Neutralino Dark Matter in Brane World Cosmology

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The thermal relic density of neutralino dark matter in the brane world cosmology is studied. Since the expansion law at a high energy regime in the brane world cosmology is modified from the one in the standard cosmology, the resultant relic density can be altered. It has been found that, if the five-dimensional Planck mass M_5 is lower than 10^4 TeV, the brane world cosmological effect is significant at the decoupling time, and the resultant relic density is enhanced. We calculate the neutralino relic density in the constrained minimal supersymmetric standard model and show that the allowed region is dramatically modified from the one in the standard cosmology and eventually disappears as M_5 is decreasing. We also find a new lower bound on $M_5 \gtrsim 600$ TeV based on the neutralino dark matter hypothesis, namely, the lower bound in order for the allowed region of neutralino dark matter to exist.

I. INTRODUCTION

Recent cosmological observations, especially the Wilkinson Microwave Anisotropy Probe (WMAP) satellite [1], have established the ΛCDM cosmological model with great accuracy, and the relic abundance of the cold dark matter is estimated as (in the 2σ range)

$$\Omega_{\rm CDM}h^2 = 0.1126^{+0.0161}_{-0.0181}. \qquad (1)$$

To clarify the identity of a particle as cold dark matter is still an open prime problem in cosmology and particle physics. The lightest supersymmetric particle (LSP) is suitable for cold dark matter, because they are stable owing to the conservation of R parity. In the minimal supersymmetric standard model (MSSM), the lightest neutralino is typically the LSP and the promising candidate for cold dark matter. In light of the WMAP data, the parameter space in the constrained MSSM (CMSSM) which allows the neutralino relic density suitable for cold dark matter has been recently reanalyzed [2]. It has been shown that the resultant allowed region is dramatically reduced due to the great accuracy of the WMAP data.

Note that the thermal relic density of the matter depends on the underlying cosmological model as well as its annihilation cross section. If a nonstandard cosmological model is taken into account, the resultant relic density of dark matter can be altered from the one in the standard

cosmology. A brane world cosmological model which has been intensively investigated [3] is a well-known example of such a nonstandard cosmological model. The model is a cosmological version of the so-called "RS II" model first proposed by Randall and Sundrum [4], where our four-dimensional Universe is realized on the "3-brane" located at the ultraviolet boundary of a five-dimensional anti-de Sitter spacetime. In this setup, the Friedmann equation for a spatially flat spacetime is found to be

$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{\rho_0}\right), \qquad (2)$$

where

$$\rho_0 = 96\pi G M_5^6, \qquad (3)$$

H is the Hubble parameter, ρ is the energy density of matters, G is Newton's gravitational constant with M_5 being the five-dimensional Planck mass, and the four-dimensional cosmological constant has been tuned to be almost zero. Here we have omitted the term so-called "dark radiation," since it is severely constrained by big bang nucleosynthesis (BBN) [5]. The coefficient ρ_0 is also constrained by the BBN, which is roughly given by $\rho_0^{1/4} \gtrsim 1$ MeV (or, equivalently, $M_5 \gtrsim 8.8$ TeV) [3]. This is a model-independent cosmological constraint. On the other hand, as discussed in the original paper by Randall and Sundrum [4], the precision measurements of the gravitational law in submillimeter range lead to a more stringent constraint $\rho_0^{1/4} \gtrsim 1.3$ TeV (or $M_5 \gtrsim 1.1 \times 10^8$ GeV) through the vanishing cosmological constant condition. However, note that this constraint, in general, is quite

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model dependent. In fact, if we consider an extension of the model so as to introduce a bulk scalar field, the constraint can be moderated because of the change of the vanishing cosmological constant condition [6]. Hence, we care about only the BBN constraint on ρ_0 in this paper.

Note that the ρ^2 term in Eq. (2) is a new ingredient in the brane world cosmology. Since at a high energy regime this term dominates and the Universe obeys a nonstandard expansion law, some results previously obtained in the standard cosmology can be altered. In fact, some interesting consequences in the brane world cosmology have been recently pointed out [7,8]. Especially, the thermal relic abundance of dark matter can be considerably enhanced compared to that in the standard cosmology [7].

In this paper, we investigate the brane world cosmological effect for the relic density of the neutralino dark matter in detail by numerical analysis. We will show that the allowed region for the neutralino dark matter in the CMSSM is dramatically modified in the brane world cosmology. In the next section, we give a brief review of Ref. [7]. In Sec. III, we present our numerical results for the neutralino relic density in the brane world cosmology. We will find some interesting consequences. Section IV is devoted to conclusions.

II. ENHANCEMENT OF RELIC DENSITY IN BRANE WORLD COSMOLOGY

In this section, we give a brief review on the relic density of dark matter in the brane world cosmology with a low five-dimensional Planck mass [7].

In the context of the brane world cosmology, we estimate the thermal relic density of a dark matter particle by solving the Boltzmann equation

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{EQ}^2),\tag{4}$$

with the modified Friedmann equation (2), where n is the actual number density of the dark matter particle, n_{EQ} is the equilibrium number density, and $\langle \sigma v \rangle$ is the thermal averaged product of the annihilation cross section σ and the relative velocity v. It is useful to rewrite Eq. (4) into the form

$$\frac{dY}{dx} = -\frac{s}{xH}\langle \sigma v \rangle (Y^2 - Y_{EQ}^2)$$

$$= -\lambda \frac{x^{-2}}{\sqrt{1 + (\frac{x_t}{x})^4}}\langle \sigma v \rangle (Y^2 - Y_{EQ}^2),\tag{5}$$

in terms of the number density to entropy ratio $Y = n/s$ and $x = m/T$, where m is a dark matter particle mass, $\lambda = 0.26(g_{*S}/g_*^{1/2})M_P m$, $M_P \simeq 1.2 \times 10^{19}$ GeV is the Planck mass, and x_t is defined as

$$x_t^4 \equiv \frac{\rho}{\rho_0}\bigg|_{T=m}.\tag{6}$$

At the era $x \ll x_t$ the ρ^2 term dominates in Eq. (2), while the ρ^2 term becomes negligible after $x \gg x_t$ and the expansion law in the standard cosmology is realized. Hereafter, we call the temperature defined as $T_t = mx_t^{-1}$ (or x_t itself) "transition temperature" at which the expansion law of the early Universe changes from the nonstandard one to the standard one. Since we are interested in the effect of the ρ^2 term for the dark matter relic density, we consider the case that the decoupling temperature of dark matter (T_d) is higher than the transition temperature, namely, $x_t \geq x_d = m/T_d$. Using Eqs. (3) and (6), this condition leads to

$$M_5 \leq \left(\frac{\pi^2 g_*}{30} m^4 \frac{1}{96\pi G} x_d^{-4}\right)^{1/6}\tag{7}$$

$$\simeq 4.6 \times 10^3 \text{ TeV}\left(\frac{m}{100 \text{ GeV}}\right)^{2/3}\left(\frac{20}{x_d}\right)^{2/3}.\tag{8}$$

Here we have normalized the decoupling temperature by its typical value in our case estimated as $x_d \equiv m/T_d \simeq 20$, which is the same scale as in the standard cosmology [9], as is shown in the appendix. For $M_5 \lesssim 10^3$ TeV, we can expect significant brane world cosmological effects. This, indeed, will be confirmed by numerical calculations in the next section (see Fig. 3).

It is easy to numerically solve the Boltzmann equation (5) with a given $\langle \sigma v \rangle$ and x_t. Using appropriate approximations, we can drive analytic formulas for the relic number density of dark matter [7]. When we parametrize $\langle \sigma v \rangle$ as $\langle \sigma v \rangle = \sigma_n x^{-n}$ with fixed $n = 0, 1, \cdots$, for simplicity, we can obtain simple formulas for the resultant relic densities such that

$$Y(x \to \infty) \simeq 0.54 \frac{x_t}{\lambda \sigma_0} \quad \text{for } n = 0,$$

$$\frac{x_t^2}{\lambda \sigma_1 \ln x_t} \quad \text{for } n = 1,\tag{9}$$

in the limit $x_d \ll x_t$, where x_d is the decoupling temperature. Note that the results are characterized by the transition temperature rather than the decoupling temperature. It is interesting to compare these results to that in the standard cosmology. Using the well-known approximate formulas in the standard cosmology [9],

$$Y(x \to \infty) \simeq \frac{x_d}{\lambda \sigma_0} \quad \text{for } n = 0,$$

$$\frac{2x_d^2}{\lambda \sigma_1} \quad \text{for } n = 1,\tag{10}$$

we obtain the ratio of the relic energy density of dark matter in the brane world cosmology $(\Omega_{(b)})$ to the one in the standard cosmology $(\Omega_{(s)})$ such that

$$\frac{\Omega_{(b)}}{\Omega_{(s)}} \simeq 0.54\left(\frac{x_t}{x_{d(s)}}\right) \quad \text{for } n = 0,$$

$$\frac{1}{2\ln x_t}\left(\frac{x_t}{x_{d(s)}}\right)^2 \quad \text{for } n = 1,$$

(11)

where $x_{d(s)}$ denotes the decoupling temperature in the standard cosmology. Similar results can be obtained for $n \geq 2$. Note that the relic energy density in the brane world cosmology can be enhanced from the one in the standard cosmology if the transition temperature is low enough. With the help of Eqs. (3) and (6), we find that the enhanced energy density is proportional to $M_5^{-3/2}$ (roughly M_5^{-3}) for $n = 0$ ($n = 1$).

This is the main point discussed in Ref. [7]. If the above discussion is applied to detailed analysis of the relic abundance of neutralino dark matter, we can expect a dramatic modification of the allowed region in the CMSSM.

III. NUMERICAL RESULTS FOR THE NEUTRALINO RELIC DENSITY

In this section, we present the results of our numerical analysis. We calculate the relic density of the neutralino $\Omega_\chi h^2$ in the CMSSM with the modified Friedmann equation (2). For this purpose, we have modified the code DARKSUSY [10] so that the modified Friedmann equation is implemented. In evaluating the relic density, coannihilations with neutralinos, charginos, and sfermions are taken into account.

The mass spectra in the CMSSM are determined by the following input parameters:

$$m_0, \quad m_{1/2}, \quad A, \quad \tan\beta, \quad \text{sgn}(\mu), \quad (12)$$

where m_0 is the universal scalar mass, $m_{1/2}$ is the universal gaugino mass, A is the universal coefficient of scalar trilinear couplings, $\tan\beta$ is the ratio of the vacuum expectation values of the two neutral Higgs fields, and $\text{sgn}(\mu)$ is the sign of the Higgsino mass parameter μ. With these input parameters, renormalization group equations for the CMSSM parameters are solved using the code ISASUGRA [11] to obtain the mass spectra at the weak scale. In the present analysis, we take $A = 0$ and $\mu > 0$ with $m_t = 178$ GeV (top quark pole mass) and $m_b = 4.25$ GeV (bottom quark $\overline{\text{MS}}$ mass at m_b) and investigate the M_5 dependence of the relic density $\Omega_\chi h^2$ in the three-dimensional CMSSM parameter space $(m_0, m_{1/2}, \tan\beta)$.

In Figs. 1 and 2, we show the allowed region in the $(m_{1/2}, m_0)$ plane consistent with the WMAP 2σ allowed range $0.094 < \Omega_\chi h^2 < 0.129$. Figure 1 contains the contour plots of $\Omega_\chi h^2$ for $\tan\beta = 50$, $A = 0$, and $\mu > 0$. The upper left window corresponds to the usual result in the standard cosmology ($M_5 = \infty$). The dotted, dashed, solid, and short-dashed–long-dashed lines correspond to $\Omega_\chi h^2 = 1.0$, 0.3, 0.1, and 0.05, respectively. The region
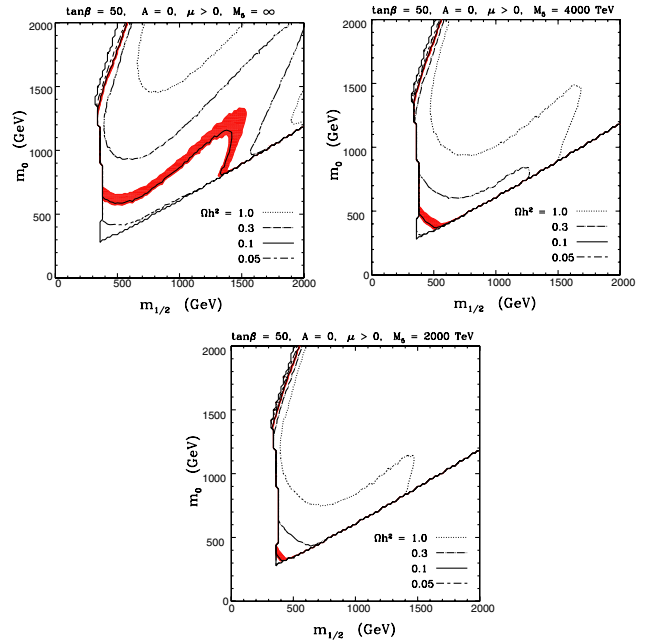

FIG. 1 (color online). Contours of the neutralino relic density $\Omega_\chi h^2$ in the $(m_{1/2}, m_0)$ plane for $M_5 = \infty$ (upper left window), 4000 TeV (upper right window), and 2000 TeV (lower window) in the case of $\tan\beta = 50$, $A = 0$, and $\mu > 0$. The dotted, dashed, solid, and short-dashed–long-dashed lines correspond to $\Omega_\chi h^2 = 1.0$, 0.3, 0.1, and 0.05, respectively. The shaded regions (in red) are allowed by the WMAP constraint. The region outside the bold line, including the two axes, is excluded by experimental constraints or the condition for successful electroweak symmetry breaking.
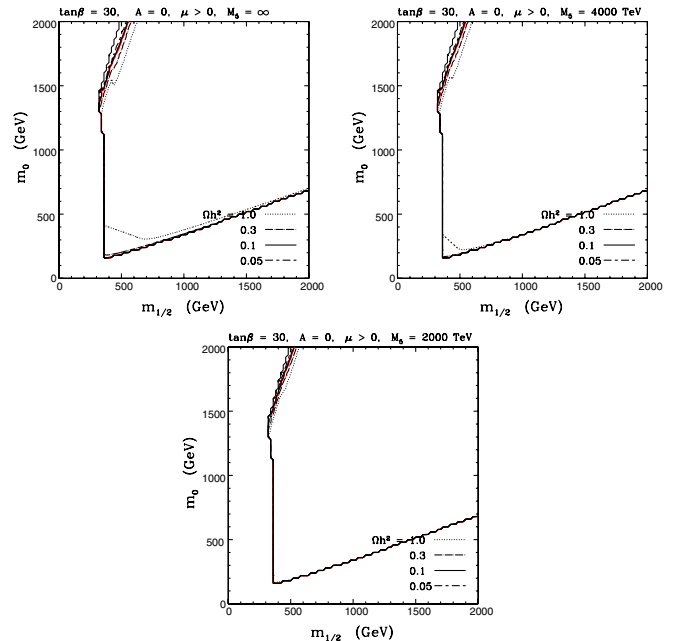

FIG. 2 (color online). The same as in Fig. 1 but for $\tan\beta = 30$.

between the bold line and the two coordinate axes is excluded by various experimental constraints (the lightest Higgs mass bound, $b \rightarrow s\gamma$ constraint, the lightest chargino mass bound, etc.) [2,12] or the condition for successful electroweak symmetry breaking. In particular, the region $m_0 \ll m_{1/2}$ is excluded since the lighter stau is the LSP, while the region $m_0 \gg m_{1/2}$ is excluded since successful electroweak symmetry breaking (EWSB) does not occur in this region (no-EWSB region).

The shaded regions (in red) are allowed by the WMAP constraint. The allowed regions include (i) a resonance region which appears in the bulk region, (ii) a stau coannihilation region which appears as a narrow strip along the boundary of the stau LSP region, and (iii) a Higgsino-like region which appears near the no-EWSB region. In the resonance region, the annihilation cross section is enhanced via heavy Higgs resonances. In the stau coannihilation region, the lighter stau is nearly degenerated with the lightest neutralino so that the cross section is enhanced by coannihilation effects with the lighter stau. In the Higgsino-like region, the lightest neutralino is Higgsino-like, and the cross section is enhanced through coannihilation effects with the second-lightest neutralino and the lighter chargino, since μ is relatively small in this region so that these particles are nearly degenerated with the lightest neutralino.

The upper right window and the lower window in Fig. 1 are the corresponding results for $M_5 = 4000$ and 2000 TeV, respectively. These figures clearly indicate that, as M_5 decreases, the allowed regions shrink significantly. The allowed region eventually disappears as M_5 decreases further.

Figure 2 is the similar result for $\tan\beta = 30$. In this case, a resonance region does not exist, and only narrow areas in the Higgsino-like region and the stau coannihilation region are allowed even in the standard case ($M_5 = \infty$).[1] As M_5 decreases, the allowed regions shrink significantly and disappear eventually as in the case of $\tan\beta = 50$.

Finally, we present sensitivity of the relic density to M_5 in Fig. 3, fixing m_0 and $m_{1/2}$ as well as $\tan\beta$, A, and sgn(μ). The left window, where we take $\tan\beta = 50$, $m_0 = 280$ GeV, and $m_{1/2} = 360$ GeV, corresponds to the point giving the smallest value of $\Omega_\chi h^2$ in the small mass region ($m_0, m_{1/2} < 1$ TeV) for $\tan\beta = 50$. The range of $\Omega_\chi h^2$ between the two dashed–dotted lines satisfies the WMAP constraint. For large $M_5 \gtrsim 10^4$ TeV, the (too small) relic density ($\Omega_\chi h^2 \approx 0.02$) in the standard case is reproduced independently of M_5. This is because $x_t \lesssim x_{d(s)}$ is obtained for M_5 ($\gtrsim 10^4$ TeV). As M_5 decreases, however, the relic density $\Omega_\chi h^2$ starts to be enhanced signifi-
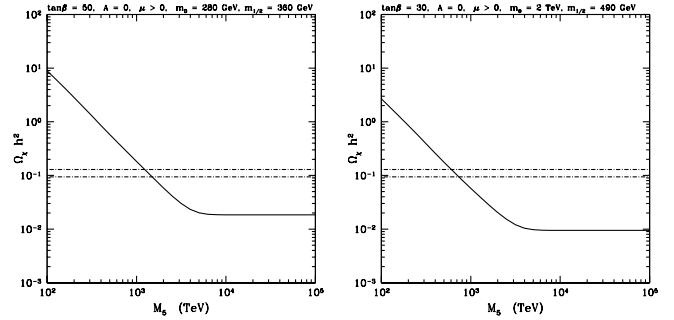

FIG. 3. $\Omega_\chi h^2$ vs M_5 for $\tan\beta = 50$, $m_0 = 280$ GeV, $m_{1/2} = 360$ GeV (left window) and $\tan\beta = 30$, $m_0 = 2TeV$, $m_{1/2} = 490$ GeV (right window) with $A = 0$ and $\mu > 0$. The range of $\Omega_\chi h^2$ between the two dashed–dotted lines satisfies the WMAP constraint.

cantly, and it amounts to ≈ 10 for $M_5 \approx 100$ TeV. It is found that, through the enhancement, an allowed region comes out for M_5 in the range of 1000 TeV $\lesssim M_5 \lesssim$ 1500 TeV.

On the other hand, the right window, where we take $\tan\beta = 30$, $m_0 = 2$ TeV, and $m_{1/2} = 490$ GeV, represents the point giving the smallest value of $\Omega_\chi h^2$ in the Higgsino-like region for $\tan\beta = 30$. The M_5 dependence is quite similar to the case in the left window. However, since the relic density $\Omega_\chi h^2 \approx 0.01$ in the standard case is smaller than that in the left window, the WMAP allowed region is shifted to a smaller M_5 region as 600 TeV $\lesssim M_5 \lesssim 800$ TeV.

In Fig. 3, we can see that Ω_χ is proportional to about M_5^{-2} in the left window and $M_5^{-1.6}$ in the right window for a small M_5 region where neutralino abundance is enhanced. These M_5 dependences of Ω_χ are consistent with the observation in the previous section, where $\Omega_\chi \propto M_5^{-3/2}$ for S wave ($n = 0$) and $\propto M_5^{-3}$ for P wave ($n = 1$). This is because in the left window the neutralino is Higgsino-like and its pair annihilation process is dominated by the S wave ($n = 0$). On the other hand, in the right window, the neutralino is binolike, so that (for the parameters we have used in our analysis) the S-wave annihilation process is somewhat suppressed and the resultant M_5 dependence is expected to be in the middle range between $M_5^{-1.5}$ and M_5^{-3}.

We have produced the same contour plots as Figs. 1 and 2 for $\tan\beta = 5, 10, 20$, and 40 as well. As in Figs. 1 and 2, the WMAP allowed regions for each $\tan\beta$ shrink and eventually disappear as M_5 decreases. It is found that, almost independently of $\tan\beta$, the allowed region which disappears last is always the Higgsino-like region. Then, for the Higgsino-like region, we have plotted the same figures as Fig. 3 for various $\tan\beta = 5, 10, 20, 40$, and 50. It turns out that the WMAP allowed range of M_5 is not so sensitive to $\tan\beta$ and comes out around 600 TeV $\lesssim M_5 \lesssim$ 1000 TeV. Note that, since for M_5 smaller than this range,

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[1]Unfortunately, the allowed coannihilation region is too narrow to be clearly seen even in the upper right window. See Ref. [2] for figures of this region. In the other two windows, there is no allowed coannihilation region.

we cannot find the WMAP allowed region, $M_5 \gtrsim 600$ TeV is the lower bound on M_5 in the brane world cosmology based on the neutralino dark matter hypothesis, namely, the lower bound in order for the allowed region of neutralino dark matter to exist.

IV. CONCLUSIONS

We have studied the neutralino relic density in the CMSSM in the context of the brane world cosmology. If the five-dimensional Planck mass is low enough, $M_5 \lesssim 10^4$ TeV, the ρ^2 term in the modified Friedmann equation can be effective at the decoupling time and the relic density can be enhanced. Therefore, we can expect that the allowed region in the parameter space is modified from the one in the standard cosmology. We have presented our numerical results and shown that the allowed region shrinks and eventually disappears as M_5 decreases. Through the numerical analysis, we have found a new lower bound on $M_5 \gtrsim 600$ TeV in the brane world cosmology based on the neutralino dark matter hypothesis. This lower bound is obtained by concerning the Higgsino-like region which disappears last. If we consider the resonance region or the stau coannihilation region individually for each $\tan\beta$ fixed, we can obtain a more stringent lower bound.

ACKNOWLEDGMENTS

APPENDIX

In this appendix, we derive an approximate formula for a decoupling temperature of dark matter particles in brane world cosmology and present the effect of the ρ^2 term on the decoupling of dark matter. It is shown that the decou-

pling temperature weakly depends on the transition temperature, and, thus, the resultant decoupling temperature in the brane world cosmology is not so much different from the one in the standard cosmology.

Our discussion follows the strategy in Ref. [9]. We define the decoupling temperature x_d, in terms of x, as a temperature which satisfies $\Delta(x_d) = Y_{EQ}(x_d)$, where $\Delta \equiv Y - Y_{EQ}$ is the deviation of the abundance from its thermal equilibrium value. Here we examine the case where the annihilation process is dominated by the S wave ($n = 0$), for simplicity. Then, the condition (in the ρ^2 term dominated era) is expressed as

$$\Delta(x_d) \simeq \frac{\sqrt{\frac{\rho}{\rho_0}\big|_{T=m}}}{2\lambda\sigma_0} = Y_{EQ} \simeq 0.145 \frac{g}{g_{*S}} x^{3/2} e^{-x}. \quad \text{(A1)}$$

Thus, x_d is roughly written as

$$x_d \sim \ln\left[0.145 \frac{g}{g_{*S}} 2\lambda\sigma_0 \frac{1}{\sqrt{\frac{\rho}{\rho_0}\big|_{T=m}}} \right] = x_{d(s)} - 2\ln x_t, \quad \text{(A2)}$$

where

$$x_{d(s)} \equiv \ln\left[0.145 \frac{g}{g_{*S}} 2\lambda\sigma_0 \right] \quad \text{(A3)}$$

is the decoupling temperature in the standard cosmology [9]. The second term, $-2\ln x_t$, is nothing but the effect of the ρ^2 term, and the minus sign indicates that the brane world effect advances the decoupling time.

In the case that we are interested in, $x_{d(s)} < x_t < m/1$ MeV. Using the neutralino decoupling temperature $x_{d(s)} \simeq 30$, normally used in the standard cosmology, we obtain $7.0 < x_d < 23$ for the neutralino with mass $m \simeq 100$ GeV. The normalization used in Eq. (8) corresponds to

$$x_t \simeq \frac{g_*^{1/4} m}{\rho_0^{1/4}} \simeq 150\left(\frac{m}{100 \text{ GeV}}\right)\left(\frac{10^3 \text{ TeV}}{M_5}\right)^{3/2}. \quad \text{(A4)}$$

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