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Abstract.
The Standard Model Higgs boson with the nonminimal coupling to the gravitational curvature can drive cosmological inflation. We study this type of inflationary scenario in the context of supergravity. We first point out that it is naturally implemented in the minimal supersymmetric SU(5) model, and hence virtually in any GUT models. Next we propose another scenario based on the Minimal Supersymmetric Standard Model supplemented by the right-handed neutrinos. These models can be tested by new observational data from the Planck satellite experiments within a few years.

Keywords: inflation, supersymmetry, grand unification

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Introduction

Recently the idea that the Standard Model (SM) Higgs field may be identified with an inflaton field, has attracted much attention [1]. The major role is played by the nonminimal coupling to gravity, which renders the Higgs mass to be within the range of $126 - 194$ GeV, while keeping the amplitude of the primordial curvature perturbation at the scale of $\sim 10^{-5}$. The idea of inflation by nonminimally coupled inflaton field itself is certainly not new. Nevertheless, the striking agreement with the present-day cosmological data, combined with the minimalistic nature of the model, makes this type of scenario very attractive. The predicted mass range of the Higgs particle is also interesting for the physics of the Large Hadron Collider.

The Higgs potential in the SM is unstable against quantum corrections (the hierarchy problem) and it therefore is reasonable to reconsider Higgs inflation in supersymmetric theory [2, 3]. It is shown in [2] that Higgs inflation cannot be implemented within the minimal supersymmetric Standard Model (MSSM), as the field content of the latter is too restrictive. Instead, with an extra gauge singlet field (i.e. in the next-to-minimal supersymmetric Standard Model, NMSSM) a sensible scenario of Higgs inflation is found to be possible.

We first discuss the possibility of Higgs inflation in supersymmetric grand unified theory (GUT). One obvious reason to motivate this study is that the energy scale of inflation is typically above the grand unification scale, and it is unnatural to suppose that the SM Lagrangian and it seems that supersymmetric GUT is an appropriate theory to start with. Since the NMSSM is structurally similar to the SU(5) GUT model, it seems natural to conjecture that the SU(5) GUT, rather than the NMSSM, may be a more appropriate minimal supersymmetric theory that accommodates Higgs inflation. We will see that a viable Higgs inflationary scenario nicely fits into the minimal SU(5) model. Next we propose another inflationary scenario based on the MSSM supplemented by the right-handed neutrinos. This model is the simplest extension of the MSSM in order to incorporate the observed neutrino masses and flavor mixings via the seesaw mechanism. The structure of the model is also similar to NMSSM. Because of the seesaw mechanism, the inflationary prediction depends on the mass scale of the right-handed neutrinos.

Supersymmetric SU(5) GUT [4]

The minimal supersymmetric SU(5) model consists of a vector super-multiplet transforming as an adjoint 24 of the SU(5), as well as 5 types of chiral super-multiplets, namely $N_f$ (the number of flavors) multiplets in 5 (that include $\bar{d}$ and $L$ of the MSSM), $N_f$ multiplets in 10 (include $Q$, $\bar{u}$, and $\bar{e}$), one each in 24 (denoted $\Sigma$), 5 ($H$) and 5 ($\bar{H}$). $\Sigma$ is the Higgs multiplet responsible for breaking the GUT symmetry, while $H$ and $\bar{H}$ respectively include the up- and down-type MSSM Higgs multiplets. Among these, only the three Higgs chiriral multiplets $\Sigma$, $H$ and $\bar{H}$ play roles in the
In order to realize the triplet-doublet Higgs mass splitting, we set $H_u \equiv H_c$ and $H_d$ is the GUT scale given by $\langle H_d \rangle \equiv \langle H_c \rangle = 0$, from the onset of the inflation. During inflation the dominant rôle is played by the MSSM Higgs fields $H_u$ and $H_d$, which settle down to the present values after the inflation. The charged Higgs can be consistently set to be zero, $H_u = (0 H_d^0)^T$, $H_d = (H_u^0)^T$, and parameterizing $S = s e^{i\beta}$, $H_u^0 = \frac{1}{\sqrt{2}} h_1 e^{i\alpha}$, $H_d^0 = \frac{1}{\sqrt{2}} h_2 e^{i\alpha}$, with $s, h_1, h_2, \alpha, \alpha_1, \alpha_2 \in \mathbb{R}$, and further setting $h_1 = h \sin \beta$ and $h_2 = h \cos \beta$, the model depends on five parameters $\rho, \lambda, \gamma, \delta, \zeta$, and six real scalar fields $s, h, \alpha, \beta, \alpha_1, \alpha_2$. Note that $\rho$ and $\lambda$ are parameters appearing in the GUT superpotential and are typically of order one, while there is no such restriction for $\gamma, \delta$, and $\zeta$. Analyzing the scalar potential, we find stability at $\alpha = \alpha_1 = \alpha_2 = 0$. Furthermore, the D-flat condition sets the value of $\beta$ to be $\pi/4$. Thus the model reduces to a system of two real scalars $h$ and $s$, with the scalar-gravity part of the Jordan frame Lagrangian (cf. [3]),

$$\mathcal{L}_J = -\frac{1}{2} \Phi R_J - \frac{1}{2} g_s^2 \partial_\mu h \partial^\mu h - \kappa g_s^2 \partial_\mu s \partial^\mu s - V_J \right].$$

The subscript J denotes quantities in the Jordan frame, $\kappa \equiv K_{SSS} = 1 - 4 \omega s - 4 \zeta s^2$ is the nontrivial component of the Kähler metric, $\omega \equiv -\delta/\sqrt{30}$, and

$$\Phi = 1 - \frac{1}{2} s^2 + \frac{2}{3} \omega s^3 + \frac{\zeta}{3} s^4 + \left(\frac{\gamma}{4} - \frac{1}{6}\right) h^2.$$
FIGURE 2. The tensor-to-scalar ratio $r$ and the scalar spectral index $n_s$, with the 68% and 95% confidence level contours from the WMAP7+BAO+H0 data [5]. The Harrison-Zel’dovich (HZ) values as well as the predictions of the $\phi^4$ and $\phi^2$ chaotic inflation models are also shown for comparison.

$V_j$ is the F-term scalar potential in the Jordan frame as

$$V_j = \frac{3}{10} \left( \frac{\rho^2}{2} (s-v)^2 h^2 + \frac{1}{\kappa} \left[ \rho \frac{\rho}{4} h^2 - \frac{\lambda}{3} (s-v) \right]^2 \right) - \frac{\left( \frac{2}{3} \gamma + \omega \right) \rho \left( \frac{\rho}{4} h^2 - \frac{\lambda}{3} (s-v) \right) \left[ \frac{\rho \phi^2}{4} - \frac{\lambda \phi^2}{3} \right] s^2 + \frac{\rho \phi^2}{4} - \frac{\lambda \phi^2}{6} - \frac{3 \gamma \phi^2}{4} \right)^2}{10 \left[ 1 + \frac{\gamma}{4} (2 \gamma - 1) h^2 + \frac{\gamma}{4} (2 \gamma - 1) \right]^2}. \quad (6)$$

The dynamics of inflation is encoded in the scalar potential $V_E = \Phi^{-2} V_j$ in the Einstein frame. If we take the canonical form of the Kähler potential (i.e. $\omega = \zeta = 0$), the potential exhibits tachyonic instability in the direction of the field $s$, which occurs also in the case of the NMSSM Higgs inflation [2, 3]. The instability is controlled by introducing a cubic term ($\omega \neq 0$) and a quartic term ($\zeta \neq 0$) in the Kähler potential. Note that these terms are perfectly consistent with the supergravity embedding. The bottom line is that for a wide range of the parameter space with up to quartic order terms in the Kähler potential, there exist reasonable trajectories of the inflaton field. In Fig.1 we show the shape of the scalar potential $V_E$ (the left panel), the inflaton trajectory (center), and the values of $V_E$ at local minima (bottom of the valley) for given $h$ (right). In this example we have taken $\rho = \lambda = 0.5$, $\omega = -100$, $\zeta = 10000$, and $\gamma = 1.86 \times 10^4$. The plateau of the potential at the large $h$ values is a characteristic feature of Higgs inflation. As the field $s$ controls breaking of the GUT symmetry, the trajectory shows that $SU(5)$ is broken from the onset, indicating that problematic topological defects are not produced during inflation. For this parameter set the dynamics of the slow roll inflation is dominated by the field $h$, as the displacement of $s$ is negligibly small ($\Delta s/\Delta h \lesssim 2\%$, with suitable normalization $d\tilde{s} = \sqrt{2\kappa} ds$). Assuming that $s$ is nearly constant, the model simplifies to single field inflation. The Lagrangian (4) can then be written in a form similar to the SM Higgs inflation [1],

$$\mathcal{L}_j = \sqrt{-g} \left[ \frac{M^2 + \xi h^2}{2} R_j - \frac{1}{2} \partial_\mu h \partial_\nu h - V_j \right], \quad (7)$$

with $M^2 = 1 - \frac{1}{2}s^2 + \frac{\gamma}{6} s^4$ and $\xi = \frac{1}{2} \gamma - \frac{1}{b}$.

The slow roll parameters,

$$\epsilon = \frac{1}{2} \left( \frac{1}{V_E} \frac{dV_E}{dh} \right), \quad \eta = \frac{1}{2} \frac{d^2V_E}{V_E dh^2}, \quad (8)$$

are defined for the scalar potential $V_E$ and the canonically normalized inflaton field $\hat{h}$ in the Einstein frame. The latter is related to $h$ by

$$d\hat{h} = \sqrt{M^2 + \xi h^2 + \frac{6 \xi}{2} h^2} dh. \quad (9)$$

For given $(\lambda, \rho, \omega, \zeta)$, the nonminimal coupling $\xi$ is determined from the power spectrum of the curvature perturbation $\mathcal{P}_R = \frac{V_E}{24 \pi^2} \epsilon$. The slow roll terminates when either of the slow roll parameters ($\epsilon$ in the present case) becomes
non-supersymmetric case, the inflationary dynamics does not constrain the Higgs mass at the electroweak scale.

Supersymmetric SM with right-handed neutrinos [6]

Next we propose another inflation model which is based on the MSSM extended with the right-handed neutrinos. The superpotential is

$$W = W_{\text{MSSM}} + \frac{1}{2} M_R N_R^2 + y_D N_R^c L H_u,$$

where $W_{\text{MSSM}}$ is the MSSM superpotential part, $N_R$ is the right-handed neutrino superfield, $M_R$ is the mass parameter for $N_R$, and $y_D$ is the neutrino Dirac Yukawa coupling (the family indices are suppressed). As noted in [2], successful nonminimally coupled Higgs inflation requires at least an extra field besides those in the MSSM. Our crucial observation here is that the model (10) is already such an extension, with the $L$-$H_u$ direction playing the rôle of inflaton. Parameterizing the D-flat direction along $L$-$H_u$ as $L = (\phi \ 0)^T / \sqrt{2}$ and $H_u = (0 \ \phi)^T / \sqrt{2}$, the superpotential becomes

$$W = \frac{1}{2} M_R N_R^2 + \frac{1}{2} y_D N_R^c \phi^2.$$  

We assume supergravity embedding and choose the Kähler potential $K = -3\Phi$ as

$$\Phi = 1 - \frac{1}{3} (|N_R|^2 + |\phi|^2) + \frac{1}{4} \chi (\phi^2 + \text{c.c.}) + \frac{1}{3} |N_R^c|^4,$$

with $\gamma$ and $\zeta$ real parameters. Again, the reduced Planck scale $M_P = 2.4 \times 10^{18}$ GeV has been set to be unity. We shall, for simplicity, take $y_D$ also to be real and consider only one generation below.

We introduce real scalar fields $\chi, N, \alpha_1, \alpha_2$ by $\Phi = \frac{1}{\sqrt{2}} \chi e^{i\alpha_1}, N_R = N e^{i\alpha_2}$. It can be checked that the scalar potential is stable along the real axes of $\phi$ and $N^c_R$, and thus we shall assume $\alpha_1 = \alpha_2 = 0$. The scalar-gravity part of the Lagrangian in the Jordan frame (cf. [3])

$$\mathcal{L}_J = \sqrt{-g_J} \left[ \frac{1}{2} \Phi_R - \frac{1}{2} \delta_{\mu}^\nu \partial_\mu \chi \partial_\nu \chi - \kappa \delta_{\mu}^\nu \partial_\mu N \partial_\nu N - V_J \right].$$

where

$$\Phi = M^2 + \zeta \chi^2, \quad M^2 \equiv 1 - \frac{1}{3} N^2 + \frac{\zeta}{3} N^4, \quad \zeta \equiv \frac{\gamma}{4} - \frac{1}{6}.$$  

The subscripts $J$ indicate quantities in the Jordan frame, and $\kappa = 1 - 4\zeta N^2$ is the nontrivial component of the Kähler metric. The F-term scalar potential reads

$$V_J = \frac{1}{2} y_D^2 N^2 \chi^2 + \frac{(M_R N + \frac{1}{2} y_D \chi^2)^2}{1 - 4\zeta N^2} - \frac{N^2 \left( \frac{1}{2} M_R N + \frac{3}{2} y_D \chi^2 - \frac{\zeta N^2 (21 - 4\zeta N^2)}{21 - 4\zeta N^2} \right)^2}{3 + \frac{\zeta N^4}{1 - 4\zeta N^2} + \frac{3}{4} \chi^4 (\frac{1}{2} \gamma - 1)}.$$  

The scalar potential in the Einstein frame is $V_E = \Phi^{-2} V_J$.  

179
In this model the Dirac Yukawa coupling $y_D$ and the right-handed neutrino mass $M_R$ are not independent. They are related by the seesaw relation [7],

$$m_\nu = \frac{y_D^2 H_0^2}{M_R},$$

where $m_\nu$ is the mass scale of the light (left-handed) neutrinos. Using the neutrino oscillation data $m_\nu^2 \approx \Delta m_{32}^2 = 2.43 \times 10^{-3}$ eV$^2$ [8] and the Higgs VEV at low energy $\langle H_0 \rangle \approx 174$ GeV, we find

$$y_D = \left( \frac{M_R}{6.14 \times 10^{14} \text{ GeV}} \right)^{\frac{1}{2}}.$$  

(17)

This puts an upper bound on $M_R$ since $y_D \lesssim \mathcal{O}(1)$.

For large $y_D$ (and thus large $M_R$) the inflationary model is very similar to the next-to-minimal supersymmetric SM [2, 3] or the supersymmetric grand unified theory model [4]. As in the SUSY SU(5) GUT model discussed in the previous sections, we allow the quartic Kähler term in (12) to control the instability in the $N$-direction. For $M_R = 10^{13}$ GeV we find $\xi = 100$ keeps the deviation of $N$ from $N = 0$ negligibly small ($\sqrt{2} \Delta N / \Delta \chi \lesssim 1\%$ throughout the slow roll of $N_c = 60$ e-folds). For $M_R \lesssim 10^{11}$ GeV, $\xi = 1$ is enough. The plots of the scalar potential and the inflation trajectory for, say, $N_c = 60, M_R = 10^{13}$ GeV and $\xi = 100$ are quite similar to the ones in Fig. 1 with the identification $s = 0.05 \rightarrow N$. Once the trajectory is stabilized the cosmological parameters are insensitive to the value of $\xi$, and as the trajectory is nearly straight the model simplifies to single field inflation with the inflaton $\chi$. The Lagrangian then becomes

$$\mathcal{L}_I = \sqrt{-g} \left[ \frac{M^2 + \xi \chi^2}{2} \mathcal{R}_I - \frac{1}{2} \partial_\mu \chi \partial^\mu \phi - V_{\text{Kähler}} \right].$$

(18)

For analysis of inflationary predictions, the model contains only two parameters: $\xi$ and $y_D$. The former is fixed by the curvature perturbation $\mathcal{P}_R$, and the latter is related to the right-handed neutrino mass $M_R$, through (17). Note that there exists a lower bound on $y_D$, set by the minimal coupling limit $\xi \rightarrow 0$. In this limit our model is essentially the chaotic inflation with quartic potential $V_E = \frac{1}{2} \xi \phi^2 \chi^4$, with $y_D$ fixed by $\mathcal{P}_R$. The corresponding value of $M_R$ at $\xi = 0$ is 644 GeV for $N_c = 50$ and 378 GeV for $N_c = 60$. For a given value of $M_R$ the scalar spectral index $n_\chi \equiv d \ln \mathcal{P}_R / d \ln k = 1 - 6e + 2\eta$ and the tensor-to-scalar ratio $r \equiv \mathcal{P}_T / \mathcal{P}_R = 16\eta$ can be computed. We find that the nonminimal coupling is $\xi \lesssim \mathcal{O}(1)$ when $M_R \lesssim 10^6$ GeV. This shows that in the wide parameter region our model is free from the dangers [10] arising from the large nonminimal coupling. This feature is similar to the model studied in [11]. The prediction of $n_\chi$ and $r$ in our model is shown in Fig. 3, along with the 68% and 95% confidence level contours from the WMAP7+BAO+H$_0$ data [5]. Also indicated are the predictions of two other inflationary models arising from the same Lagrangian (10), namely the $N_R$ chaotic inflation model [12], marked with ●, and the A-term inflation models [13] marked with ■ (AFD). The former is essentially the standard $m^2 \phi^2$ chaotic inflation. In the latter, the inflaton is $\eta \equiv d^4^c d^4^c$, $e^{-L_L}$, or $N_R^c L_R$, direction in the ($N_R$-extended) MSSM, and its typical prediction is very small and $n_\chi \approx 1 - 4/N_c$; we used $N_c = 50$ (thus $n_\chi = 0.92$) as the e-folding cannot be large ($N_c \gtrsim 50$) in such low-scale inflation models. We see that our model fits well with the present data unless $M_R$ is small. The 2-$\sigma$ constraints roughly give $M_R \gtrsim 1$ TeV, depending on the e-folding number (and thus on the reheating temperature). In the near future detailed data from the Planck satellite experiments [14] will be available, with the expected resolution $\Delta n_\chi \approx 0.0045$, also indicated in Fig.3. With such high precision the three inflation models arising from the ($N_R$-extended) MSSM would clearly be discriminated. If our model turns out to be the likely scenario, the Planck data would also constrain the mass scale of the right-handed neutrinos.

**Conclusions**

We have discussed Higgs inflation in the context of supergravity. First we have studied the implementation of the scenario in the SUSY GUT model, in particular, the minimal SU(5) GUT model. We have shown that with an appropriate Kähler potential a viable Higgs inflationary scenario fits into the minimal SU(5) model and the inflationary prediction is consistent with the present WMAP7-BAO-$H_0$ data. The scenario can also be extended to other GUT models whose gauge group contains $SU(5)$ as a subgroup. When the Higgs multiplets of the GUT model contain 5, 5 and 24 of the minimal SU(5) GUT, a superpotential like (1) can be introduced. Then a viable model of Higgs inflation is implemented as described for the minimal SU(5) model. One such simple example is the $SO(10)$ GUT with Higgs multiplets in 10 and 54 representations.
FIGURE 3. The scalar spectral index $n_s$ and the tensor-to-scalar ratio $r$, with the 68% and 95% confidence level contours from the WMAP7+BAO+H0 data [5]. The prediction of our model (NM-LH0) is indicated by • with corresponding $M_R$ values. The predictions of the Harrison-Zel’dovich (HZ), the $\lambda\phi^4$ and $m^2\phi^2$ chaotic inflation models, as well as the A-term MSSM flat-direction (AFD) inflation models, are also shown for comparison. $\Delta n_s$ is the expected Planck accuracy [14].

Next we have proposed another cosmological inflationary scenario based on the MSSM supplemented by the right-handed neutrinos. We have shown that with an appropriate Kähler potential the $L$-$H_u$ direction gives rise to successful inflation. The mass scale $M_R$ of the right-handed neutrinos is subject to the seesaw relation and the present 2-σ constraint from the WMAP7+BAO-H0 data sets its lower bound $M_R \gtrsim 1$ TeV. We expect within a few years new observational data from the Planck satellite clearly discriminates this model from other existing inflationary models arising from the same Lagrangian, and possibly yields stringent constraints on $M_R$.

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