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N. Okada – KEK

Deposited 06/06/2019

Citation of published version:

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Nobuchika Okada
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Nobuchika Okada

Theory Division, KEK
Tsukuba, Ibaraki 305-0801, Japan

Abstract. Future collider experiments are expected to discover new physics beyond the Standard Model and Higgs boson. As a new physics beyond the Standard Model, we consider unified models which give some unified picture for some parts of the Standard Model. Although unified models usually take place at very high energies, the future collider experiments may provide us some indirect information by precise measurements of particle masses and couplings. A class of unified models predicts some exotic particles. Once the future colliders discover such exotic particles, we obtain the direct information on the unified models.

Keywords: Unification, Collider
PACS: 12.10.Dm, 12.60.Jv

INTRODUCTION

The Standard Model (SM) of particle physics has been in excellent agreement with almost of all current experiments. However, some theoretical problems and several experimental observations suggest the existence of new physics beyond the SM. There have been a lot of interesting new physics models beyond the SM proposed during the passed decays. Here we focus on “unified models” as an example of new physics models, which provide a unified picture for some parts in the SM, such as the grand unified theories (GUTs) and the gauge-Higgs unification scenario.

The Large Hadron Collider (LHC) will start its operation soon and the collider phenomenologies on various new physics models and Higgs boson have been extensively studied. We have been expecting the discovery of new physics and also Higgs boson at the LHC. The LHC would be followed by a further future collider experiment such as the International Linear Collider (ILC), where new physics and Higgs boson properties can be more precisely measured.

The typical scale of unified models is usually very high and it seems very difficult to obtain information on unified models by future collider experiments with the beam energy around 1 TeV. However, once new particles and Higgs boson have been discovered at the colliders and their masses and couplings have been precisely measured, we can extract indirect information on an unified model through the extrapolation of the measured physics parameters toward high energies according to their renormalization group equations (RGEs). A class of unified models predicts some exotic particles with a mass scale accessible to the LHC and ILC. If the LHC and/or ILC discover some exotic particles, we can obtain the direct information on the unified models. In the following sections, we discuss indirect and direct informations about unified models that could be provided by the future collider experiments.

INDIRECT INFORMATION

Soft mass spectrum as the information of UV theory

Let us first remind the well-known indirect information of the GUTs, which has been discussed for passed two decays, namely, the gauge coupling unification. Assuming the weak scale supersymmetry (SUSY), the precise measurements of the three gauge couplings of the SM and their RGE extrapolations suggest the existence of the GUT through the successful gauge coupling unification around the GUT scale, $M_{\text{GUT}} \simeq 2 \times 10^{15}$ GeV [1]. In order to achieve the gauge coupling unification, the weak scale SUSY is crucial. Therefore, this scenario also suggests the possibility of the discovery of SUSY at the future colliders. Once sparticles have been discovered and their mass spectrum are measured in some precision, we can obtain more information on GUTs.

Suppose that SUSY breaking is mediated at a scale higher than the GUT scale. In this case, soft mass spectrum at the GUT scale should be unified for the particles in the same GUT multiplets. Note that the way of unification of the
SM matter multiplets depends on GUT models. Here, we consider two GUT models based on the gauge group SU(5) and SO(10), as an example. In the SU(5) GUT model, the SM matters are embedded into two representations, $5^* + 10$, while in the SO(10) GUT model, the whole SM matter multiplets are embedded into one 16 multiplet, together with the right-handed neutrino. This difference of the unification of the SM matter multiplets reflects the sfermion soft mass spectrum at the weak scale.

For simplicity, we consider the minimal supergravity for the SUSY breaking mediation mechanism. In the SU(5) GUT model, the right-handed down squark ($D$) and the left-handed slepton ($\tilde{L}$) are in the $5^*$ multiplet, while the left-handed squark ($Q$), the right-handed up squark ($\tilde{U}$) and the right-handed slepton ($\tilde{E}$) are unified into the 10 multiplet. Therefore, the soft mass spectrum at the GUT scale should satisfy the mass relations,

$$m_D = m_L, \quad m_{\tilde{D}} = m_{\tilde{L}} = m_{\tilde{E}}. \quad (1)$$

Note that in general, the soft mass of the $5^*$ multiplet is not necessary the same as the soft mass of the 10 multiplet. On the other hand, in the SO(10) GUT model, the soft mass spectrum should be universal,

$$m_D = m_L = m_{\tilde{D}} = m_{\tilde{L}} = m_{\tilde{E}}, \quad (2)$$

since all the multiplets are unified into only one 16 multiplet.

Figure 1 depicts the RGE running of the soft mass spectrum in the SU(5) model with different mass inputs for the $5^*$ and 10 multiplets in the SU(5) GUT model. The same figure for the SO(10) model is shown in Figure 2, where the soft masses at the GUT scale are completely unified. The difference of the resultant mass spectra between the SU(5) and SO(10) GUTs appears in the mass difference between $m_D$ and $m_{\tilde{L}}$ and in the mass difference between $m_D$ and $m_{\tilde{E}}$.

Precise measurements of the soft mass spectrum can give us the cure how to distinguish the different GUT models. The same strategy is applicable to other SUSY breaking mediation scenarios. As an example, we consider the gauge mediated scenario (GMSB) [2]. In order to keep the successful gauge coupling unification, we introduce the messenger fields as the SU(5) multiplets. Soft masses for the gauginos and sfermions at the messenger scale ($M_{mess}$) are given by

$$M_i(M_{mess}) = \frac{\alpha_i(M_{mess})}{4\pi} \frac{F}{M_{mess}} N_{mess}$$

$$\tilde{m}_i(M_{mess})^2 = 2c_i \left( \frac{\alpha_i(M_{mess})}{4\pi} \right)^2 \left( \frac{F}{M_{mess}} \right)^2 N_{mess}, \quad (3)$$

where $c_i$ is the quadratic Casimir of corresponding gauge interactions, $F$ is the SUSY breaking order parameter, and $N_{mess}$ is the Dynkin index from the messenger fields ($N_{mess} = 1$ for a $5^* + 5$ pair of messenger fields). Note that the ratio of the gaugino and sfermion masses depends on $N_{mess}$ such as

$$\left( \frac{\tilde{m}_i}{M_i} \right)^2 = \frac{2c_i}{N_{mess}}. \quad (4)$$
FIGURE 2. The RGE running of sfermion masses in the SO(10) GUT model for the inputs $m_{\tilde{D}} = m_{\tilde{L}} = m_{\tilde{Q}} = m_{\tilde{R}} = m_{\tilde{E}} = 300$ GeV at the GUT scale. Each line corresponds to $m_{\tilde{Q}}$, $m_{\tilde{D}}$, $m_{\tilde{R}}$, and $m_{\tilde{E}}$ from top to bottom at $\log_{10}(\mu/\text{GeV}) = 2$. $m_{\tilde{D}}$ and $m_{\tilde{R}}$ are well-overlapped.

FIGURE 3. The ratios of the running soft masses in the type II (solid) and type III (dashed) seesaw scenarios. The ratios, $\frac{m_{\tilde{Q}}^{2} - m_{\tilde{D}}^{2}}{M_{1}^{2}}$ and $\frac{m_{\tilde{D}}^{2} - m_{\tilde{E}}^{2}}{M_{1}^{2}}$, are depicted from top to bottom, where $M_{1} = M_{F}(\mu = 1 \text{ TeV})$.

In general, the number of messengers and the messenger scale are free parameters in GMSB scenario.

Recently, an economical GMSB scenario has been proposed [3], where the so-called type II seesaw scenario [4] is combined with the GMSB scenario and the $15^* + 15$ multiplets (in the SU(5) GUT notation) play the role of the messenger fields. As the result, the messenger scale is the seesaw scale around $M_{\text{mess}} = 10^{12-14}$ GeV. As a simple extension of the scenario, it would be interesting to consider another possibility, namely, the type III seesaw scenario. We can introduce 24 multiplets as the messenger fields [5] for both the GMSB scenario and the type III seesaw mechanism.

Note that the difference between these two scenarios appears in the soft mass spectrum through the Dynkin index. In the type II seesaw scenario with the $15^* + 15$ messenger fields, $N_{\text{mess}} = 7$, while $N_{\text{mess}} = 15$ for the type III seesaw scenario with three 24 messenger multiplets. The mass differences in the type II and III seesaw scenario are depicted in Figure 3. Here we have taken $M_{\text{mess}} = 10^{13}$ GeV. Precision measurements of the soft mass spectrum can make it possible to distinguish the type II seesaw scenario from the type III and vise versa. In the ILC, the measurements of the soft mass spectrum in a few % accuracy and more can be possible [6].
FIGURE 4. The RGE running of Higgs boson mass for various inputs at the weak scale.

Higgs mass measurement and implication for UV theory

One of the main purposes of the future collider experiments is to find the Higgs boson, that is the last particle in the SM to be directly observed. Measurements of Higgs boson mass and its coupling to the SM particles will reveal the origin of the electroweak symmetry breaking and of the mass generation mechanism. In this subsection, we discuss the implication of the Higgs boson mass measurement for an UV theory behind the SM.

Providing the correct electroweak symmetry breaking, the Higgs boson mass is determined by its quartic coupling as

$$M_H^2 = \frac{\lambda v^2}{2}$$

where $\lambda$ is the quartic Higgs coupling in the Higgs potential, and $v = 246$ GeV is the Higgs VEV. Once the Higgs boson mass is measured, we can obtain some information about an UV theory through its RGE running towards high energies. Figure 4 shows the RGE running of the Higgs boson mass for various Higgs boson mass inputs at the weak scale. The upper horizontal line corresponds to the Higgs mass with a large quartic coupling beyond perturbation (we took $\lambda = \sqrt{4\pi}$ as a concrete value). We can see that for the Higgs mass $\gtrsim 170$ GeV, the Higgs quartic coupling exceeds the perturbative value below the Planck scale $M_{Pl} = 1.2 \times 10^{19}$ GeV (perturbativity bound). On the other hand, for the Higgs mass $\lesssim 125$ GeV, the quartic coupling becomes negative at a scale below the Planck scale, so that the instability happen in the effective Higgs potential (stability bound). If we assume that the next energy frontier lies at the Planck scale, the Higgs boson mass be in the range $125$ GeV $\lesssim M_H \lesssim 170$ GeV [7]. In other words, the Higgs boson mass outside of this range, once measured, provides some implication for an UV theory which takes place below the Planck scale.

Here we introduce the so-called gauge-Higgs unification (GHU) scenario [8] as a new physics model behind the SM for the case that Higgs boson mass is lower than the stability bound. The GHU model is the higher dimensional gauge theory, where the SM Higgs doublet is identified, after compactification of the extra dimension(s) on some orbifold, with the zero mode of the fifth component of the gauge field. Thus, the GHU model provides an unified picture between the gauge bosons and the SM Higgs doublet.

As a concrete GHU model, here we consider the five dimensional model with the fifth dimension compactified on the $S^1/Z_2$ orbifold. The five dimensional gauge invariance protects the Higgs doublet from having a potential at tree level, while an effective potential for the Higgs is generated via quantum corrections, triggered by the breaking of the underlying gauge symmetry through boundary conditions. Recently, a new phenomenological treatment of GHU models has been proposed [9]. It has been shown that the effective SM Higgs quartic coupling $\lambda$ calculated in a given GHU model coincides with the one radiatively generated in the effective low energy theory (without a quartic coupling at tree level) with the compactification scale $\Lambda = 1/(2\pi R)$ identified with the cutoff scale in evaluating quantum corrections. Thus, using RGEs, we can evaluate the SM quartic Higgs coupling by requiring that

$$\lambda(\Lambda) = 0,$$
where the cutoff scale $\Lambda$ is identified as the compactification scale $\Lambda = 1/(2\pi R)$. This boundary condition for $\lambda$ is called [9] the “gauge-Higgs condition”. The resultant Higgs boson mass in this way is the same as the stability bound on $M_H$ with a given compactification scale.

Figure 5 [10] shows the Higgs boson mass as a function of the given compactification scale with the gauge-Higgs condition for the top quark pole mass $M_t = 170.9 \pm 1.8$ GeV [11]. The horizontal dashed line corresponds to the LEP2 bound [12] on the Higgs boson mass $M_H > 114.4$ GeV. To satisfy this bound, the compactification scale should be $\Lambda > 10^{6.5}$ GeV. Note that this result is obtained when only the SM matter contents are assumed to appear in the scale $\Lambda$. If some vector-like fermions are introduced below the compactification scale, the Higgs boson mass can be pushed up [9].

**New particle effects on Higgs phenomenology at the LHC**

In GHU models, SM matters are accompanied by their Kaluza-Klein (KK) modes, which have interesting indirect effects on Higgs boson phenomenology at the LHC. The gluon fusion process is the dominant Higgs boson production process at the LHC and for light Higgs boson with mass $M_H \lesssim 150$ GeV, two photon decay mode of Higgs boson becomes the primary discovery mode [13] nevertheless its branching ratio is very small, $\mathcal{O}(10^{-3})$. Since the coupling between Higgs boson and these gauge bosons are induced through quantum corrections at one-loop level even in the Standard Model, we can expect a sizable effect from KK particles if their masses are low enough. Here, in a five dimensional GHU model, we calculate one-loop diagrams with KK top quarks for the effective couplings between Higgs boson and the gauge bosons (gluons and photons).

In a simple GHU model in five dimensions, the KK mode mass eigenvalues are found to be (see [14] for details of the discussion in this section)

$$m_{\pm}^{(n)} = \frac{n}{R} \pm m_t,$$

where $n = 1, 2, \cdots$. Each KK mode has the Yukawa coupling with the Higgs boson as $\mp \frac{m_t}{\sqrt{2}}$. With these mass eigenvalues and Yukawa couplings, the KK mode contributions to the effective coupling between the Higgs boson and gluons are
calculated,

\[ L_{\text{eff}} = C_{g}^{\text{KK}(GH)} \ h \ G^{\mu \nu} G_{\mu \nu}^{\alpha} \]

\[ C_{g}^{\text{KK}(GH)} = -\sum_{n=1}^{\infty} \left[ \frac{m_{t}}{v} \times \frac{\alpha_{s} F_{1/2}(4 m_{n}^{(n)} / M_{H}^{2})}{8 \pi m_{n}^{(n)} M_{H}^{2}} \times \frac{1}{2} \right] + \sum_{n=1}^{\infty} \left[ \frac{m_{t}}{v} \times \frac{\alpha_{s} F_{1/2}(4 m_{n}^{(n)} / M_{H}^{2})}{8 \pi m_{n}^{(n)} M_{H}^{2}} \times \frac{1}{2} \right] \]

\[ \simeq m_{t} \alpha_{s} \frac{12 \pi v}{2} \sum_{n=1}^{\infty} \left[ \frac{1}{m_{n}^{(n)}} - \frac{1}{m_{n}^{(n)}} \right] \simeq -\frac{\alpha_{s}}{6 \pi v} \sum_{n=1}^{\infty} \frac{m_{n}^{2}}{m_{n}^{2}} \]  

(8)

where we have taken the limit \( M_{H}^{2}, m_{t}^{2} \ll m_{n}^{2} \), to simplify the results. Note that this result is finite and this finiteness is a consequence of cancellation between two divergent corrections with opposite signs. Also, note that the KK mode contribution is subtractive against the top quark contribution in the SM.

The contribution of top quark KK modes to the effective coupling between Higgs boson and photons are calculated in the same way. In fact, the final result can be obtained by the replacements, \( \alpha_{s} \rightarrow \alpha_{em} \) and the group factor \( 1/2 \rightarrow 3 \), top quark electric charge \( 2 \) x number of colors:

\[ L_{\text{eff}} = C_{g}^{\text{KK}(GH)} \ h \ F^{\mu \nu} F_{\mu \nu} \]

\[ C_{g}^{\text{KK}(GH)} = -\sum_{n=1}^{\infty} \left[ \frac{m_{t}}{v} \times \frac{\alpha_{em} F_{1/2}(4 m_{n}^{(n)} / M_{H}^{2})}{8 \pi m_{n}^{(n)} M_{H}^{2}} \times \frac{4}{3} \right] + \sum_{n=1}^{\infty} \left[ \frac{m_{t}}{v} \times \frac{\alpha_{em} F_{1/2}(4 m_{n}^{(n)} / M_{H}^{2})}{8 \pi m_{n}^{(n)} M_{H}^{2}} \times \frac{4}{3} \right] \]

\[ \simeq \frac{2 m_{t} \alpha_{em}}{9 \pi v} \sum_{n=1}^{\infty} \left[ \frac{1}{m_{n}^{(n)}} - \frac{1}{m_{n}^{(n)}} \right] \simeq -\frac{\alpha_{em}}{9 \pi v} \sum_{n=1}^{\infty} \frac{m_{n}^{2}}{m_{n}^{2}} \]  

(9)

where we have, again, taken the limit \( M_{H}^{2}, m_{t}^{2} \ll m_{n}^{2} \), to simplify the results.

It is interesting to compare these results to that in the Universal Extra Dimension (UED) scenario [15, 16], which also provides the top quark KK modes with different KK mode mass spectrum and Yukawa couplings given by

\[ m_{n}^{(n)} = \sqrt{\left( \frac{n}{R} \right)^{2} + m_{t}^{2}} \]

(10)

without mass splitting and \(- (m_{t} / v) \times (m_{t} / M_{n})\), respectively. In the UED, we find the effective coupling as [17]

\[ L_{\text{eff}} = C_{g}^{\text{KK}(UED)} \ h \ G^{\mu \nu} G_{\mu \nu}^{\alpha} \]

\[ C_{g}^{\text{KK}(UED)} = -\sum_{n=1}^{\infty} \left[ \frac{m_{t}}{v} \times \frac{\alpha_{s} F_{1/2}(4 M_{n}^{2} / M_{H}^{2})}{8 \pi M_{n}} \times \frac{1}{2} \right] \times 2 \]

\[ \simeq \frac{\alpha_{s}}{6 \pi v} \sum_{n=1}^{\infty} \frac{m_{n}^{2}}{m_{n}^{2}} \]

(11)

where we have, again, taken the limit \( M_{H}^{2}, m_{t}^{2} \ll m_{n}^{2} \), to simplify the result. In the limit, we arrive at the same result as the one in the GHU model, except for the sign. This KK mode contribution is constructive to the top quark one in the SM.

Now let us evaluate the ratio of the Higgs boson production cross section in the GHU model to the SM one, which is described as

\[ \frac{\sigma(gg \rightarrow h; \ SM + KK)}{\sigma(gg \rightarrow h; \ SM)} = \left( 1 + \frac{C_{g}^{\text{KK}(GH)}}{C_{g}^{\text{SM}}} \right)^{2} \]

(12)

where \( C_{g}^{\text{SM}} \) is the effective coupling between the Higgs boson and gluons in the SM only from top quark loop corrections. The results are depicted in Figure 6 [14] as a function of the mass of the lightest KK mode (diagonal) mass eigenvalue \( (m_{1} = 1/R) \). In this analysis, we have taken \( M_{H} = 120 \text{ GeV} \). As a reference, the result in the UED scenario is also shown. The KK fermion contribution is subtractive and the Higgs production cross section is reduced in the GHU scenario, while it is increased in the UED scenario. This is a crucial point to distinguish the GHU scenario from the UED scenario. Interestingly, even for \( m_{1} = 1 \text{ TeV} \), the KK fermion contribution is sizable and the production cross section is reduced by about 18%.
A class of unified models predicts some exotic particles with the mass scale accessible to the future colliders. In this case, the discovery of such particles gives us direct information of the new physics model. In this section, we consider “diquark Higgs” production at the LHC [18].

The diquark Higgs (Δ_dq) is the color sextet Higgs field, which can naturally arise in a class of supersymmetric seesaw models for neutrino masses [19]. The seesaw mechanism extends the standard model with three right handed neutrinos and add large Majorana masses for them. The fact that the seesaw scale is much lower than the Planck scale suggests that there may be a symmetry protecting this scale. A natural symmetry is local B-L symmetry whose breaking leads to the right-handed Majorana neutrino masses. A gauge theory that accommodates this scenario is the left-right symmetric model based on the gauge group SU(2)_L x SU(2)_R x U(1)_{B-L} x SU(3)_c. This model being quark lepton symmetric easily lends itself to quark-lepton unification a la Pati-Salam into the gauge group SU(2)_L x SU(2)_R x SU(4)_c [20]. It has already been shown [21] that within a supersymmetric Pati-Salam scheme, if SU(4)_c color is broken not by SU(2)_L,R doublet fields as was suggested in [20] but rather by triplets as proposed in [22], then despite the high seesaw scale of around 10^{11} GeV or so, there are light (TeV mass) sextet diquark of the type Δ_dq.

To show this more explicitly, recall that the quarks and leptons in this model are unified and transform as l/f: (2, 1, 4)_{EB} l/fc: (1, 2, 4)_{EB} representations of SU(2)_L x SU(2)_R x SU(4)_c. For the Higgs sector, we choose, φ_1: (2, 2, 1)_{EB} and φ_15: (2, 2, 15)_{EB} to give mass to the fermions and the Δ_dq: (1, 3, 10)_{EB} to break the B-L symmetry. The diquarks mentioned above are contained in the Δ_dq: (1, 3, 10)_{EB} multiplet.

The renormalizable superpotential for this model has a large global symmetry of U(30, c) and on gauge symmetry breaking, leads to all diquark Higgs fields and a pair of doubly charged Higgs bosons remaining light. In this theory, the gauge couplings become non-perturbative in the 10-100 TeV range and do not yield a high seesaw scale, as may be desirable. On the other hand, if we add an extra B-L neutral triplet Higgs field Φ: (1, 3, 1)_{EB} to this theory, the symmetry of the theory gets lowered and this helps to greatly reduce the number of light diquark states. The reduction of the global symmetry can be seen from the superpotential of this model W = W_H + W_Y, where

\begin{align*}
W_H &= \lambda_4 (\Delta_d^\dagger \Delta_y^\dagger - M_\Delta^2) + \mu_1 \text{Tr}(\phi_i \phi_i), \\
W_Y &= h_1 \psi_i \psi_i \bar{\psi} + h_{15} \psi_i \phi_i \bar{\psi} + f \psi^\dagger \Delta_d \psi^y. \quad (13)
\end{align*}

Note that since we do not have parity symmetry in the model, the Yukawa couplings h_1 and h_{15} need not be symmetric matrices. This superpotential has U(10, c) x SU(2) global symmetry. When the neutral component of \( (1, 3, 10 + 10) \) picks up VEV, this symmetry breaks down to U(9, c) x U(1), leaving 21 complex massless scalar fields. Since the gauge symmetry also breaks down from SU(2)_R x SU(4)_c to SU(3)_c x U(1)_y, nine of these are absorbed leaving 12 complex massless states, which are the sextet Δ_d (the submultiplet of the Δ_d in Eq. (14)) plus its complex conjugate states from the 10 representation above. Once supersymmetry breaking effects are included and higher dimensional
terms
\[ \lambda_{A} \frac{(\Delta' \Delta')^{2}}{M_{p}} + \lambda_{B} \frac{(\Delta' \Delta')(\Delta' \Delta)}{M_{p}} + \lambda_{C} \Delta' \Delta \Omega + \frac{\text{Tr}(\phi_{1} \Delta' \Delta \phi_{15})}{M_{p}}. \] (15)
are included, these \( \Delta_{\nu' \nu'} \) fields pick up mass of order \( \frac{\lambda_{B} \Delta' \Delta}{M_{p}} \) which for \( v_{BL} \sim 10^{11} \) GeV is in the 100 GeV to TeV range naturally. We denote the mass of \( \Delta_{\nu' \nu'} \) by \( m_{\Delta} \).

The magnitudes of the couplings of diquark Higgs to up-type quarks are important for its LHC signal as well as other manifestations in the domain of rare processes. As is clear from Eq. (14), the sextet \( \Delta_{\nu' \nu'} \) couplings to quarks, \( f_{ij} \) are also directly related to the neutrino masses, which provides a way to probe neutrino masses from LHC observations. Due to the existence of other parameters, current neutrino observations do not precisely pin down the \( f_{ij} \). There are however other constraints on them.

To study these constraints, we define the \( \Delta_{\nu' \nu'} \) couplings \( (f_{ij}) \) in a basis where the up-type quarks are mass eigenstates. A major constraint on them comes from the \( D^{0} - \bar{D}^{0} \) mixing which is caused by the exchange of \( \Delta_{\nu' \nu'} \) field:

\[ M_{D^{0} - \bar{D}^{0}} = \frac{f_{11} f_{22}}{4 m_{\Delta}} c_{\gamma}(1 - y_{\gamma}) u c^{\gamma}(1 - y_{\gamma}) u; \] (16)
The present observations [23] imply that the transition mass \( \Delta M_{D} \) for \( D^{0} - \bar{D}^{0} \) to be \( 8.5 \times 10^{-15} \leq \Delta M_{D} \leq 1.9 \times 10^{-14} \) in GeV units. In our model, we can estimate this to be

\[ \Delta M_{D} \approx \frac{f_{11} f_{22}}{4 m_{\Delta}} \beta_{3} M_{D} \] (17)
which implies that \( \frac{f_{11} f_{22}}{4 m_{\Delta}} \leq 10^{-12} \) GeV\(^{-2} \); for a TeV delta mass, which is in the range of our interest, this implies \( f_{11} f_{22} \leq 4 \times 10^{-6} \). If we assume that \( f_{11} \gg f_{22} \), then for \( f_{11} \sim 0.1 \) or so, \( f_{22} \) is close to zero, which assume to be the case in our phenomenological analysis [24].

Next constraint comes from non-strange pion decays e.g. \( D \rightarrow \pi \pi \) which are suppressed compared to the decays with strange final states. This bound however is weak. The present limits on such non-strange final states are at the level of \( B \leq 10^{-4} \) [25], which implies \( f_{11} f_{12} \leq 4 \times 10^{-2} \) for \( m_{\Delta} \approx \) few hundred GeV to TeV range. This will be easily satisfied if \( f_{11} \sim f_{12} \sim 0.2 \).

Due to the diquark Higgs coupling to a pair of up-type quarks, it can be produced at high energy hadron colliders such as Tevatron and LHC through the annihilation of a pair of up quarks. Clearly, a proton-antiproton collider leads to a higher production rate for \( \Delta_{\nu' \nu'} \) compared to the proton-anti-proton colliding machine. As a signature of diquark productions at hadron colliders, we concentrate on its decay channel which includes at least one anti-top quark (top quark for anti-diquark Higgs case) in the final state. Top quark has large mass and decays electroweakly before hadronizing. Due to this characteristic feature distinguishable from other quarks, top quarks can be used as an ideal tool [26] to probe other new physics beyond the Standard Model [27].

Since diquark couples with only up-type quarks, once it is produced, its decay give rise to production of double top quarks \( (\Delta_{\nu' \nu'} \rightarrow tt) \) and a single top quark + jet \( (\Delta_{\nu' \nu'} \rightarrow tu \text{or} tc) \). These processes have no SM counterpart, and the signature of diquark production would be cleanly distinguished from the SM background. We leave detailed collider studies on signal event of diquark (anti-diquark) Higgs production and the SM background event for future works.

First, we give basic formulas for our study on diquark Higgs production at hadron colliders. The fundamental processes in question are \( uu \rightarrow \Delta_{\nu' \nu'} \rightarrow tt, tu, tc \) (\( \bar{u}u \rightarrow \Delta_{\nu' \nu'} \rightarrow \bar{t}t, \bar{u}c, \bar{u}c \) for anti-diquark Higgs production). From
Eq. (14), the cross section is found to be

\[
\frac{d\sigma(uu \rightarrow \Delta_{u'c'} \rightarrow uu, ct)}{d \cos \theta} = \frac{|f_{11}|^2 |f_{33}|^2}{16\pi} \frac{(\hat{s} - 2m_{\Delta}^2)}{(\hat{s} - m_{\Delta}^2)^2 + m_{\Delta}^4 \Gamma_{\text{tot}}^2} \sqrt{1 - \frac{4m_{\Delta}^2}{\hat{s}}},
\]

\[
\frac{d\sigma(uu \rightarrow \Delta_{u'c'} \rightarrow uu, cc)}{d \cos \theta} = \frac{|f_{11}|^2 |f_{33}|^2}{8\pi\hat{s}} \frac{(\hat{s} - m_{\Delta}^2)}{(\hat{s} - m_{\Delta}^2)^2 + m_{\Delta}^4 \Gamma_{\text{tot}}^2}.
\]

Here, we have neglected all quark masses except for top quark mass \(m_t\), \(\cos \theta\) is the scattering angle, and \(\Gamma_{\text{tot}}\) is the total decay width of diquark Higgs, which is the sum of each partial decay width,

\[
\Gamma(\Delta_{u'c'} \rightarrow uu, cc) = \frac{3}{16\pi} |f_{11,22}|^2 m_{\Delta},
\]

\[
\Gamma(\Delta_{u'c'} \rightarrow uu, tt) = \frac{3}{16\pi} |f_{33}|^2 m_{\Delta} \sqrt{1 - \frac{4m_{\Delta}^2}{m_t^2}} \left(1 - \frac{2m_{\Delta}^2}{m_t^2}\right),
\]

\[
\Gamma(\Delta_{u'c'} \rightarrow uu, uc) = \frac{3}{8\pi} |f_{12}|^2 m_{\Delta},
\]

\[
\Gamma(\Delta_{u'c'} \rightarrow uu, ct) = \frac{3}{8\pi} |f_{13,23}|^2 m_{\Delta} \left(1 - \frac{m_{\Delta}^2}{m_{\Delta}^2}\right)^2.
\]

Note that the cross section is independent of the scattering angle because the diquark Higgs is a scalar.

With these cross sections at the parton level, we study the diquark production at Tevatron and LHC. At Tevatron, the total production cross section of an up-type quark pair \((u, u)\) where \(u_{1,2,3} = u, c, t\) through diquark Higgs in the s-channel is given by

\[
\sigma(pp \rightarrow u(u)) = \int dx_1 \int dx_2 \int d\cos \theta f_u(x_1, Q^2) f_{\bar{u}}(x_2, Q^2) \frac{d\sigma(uu \rightarrow \Delta_{u'c'} \rightarrow uu; \hat{s} = x_1 x_2 E_{\text{CMS}}^2)}{d \cos \theta},
\]

where \(f_u(x_1, Q^2)\) and \(f_{\bar{u}}(x_2, Q^2)\) denote the parton distribution function, and \(E_{\text{CMS}}\) is the collider energy. Note that one parton distribution function is for up quark and the other is for the sea up quark, since it comes from an anti-proton (for a proton-anti-proton system such as at Tevatron). This fact indicates that at Tevatron the production cross section of diquark Higgs is the same as the one of anti-diquark Higgs, reflecting that the total baryon number of initial \(pp\) state is zero.

At LHC, the total production cross section of an up-type quark pair is given by

\[
\sigma(pp \rightarrow u(u)) = \int dx_1 \int dx_2 \int d\cos \theta f_u(x_1, Q^2) f_{\bar{u}}(x_2, Q^2) \frac{d\sigma(uu \rightarrow \Delta_{u'c'} \rightarrow uu; \hat{s} = x_1 x_2 E_{\text{CMS}}^2)}{d \cos \theta}.
\]

Here, both of parton distribution functions are for up quark in proton (both valence quarks), corresponding to a proton-proton system at LHC. Total production cross section of an up-type anti-quark pair \((\bar{u}, \bar{u})\) is obtained by replacing the parton distribution function into the one for anti-quark. The initial \(pp\) state has a positive baryon number, so that the production cross section of diquark Higgs is much larger than the one of anti-diquark Higgs at LHC. The dependence of the cross section on the final state invariant mass \(M_{u'uj}\) is described as

\[
\frac{d\sigma(pp \rightarrow u(u))}{dM_{u'uj}} = \int d\cos \theta \frac{1}{E_{\text{CMS}}} \int_{0}^{1} dx_1 \frac{2M_{u'uj}}{x_1 E_{\text{CMS}}^2} f_u(x_1, Q^2) f_{\bar{u}} \left(\frac{M_{u'uj}^2}{x_1 E_{\text{CMS}}^2} - Q^2\right) \frac{d\sigma(uu \rightarrow \Delta_{u'c'} \rightarrow uu; \hat{s})}{d \cos \theta}.
\]

The production cross section of the diquark Higgs and its branching ratio to final state up-type quarks depends on the coupling \(f_{ij}\). This coupling is, in general, a free parameter in the model, and in our following analysis, we take an example for \(f_{ij}\),

\[
f_{ij} = \begin{bmatrix} 0.3 & 0 & 0.3 \\ 0 & 0 & 0 \\ 0.3 & 0 & 0.3 \end{bmatrix}.
\]
FIGURE 7. The cross sections of $tt$ (dotted line) and $tj$ (dashed line) productions mediated by the diquark Higgs in s-channel at Tevatron with $E_{CMS} = 1.96$ TeV.

In this example, the phenomenological constraints on $f_{ij}$ discussed above are satisfied with $f_{12} = f_{22} = 0$. This example gives rise to processes, $uu \rightarrow tt, ut$, that we are interested in.

Let us first examine the lower bound on the diquark Higgs mass from Tevatron experiments. We refer the current experimental data of the cross section of top quark pair production [29],

$$\sigma(tt) = 7.3 \pm 0.5(\text{stat}) \pm 0.6(\text{syst}) \pm 0.4(\text{lum}) \, \text{pb},$$

and impose a constraint for the double top quark and a single top quark production cross sections through diquark Higgs in the s-channel. Since most of the $\sigma_{tt}$ value can be understood as the SM effect, the possible new physics should be in the uncertainty range of $\sigma_{tt}$, we take the following conservative bound as

$$\sigma(p\bar{p} \rightarrow \Delta uu \rightarrow tt, ut) \lesssim 1.5 \, \text{pb}.$$  \hspace{1cm} (25)

In our numerical analysis, we employ CTEQ5M [30] for the parton distribution functions with the factorization scale $Q = m_t = 172$ GeV. Figure 7 shows the total cross section of $tt$ and $tu$ productions as a function of the diquark Higgs mass, with $E_{CMS} = 1.96$ TeV. The lower bound is found to be $m_\Delta \gtrsim 470$ GeV.

Next we investigate the diquark and anti-diquark Higgs production at LHC with $E_{CMS} = 14$ TeV. The differential cross sections for each process with $m_\Delta = 600$ GeV and 1 TeV are depicted in Figure 8, together with the $tt$ production cross section in the SM. We can see that the peak cross sections for the $tt$ and $tu$ productions exceed the SM cross section while the $\bar{t}\bar{t}$ and $\bar{t}u$ cross sections are lower than it. This discrepancy between the production cross sections of diquark and anti-diquark Higgs at LHC is the direct evidence of the non-zero baryon number of diquark Higgs. The charge of the lepton from leptonic decay of top quark or anti-top quark can distinguish top quark from anti-top quark. Counting the number of top quark events and anti-top quark events from their leptonic decay modes would reveal non-zero baryon number of diquark Higgs.

The angular distribution of the final states carries the information of the spin of the intermediate states. As shown in Eq. (18), there is no angular dependence on the diquark Higgs production cross section, because the diquark Higgs is a scalar particle. On the other hand, the top quark pair production in the Standard Model is dominated by the gluon fusion process, and the differential cross section shows peaks in the forward and backward region. Therefore, the signal of the diquark Higgs production is enhanced at the region with a large scattering angle (in center of mass frame of colliding partons). Imposing a lower cut on the invariant mass $M_{cut}$, the angular dependence of the cross section is
FIGURE 8. The differential cross sections for $t\bar{t}$ (dashed line), $tt$ (dotted line), $\tilde{t}\bar{t}$ (dashed-dotted line) and $t\bar{t}$ (dashed-dotted-dotted line) as a function of the invariant mass of final state $M_{u\bar{u}}$. The left peak corresponds to $m_\Delta = 600\text{(GeV)}$ and the right one to $m_\Delta = 1\text{ TeV}$. The solid line is the Standard Model $t\bar{t}$ background.

FIGURE 9. Angular distribution of the cross section for $m_\Delta = 600\text{ GeV}$ with $M_{\text{cut}} = 550\text{ GeV}$, together with the $t\bar{t}$ production in the SM. The same line convention as in the Figure 8 has been used.

Described as

$$\frac{d\sigma(pp\rightarrow u_ju_j)}{d\cos\theta} = \int_{M_{\text{cut}}}^{E_{\text{CMS}}} dM_{u_ju_j} \int_{0}^{1} dx_1 \frac{2M_{u_ju_j}}{x_1E_{\text{CMS}}^2} f_a(x_1,Q^2) f_a \left( \frac{M_{u_ju_j}^2}{x_1E_{\text{CMS}}^2}, Q^2 \right) \frac{d\sigma(\mu\mu \rightarrow \Delta_{\mu\mu} \rightarrow u_ju_j)}{d\cos\theta}. \quad (26)$$

The results for $m_\Delta = 600\text{ GeV}$ with $M_{\text{cut}} = 550\text{ GeV}$ are depicted in Figure 9, together with the Standard Model result. Here the lower cut on the invariant mass close to the diquark Higgs mass dramatically reduces the Standard Model.
cross section compared to the diquark Higgs signal.

We now discuss the connection of the coupling $f_{ij}$ to the neutrino mass. Once the B-L symmetry is broken by $\langle \Delta^c \rangle$ along the $\nu^c \nu^c$ direction, right-handed neutrinos acquire masses through the Yukawa coupling in Eq. (14) and their mass matrix is proportional to $f_{ij}$. Therefore, $f_{ij}$ is related to neutrino oscillation data though the (type I) seesaw mechanism which unfortunately involves unknown Dirac Yukawa couplings. When we impose the left-right symmetry on a model, $\Delta$ is accompanied by $\Delta: (3, 1, 10)$, which adds a new term to the superpotential $f \psi \Delta^c \psi$ with the same Yukawa coupling $f_{ij}$. Through this Yukawa coupling, the type II seesaw mechanism can generates Majorana masses for left handed neutrinos. When the type II see-saw contributions dominate the light neutrino mass matrix becomes proportional to $f_{ij}$. In this case, there is a direct relation between the collider physics involving diquark Higgs production and neutrino oscillation data.

For the type II see-saw dominance, a sample value for $f_{ij}$ that fits neutrino observations is given by,

$$f_{ij} = \begin{pmatrix} 0.27 & -0.48 & -0.47 \\ -0.48 & 0 & -0.38 \\ -0.47 & -0.38 & 0.2 \end{pmatrix}.$$

Again, this Yukawa coupling matrix is consistent with phenomenological constraints discussed above. The type II seesaw gives the light neutrino mass matrix via $m_\nu = f \nu_T$ with $\nu_T = \langle \Delta \rangle$. For $\nu_T = 0.1$ eV, it predicts neutrino oscillation parameters to be:

$$\Delta m^2_{12} = 8.9 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2_{23} = 3 \times 10^{-3} \text{ eV}^2, \quad \sin^2 \theta_{12} = 0.32, \quad \sin^2 2\theta_{23} = 0.99, \quad |U_{e3}| = 0.2,$$

which are all consistent with the current neutrino oscillation data [25]. Here the resultant light neutrino mass spectrum is the inverse hierarchical. For $f_{22} \ll 1$ as required by $D^0 - \bar{D}^0$ mixing data, analytic and numerical studies show that only the inverse hierarchical mass spectrum can reproduce the observed neutrino oscillation data for the type II seesaw case.

We have performed the same analysis as before for this case and find the lower bound on the diquark Higgs mass from Tevatron data to be $m_\Delta \gtrsim 450$ GeV, which is a little milder than before. In this case, the peak cross section of only the single top + jet production exceeds the $t \bar{t}$ production cross section of the standard model. The differential cross section of Eq. (26) is independent of the scattering angle, and we find $d\sigma/d\cos \theta = 60.6$ pb for the single top + jet production for $m_\Delta = 600$ GeV with $M_{cut} = 550$ GeV.

Finally, we comment on spin polarization of the final state top (anti-top) quark. Because of its large mass, top quark decays before hadronizing and the information of the top quark spin polarization is directly transferred to its decay products and results in significant angular correlations between the top quark polarization axes and the direction of motion of the decay products [31]. Measuring the top spin polarization provides the information on the chirality nature of top quark in its interaction vertex. It has been shown that measuring top spin correlations can increase the sensitivity to a new particle at Tevatron [32] and LHC [33]. In the diquark Higgs production, it is very interesting to measure the polarization of top (anti-top) quark in the single top production. Only the right-handed top quark couples to diquark Higgs and the top quark produced from diquark Higgs decay is right-handed state, while top quark from the single top production through electroweak processes in the standard model is purely left-handed.

**SUMMARY**

It has been expected that future collider experiments discover new physics beyond the Standard Model and Higgs boson. Precision measurements of masses and couplings of new particles and Higgs boson allow us to obtain indirect informations of a possible unified model that takes place at much higher scale, through the RGE extrapolations of the experimental data. A class of unified models predicts some exotic particles with masses accessible to the future colliders. Once such particles are observed, we obtain the direct information of the new physics model.

**ACKNOWLEDGMENTS**

The author would like to thank Ilia Gogoladze, Naoyuki Haba, Nobuhito Maru, Shigeki Matsumoto, Rabi Mohapatra, Qaisar Shafi, Toshifumi Yamashita, and Hai-Bo Yu for collaborations of the works presented in this talk. This work is
supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan (No. 18740170).

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