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Window for Higgs boson mass from gauge-Higgs unification

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Abstract

We consider six-dimensional gauge models compactified on the orbifold T^2/Z_N ($N = 2, 3, 4, 6$) such that the Standard Model (SM) Higgs doublet arises from the extra-dimensional components of the gauge field. For $\Lambda \leqslant 10^{19}$ GeV, where Λ denotes the compactification scale, we obtain 114.4 GeV $\leqslant m_H \leqslant$ 164 GeV for the SM Higgs boson mass. We also consider gauge-Higgs-top and gauge-Higgs-bottom Yukawa unification which respectively yield $m_H = 131^{+4}_{-5}$ GeV and $m_H = 150^{+2}_{-2}$ GeV for a top quark pole mass $M_t = 170.9^{+1.8}_{-1.8}$ GeV. As a special case we recover the result $m_H \leqslant 132$ GeV previously obtained for five-dimensional models.

In a recent paper [1], hereafter called I, we investigated five-dimensional (5D) gauge models compactified on the orbifold S^1/Z_2 in which the zero mode of the fifth component of the gauge field could be identified with the SM Higgs doublet H. The five-dimensional gauge invariance requires that the Higgs quartic coupling vanish at tree level. By imposing this condition at the compactification scale Λ [2], the SM Higgs boson mass was estimated using two-loop renormalization group equations (RGE). For 10^6 GeV $\leqslant \Lambda \leqslant 10^{19}$ GeV, and for a top quark pole mass of 170.9 ± 1.8 GeV [3], the mass is in the range 114.4 GeV $\leqslant m_H \leqslant$ 132 GeV. [In I the upper bound was found to be 129 GeV. A more careful treatment here of the top quark pole mass yields the slightly larger value of 132 GeV.] In the SM with the standard particle content, the SU(2) gauge coupling and the top quark Yukawa coupling have the same magnitude at scales of order 10^8 GeV. If the latter scale is identified with Λ, one obtains $m_H = 117 \pm 4$ GeV [1]. It is amusing to note that the Higgs boson mass predictions in this 5D model have a great deal of overlap with the minimal supersymmetric Standard Model (MSSM) prediction for the mass of the lightest CP-even Higgs boson. The uncertainty in the Higgs mass predictions are largely due to the experimental uncertainty in the determination of the top quark mass.

In this Letter we extend our earlier results in I by considering 6D gauge-Higgs unification (GHU) models compactified on the orbifold T^2/Z_N, with $N = 2, 3, 4$ and 6. For $N = 2$, two SU(2) doublets appear under appropriate boundary conditions as the zero modes of the extra-dimensional components of the gauge field [4]. Since our goal here is to predict the mass of the SM Higgs boson H, we will assume that a suitable linear combination of these two doublets corresponds to H, while the orthogonal combination acquires mass of order Λ. The six-dimensional gauge invariance determines the quartic tree level coupling of H in terms of the SU(2) gauge coupling and $\tan\beta$, where $\tan\beta$ is defined, as usual, as the ratio of the vacuum expectation value of the two Higgs doublets. With this value of the quartic coupling as the boundary condition at a given scale Λ, and for a given $\tan\beta$, the Higgs boson mass is estimated using two-loop RGEs. We find 114.4 GeV $\leqslant m_H \leqslant$ 164 GeV, for $\Lambda \leqslant 10^{19}$ GeV, with $0 \leqslant \tan\beta < \infty$. Compactification on T^2/Z_N orbifolds with $N = 3, 4, 6$ leaves only a single Higgs doublet [5], as desired. The Higgs mass for these

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cases is realized in the limit $\tan\beta = 0$ or ∞. We also discuss gauge-Higgs–Yukawa unification for which a more precise prediction for m_H is found. Thus, $m_H = 131$ GeV (150 GeV) with top (bottom) quark unification for a top quark pole mass $M_t = 170.9$ GeV. Finally, for $\tan\beta = 1$, we recover our earlier result for $m_H (\leqslant 132$ GeV) obtained in I with 5D gauge-Higgs unification.

We begin with a very brief review of the basic structure of 6D GHU models. As a simple example, consider an SU(3) GHU model compactified on the orbifold T^2/Z_N ($N = 2, 3, 4, 6$) [4,5]. Note that at least one additional $\mathrm{U}(1)'$ factor is needed to recover the 4D electroweak gauge symmetry. Without the additional U(1) the weak mixing angle is too large when gauge couplings run according to the SM RGE and if an additional brane localized gauge kinetic terms are suppressed. The SM $\mathrm{U}(1)_Y$ gauge coupling is realized as the mixture of the U(1) in SU(3) and the additional $\mathrm{U}(1)'$, so that the weak mixing angle can be a free parameter. Therefore, the compactification scale can also remain free, while we retain the predictivity of the Higgs quartic coupling.

The six-dimensional Lagrangian for the SU(3) gauge field, $A_{\hat\mu} = A^a_{\hat\mu}\lambda^a/2$, with λ^a the Gell-Mann matrix, is given by

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{tr}\left[F^{\hat\mu\hat\nu}F_{\hat\mu\hat\nu}\right], \tag{1}$$

where $F_{\hat\mu\hat\nu} = \partial_{\hat\mu}A_{\hat\nu} - \partial_{\hat\nu}A_{\hat\mu} - ig_6[A_{\hat\mu}, A_{\hat\nu}]$, g_6 is the 6D gauge coupling, and $\hat\mu, \hat\nu = 0, 1, 2, 3, 5, 6$. In the following, we use the notation $\mu, \nu = 0, 1, 2, 3$ and $M, N = 5, 6$ for the usual four dimensions and the extra two dimensions, respectively. In terms of four-dimensional effective theory, A_μ corresponds to the vector field while A_M denotes the scalar fields.

In order to describe the model, it is useful to introduce a complex coordinate $z = (x^5 + ix^6)/\sqrt{2}$ on the torus T_2 and a corresponding gauge field $A_z = (A_5 - iA_6)/\sqrt{2}$. Under the T^2/Z_N orbifold transformation, $z \to \tau z$ with $\tau = e^{i2\pi/N}$, the transformation law, which keeps the action invariant, for the four-dimensional and extra-dimensional components of the gauge field is given as

$$\begin{aligned} A_\mu\left(x^\mu, \tau z\right) &= \hat P A_\mu\left(x^\mu, x^5, x^6\right)\hat P^\dagger, \\ A_z\left(x^\mu, \tau z\right) &= \tau^{-1}\hat P A_z\left(x^\mu, z\right)\hat P^\dagger, \end{aligned} \tag{2}$$

with $\hat P = \mathrm{diag}(\tau, \tau, 1)$. With this boundary condition, the SU(3) gauge symmetry is broken down to $\mathrm{SU}(2)\times\mathrm{U}(1)$.

The zero modes of the vector (gauge) fields can be explicitly written as

$$\begin{aligned} A_\mu &= \frac{1}{2\sqrt{V_2}}\begin{pmatrix} A^{(3)}_\mu + \frac{1}{\sqrt3}A^{(8)}_\mu & A^{(1)}_\mu - iA^{(2)}_\mu & 0 \\ A^{(1)}_\mu + iA^{(2)}_\mu & -A^{(3)}_\mu + \frac{1}{\sqrt3}A^{(8)}_\mu & 0 \\ 0 & 0 & -\frac{2}{\sqrt3}A^{(8)}_\mu \end{pmatrix} \\ &= \frac{1}{2\sqrt{V_2}}\begin{pmatrix} W^{(3)}_\mu + \frac{1}{\sqrt3}B^{(8)}_\mu & \sqrt2 W^+_\mu & 0 \\ \sqrt2 W^-_\mu & -W^{(3)}_\mu + \frac{1}{\sqrt3}B^{(8)}_\mu & 0 \\ 0 & 0 & -\frac{2}{\sqrt3}B^{(8)}_\mu \end{pmatrix}, \end{aligned} \tag{3}$$

where V_2 is the volume of the extra-dimensions and we have used the usual SU(2) notation. The off-diagonal components of A_z, which form the SU(2) doublet fields, include candidates for zero-modes,

$$A_z = \frac{1}{2\sqrt{V_2}}\begin{pmatrix} 0 & \sqrt2 H_2 \\ \sqrt2 H_1^T & 0 \end{pmatrix}, \tag{4}$$

where we have defined the two independent doublet fields as

$$\begin{aligned} H_1 &= \frac{1}{\sqrt2}\begin{pmatrix} A^{(4)}_z + iA^{(5)}_z \\ A^{(6)}_z + iA^{(7)}_z \end{pmatrix}, \\ H_2 &= \frac{1}{\sqrt2}\begin{pmatrix} A^{(4)}_z - iA^{(5)}_z \\ A^{(6)}_z - iA^{(7)}_z \end{pmatrix}. \end{aligned} \tag{5}$$

According to Eq. (2), the transformation law for these SU(2) doublets is explicitly written as

$$\begin{aligned} H_1\left(x^\mu, \tau z\right) &= \tau^{-2}H_1\left(x^\mu, z\right), \\ H_2\left(x^\mu, \tau z\right) &= H_2\left(x^\mu, z\right). \end{aligned} \tag{6}$$

Therefore, under this orbifold boundary condition, the doublet H_2 always has a zero-mode, while the doublet H_1 has a zero-mode only when $N = 2$ and $\tau^{-2} = 1$.

Substituting these various expressions in Eq. (1) and integrating over the extra-dimensional coordinates (x_5, x_6), we obtain the Lagrangian of $\mathrm{SU}(2)\times\mathrm{U}(1)$ with two Higgs doublets in four-dimensions. In particular, the Higgs potential is obtained from the term,

$$\begin{aligned} V &= \int dx^5\,dx^6\,\frac{1}{2}\,\mathrm{tr}\left[F^{MN}F_{MN}\right] = \int dx^5\,dx^6\,\mathrm{tr}\left[(F_{56})^2\right] \\ &= -g_6^2\int dx^5\,dx^6\,\mathrm{tr}[A_5, A_6]^2 = g_6^2\int dx^5\,dx^6\,\mathrm{tr}\left[A_z, A_z^\dagger\right]^2, \end{aligned} \tag{7}$$

where we have used $F_{MN} = \partial_M A_N - \partial_N A_M - ig_6[A_M, A_N] = -ig_6[A_M, A_N]$ for the zero modes, and the definition $A_z = (A_5 - iA_6)/\sqrt2$ in the last expression.

In the $N = 2$ case, the model provides two scalar SU(2) doublets as zero modes. We define the up-type and down-type Higgs doublets as

$$H_u = H_2^0, \qquad H_d = -i\sigma_2 H_1^0, \tag{8}$$

where $H^0_{1,2}$ denotes the zero-mode of $H_{1,2}$. Substituting the explicit matrix expression of Eq. (4) together with Eqs. (5) and (8) into Eq. (7), we obtain the following Higgs potential [4]

$$V = \frac{g^2}{2}\left(|H_d|^2 - |H_u|^2\right)^2 + \frac{g^2}{2}\left(H_u^\dagger H_d\right)\left(H_d^\dagger H_u\right), \tag{9}$$

where $g = g_6/\sqrt{V_2}$ is the SU(2) gauge coupling. The quadratic mass term is forbidden at tree level by the six-dimensional gauge invariance. This formula corresponds to the D-term potential in the MSSM with the identification $g'^2 = 3g^2$ for the $\mathrm{U}(1)_Y$ gauge coupling.

In the case of $N = 3, 4, 6$, only a single Higgs doublet field emerges as the zero-mode of H_2, which is identified as the SM Higgs doublet. The quartic Higgs potential in this case is given by [5],

$$V = \frac{1}{2}g^2|H|^4, \tag{10}$$

where we have defined the SM Higgs doublet as $H = H_2^0$.

As has been explicitly shown in Ref. [2] (for a 5D GHU model), the effective SM Higgs quartic coupling calculated in a given GHU model coincides with the one generated through the RGEs in the SM by imposing a special boundary condition at the compactification scale. This boundary condition is the gauge-Higgs condition, where the quartic coupling at the compactification scale is set to be the tree level one required by the higher-dimensional gauge invariance. This treatment is natural from an effective field theoretical point of view, where Kaluza–Klein modes should decouple at low energy, and (dimensionless) couplings in the low energy effective theory should be matched with the one from high energy theory at the compactification scale. Corrections to the Higgs mass squared in GHU models, on the other hand, are very much dependent on the particle contents and imposed boundary conditions (see for instance [2]). The fine tuning condition is not as severe as in the SM case but it is there when we consider compactification scales larger than a few TeV. Our point is that the fine tuning condition (the SM Higgs mass cancelation mechanism) is model dependent, and there can be different possibilities to achieve it (by introducing appropriate bulk masses and bulk fields), and still retain gauge-Higgs condition. We calculate the Higgs mass range from gauge-Higgs condition in general, which can be applicable for a variety of orbifold models with distinct gauge symmetries and varying particle content. In this Letter we will therefore treat the Higgs mass squared as a free parameter in the low energy effective theory, to be suitably adjusted to yield the desired electroweak symmetry breaking. Once the electroweak symmetry breaking is correctly achieved, the Higgs boson mass is determined by its quartic coupling at the electroweak scale.

We are now ready to discuss the physical Higgs boson mass m_H. For compactification on T^2/Z_2, only one combination of the Higgs doublets is assumed to be light and identified as the SM Higgs doublet, while the orthogonal combination has a compactification mass scale and decouples at low energies. Under this assumption, the Higgs doublets H_u, H_d are described in terms of the SM(-like) Higgs doublet (H), the heavy Higgs doublet ($\tilde{H}$) and $\tan\beta = \langle H_u\rangle/\langle H_d\rangle$,

$$H_u = H\sin\beta + \tilde{H}\cos\beta,$$
$$i\sigma_2 H_d^* = H\cos\beta - \tilde{H}\sin\beta. \tag{11}$$

The quartic coupling of the SM Higgs boson is read off from Eq. (9) as

$$V = \frac{1}{2}g^2\cos^2(2\beta)|H|^4. \tag{12}$$

Note that the one Higgs doublet case corresponding to compactification on the orbifold T^2/Z_N with $N \neq 2$, can be realized as the limit $\tan\beta = 0/\infty$, or equivalently $\cos^2(2\beta) = 1$. In this case $m_H = 2m_W$ at tree level [5]. Also, for $\tan\beta = 1$ or $\cos(2\beta) = 0$, we recover the condition analyzed in Ref. [1] for 5D GHU models. Therefore, the 6D GHU model with varying $\tan\beta$ provides the general results for the SM Higgs boson mass in this class of models.

Imposing the gauge-Higgs condition for the Higgs quartic coupling, $\lambda = g^2\cos^2(2\beta)$, at a given compactification scale Λ, and for a given $\tan\beta$, we solve the two loop RGEs [6], to obtain the Higgs boson mass. Namely,

$$m_H(m_H) = \sqrt{\lambda(m_H)}v. \tag{13}$$

For the three SM gauge couplings, we have

$$\frac{dg_i}{d\ln\mu} = \frac{b_i}{16\pi^2}g_i^3 + \frac{g_i^3}{(16\pi^2)^2}\sum_{j=1}^{3}B_{ij}g_j^2, \tag{14}$$

where μ is the renormalization scale, g_i ($i = 1, 2, 3$) are the SM gauge couplings and

$$b_i = \left(\frac{41}{10}, -\frac{19}{6}, -7\right), \qquad b_{ij} = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix}. \tag{15}$$

The top quark pole mass is taken to be $M_t = 170.9 \pm 1.8$ GeV, [3], with $(\alpha_1, \alpha_2, \alpha_3) = (0.01681, 0.03354, 0.1176)$ at the Z-pole (M_Z) [7]. For the top Yukawa coupling y_t, we have [6],

$$\frac{dy_t}{d\ln\mu} = y_t\left(\frac{1}{16\pi^2}\beta_t^{(1)} + \frac{1}{(16\pi^2)^2}\beta_t^{(2)}\right). \tag{16}$$

Here the one-loop contribution is

$$\beta_t^{(1)} = \frac{9}{2}y_t^2 - \left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right), \tag{17}$$

while the two-loop contribution is given by

$$\begin{aligned}\beta_t^{(2)} = {} & -12y_t^4 + \left(\frac{393}{80}g_1^2 + \frac{225}{16}g_2^2 + 36g_3^2\right)y_t^2 \\ & + \frac{1187}{600}g_1^4 - \frac{9}{20}g_1^2g_2^2 + \frac{19}{15}g_1^2g_3^2 - \frac{23}{4}g_2^4 \\ & + 9g_2^2g_3^2 - 108g_3^4 + \frac{3}{2}\lambda^2 - 6\lambda y_t^2.\end{aligned} \tag{18}$$

In solving Eq. (16) from M_t to the compactification scale Λ, the initial top Yukawa coupling at $\mu = M_t$ is determined from the relation between the pole mass and the running Yukawa coupling [8,9],

$$\begin{aligned}M_t \simeq m_t(M_t)\Bigg(1 & + \frac{4}{3}\frac{\alpha_3(M_t)}{\pi} + 11\left(\frac{\alpha_3(M_t)}{\pi}\right)^2 \\ & - \left(\frac{m_t(M_t)}{2\pi v}\right)^2\Bigg),\end{aligned} \tag{19}$$

with $y_t(M_t) = \sqrt{2}m_t(M_t)/v$, where $v = 246.2$ GeV. Here, the second and third terms in the parentheses correspond to one- and two-loop QCD corrections, respectively, while the fourth term comes from the electroweak corrections at one-loop level. The numerical values of the third and fourth terms are comparable (signs are opposite). The electroweak corrections at two-loop level and the three-loop QCD corrections [9], are of comparable and sufficiently small magnitude [9] to be safely ignored.

The RGE for the Higgs quartic coupling is given by [6],

$$\frac{d\lambda}{d\ln\mu} = \frac{1}{16\pi^2}\beta_\lambda^{(1)} + \frac{1}{(16\pi^2)^2}\beta_\lambda^{(2)}, \tag{20}$$

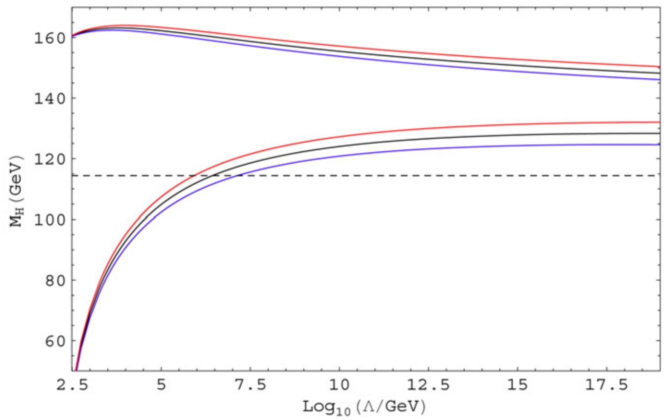

Fig. 1. Higgs boson mass prediction versus the compactification scale for a given $\cos(2\beta)$. The upper three lines (in blue, black and red) from bottom to top correspond to input top quark pole masses, $M_t = 169.1$, 170.9 and 172.7 GeV, for $\tan\beta = 0/\infty$ or equivalently $\cos^2(2\beta) = 1$. The lower three lines (in blue, black and red) from bottom to top correspond to input top quark pole masses, $M_t = 169.1$, 170.9 and 172.7 GeV, for $\tan\beta = 1$ or equivalently $\cos^2(2\beta) = 0$. The lower lines show the same results as in 5D GHU models [1]. The horizontal line shows the current Higgs boson mass bound, $m_H \geqslant 114.4$ GeV, from LEP2 [10]. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

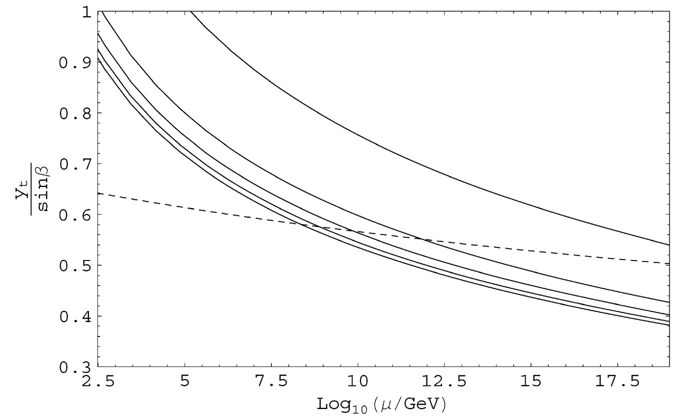

Fig. 2. Plot of the running SU(2) gauge coupling $g(\mu)$ (dashed line) and the running top Yukawa coupling (divided by a given $\sin\beta$), $y_t(\mu)/\sin\beta$ (solid line), versus $\mathrm{Log}_{10}(\mu/\mathrm{GeV})$ for a given $\tan\beta$. Solid lines correspond to $\tan\beta = 1$, 2, 3, 5, and 30, respectively, from top to bottom. Here we took top quark pole mass $M_t = 170.9$ GeV.

with

$$\beta_\lambda^{(1)} = 12\lambda^2 - \left(\frac{9}{5}g_1^2 + 9g_2^2\right)\lambda + \frac{9}{4}\left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2g_2^2 + g_2^4\right) + 12y_t^2\lambda - 12y_t^4, \tag{21}$$

and

$$\begin{aligned}\beta_\lambda^{(2)} = {} & -78\lambda^3 + 18\left(\frac{3}{5}g_1^2 + 3g_2^2\right)\lambda^2 \\ & - \left(\frac{73}{8}g_2^4 - \frac{117}{20}g_1^2g_2^2 + \frac{2661}{100}g_1^4\right)\lambda - 3\lambda y_t^4 \\ & + \frac{305}{8}g_2^6 - \frac{289}{40}g_1^2g_2^4 - \frac{1677}{200}g_1^4g_2^2 - \frac{3411}{1000}g_1^6 \\ & - 64g_3^2y_t^4 - \frac{16}{5}g_1^2y_t^4 - \frac{9}{2}g_2^4y_t^2 \\ & + 10\lambda\left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right)y_t^2 \\ & - \frac{3}{5}g_1^2\left(\frac{57}{10}g_1^2 - 21g_2^2\right)y_t^2 - 72\lambda^2y_t^2 + 60y_t^6.\end{aligned} \tag{22}$$

In Fig. 1, we plot the Higgs boson mass m_H as a function of the compactification scale for a given $\tan\beta$ with an input top quark pole mass $M_t = 170.9 \pm 1.8$ GeV. Each set of three lines (in red, black and blue) corresponds to $M_t = 172.7$, 170.9 and 169.1 GeV, from top to bottom. The upper three lines (in red, black and blue) are the results for the gauge-Higgs condition $\lambda = g^2$ (with $\tan\beta = 0/\infty$ or equivalently $|\cos(2\beta)| = 1$), which also correspond to the results in 6D GHU models with the orbifold compactifications T^2/Z_N ($N \neq 2$), as previously mentioned. The lower three lines are the results for the gauge-Higgs condition $\lambda = 0$ (with $\tan\beta = 1$ or equivalently $\cos(2\beta) = 0$), which are exactly the same ones presented in Ref. [1]. For general values of $\tan\beta$ (or $\cos(2\beta)$), the resultant Higgs mass should appear between the top and bottom lines. Therefore, in this class of models, the Higgs mass is predicted in the range 114.4 GeV $\leqslant m_H \leqslant$ 164 GeV, for a cutoff scale lower than the Planck scale. The range is narrow compared to the one obtained from the stability and triviality bounds on the Higgs quartic coupling [11].

In the GHU model with fermions in the bulk one would expect unification of gauge and Yukawa interactions at the compactification scale Λ. This is clearly not plausible for fermions in the first two generations. (To realize the hierarchy of fermion masses of the SM, a more elaborate GHU model must be considered. There have been various efforts along this direction [12].) However, for a fermion in the third generation, its running Yukawa coupling and the running SU(2) gauge coupling can meet at some scale, which depends on $\tan\beta$, and it would be natural to identify this 'merger' point with the compactification scale. Let us first assume that this holds for the top quark Yukawa coupling. We have

$$\mathcal{L}_Y = -g\overline{q_{3L}}H_u t_R, \tag{23}$$

where q_{3L} is the quark doublet of the third generation, and the up-type Higgs $H_u = H\sin\beta + \tilde{H}\cos\beta$. The SM top Yukawa coupling is defined as $y_t = g\sin\beta$. We can fix the compactification scale as the point where the relation, $y_t(\Lambda) = g(\Lambda)\sin\beta$, is satisfied, with a given $\tan\beta$ (see Fig. 2). The compactification scale as a function of $\tan\beta$ is depicted in Fig. 3 for input top quark pole masses, $M_t = 169.1$, 170.9 and 172.7 GeV. Fig. 4 shows the Higgs mass as a function of the compactification scale which has been fixed for a given $\tan\beta$. This observation allows us to realize gauge-Higgs and gauge-top Yukawa coupling unification at Λ, [13], and to narrow down the Higgs boson mass window from Fig. 1 to 125 GeV $\leqslant m_H \leqslant$ 158 GeV in Fig. 4.

In T^2/Z_2 models with large $\tan\beta$, the bottom quark and tau Yukawa couplings can be large, and it is possible for the running bottom quark (or tau) Yukawa coupling to meet the running gauge coupling at a high scale. As an example, we examine the unification of gauge and bottom quark Yukawa couplings,

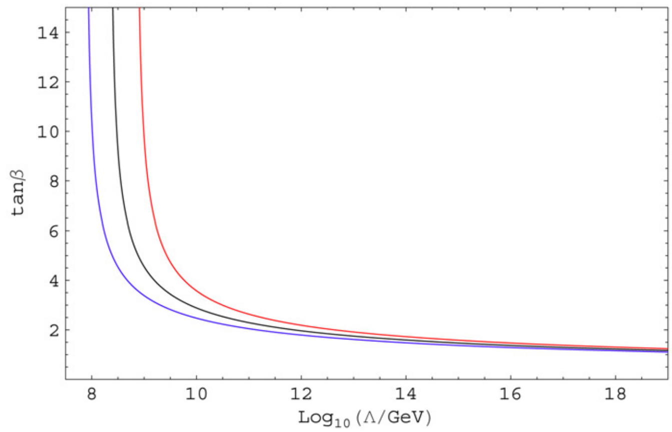

Fig. 3. Plot of $\tan\beta$ versus the compactification scale, $\mathrm{Log}_{10}(\Lambda/\mathrm{GeV})$, determined as the merger point of the running top Yukawa and the SU(2) gauge couplings. Three solid lines (in red, black and blue) from top to bottom correspond to top quark pole masses $M_t = 172.7$, 170.9 and 169.1 GeV, respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

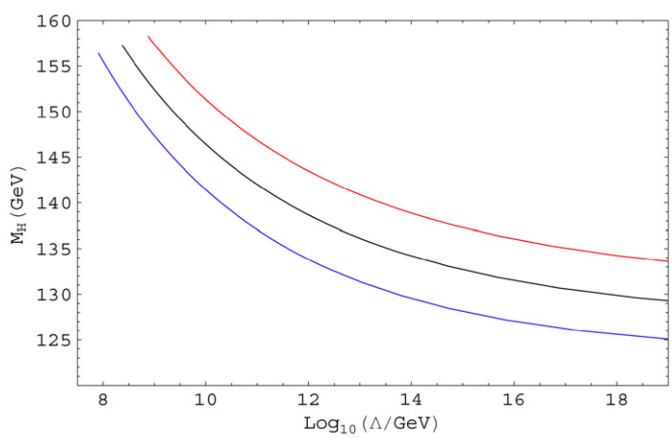

Fig. 4. Plot of Higgs boson mass versus compactification scale, $\mathrm{Log}_{10}(\Lambda/\mathrm{GeV})$, determined as the unification scale of the running top Yukawa and the SU(2) gauge couplings. Three solid lines (in red, black and blue) from top to bottom correspond to top pole masses $M_t = 172.7$, 170.9 and 169.1 GeV, respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

with

$$\mathcal{L}_Y = -g\overline{q_{3L}}H_d b_R \tag{24}$$

with the down-type Higgs defined by Eq. (11), so that the SM bottom Yukawa coupling is given as $y_b = g\cos\beta$. In considering gauge-bottom Yukawa coupling unification we assume that there is some mechanism which generates the required top Yukawa coupling from a brane localized interaction, without significantly affecting the other SM parameters. This scenario may seem technically unnatural but it can be realized.

For our analysis, we employ the two-loop RGE of bottom Yukawa coupling [6],

$$\frac{dy_b}{d\ln\mu} = y_b\left(\frac{1}{16\pi^2}\beta_b^{(1)} + \frac{1}{(16\pi^2)^2}\beta_b^{(2)}\right). \tag{25}$$

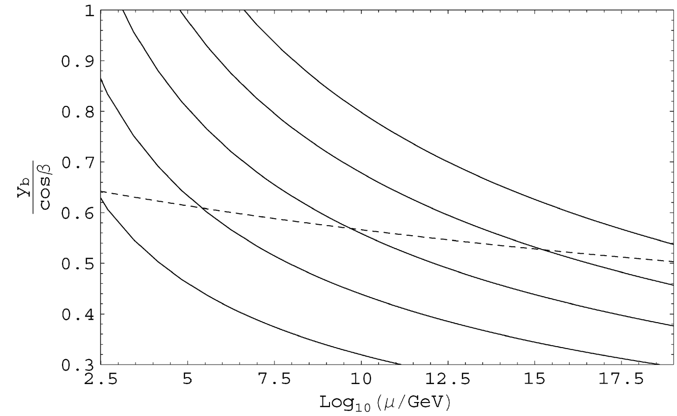

Fig. 5. Plot of the running SU(2) gauge coupling $g(\mu)$ (dashed line) and the running bottom Yukawa coupling (divided by a given $\cos\beta$), $y_b(\mu)/\cos\beta$ (solid line), versus $\mathrm{Log}_{10}(\mu/\mathrm{GeV})$ for a given $\tan\beta$. Solid lines correspond to $\tan\beta = 40$, 55, 70, 85, and 100, respectively, from bottom to top. Here we took the top quark pole mass $M_t = 170.9$ GeV.

Here the one-loop contribution is

$$\beta_b^{(1)} = \frac{3}{2}y_t^2 - \left(\frac{1}{4}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right), \tag{26}$$

while the two-loop contribution is given by

$$\beta_b^{(2)} = -\frac{1}{4}y_t^4 + \left(\frac{91}{80}g_1^2 + \frac{99}{16}g_2^2 + 4g_3^2\right)y_t^2 - \frac{127}{600}g_1^4 - \frac{23}{4}g_2^4 - 108g_3^4 - \frac{27}{20}g_1^2g_2^2 + \frac{31}{15}g_1^2g_3^2 + 9g_2^2g_3^2. \tag{27}$$

In this RGE, we have considered only terms involving gauge couplings and top Yukawa coupling in $\beta_b^{(1)}$ and $\beta_b^{(2)}$, as a good approximation. Solving this RGE with the running bottom quark mass at the Z-pole as $m_b(M_Z) = 3$ GeV [14], for simplicity, we can fix the compactification scale as the point where the relation, $y_b(\Lambda) = g(\Lambda)\cos\beta$, is satisfied, with a given $\tan\beta$ (see Fig. 5). The compactification scale as a function of $\tan\beta$ is depicted in Fig. 6 for input top quark pole masses, $M_t = 169.1$, 170.9 and 172.7 GeV. Fig. 7 shows the Higgs mass as a function of the compactification scale which has been fixed for a given $\tan\beta$. The resultant Higgs boson mass is found in the range 146 GeV $\leqslant m_H \leqslant$ 164 GeV. We have verified that our results for the Higgs boson mass show only a very mild dependence on the few percent uncertainty present in the bottom quark mass.

Finally, it is tempting to simultaneously impose both gauge coupling and gauge-Yukawa unification at the compactification scale. [In general, embedding $SU(3)_w$ into an orbifold GUT does not affect the GHU condition. There are many possibilities to realize gauge and Yukawa coupling unification within orbifold GUTs (see for instance [13]). Since our goal here is to derive a range for the SM Higgs mass from general orbifold GUTs, so we do not specify any specific model in this Letter.] Although the three SM gauge couplings do not meet with a canonical normalization of 5/3 for $U(1)_Y$, a different choice, for example, 4/3, can lead to gauge coupling unification at $M_{\mathrm{GUT}} = 4 \times 10^{16}$ GeV [15,16]. If we identify this gauge coupling unification scale with the compactification scale Λ, the SM Higgs boson mass and $\tan\beta$ can both be

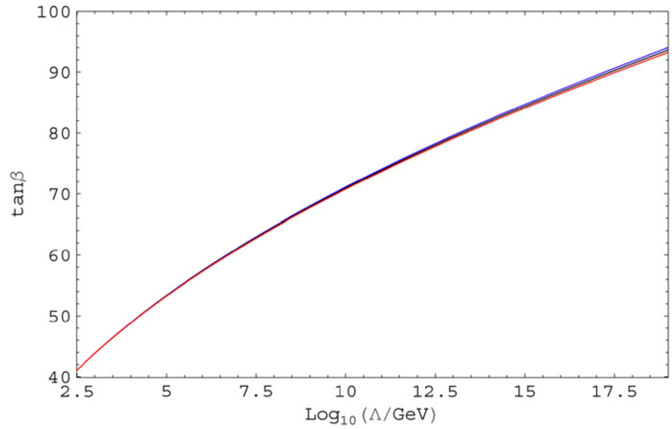

Fig. 6. Plot of $\tan\beta$ versus the compactification scale, $\mathrm{Log}_{10}(\Lambda/\mathrm{GeV})$, determined as the unification scale of the running bottom Yukawa and the SU(2) gauge couplings. Three solid lines (in blue, black and red) from top to bottom correspond to top quark pole masses $M_t = 169.1$, 170.9 and 172.7 GeV, respectively. Three lines are well degenerate because their differences originate from the running of gauge coupling from M_Z to M_t. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

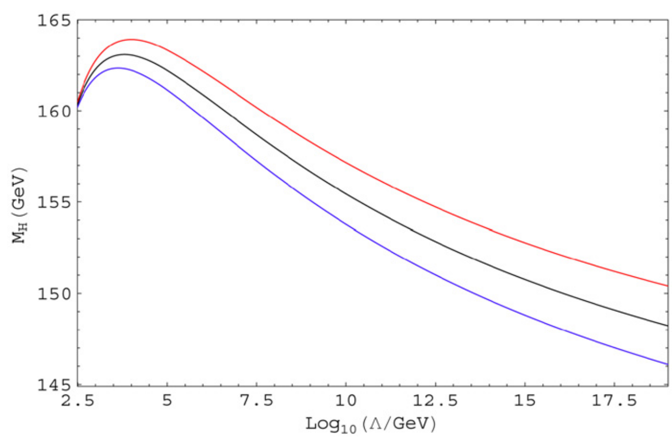

Fig. 7. Plot of Higgs boson mass versus compactification scale, $\mathrm{Log}_{10}(\Lambda/\mathrm{GeV})$, determined as the unification scale of the running bottom Yukawa and the SU(2) gauge couplings. Three solid lines (in red, black and blue) from top to bottom correspond to top pole masses $M_t = 172.7$, 170.9 and 169.1 GeV, respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

fixed. For a top quark pole mass $M_t = 170.9^{+1.8}_{-1.8}$ GeV, we find $m_H = 131^{+4}_{-5}$ GeV and $\tan\beta = 1.32^{+0.09}_{-0.08}$ in the case of gauge-top Yukawa coupling unification, while $m_H = 150^{+2}_{-2}$ GeV and $\tan\beta = 88.2^{-0.3}_{+0.4}$ for gauge-bottom Yukawa coupling unification.

In conclusion, we have considered gauge-Higgs unification models in six-dimensions with T^2/Z_N orbifold compactification, such that up to two SM Higgs doublets emerge as the extra-dimensional components of the higher-dimensional gauge field. The effective quartic Higgs coupling at tree level is determined from higher-dimensional gauge invariance, that should be matched with the one from the low energy effective theory below the compactification scale (the gauge-Higgs condition). In the $N = 2$ case, the model provides two scalar doublets as zero-modes of the extra-dimensional components of the higher-dimensional gauge field. We have assumed that one linear combination (the SM Higgs doublet) survives below the compactification scale, and that the low energy effective theory coincides with the Standard Model. Imposing the gauge-Higgs condition and solving the RGEs in the SM, we have obtained the Higgs boson mass in the range, 114.4 GeV $\leqslant m_H \leqslant$ 164 GeV, for given compactification scale and $\tan\beta$, with the top quark pole mass $M_t = 170.9 \pm 1.8$ GeV. The Higgs boson mass in 5D GHU models with S^1/Z_2 orbifold compactification and for 6D GHU models compactified on T^2/Z_N, with $N \neq 2$, can be obtained by setting $\tan\beta = 1$ and $\tan\beta = 0/\infty$, respectively. Interestingly, these two limits fix the lower and upper boundaries of the resultant Higgs boson mass window. Therefore, the Higgs boson mass we have found in this Letter is the general result in this class of models. Imposing further conditions such as gauge-Yukawa coupling unification enables us to confine the Higgs boson mass in a much more narrow range.

Finally, we offer some comments concerning non-baryonic dark matter whose presence has been established from various observations of the present universe. Except for the Higgs sector, the gauge-Higgs unification model shares the same structure as the universal extra dimension (UED) model [17,18], in which, due to a conserved KK parity, the lightest KK mode is a plausible dark matter candidate [19]. If the compactification scale is around 1 TeV, KK dark matter as a thermal relic is found to be consistent with the observed dark matter density in the present universe [20]. In our case, for $|\cos(2\beta)| \geqslant 0.71$ (see Eq. (12)), the compactification scale can be somewhat less than 1 TeV with the resultant Higgs boson mass $m_H \geqslant 114.4$ GeV, so that the KK dark matter scenario can be realized. If the compactification scale exceeds the unitarity limit on the mass of cold dark matter as a thermal relic, a superheavy KK dark matter scenario may still be realized through another mechanism, such as its production through inflaton decay.

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