{cZ}{(1+Z)} ,$$

$$\text{or} \quad H_0(-1)/H_c = (1+Z)^{-1} , \quad \text{for} \quad q_0 = -1 , \qquad (9c)$$

where $H_c = (d_r/D_r)cZ$ is the Euclidean limit valid for $Z \to 0$, the definition used in § V*a*. The corresponding correction factors for $Z = 0.037$ are, respectively, 0.930, 0.947, and 0.964. However, because our calibrating sample (Paper II, Table 2C) has an appreciable mean redshift, $\langle Z \rangle \sim 0.006$, for which the correction factors, 0.988, 0.991, and 0.994, are implied in the zero point of the $\mu_0(r)$ scale, only differential corrections need be applied to the Hercules Supercluster, viz., 0.941, 0.956, and 0.970, respectively. Then, with $H_c = 98.6 \pm 5.5$ (§ V*a*), $H_0(+1) = 92.8 \pm 5.2$, $H_0(0) = 94.3 \pm 5.3$, and $H_0(-1) = 95.6 \pm 5.3$, for a possible mean $H_0 = 94 \pm 5$. The corresponding values for the IC 4329 Group, with $\langle Z \rangle = 0.014$, are 0.984, 0.988, and 0.992; then with $H_c = 94.1 \pm 9.2$ (Table 13B), $H_0(+1) = 92.6 \pm 9.1$, $H_0(0) = 93.0 \pm 9.1$, and $H_0(-1) = 93.3 \pm 9.1$, for a possible mean $H_0 = 93 \pm 9$.

An added difficulty is that over the short distances involved (compared with the mean radius of curvature) the universe is neither homogeneous nor isotropic, and a rigorous solution of the light propagation problem is essentially impossible in the absence of detailed data on the actual, clumpy mass distributions. In particular, because of the "cellular" large scale structure of the matter distribution (Einasto, Jôeveer, and Saar 1980), clearly in evidence in the redshift data in the direction of the Hercules Supercluster (Tarenghi *et al.* 1979), it is quite possible that Euclidean straight lines may be as good an approximation of the light rays as the Robertson-Walker geodesics. We note the difficulty but abstain from making a choice.

VI. CONCLUSIONS

We conclude that (1) the values of H_0 derived from the two distant clusters ($45 \leq \Delta \leq 112$ Mpc), $\langle H_c \rangle = 96 \pm 5$ (without cosmological corrections) or $\langle H_0 \rangle = 93 \pm 5$ (with plausible cosmological corrections) are in excellent agreement with the mean value $\langle H_c \rangle = 92 \pm 4$ derived from the nearer groups and clusters ($10 \leq \Delta \leq 16$ Mpc) after correction for solar motion; (2) consequently, the distance scale defined by the rings gives no support to the concept of a quadratic redshift-distance relation (Segal 1976) in the distance interval $10 \leq \Delta \leq 112$ Mpc; and (3) $\langle H_0 \rangle = 93 \pm 4$ (internal mean error or ± 10 including zero point errors) is the value of the Hubble constant on this scale.

We will report elsewhere on a simultaneous solution for solar motion and the Hubble constant derived from the all-sky sample of mainly field galaxies with the best determined ring distance moduli.

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