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Portal Dark Matter

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SU(5) × U(1)_X grand unification with minimal seesaw and Z'-portal dark matter



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ABSTRACT

We propose a grand unified SU(5) × U(1)_X model, where the standard SU(5) grand unified theory is supplemented by minimal seesaw and a right-handed neutrino dark matter with an introduction of a global Z₂-parity. In the presence of three right-handed neutrinos (RHNs), the model is free from all gauge and mixed-gravitational anomalies. The SU(5) symmetry is broken into the Standard Model (SM) gauge group at $M_{\text{GUT}} \simeq 4 \times 10^{16}$ GeV in the standard manner, while the U(1)_X symmetry breaking occurs at the TeV scale, which generates the TeV-scale mass of the U(1)_X gauge boson (Z' boson) and the three Majorana RHNs. A unique Z₂-odd RHN is stable and serves as the dark matter (DM) in the present Universe, while the remaining two RHNs work to generate the SM neutrino masses through the minimal seesaw. We investigate the Z'-portal RHN DM scenario in this model context. We find that the constraints from the DM relic abundance, and the Z' boson search at the Large Hadron Collider (LHC), and the perturbativity bound on the U(1)_X gauge coupling are complementary to narrow down the allowed parameter region in the range of $3.0 \leq m_{Z'} [\text{TeV}] \leq 9.2$ for the Z' boson mass. The allowed region for $m_{Z'} \leq 5$ TeV will be fully covered by the future LHC experiments. We also briefly discuss the successful implementation of Baryogenesis and cosmological inflation scenarios in the present model.

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Despite its great success, the Standard Model (SM) suffers from several problems. The neutrino mass matrix and a candidate of dark matter (DM) are two major missing pieces of the SM, and they must be supplemented by a framework beyond the SM. The minimal U(1)_{B-L} model [1] is a very simple extension of the SM, where the anomaly-free global B-L (baryon number minus lepton number) symmetry in the SM is gauged and only the three right-handed neutrinos (RHNs) and the U(1)_{B-L} Higgs field in addition to the SM particle content. In the presence of the RHNs, the seesaw mechanism [2] is automatically implemented. Associated with the spontaneous U(1)_{B-L} symmetry breaking by a vacuum expectation value (VEV) of the U(1)_{B-L} Higgs field, the U(1)_{B-L} gauge boson (Z' boson) and the three Majorana RHNs acquire their masses. The SM neutrino mass matrix is generated through the seesaw mechanism after the electroweak symmetry breaking.

Among various possibilities of introducing a DM candidate into the minimal U(1)_{B-L} model, a way proposed in Ref. [3] would be the simplest, where instead of extending the particle content, a

Z₂-parity is introduced and a unique Z₂-odd RHN plays the role of DM. The remaining two RHNs work to generate the SM neutrino mass matrix through the seesaw mechanism. Therefore, in this framework, the three RHNs are categorized into one Z₂-odd RHN DM and two Z₂-even RHNs for the so-called Minimal Seesaw [4], which is the minimal setup to reproduce the neutrino oscillation data with a prediction of one massless eigenstate. In this way, the two missing pieces of the SM are supplemented with no extension of the particle content of the minimal U(1)_{B-L} model.

The RHN DM communicates with the SM particles in two ways: One is through exchanges of Higgs bosons in their mass basis (Higgs-portal RHN DM), where two physical Higgs bosons are realized as linear combinations of the U(1)_{B-L} and the SM Higgs bosons. The other is through Z' boson exchange (Z'-portal RHN DM). Phenomenology of the RHN DM has been extensively studied [3,5,6], in particular, a complementarity between the RHN DM physics and the LHC physics for the Z'-portal RHN DM scenario has been pointed out in Ref. [6].

The minimal U(1)_{B-L} model is easily generalized to the minimal U(1)_X model [7], whose particle content is the same as the one of the minimal U(1)_{B-L} model. The generalization appears in

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the $U(1)_X$ charge assignment for the SM fields: the $U(1)_X$ charge of an SM field (f) is defined as $Q_X = Y_f x_H + Q_{B-L}^f$, where Y_f and Q_{B-L}^f are the hypercharge and the $B-L$ charge of the field, and x_H is a new real parameter (see Table 1). With the charge assignment, the minimal $U(1)_X$ model is free from all the gauge and mixed-gravitational anomalies (see, for example, Ref. [8] for detailed calculations of the anomaly coefficients). The minimal $U(1)_{B-L}$ model is defined as the limit of $x_H \rightarrow 0$. The RHN DM is introduced in exactly the same way for the $U(1)_{B-L}$ model. In Ref. [9], the minimal $U(1)_X$ model with the Z' -portal RHN DM has been extensively studied. In the analysis, only four free parameters are involved: the $U(1)_X$ gauge coupling (α_X), the Z' boson mass ($m_{Z'}$), x_H , and the RHN DM mass (m_{DM}). It has been found in Ref. [9] that $m_{DM} \simeq m_{Z'}/2$ is required to reproduce the observed DM relic density, and the number of free parameters is effectively reduced to three: α_X , $m_{Z'}$ and x_H . The cosmological constraint from the DM relic density leads to a lower bound on α_X for fixed values of $m_{Z'}$ and x_H . On the other hand, the LHC Run-2 results from the search for a Z' boson resonance provide an upper bound on α_X for fixed values of $m_{Z'}$ and x_H . Therefore, the DM physics and the LHC Run-2 phenomenology are complementary to narrow down the model parameter space.

In this letter, we propose a grand unified $SU(5) \times U(1)_X$ model,¹ into which the minimal $U(1)_X$ model with the RHN DM is embedded. The grand unified theory (GUT) has been attracting a lot of attention since its first proposal in Ref. [11], where all the three gauge interactions in the SM are embedded into the $SU(5)$ gauge group, and all the fermions in the SM are unified into three generations of $\mathbf{5}^*$ and $\mathbf{10}$ -representations under the $SU(5)$. The picture is not only mathematically beautiful, but also provides the charge quantization for the SM quarks and leptons. It seems natural to regard the GUT as a primary candidate of physics beyond the SM. However, if this is the case, we may require a GUT model to incorporate the neutrino mass matrix and a DM candidate. The model we propose satisfies this requirement with the RHN DM and two RHNs for the minimal type-I seesaw. As in the original proposal in Ref. [11], the SM gauge groups are embedded into the $SU(5)$ group. However, note that the unification of the quarks and leptons into $\mathbf{5}^*$ and $\mathbf{10}$ -representations is possible only if $x_H = -4/5$. Therefore, the $U(1)_X$ charge is quantized and x_H is no longer a free parameter.

In the following, we show that our GUT model is phenomenologically viable. As we will discuss below, the $SU(5)$ gauge symmetry is broken at $M_{GUT} \simeq 4 \times 10^{16}$ GeV, and the minimal $U(1)_X$ model with the RHN DM ($x_H = -4/5$) is realized as low energy effective theory. The $U(1)_X$ symmetry is assumed to be broken at the TeV scale. We first review this effective minimal $U(1)_X$ model at the TeV scale, and investigate phenomenological constraints on the model parameters. In our analysis, we follow Ref. [9] and identify an allowed parameter region, which will be found to be very narrow since $x_H = -4/5$ is no longer a free parameter. Next, we discuss that the SM gauge couplings are successfully unified at M_{GUT} with some extra fermions at the TeV scale, which originate one $\mathbf{5} + \mathbf{5}^*$ and one $\mathbf{10} + \mathbf{10}^*$ multiplets under the $SU(5)$ gauge group. After the $SU(5)$ breaking, a kinetic mixing between the $U(1)_Y$ and $U(1)_X$ gauge bosons is generated through the renormalization group (RG) evolution. We also discuss that this mixing is negligibly small and has little effect on our analysis.

We first define the minimal $U(1)_X$ model by the particle content listed in Table 1. The minimal $B-L$ model is reproduced as the limit of $x_H \rightarrow 0$. The model is free from all the gauge and the

Table 1

The particle content of the minimal $U(1)_X$ extended SM with Z_2 -parity. In addition to the SM particle content ($i = 1, 2, 3$), the three RHNs (N_R^j ($j = 1, 2$) and N_R) and the $U(1)_X$ Higgs field (Φ) are introduced. The unification into $SU(5) \times U(1)_X$ is achieved only for $x_H = -4/5$, and x_H is quantized in our model.

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$	Z_2
q_L^i	3	2	1/6	$(1/6)x_H + 1/3$	+
u_R^i	3	1	2/3	$(2/3)x_H + 1/3$	+
d_R^i	3	1	-1/3	$(-1/3)x_H + 1/3$	+
ℓ_L^i	1	2	-1/2	$(-1/2)x_H - 1$	+
e_R^i	1	1	-1	$(-1)x_H - 1$	+
H	1	2	-1/2	$(-1/2)x_H$	+
N_R^j	1	1	0	-1	+
N_R	1	1	0	-1	-
Φ	1	1	0	+2	+

gravitational anomalies in the presence of the three RHNs. Because of the Z_2 -parity assignment shown in Table 1, the N_R is a unique (cold) DM candidate. Fixing $x_H = -4/5$, we can see the unification of quarks and lepton into $SU(5) \times U(1)_X$ multiplets: $\overline{\mathbf{F}}_5^i$ of $(\mathbf{5}^*, -3/5) \supset (d_R^i)^c \oplus \ell_L^i$, and F_{10}^j of $(\mathbf{10}, 1/5) \supset q_L^j \oplus (u_R^j)^c \oplus (e_R^j)^c$.

The Yukawa sector of the SM is extended to have

$$\mathcal{L}_{Yukawa} \supset - \sum_{i=1}^3 \sum_{j=1}^2 Y_D^{ij} \overline{\ell}_L^i H N_R^j - \frac{1}{2} \sum_{k=1}^2 Y_N^k \Phi \overline{N}_R^c N_R^k - \frac{1}{2} Y_N \Phi \overline{N}_R^c N_R + \text{H.c.}, \quad (1)$$

where the first term is the neutrino Dirac Yukawa coupling, and the second and third terms are the Majorana Yukawa couplings. Without loss of generality, the Majorana Yukawa couplings are already diagonalized in our basis. Note that only the two Z_2 -even RHNs are involved in the neutrino Dirac Yukawa coupling. After the $U(1)_X$ and the electroweak symmetry breakings, the Z' boson mass, the Majorana masses for the RHNs, and the neutrino Dirac masses are generated:

$$m_{Z'} = g_X \sqrt{4v_\Phi^2 + \frac{1}{4}x_H^2 v_h^2} \simeq 2g_X v_\Phi, \quad m_{Ni} = \frac{Y_N^i}{\sqrt{2}} v_\Phi, \quad m_{DM} = \frac{Y_N}{\sqrt{2}} v_\Phi, \quad m_D^{ij} = \frac{Y_D^{ij}}{\sqrt{2}} v_h, \quad (2)$$

where g_X is the $U(1)_X$ gauge coupling, $\langle \Phi \rangle = v_\Phi/\sqrt{2}$, $v_h = 246$ GeV is the SM Higgs VEV, and we have used the LEP constraint [12] $v_\Phi^2 \gg v_h^2$.

Now we investigate phenomenological constraints for the minimal $U(1)_X$ model with the RHN DM. We follow Ref. [9] for our analysis, where only four free parameters, $\alpha_X = g_X^2/(4\pi)$, $m_{Z'}$, x_H and m_{DM} . However, the grand unification to $SU(5) \times U(1)_X$ requires to fix $x_H = -4/5$, and hence the resultant allowed parameter space is more restricted.

Let us first evaluate the DM relic density by integrating the Boltzmann equation given by [13]

$$\frac{dY}{dx} = - \frac{xS \langle \sigma v_{rel} \rangle}{H(m_{DM})} \left(Y^2 - Y_{EQ}^2 \right), \quad (3)$$

where $x = m_{DM}/T$ is the ratio of the DM mass to the temperature of the Universe (T), $H(m_{DM})$ is the Hubble parameter at $T = m_{DM}$, Y is the yield (the ratio of the DM number density to the entropy density s) of the DM, and Y_{EQ} is the yield of the DM particle in thermal equilibrium. The thermal average of the DM annihilation cross section times relative velocity, $\langle \sigma v_{rel} \rangle$, is calculated from the

¹ A similar model, but the unification into the flipped $SU(5) \times U(1)$ group has been recently proposed in Ref. [10].

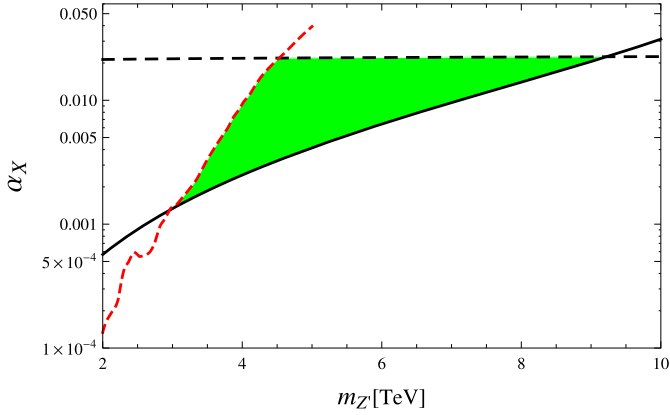


Fig. 1. Allowed parameter region (green shaded) for the Z' -portal RHN DM scenario in the context of our $SU(5) \times U(1)_X$ model ($x_H = -4/5$). The (black) solid line denotes the lower bound on α_X as a function of $m_{Z'}$ to reproduce the observed DM relic abundance. The diagonal dashed line (in red) shows the upper bound on α_X obtained from the search results for a Z' boson resonance at the LHC, which is applicable to $m_{Z'} \leq 5.0$ TeV. The perturbativity bound (see the discussion around Eq. (15)) is depicted by the horizontal dashed line. Combining all three constraints, we obtain the Z' boson mass bound in the range of $3.0 \leq m_{Z'} [\text{TeV}] \leq 9.2$. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

total cross section of the DM pair annihilation process $NN \rightarrow Z' \rightarrow f\bar{f}$ (f denotes the SM fermions) given by

$$\sigma(s) = \frac{\pi}{3} \alpha_X^2 \frac{\sqrt{s(s - 4m_{DM}^2)}}{(s - m_{Z'}^2)^2 + m_{Z'}^2 \Gamma_{Z'}^2} F(x_H), \quad (4)$$

where

$$F(x_H) = 10 \left(x_H + \frac{4}{5} \right)^2 + \frac{33}{5}, \quad (5)$$

and the total decay width of Z' boson is given by

$$\Gamma_{Z'} = \frac{\alpha_X}{6} m_{Z'} \left[F(x_H) + \beta^3 \theta(\beta^2) \right], \quad (6)$$

where $\beta = \sqrt{1 - 4m_{DM}^2/m_{Z'}^2}$, $\theta(z)$ is the unit step function, and the masses of all SM fermions are neglected. Here, we have assumed $m_{N_R^{1,2}} > m_{DM}$, $m_{Z'}/2$, for simplicity. By solving the Boltzmann equation numerically, the present DM relic density is evaluated by

$$\Omega_{DM} h^2 = \frac{m_{DM} s_0 Y(\infty)}{\rho_c/h^2}, \quad (7)$$

where $s_0 = 2890 \text{ cm}^{-3}$ is the entropy density of the present Universe, and $\rho_c/h^2 = 1.05 \times 10^{-5} \text{ GeV/cm}^3$ is the critical density. We identify a parameter region to reproduce the observed DM relic density in the range of $0.1183 \leq \Omega_\chi h^2 \leq 0.1213$ (68% confidence level), measured by the Planck satellite experiment [14]. As shown in Ref. [9], an enhancement of the DM pair annihilation cross section via the Z' boson resonance in the s -channel necessary to reproduce the observed DM density. Hence, we always find $m_{DM} \simeq m_{Z'}/2$, and thus our results are effectively described by only two free parameters: α_X and $m_{Z'}$.

Next we consider the current LHC constraints from the search for a narrow resonance with dilepton final states at the LHC Run-2. In the analysis by the ATLAS and CMS collaborations, the so-called sequential SM Z' (Z'_{SSM}) has been studied as a reference model. We interpret the current LHC constraints on the Z'_{SSM} boson into the Z' boson in our $U(1)_X$ model to identify an allowed parameter region. Although our analysis follows that in Ref. [9], we will

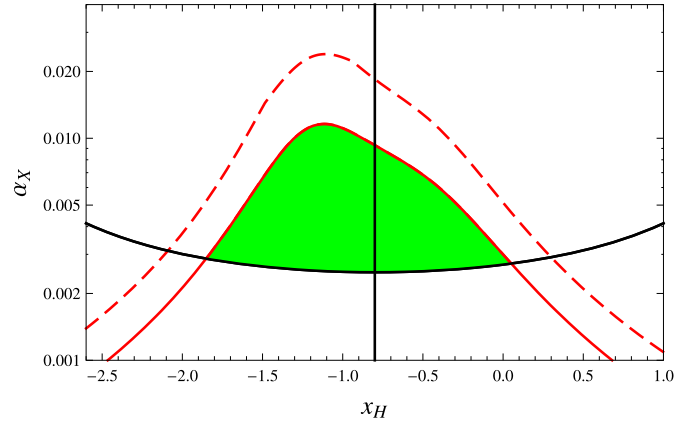


Fig. 2. Allowed parameter region (green shaded) as a function of x_H , for $m_{Z'} = 4$ TeV in the minimal $U(1)_X$ model with the Z' -portal RHN DM. For the grand unified model, $x_H = -4/5$ is indicated by a vertical solid line. The (black) solid line shows the cosmological lower bound on α_X as a function of x_H . The dashed line (in red) shows the upper bound on α_X obtained in Ref. [9] by employing the LHC data with a 13/fb luminosity. Our update of the results by employing the ATLAS results with 36/fb is shown by the (red) solid line.

update the LHC constraints presented in Ref. [9] by employing the latest ATLAS results in 2017 [15].

The cross section for the process $pp \rightarrow Z' + X \rightarrow \ell^+ \ell^- + X$ is given by

$$\sigma = 2 \sum_{q, \bar{q}} \int dM_{\ell\ell} \int_{\frac{M_{\ell\ell}^2}{s}}^1 dx \frac{2M_{\ell\ell}}{xs} f_q(x, Q^2) f_{\bar{q}} \left(\frac{M_{\ell\ell}^2}{xs}, Q^2 \right) \times \hat{\sigma}(q\bar{q} \rightarrow Z' \rightarrow \ell^+ \ell^-), \quad (8)$$

where $M_{\ell\ell}$ is the invariant mass of a final state dilepton, f_q is the parton distribution function for a parton (quark) “ q ”, and $\sqrt{s} = 13 \text{ TeV}$ is the center-of-mass energy of the LHC Run-2. In our numerical analysis, we employ CTEQ6L [16] for the parton distribution functions with the factorization scale $Q = m_{Z'}$. The cross section for the colliding partons is given by

$$\hat{\sigma}(q\bar{q} \rightarrow Z' \rightarrow \ell^+ \ell^-) = \frac{\pi}{1296} \alpha_X^2 \frac{M_{\ell\ell}^2}{(M_{\ell\ell}^2 - m_{Z'}^2)^2 + m_{Z'}^2 \Gamma_{Z'}^2} F_{q\ell}(x_H), \quad (9)$$

where the function $F_{q\ell}(x_H)$ is given by

$$F_{u\ell}(x_H) = (8 + 20x_H + 17x_H^2)(8 + 12x_H + 5x_H^2), \\ F_{d\ell}(x_H) = (8 - 4x_H + 5x_H^2)(8 + 12x_H + 5x_H^2) \quad (10)$$

for q being the up-type (u) and down-type (d) quarks, respectively. By integrating the differential cross section over a range of $M_{\ell\ell}$ set by the ATLAS analysis, we obtain the cross section to be compared with the upper bounds obtained by the ATLAS collaboration. Only two free parameters, α_X and $m_{Z'}$, are involved in our analysis.

In Fig. 1, we show our combined results from the DM relic abundance and the search results for a Z' boson resonance at the LHC. We can see these two constraints are complementary to narrow down the allowed parameter region. In Fig. 1, we also show the perturbativity bound (see the discussion around Eq. (15)), which provides the upper bound on the $U(1)_X$ gauge coupling. Combining all three constraints, we have obtained the Z' boson mass bound in the range of $3.0 \leq m_{Z'} [\text{TeV}] \leq 9.2$.

Although in our grand unified model $x_H = -4/5$ is not a free parameter, we present the combined results as a function of x_H

in Fig. 2 for $m'_{Z'} = 4$ TeV, in order to show that $x_H = -4/5$ has an interesting phenomenological implication. The (black) convex-downward solid line shows the cosmological lower bound on α_X as a function of x_H . The (red) convex-upward dashed line shows the upper bound on α_X presented in Ref. [9], where the results are obtained from the LHC 2016 data with a 13/fb luminosity. We have updated the results by employing the latest ATLAS results with 36/fb [15], and our result is shown by the (red) convex-upward solid line. The (green) shaded region is the final result for the allowed parameter space after combining the cosmological and the LHC constraints when x_H is a free parameter. Interestingly, the plot indicates that $x_H = -4/5$ required by the grand unification into $SU(5) \times U(1)_X$ is almost the best value within the allowed region.

Let us now consider the gauge coupling unification. To realize the grand unification picture, the SM gauge coupling must be unified at a high energy scale. It has been shown in Ref. [17] that the SM gauge couplings are successfully unified around $M_{\text{GUT}} \simeq 4 \times 10^{16}$ GeV in the presence of two pairs of vector-like quarks, $D_L + D_R$ and $Q_L + Q_R$, with their mass of $\mathcal{O}(1 \text{ TeV})$ in the representations of $(\mathbf{3}, \mathbf{1}, 1/3)$ and $(\mathbf{3}, \mathbf{2}, 1/6)$, respectively, under the SM gauge group of $SU(3)_C \times SU(2)_L \times U(1)_Y$. Here, we adopt this simple case to our scenario with a common mass (M) for $D_L + D_R$ and $Q_L + Q_R$. We simply take $M = m_{Z'}/2$ not to change the Z' boson decay width that we have used in our DM analysis. The unification scale leads to proton lifetime to be $\tau_p \simeq 10^{38}$ years, which is consistent with the current experimental lower bound obtained by the Super-Kamiokande [18], $\tau_p(p \rightarrow \pi^0 e^+) \gtrsim 10^{34}$ years.

In the grand unified $SU(5) \times U(1)_X$ picture, $D_L + D_R^C$ and $Q_L + Q_R^C$ are unified into $F_5 + \overline{F}_5 = (\mathbf{5}, 3/5) + (\mathbf{5}^*, -3/5)$ and $F_{10} + \overline{F}_{10} = (\mathbf{10}, 1/5) + (\mathbf{10}^*, -1/5)$, respectively. The way to realize the mass splittings among the components in the $SU(5)$ multiplets and leave only $D_L + D_R$ and $Q_L + Q_R$ light is analogous to the triplet-doublet mass splitting for the 5-plet Higgs field in the usual $SU(5)$ GUT model. We introduce the Yukawa couplings and the mass terms such as

$$\mathcal{L}_Y = \overline{F}_5(Y_5 \Sigma - M_5)F_5 + \text{tr}[\overline{F}_{10}(Y_{10} \Sigma - M_{10})F_{10}], \quad (11)$$

where $Y_{5,10}$ are Yukawa coupling constants, $M_{5,10}$ are masses for the vector-like fermions, Σ is a $U(1)_X$ charge-neutral $SU(5)$ adjoint Higgs field, whose VEV of $\langle \Sigma \rangle = v_{\text{GUT}} \text{diag}(1, 1, 1, -3/2, -3/2)$ breaks the $SU(5)$ gauge group into the SM ones, and we have used antisymmetric 5×5 matrices to express F_{10} and \overline{F}_{10} . We tune the Yukawa coupling to realize $Y_5 v_{\text{GUT}} - M_5 = \mathcal{O}(1 \text{ TeV})$ for $F_5 + \overline{F}_5$, so that the vector-like $SU(3)_C$ color triplets ($D_L + D_R$) become light, while the vector-like $SU(2)_L$ doublets in the multiplets are heavy with mass of $(5/2)M_5$. By tuning Y_{10} to realize $Y_{10} v_{\text{GUT}} + 4M_{10} = \mathcal{O}(1 \text{ TeV})$ for F_{10} and \overline{F}_{10} , we obtain $Q_L + Q_R$ light, while the rest in the $\mathbf{10}$ -plets are heavy with mass of $5M_{10}$. Taking suitable values for $Y_{5,10} = \mathcal{O}(1)$, we can obtain the common TeV scale mass for $D_L + D_R$ and $Q_L + Q_R$, while the other components have GUT-scale masses.

Once the $SU(5)$ symmetry is broken to the SM gauge groups at M_{GUT} , a kinetic mixing between the $U(1)_Y$ and the $U(1)_X$ gauge bosons is generated at low energies through the RG evolutions. Following standard techniques in Ref. [19], we generally set a basis where the gauge boson kinetic terms are diagonalized and a covariant derivative of a field is defined as

$$D_\mu = \partial_\mu - (Y Q_X) \begin{pmatrix} g_Y & g_{\text{mix}} \\ 0 & g_X \end{pmatrix} \begin{pmatrix} B_\mu \\ Z'_\mu \end{pmatrix}. \quad (12)$$

Here, Y and Q_X are $U(1)_Y$ and $U(1)_X$ charges of the field, respectively, B_μ is the SM $U(1)_Y$ gauge field, and g_Y is the $U(1)_Y$ gauge coupling. Originating from the gauge kinetic mixing, a new parameter, namely, “mixed gauge coupling” g_{mix} is introduced. In this

basis, the RG evolution of the SM $U(1)_Y$ gauge coupling remains the same as the SM one at the one-loop level, while g_X and g_{mix} evolve according to their coupled RG equations. At the one-loop level, the coupled RG equations for $\mu > \mathcal{O}(\text{TeV})$ are given by

$$\mu \frac{dg_X}{d\mu} = \frac{\beta_{g_X}}{16\pi^2}, \quad \mu \frac{dg_{\text{mix}}}{d\mu} = \frac{\beta_{g_{\text{mix}}}}{16\pi^2}, \quad (13)$$

where

$$\begin{aligned} \beta_{g_X} &= \frac{1}{6} g_X \left[(80 + 64x_H + 45x_H^2) g_X^2 + 2(32 + 45x_H) g_X g_{\text{mix}} \right. \\ &\quad \left. + 45g_{\text{mix}}^2 \right], \\ \beta_{g_{\text{mix}}} &= \frac{5}{3} g_Y^2 \left[\left(\frac{32}{5} + 9x_H \right) g_X + 9g_{\text{mix}} \right] \\ &\quad + \frac{1}{6} g_{\text{mix}} \left[(80 + 64x_H + 45x_H^2) g_X^2 \right. \\ &\quad \left. + 2(32 + 45x_H) g_X g_{\text{mix}} + 45g_{\text{mix}}^2 \right]. \end{aligned} \quad (14)$$

Here we have taken into account all particle contributions to the beta functions at the TeV scale. Numerically solving the RG equations with $g_{\text{mix}} = 0$ and various values of g_X at $\mu = M_{\text{GUT}}$, we have found that $g_{\text{mix}}/g_X \simeq 0.034$ at the TeV scale for any input values of g_X at M_{GUT} . Therefore, we can safely neglect effects of g_{mix} in our analysis and set $g_{\text{mix}} = 0$ as a good approximation.

Neglecting g_{mix} in the RG equations, we find the following analytic solution for the $U(1)_X$ gauge coupling:

$$\alpha_X(m_{Z'}) = \frac{\alpha_X(M_{\text{GUT}})}{1 + \alpha_X(M_{\text{GUT}}) \frac{b_X}{2\pi} \ln \left[\frac{M_{\text{GUT}}}{m_{Z'}} \right]}, \quad (15)$$

where $b_X = (80 + 64x_H + 45x_H^2)/6 = 48/5$ for $x_H = -4/5$. Since the running $U(1)_X$ gauge coupling $\alpha_X(\mu)$ is asymptotically non-free, we now impose the “perturbativity bound” that $\alpha_X(M_{\text{GUT}})$ must be in the perturbative regime. Adopting a condition of $\alpha_X(M_{\text{GUT}}) \leq 4\pi$, we find $\alpha_X(m_{Z'}) \leq 0.022$ for $m_{Z'} \leq 10$ TeV. In Fig. 1, we see that this perturbativity bound is more severe than the LHC bound for $m_{Z'} \gtrsim 4.5$ TeV.

Finally, our grand unified $SU(5) \times U(1)_X$ model can also account for the origin of the Baryon asymmetry in the Universe through leptogenesis [20] with two Z_2 -even RHNs if they are almost degenerate (resonant leptogenesis [21]). Introducing non-minimal gravitational couplings, the $U(1)_X$ Higgs field plays the role of inflaton. We can achieve the successful cosmological inflation scenario with a suitable choice of the non-minimal gravitational coupling constant. See, for example, Ref. [22].

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