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A 3×2 texture for neutrino oscillations and leptogenesis

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Abstract

In an economical system with only two heavy right-handed neutrinos, we postulate a new texture for 3×2 Dirac mass matrix m_D . This model implies one massless light neutrino and thus displays only two patterns of mass spectrum for light neutrinos, namely hierarchical or inverse-hierarchical. Both the cases can correctly reproduce all the current neutrino oscillation data with a unique prediction $m_{\nu_e \nu_e} = \frac{\sqrt{\Delta m_{\text{solar}}^2}}{3}$ and $\sqrt{\Delta m_{\text{atm}}^2}$ for the hierarchical and the inverse-hierarchical cases, respectively, which can be tested in next generation neutrino-less double beta decay experiments. Introducing a single physical CP phase in m_D , we examine baryon asymmetry through leptogenesis. Interestingly, through the CP phase there are correlations between the amount of baryon asymmetry and neutrino oscillation parameters. We find that for a fixed CP phase, the hierarchical case also succeeds in generating the observed baryon asymmetry in our universe, plus a non-vanishing U_{e3} which is accessible in future baseline neutrino oscillation experiments.

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1. Introduction

The origin of the observed baryon asymmetry in our universe, ratio of number of baryons to photons [1]

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = 6.1 \pm 0.2 \times 10^{-10}, \quad (1)$$

is one of the major problems in cosmology. This number has been deduced from two independent observations. (1) From the existing abundance of light elements formed after big bang [2]. (2) Precision measurements of cosmic microwave background [1].

Leptogenesis [3,4] may explain this observed asymmetry between matter and antimatter content of the universe. In explaining this asymmetry one first creates a tiny lepton asymmetry in the early universe. This lepton asymmetry is recycled into observed baryon asymmetry above the electroweak

scale via sphaleron interactions [5]. This is possible since sphaleron interactions remain in thermal equilibrium above the electroweak scale, they violate $B + L$, and since they conserve $B - L$.

It is widely believed that the lepton asymmetry is formed by out-of-equilibrium, lepton number violating, CP violating decay of heavy right-handed neutrinos. Existence of heavy right-handed neutrinos also gives a natural framework for explaining smallness of neutrino mass via see-saw [6] mechanism. If there are no symmetry structures in the theory to make the right-handed neutrinos stable, they must decay. They have non-vanishing Yukawa couplings with Higgs scalars and left-handed doublets, complex in general, for them to do so. Therefore we study lepton asymmetry generated by the CP violating decays of heavy right-handed neutrinos (with Majorana mass) at the early stage of our universe. Since leptogenesis involves no new interactions apart from those required for see-saw mechanism to succeed, we may expect that the physics of neutrino oscillations would clarify some deep mystery of cosmology such as the observed asymmetry between matter and antimatter with which it is linked.

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We know from pioneering works of Sakharov [7] that CP violation is an essential ingredient for theories of matter–antimatter asymmetry. If there is just one right-handed neutrino, the Dirac mass matrix is (3×1) -dimensional. Lepton fields can absorb all complex phases and there is no source of CP violation. Therefore one fails to have leptogenesis with just one right-handed neutrino. If there are two right-handed neutrinos $\{N_1, N_2\}$, the Dirac mass matrix is (3×2) -dimensional. Let us choose a basis where the charged lepton mass matrix as well as the heavy right-handed neutrino mass matrix are diagonal. In this case we cannot absorb all six complex phases in the Dirac matrix plus two complex phases in the Majorana matrix by re-defining five lepton fields $\{l_e, l_\mu, l_\tau, N_1, N_2\}$. After re-phasing, we have three physical CP phases in the 3×2 Dirac mass matrix. Therefore at the minimum, two right-handed neutrinos [8] are enough to bring in a CP violating decay and successful leptogenesis.

Neutrino oscillations show that neutrinos have non-zero mass and that there are off-diagonal entries in the mass matrix written in flavor basis. Solar and atmospheric neutrino oscillation experiments have explored neutrino masses and mixing patterns. Current best fit values are [9–11],

$$\begin{aligned} 7.2 \times 10^{-5} < \Delta m_{12}^2 < 9.2 \times 10^{-5} \text{ eV}^2, \\ 1.4 \times 10^{-3} < \Delta m_{23}^2 < 3.3 \times 10^{-3} \text{ eV}^2, \\ 0.25 < \sin^2 \theta_{12} < 0.39, \\ \sin^2 2\theta_{23} > 0.9, \\ |U_{e3}| < 0.22. \end{aligned} \quad (2)$$

Any model of leptogenesis is required to reproduce these masses and mixing angles. It is indeed interesting to see that, via see-saw mechanism, existing neutrino data can give desired mass spectrum of heavy right-handed neutrinos plus right magnitudes of primordial lepton asymmetry. There are many studies of this kind where there are three heavy right-handed neutrinos and generated lepton asymmetry depends on the form of the Yukawa texture [16]. In this Letter, we examine the system with only two right-handed neutrinos. As discussed above, this system is the minimum one to bring physical CP phases in the lepton sector. Number of free parameters in the neutrino sector is much reduced compared to the usual three right-handed neutrino case. However the system still contains an enough number of free parameters to reproduce the current neutrino oscillation data. We introduce a texture for 3×2 Dirac neutrino mass matrix by which the number of free parameters is further reduced. With a small number of free parameters, we investigate neutrino oscillation parameters and the amount of baryon asymmetry through leptogenesis. We will see correlations between them through a CP phase.

This Letter is organized as follows. In Section 2, we introduce 3×2 Dirac mass matrix and a texture for it. We begin with the CP invariant case and apply a simple ansatz to the light neutrino mass matrix so as to reproduce the current best fit values for the neutrino oscillation parameters. In Section 3, we introduce a single CP phase and examine baryon asymmetry generated through leptogenesis and neutrino oscillations. With

four input parameters (two light and two heavy neutrino mass eigenvalues), all neutrino oscillation parameters as well as the baryon asymmetry through leptogenesis are shown as a function of only the CP phase. Also, the averaged neutrino mass relevant to neutrino-less double beta decay experiments and the Jarlskog invariant characterizing CP violation in the lepton sector are presented as a function of the CP phase. We see correlations among these outputs, and presents some predictions for a fixed CP phase. Section 4 is devoted to conclusions.

2. Texture and a simple ansatz in CP invariant case

Without loss generality, we begin with a reference basis in which the charged lepton mass matrix m_l , Dirac mass matrix m_D and the right-handed Majorana mass matrix M_R are written as

$$\begin{aligned} m_l &= \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \\ m_D &= \begin{pmatrix} c_1 e^{i\delta_1} & c_2 e^{i\delta_2} & c_3 e^{i\delta_3} \\ & c_4 & c_5 & c_6 \end{pmatrix}, \\ M_R &= \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}, \end{aligned} \quad (3)$$

where all parameters are real and $0 < M_1 < M_2$. We have three physical CP phases in m_D by re-phasing. There is no triplet Higgs in the model, so left-handed neutrinos do not have a Majorana mass at the beginning.

After see-saw mechanism, the light neutrino mass matrix becomes

$$m_\nu = m_D^T M_R^{-1} m_D. \quad (4)$$

Note that the system with 3×2 Dirac mass matrix leads to

$$\text{Det}(m_\nu) = 0. \quad (5)$$

Therefore, at least, one mass eigenvalue of light neutrinos is zero. Concerning the current best fit values of neutrino oscillation data, we can conclude that only two patterns of diagonalized mass matrix for light neutrinos are possible. One is the so-called hierarchical case,

$$D_\nu = \text{diag}(m_1 = 0, m_2, m_3), \quad (6)$$

with $m_2 = \sqrt{\Delta m_{12}^2}$ and $m_3 = \sqrt{\Delta m_{12}^2 + \Delta m_{23}^2}$. The other is the so-called inverse-hierarchical case,

$$D_\nu = \text{diag}(m_1, m_2, m_3 = 0), \quad (7)$$

with $m_1 = \sqrt{-\Delta m_{12}^2 + \Delta m_{23}^2}$ and $m_2 = \sqrt{\Delta m_{23}^2}$.

Now we introduce a texture for the Dirac mass matrix as $c_1 = 0$ and $\delta_2 = 0$, and m_D becomes a more simple form,

$$m_D = \begin{pmatrix} 0 & c_2 & c_3 e^{-i\phi} \\ c_4 & c_5 & c_6 \end{pmatrix}, \quad (8)$$

with a single CP phase ϕ . The texture reduces the number of free parameters into six and allows us to analyze the correlations between the amount of baryon asymmetry and neutrino

oscillation parameters with the single CP-phase. Similar textures have been discussed in Ref. [8]. The explicit form of the light neutrino mass matrix is given by

$$m_\nu = m_D^T M_R^{-1} m_D = \begin{pmatrix} \frac{c_4^2}{M_2} & \frac{c_4 c_5}{M_2} & \frac{c_4 c_6}{M_2} \\ \frac{c_4 c_5}{M_2} & \frac{c_2^2}{M_1} + \frac{c_5^2}{M_2} & \frac{c_2 c_3}{M_1} e^{-i\phi} + \frac{c_5 c_6}{M_2} \\ \frac{c_4 c_6}{M_2} & \frac{c_2 c_3}{M_1} e^{-i\phi} + \frac{c_5 c_6}{M_2} & \frac{c_3^2}{M_1} e^{-2i\phi} + \frac{c_6^2}{M_2} \end{pmatrix}. \quad (9)$$

Six parameters in the Dirac mass matrix and two heavy neutrino masses, eight parameters in total, correspond to physics of neutrinos.

We first tackle only neutrino oscillations in the CP invariant case, $\phi = 0$. CP violation will be introduced in the next section. As our starting point, we impose a simple ansatz that m_ν is diagonalized by the so-called tri-bimaximal mixing matrix [12],

$$D_\nu = U_{TB}^T m_\nu U_{TB},$$

$$\text{where } U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}. \quad (10)$$

In fact, the tri-bimaximal mixing matrix is in excellent agreement with the current best fit values in Eq. (2). This ansatz strongly constrains the parameters in Eq. (8). For the hierarchical case, solving Eq. (10) with Eq. (9), we can describe each component in the Dirac mass matrix in terms of m_2 , m_3 , M_1 and M_2 ,

$$c_2 = -c_3 = \sqrt{\frac{M_1 m_3}{2}},$$

$$c_4 = c_5 = c_6 = \sqrt{\frac{M_2 m_2}{3}}. \quad (11)$$

For the inverse-hierarchical case, we find

$$c_2 = c_3 = \sqrt{\frac{3M_1 m_1 m_2}{2(2m_1 + m_2)}},$$

$$c_4 = \sqrt{\frac{M_2(2m_1 + m_2)}{3}},$$

$$c_5 = c_6 = (-m_1 + m_2) \sqrt{\frac{M_2}{3(2m_1 + m_2)}}, \quad (12)$$

in terms of m_1 , m_2 , M_1 and M_2 . Thanks to our ansatz, only four parameters, m_1 (or m_3), m_2 , M_1 and M_2 are left free. These parameters will be used as inputs for our analysis in the next section.

Now we can discuss experimental tests of the model. Neutrino-less double beta decay experiments give upper bounds on the averaged neutrino mass, which can be extracted from the $\nu_e \nu_e$ element of the Majorana mass matrix in the flavor basis. Here we see that for the hierarchical case,

$$m_{\nu_e \nu_e} = (U_{TB} D_\nu U_{TB}^T)_{11} = \frac{m_2}{3} = \frac{\sqrt{\Delta m_{12}^2}}{3}, \quad (13)$$

while for the inverse-hierarchical case,

$$m_{\nu_e \nu_e} = (U_{TB} D_\nu U_{TB}^T)_{11} = \frac{2m_1 + m_2}{3} \simeq \sqrt{\Delta m_{23}^2}. \quad (14)$$

Here we have used an approximation $m_1 \simeq m_2 \simeq \sqrt{\Delta m_{23}^2}$ for $\Delta m_{12}^2 \ll \Delta m_{23}^2$, in the inverse-hierarchical case. Therefore we have a chance of testing this relations in future neutrino-less double beta decay experiments with the sensitivity $m_{\nu_e \nu_e} \geq 10^{-3}$ eV.

3. Numerical analysis in CP violating case

In this section we will introduce non-zero CP phase ϕ in the texture of Eq. (8). Except the CP phase, parameters in the Dirac mass matrix are described in terms of four free parameters as in Eq. (11) or in Eq. (12). With non-zero CP phase, we can obtain baryon asymmetry through leptogenesis. In addition, the Dirac mass matrix becomes complex and resultant neutrino oscillation parameters are deviating from those in the CP invariant case. We will see correlations among resultant neutrino oscillation parameters and the amount of baryon asymmetry created via leptogenesis.

Let us first consider leptogenesis. Primordial lepton asymmetry in the universe is generated through CP violating out-of-equilibrium decay of the lightest heavy neutrinos, which is characterized by the CP violating parameter ϵ [13],

$$\epsilon = -\frac{3}{4\pi v^2} \frac{1}{[m_D m_D^\dagger]_{11}} \text{Im}[(m_D m_D^\dagger)_{12}^2] F\left(\frac{M_2^2}{M_1^2}\right). \quad (15)$$

This formula for asymmetry is valid in the basis where the right-handed neutrino is diagonal. Here, $v = 246$ GeV is the vacuum expectation value of Higgs field, and $F(x) = \sqrt{x} \left[\frac{2}{x-1} + \ln\left[\frac{1+x}{x}\right] \right]$ and $F(x) \simeq 3/\sqrt{x}$ for $x \gg 1$. Sphaleron processes will convert this lepton asymmetry into baryon asymmetry and, as a result, the baryon asymmetry is approximately described as

$$\eta_B = 0.96 \times 10^{-2} (-\epsilon) \kappa. \quad (16)$$

Here $\kappa < 1$ is the efficiency factor, that parameterizes dilution effects for generated lepton asymmetry through washing-out processes. To evaluate the baryon asymmetry precisely, numerical calculations [14] are necessary. We use a fitting formula of the efficiency factor given in terms of effective light neutrino mass \tilde{m} such that [15]

$$\kappa = 2 \times 10^{-2} \left(\frac{0.01 \text{ eV}}{\tilde{m}} \right)^{1.1}, \quad \text{where } \tilde{m} = \frac{(m_D m_D^\dagger)_{11}}{M_1}. \quad (17)$$

Using the above formulas, we estimate the baryon asymmetry as a function of only the single CP phase ϕ with inputs m_1 (or m_3), m_2 , M_1 and M_2 . Numerical results are shown in Fig. 1 for the hierarchical and the inverse-hierarchical cases, respectively. Here we have taken $m_2 = 9.59 \times 10^{-3}$ eV, $m_3 = 4.56 \times 10^{-2}$ eV, $M_1 = 10^{13}$ GeV and $M_2 = 10^{14}$ GeV for the hierarchical case, while $m_1 = 4.46 \times 10^{-3}$ eV, $m_2 = 4.56 \times 10^{-2}$ eV, $M_1 = 10^{13}$ GeV and $M_2 = 10^{14}$ GeV for the inverse-hierarchical case. We can see that in the hierarchical

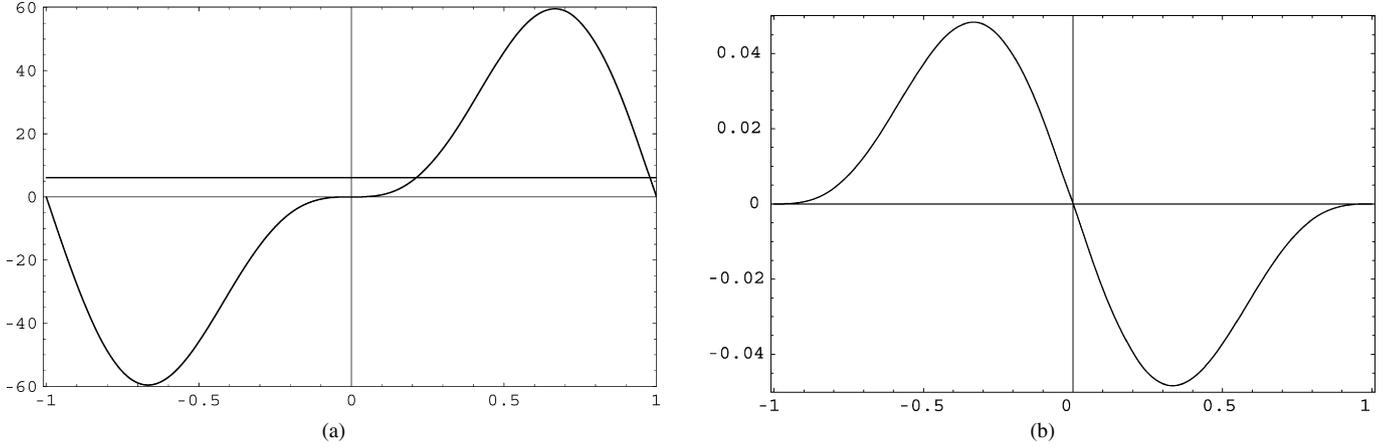


Fig. 1. Baryon asymmetry as a function of the CP phase (in units of π) for (a) hierarchical case and (b) inverse-hierarchical case. We have used input values $m_2 = 9.59 \times 10^{-3}$ eV, $m_3 = 4.56 \times 10^{-2}$ eV, $M_1 = 10^{13}$ GeV and $M_2 = 10^{14}$ GeV for the hierarchical case, while $m_1 = 4.46 \times 10^{-3}$ eV, $m_2 = 4.56 \times 10^{-2}$ eV, $M_1 = 10^{13}$ GeV and $M_2 = 10^{14}$ GeV for the inverse-hierarchical case. In (a), the horizontal line corresponds to the observed baryon asymmetry $\eta_B = 6.1 \times 10^{-10}$. (a) $\eta_B \times 10^{10}$ as a function of ϕ/π in the hierarchical case. (b) $\eta_B \times 10^{10}$ as a function of ϕ/π in the inverse-hierarchical case.

case, $\phi = 0.668$ or 3.075 (rad) provides the baryon asymmetry consistent with the current observations. On the other hand, the inverse-hierarchical case cannot provide sufficient baryon asymmetry.

To understand these results, it is useful to give explicit formulas for leptogenesis in terms of parameters in the texture of Eq. (8). The CP violating parameter and the effective mass \tilde{m} are, respectively, written as

$$\epsilon = \frac{1}{2\pi v^2} \frac{c_3 c_6 (c_2 c_4 + c_3 c_6 \cos \phi) \sin \phi}{c_2^2 + c_3^2},$$

$$\tilde{m} = \frac{c_2^2 + c_3^2}{M_1}. \quad (18)$$

In the hierarchical case, the parameters fixed in Eq. (11) gives

$$\eta_B \simeq 2.5 \times 10^{-8} \left(\frac{M_1}{10^{13} \text{ GeV}} \right) \left(\frac{m_2}{0.01 \text{ eV}} \right) \left(\frac{0.01 \text{ eV}}{m_3} \right)^{1.1} \times (1 - \cos \phi) \sin \phi. \quad (19)$$

Here we have used an approximation formula $F(M_2^2/M_1^2) \simeq 3M_1/M_2$, assuming $M_1 \ll M_2$. Our result is independent of M_2 as long as $M_1 \ll M_2$. To obtain a formula for the inverse-hierarchical case, we use an approximation, $m_1 = \sqrt{-\Delta m_{12}^2 + \Delta m_{23}^2} \simeq \sqrt{\Delta m_{23}^2} (1 + 0.5(\Delta m_{12}^2/\Delta m_{23}^2))$. Thus the parameters in Eq. (12) lead to

$$\eta_B \simeq -2.1 \times 10^{-9} \left(\frac{M_1}{10^{13} \text{ GeV}} \right) \left(\frac{m_2}{0.01 \text{ eV}} \right) \left(\frac{\Delta m_{12}^2}{\Delta m_{23}^2} \right)^2 \times \left(\frac{0.01 \text{ eV}}{m_2} \right)^{1.1} (1 - \cos \phi) \sin \phi. \quad (20)$$

Here we have again used $F(M_2^2/M_1^2) \simeq 3M_1/M_2$, assuming $M_1 \ll M_2$, and the result is independent of M_2 . The baryon asymmetry is suppressed by the factor, $(\Delta m_{12}^2/\Delta m_{23}^2)^2$. If M_1 is very large, for example $M_1 \geq 10^{15}$ GeV, we can give sufficient baryon asymmetry even with the suppression. However, in thermal leptogenesis re-heating temperature after inflation

would be larger than the lightest heavy neutrino mass. It would be difficult to achieve such a quite high reheating temperature in usual reheating scenarios. We need some other mechanism such as a resonant leptogenesis [17] to enhance the primordial lepton asymmetry.

Now we analyze the neutrino oscillation parameters in the case of non-zero CP phase. Parameters c_i in the texture are fixed as discussed in the previous section, and lead to the tribimaximal mixing matrix in the CP invariant case. When we switch CP phase on, the Dirac mass matrix becomes complex and, as a result, output oscillation parameters are deviating from the CP invariant case. In particular, we will find non-vanishing U_{e3} .

Substituting parameters given in Eq. (11) or Eq. (12) into the light neutrino mass matrix of Eq. (9), we find that m_ν is independent of M_1 and M_2 even for non-zero CP phase. Therefore, with input parameters m_1 (or m_3) and m_2 , output oscillation parameters are functions of only the CP phase ϕ .

In the hierarchical case with inputs $m_2 = 9.59 \times 10^{-3}$ eV and $m_3 = 4.56 \times 10^{-2}$ eV, resultant oscillation parameters are depicted in Figs. 2–4. For CP phase $\phi \leq 0.898$ (rad), outputs corresponding to the solar neutrino oscillation are consistent with the best fit values, while other outputs are within the best fit region for any values of CP phase. For $\phi = 0.668$ (rad), which provides the observed baryon asymmetry $\eta_b = 6.1 \times 10^{-10}$, we find the following neutrino oscillation parameters:

$$\begin{aligned} \Delta m_{12}^2 &= 8.1 \times 10^{-5} \text{ eV}^2, \\ \Delta m_{23}^2 &= 2.0 \times 10^{-3} \text{ eV}^2, \\ \sin^2 \theta_{12} &= 0.36, \\ \sin^2 2\theta_{23} &= 1.0, \\ |U_{e3}| &= 0.029. \end{aligned} \quad (21)$$

They are all consistent with observations. Non-vanishing U_{e3} is our prediction, whose value would be covered in future baseline neutrino oscillation experiments. As can be seen from Eq. (9), the $\nu_e \nu_e$ element of m_ν is independent of the CP phase and we

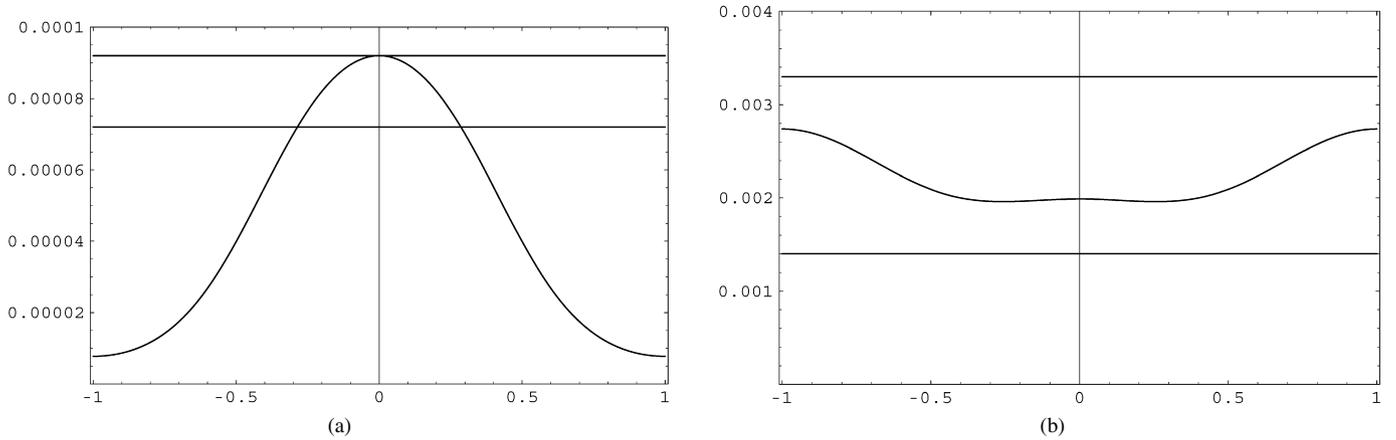


Fig. 2. Mass squared differences as a function of the CP phase (in units of π). The region between two horizontal lines in each figure are consistent with the current best fit values in Eq. (2). (a) Δm_{12}^2 (eV^2) as a function of ϕ/π . (b) Δm_{23}^2 (eV^2) as a function of ϕ/π .

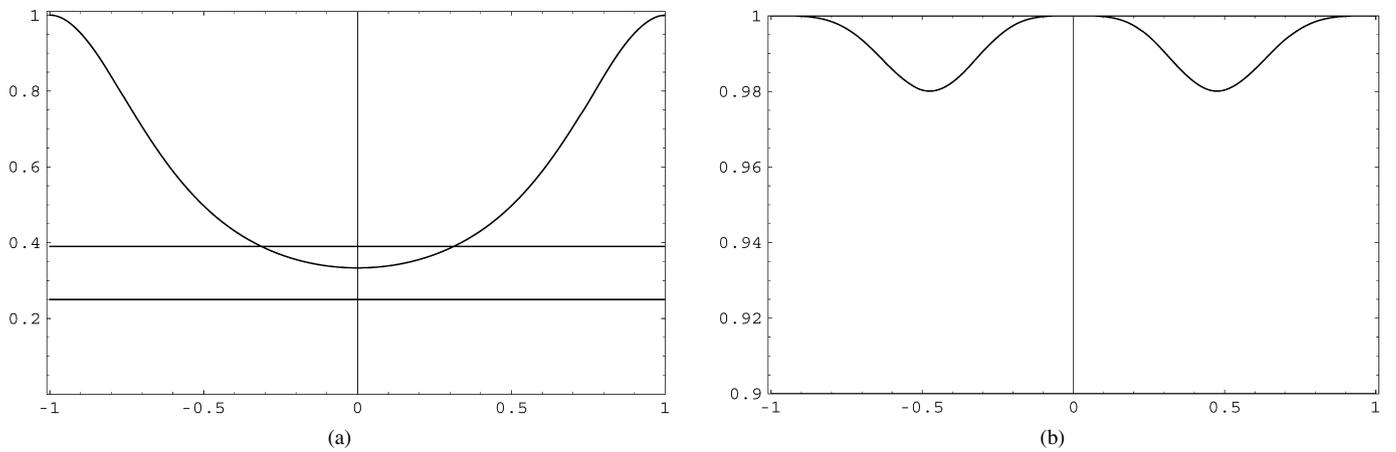


Fig. 3. Neutrino mixing angles as a function of the CP phase in units of π . The region between two horizontal lines in (a) is consistent with the current best fit values in Eq. (2). The entire region shown in (b) is allowed. (a) $\sin^2 \theta_{12}$ as a function of ϕ/π . (b) $\sin^2 2\theta_{23}$ as a function of ϕ/π .

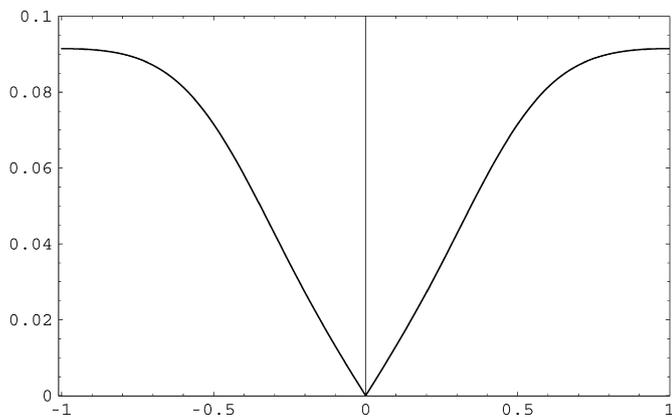


Fig. 4. $|U_{e3}|$ as a function of the CP phase in units of π .

obtain the same result as Eq. (13), numerically,

$$m_{\nu_e \nu_e} = 3.2 \times 10^{-3} \text{ eV}. \quad (22)$$

In the inverse-hierarchical case with input parameters $m_1 = 4.46 \times 10^{-3} \text{ eV}$ and $m_2 = 4.56 \times 10^{-2} \text{ eV}$, resultant oscillation parameters are depicted in Figs. 5 and 6. In this case, we find $\Delta m_{23}^2 \simeq 2.05 \times 10^{-3} \text{ eV}^2$ and $\sin^2 \theta_{23} = 1$, (almost) independent

of the CP-phase. Although the inverse-hierarchical case cannot provide the observed baryon asymmetry, output oscillation parameters are consistent with the current data for a small CP phase $\phi \leq 1.04$ (rad). Again, the $\nu_e \nu_e$ element of m_ν is independent of the CP phase, and we obtain the same result as Eq. (14), numerically,

$$m_{\nu_e \nu_e} = 4.5 \times 10^{-2} \text{ eV}. \quad (23)$$

This is an order of magnitude larger than the value in the hierarchical case.

It is also interesting to see a correlation between the baryon asymmetry through leptogenesis and the leptonic CP violating phase (Dirac phase) [18]. CP violation in the lepton sector is characterized by the Jarlskog invariant [19],

$$J_{\text{CP}} = \text{Im}[U_{e2}U_{\mu 2}^*U_{e3}^*U_{\mu 3}] \\ = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta, \quad (24)$$

where δ is the Dirac phase. The Jarlskog invariant as a function of the CP phase ϕ is depicted in Fig. 7 for (a) the hierarchical and (b) the inverse-hierarchical cases, respectively. We obtain a small but non-vanishing J_{CP} correlating with other outputs. In

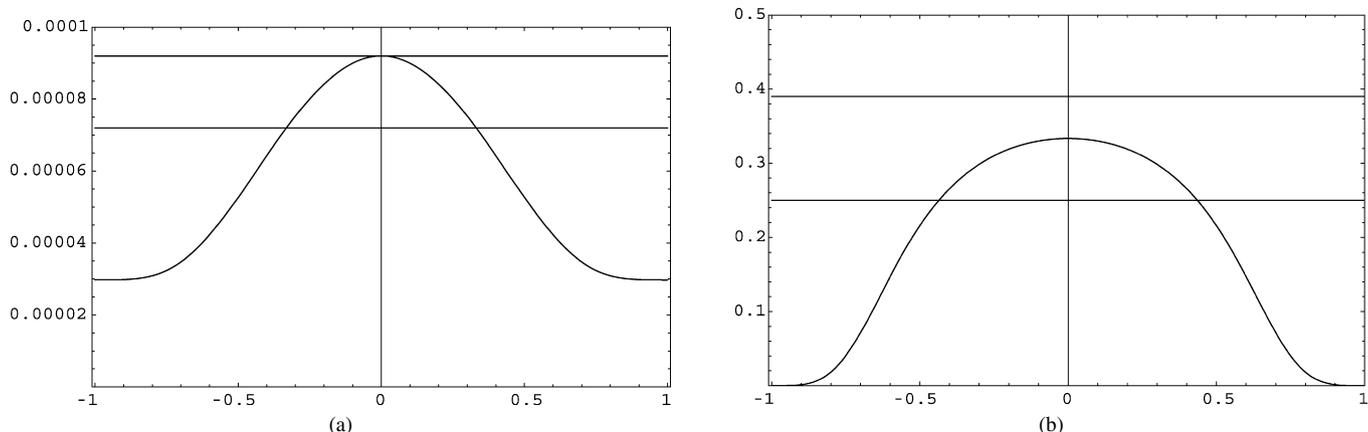


Fig. 5. Mass squared difference and mixing angle relevant for the solar neutrino oscillation as a function of the CP phase in unites of π . The region between two horizontal lines in each figure are consistent with the current best fit values in Eq. (2). (a) Δm_{12}^2 (eV^2) as a function of ϕ/π . (b) $\sin^2 \theta_{12}$ as a function of ϕ/π .

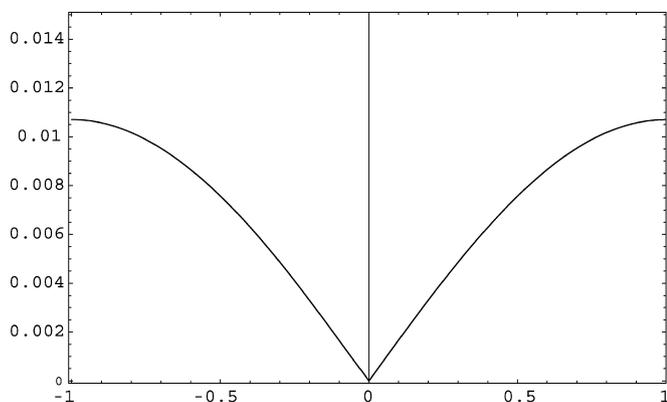


Fig. 6. $|U_{e3}|$ as a function of the CP phase in unites of π .

the hierarchical case, we find

$$J_{\text{CP}} = -4.8 \times 10^{-3} \quad (25)$$

for $\phi = 0.668$ (rad).

4. Conclusions

Neutrino oscillation experiments have explored neutrino masses and mixing patterns. Tiny neutrino masses compared to the ordinary quark masses are naturally explained by the see-saw mechanism with heavy right-handed neutrinos. Right-handed neutrinos play the important role to generate the baryon asymmetry in our universe through leptogenesis. Leptogenesis requires CP violation in the lepton sector. For CP to violate we must have at least two right-handed neutrinos. Keeping this minimal possibility in mind we have introduced only two heavy right-handed neutrinos and studied a 3×2 Dirac type mass matrix m_D . Without loss of generality one can choose a reference basis where both charged lepton mass matrix as well as the heavy right-handed Majorana mass matrices are real and diagonal. In this basis, three physical CP phases appear in m_D .

After the see-saw mechanism we obtain an effective 3×3 Majorana mass matrix for light neutrinos. As a result from the 3×2 Dirac mass matrix m_D , light neutrino mass spectrum should contain (at least) one zero mass eigenvalue. This fact

allows only two patterns for neutrino mass spectrum, normal hierarchical or inverse-hierarchical.

We have chosen a simple texture for m_D in our reference basis. To start with we have set all CP phases in m_D to be zero. Although there is no CP violation in this case, one can study this real texture in the context of ongoing neutrino experiments. We have imposed an ansatz that the light neutrino mass matrix is diagonalized by the tri-bimaximal mixing matrix. This ansatz strongly constrains model parameters, and only four parameters (two light neutrino mass eigenvalues and two heavy neutrino mass eigenvalues) have been left free. Appropriate choice of two light neutrino mass eigenvalues reproduces the current neutrino oscillation data.

Next, we have introduced a single CP phase in m_D . With four input parameters, we have examined baryon asymmetry generated through leptogenesis as well as neutrino oscillations, as a function of only the CP phase. We can see interesting correlations between resultant baryon asymmetry and neutrino oscillation parameters. For a special choice of the CP phase, the hierarchical case can reproduce both the observed baryon asymmetry and neutrino oscillation data. For a fixed CP phase reproducing the observed baryon asymmetry, we have a prediction for a non-vanishing $|U_{e3}|$ which is accessible in future baseline neutrino oscillation experiments. In the inverse-hierarchical case, we have not obtained sufficient baryon asymmetry while resultant neutrino oscillation parameters can be consistent with the current data.

Independently of the CP phase, our texture leads to a unique relation for the averaged neutrino mass relevant to neutrino-less double beta decay experiments: $m_{\nu_e \nu_e} = \frac{\sqrt{\Delta m_{12}^2}}{3}$ and $\sqrt{\Delta m_{23}^2}$ in the hierarchical and the inverse-hierarchical cases, respectively. These results can be tested in next generation experiments of neutrino-less double beta decay. We have worked in the context of a specific texture. However, as an extension to our approach one can introduce small c_1 and check whether the solutions reported in this article get drastically modified. This is so because, often in real world models, one may be able to restrict c_1 such that it is very small yet not exactly zero. If a real c_1 is introduced in the complex case ($\phi \neq 0$), and its magnitude is of order 10%

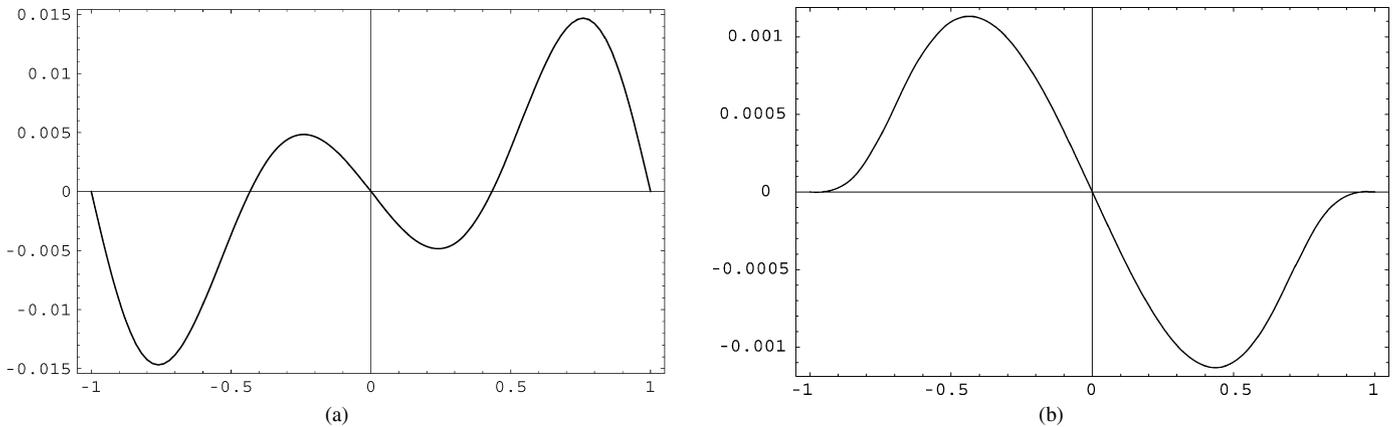


Fig. 7. The Jarlskog parameter as a function of the CP phase (in units of π) for (a) hierarchical case and (b) inverse-hierarchical case. (a) J_{CP} as a function of ϕ/π in the hierarchical case. (b) J_{CP} as a function of ϕ/π in the inverse-hierarchical case.

of the rest of the c_i s, we see a 50% variation in ϵ and U_{e3} . However, $\Delta m_{\text{solar}}^2$, Δm_{atm}^2 , θ_{12} , θ_{23} remain almost the same.

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