

**SU(2)_L Doublet Vector Dark Matter from Gauge-Higgs
Unification**

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$SU(2)_L$ doublet vector dark matter from gauge-Higgs unificationNobuhito Maru,¹ Nobuchika Okada,² and Satomi Okada³¹*Department of Mathematics and Physics, Osaka City University, Osaka 558-8585, Japan*²*Department of Physics and Astronomy, University of Alabama, Tuscaloosa, Alabama 35487, USA*³*Graduate School of Science and Engineering, Yamagata University, Yamagata 990-8560, Japan*

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A new vector dark matter (DM) scenario in the context of the gauge-Higgs unification (GHU) is proposed. The DM particle is identified with an electric-charge neutral component in an $SU(2)_L$ doublet vector field with the same quantum number as the Standard Model Higgs doublet. Since such an $SU(2)_L$ doublet vector field is incorporated in all models of the GHU scenario, it is always a primary and model-independent candidate for the DM in the scenario. The observed relic density is reproduced through DM pair annihilations into the weak gauge bosons with a TeV-scale DM mass, which is nothing but the compactification scale of extra dimensions. Due to the higher-dimensional gauge structure of the GHU scenario, a pair of the DM particles has no direct coupling with a single Z-boson/Higgs boson, so that the DM particle evades the severe constraint from the current direct DM search experiments.

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The existence of the dark matter (DM) in our Universe is undoubtedly believed from various cosmological observations, and the DM particle is one of the key ingredients in exploring physics beyond the Standard Model (SM). Identification with the DM particle is still one of the unsolved problems in particle physics and cosmology. In this paper, we consider the DM physics in the gauge-Higgs unification (GHU) scenario [1]. This scenario can provide us with a solution to the gauge hierarchy problem without invoking supersymmetry, where the SM Higgs doublet is identified with an extra spatial component of the gauge field in higher dimensions. It is remarkable that irrespective of the nonrenormalizability of the theory, the GHU scenario predicts various finite physical observables, such as Higgs potential [2,3], $H \rightarrow gg, \gamma\gamma$ [4–6], the anomalous magnetic moment $g - 2$ [7] and the electric dipole moment [8], thanks to the higher-dimensional gauge symmetry.

In this paper, we propose a new possibility that an electric-charge neutral component in an $SU(2)_L$ doublet gauge field is identified with the DM particle, where $SU(2)_L$ is the gauge group of the weak interaction in the SM. The vector DM particle as the lightest Kaluza-Klein (KK) mode of the photon in the universal extra dimension (UED) model [9] or a T -odd partner of the photon in the littlest Higgs model with T -parity [10] has

been extensively studied. However, as far as we know, a vector DM in the $SU(2)_L$ doublet, in general, nonadjoint representation of $SU(2)_L$ has not been commonly studied.¹ Although we may introduce such a particle into the SM, we notice that the $SU(2)_L$ doublet vector field is automatically incorporated into all models of the GHU.

In the GHU scenario, the SM gauge group is extended and embedded into larger gauge groups in higher-dimensional space-time. The SM $SU(2)_L$ doublet Higgs is identified with a spatial component of the higher-dimensional gauge field. Because of this GHU structure, the $SU(2)_L$ doublet gauge field with the same quantum number as the SM Higgs doublet is automatically contained in a coset space of the extended gauge group as a “gauge-Higgs partner” of the Higgs boson. The existence of the $SU(2)_L$ doublet gauge field, as a gauge-Higgs partner of the SM Higgs doublet, is therefore a model-independent feature of the GHU scenario. In order to reproduce the SM gauge group, a nontrivial orbifold boundary condition is imposed in the scenario and, as a result, a vector field of the gauge-Higgs partner to the SM Higgs boson emerges as an odd particle under a Z_2 -symmetry. This is a DM candidate of the scenario. Our main purpose of this paper is to show the basic structure of the GHU scenario based on a simple $SU(3)$ model in five-dimensional (5D) space-time and propose a new vector DM scenario. One of our remarkable results is that due to the higher-dimensional gauge structure of the GHU scenario, a coupling between a pair of the DM particles and a single Z-boson/Higgs boson is absent, and

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¹See Ref. [11] for a model proposal. Collider physics for the $SU(2)_L$ doublet vector field has been investigated in Ref. [12].

as a result, the DM particle evades the severe constraint from the current direct DM detection experiments. Through its pair annihilations into the weak gauge bosons, the observed DM relic density is reproduced with a TeV-scale DM mass, which is nothing but the compactification scale of the fifth dimension. For related DM scenarios in the context of GHU, see Ref. [13].

Let us consider a simple GHU model based on the gauge group $SU(3)$ in a flat 5D Minkowski space with the fifth dimension compactified on an orbifold S^1/Z_2 with radius R of S^1 . Since reproducing the realistic Yukawa couplings for the SM fermions is not within our main scope, we simply follow Ref. [14] for our setup on bulk fermions corresponding to the SM fermions. As is well known, the simple $SU(3)$ model cannot provide a realistic weak mixing angle. Following Ref. [15], we introduce brane localized kinetic terms for the SM gauge bosons and adjust them so as to reproduce the realistic weak mixing angle. The brane localized kinetic terms also play an important role in our DM scenario as will be discussed later.

The boundary conditions should be appropriately assigned to reproduce the SM fields as the zero modes. While a boundary condition corresponding to S^1 is taken to be periodic for all of the bulk SM fields, the Z_2 parity is assigned for gauge fields and fermions in the representation \mathcal{R} by using the parity matrix $P = \text{diag}(-, -, +)$ as

$$\begin{aligned} A_\mu(-y) &= P^\dagger A_\mu(y) P, & A_y(-y) &= -P^\dagger A_y(y) P, \\ \psi(-y) &= \mathcal{R}(P) \gamma^5 \psi(y), \end{aligned} \quad (1)$$

where the subscript μ (y) denotes the fourth- (fifth-) dimensional component. The parity assignment breaks the $SU(3)$ gauge symmetry down to the SM gauge group of $SU(2)_L \times U(1)_Y$. Off-diagonal blocks in A_y , which correspond to an $SU(2)_L$ doublet, have zero modes due to the overall sign in Eq. (1). In fact, these are identified with the SM Higgs doublet [$H = (H^+ H^0)^T$]:

$$A_y^{(0)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ H^- & H^{0*} & 0 \end{pmatrix}. \quad (2)$$

The KK modes of A_y are eaten by KK modes of the SM gauge bosons and become their longitudinal degrees of freedom like the ordinary Higgs mechanism.

The 5D Lagrangian relevant to our DM physics is given by

$$\begin{aligned} \mathcal{L}_{\text{DM}} &= -\frac{1}{2} \text{Tr}[F_{MN} F^{MN}] \\ &\quad - \left(\frac{c_L}{2} \text{Tr}[W_{\mu\nu} W^{\mu\nu}] + \frac{c_Y}{4} \text{Tr}[B_{\mu\nu} B^{\mu\nu}] \right) \\ &\quad \times (\delta(y) + \delta(y - \pi R)), \end{aligned} \quad (3)$$

where M, N and μ, ν are the 5D and 4D indices, respectively. The first term in the right-hand side is a 5D gauge kinetic term of the $SU(3)$ gauge field, while the remaining terms are the brane localized kinetic terms at orbifold fixed points $y = 0$ and πR . Since the bulk $SU(3)$ gauge symmetry is explicitly broken to $SU(2)_L \times U(1)_Y$ at the fixed points, we have introduced the brane kinetic terms for the $SU(2)_L$ and the $U(1)_Y$ gauge bosons, where $W_{\mu\nu}$ and $B_{\mu\nu}$ are their field strengths, respectively. Adjusting suitable values for the parameters, c_L and c_Y , the realistic weak mixing angle can be reproduced in 4D effective theory [15]. We here emphasize that the brane localized kinetic terms for each gauge boson have been set to be exactly the same at $y = 0$ and πR to preserve the KK-parity [16].

Let us now look at the terms in the Lagrangian involving the DM candidate. Following the parity in Eq. (1), the SM gauge fields and the Higgs doublet are embedded in A_μ and A_y , respectively, as zero modes. In the following calculations, it may be convenient to express them explicitly in the matrix form:

$$A_\mu^{(0)} = \frac{1}{2} \begin{pmatrix} W_\mu^3 + \frac{1}{\sqrt{3}} B_\mu & \sqrt{2} W_\mu^+ & 0 \\ \sqrt{2} W_\mu^- & -W_\mu^3 + \frac{1}{\sqrt{3}} B_\mu & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} B_\mu \end{pmatrix}, \quad (4)$$

for the SM gauge fields² and Eq. (2) for the Higgs doublet. The DM candidate is embedded in A_μ as the first KK mode of the gauge-Higgs partner of the Higgs doublet. Its matrix form is given by

$$A_\mu^{(1)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & X_\mu^1 \\ 0 & 0 & X_\mu^2 \\ X_\mu^{1*} & X_\mu^{2*} & 0 \end{pmatrix}. \quad (5)$$

Here, X_μ^2 is an electric-charge neutral component. In the following, it turns out that the DM candidate is the real part of X_μ^2 , which is nothing but the gauge-Higgs partner of the SM Higgs boson.

The mass spectrum of the $SU(2)_L$ doublet vector in 4D effective theory is calculated from a part of the 5D gauge kinetic term $\text{Tr}(F_{\mu\nu} F^{\mu\nu})$. After the electroweak symmetry breaking by a vacuum expectation value (VEV) of $\langle H^0 \rangle = v/\sqrt{2}$, we have the mass spectrum for the electric-charge neutral component X_μ^2 :

²In the presence of the brane localized kinetic terms, the normalization of the SM gauge fields, W_μ^\pm , W_μ^3 , and B_μ , is not yet canonical. Since the manner of coupling of the DM particle with the SM particles is independent of the normalization, we proceed with our calculations with the present normalization.

$$\begin{aligned}
 \int dy \mathcal{L}_{\text{DM}} &\supset - \int dy \text{Tr}[F_{\mu\nu} F^{\mu\nu}] \\
 &\supset - \frac{g^2 v^2}{8} \eta^{\mu\nu} (X_\mu^2 - X_\mu^{2*})(X_\nu^2 - X_\nu^{2*}) \\
 &\quad + \frac{1}{2R^2} \eta^{\mu\nu} (X_\mu^{2*} X_\nu^2) \\
 &= \frac{1}{2} \left(\frac{1}{R}\right)^2 \eta_{\mu\nu} (X_{\text{DM}}^\mu X_{\text{DM}}^\nu) \\
 &\quad + \frac{1}{2} \left(\left(\frac{1}{R}\right)^2 + 4m_W^2 \right) X^\mu X_\mu, \quad (6)
 \end{aligned}$$

where $X_{\text{DM}\mu}$ and X_μ are defined as

$$X_{\text{DM}\mu} \equiv \frac{1}{\sqrt{2}} (X_\mu^2 + X_\mu^{2*}), \quad X_\mu \equiv -\frac{i}{\sqrt{2}} (X_\mu^2 - X_\mu^{2*}). \quad (7)$$

The distinctive feature can be seen as follows. The real part of X_μ^2 (denoted as X_{DM}^μ hereafter) receives no electroweak symmetry breaking mass proportional to the Higgs doublet VEV, while its imaginary part receives it: $m_{\text{DM}}^2 = (\frac{1}{R})^2$, $m_X^2 = (\frac{1}{R})^2 + 4m_W^2$. As a result, X_{DM}^μ is the lightest mode and hence the DM candidate. Note that since the DM particle receives no electroweak symmetry breaking mass, its pair has no coupling with a single Higgs boson.

One may think that the first KK mode of the photon has the mass $1/R$ and can be the DM candidate as in the UED models. In order to see which of the first KK gauge bosons is the lightest one, we must take into account the effects of the brane localized kinetic terms. Such effects on the KK-mode mass spectrum have been analyzed in Ref. [16]. When we take $c_L, c_Y < 0$ in Eq. (3), we can find that the n th KK-mode mass is larger than n/R . In fact, let us consider the eigenvalue equation of Eq. (33) in Ref. [16] with common coefficients for the brane localized kinetic terms ($r_a = r_b$). For $|r_a m_n| \ll 1$, the eigenvalue equation is approximately given by

$$\tan(m_n \pi R) \simeq -r_a, \quad (8)$$

which means that the n th KK-mode mass is larger than n/R for $r_a < 0$.³ Hence, the lightest KK gauge boson is found as X_{DM}^μ .

One may also think that the first KK modes of the SM fermions have the mass of $1/R$. This is not true since the Z_2 -odd bulk mass terms are introduced to obtain the realistic Yukawa couplings except for the top quark [14] and the masses for the first KK modes become heavier by the bulk mass (M) contributions such as $\sqrt{M^2 + (\frac{1}{R} \pm m_f)^2}$, where m_f is a fermion mass generated by the Higgs doublet

³ $|r_a|$ cannot be very large in order to keep 4D effective gauge coupling squared positive.

VEV [17]. One should note that the Z_2 -odd bulk mass terms preserve the KK-parity [18]. For a top quark, the bulk mass must be zero to reproduce the top quark mass. Therefore, one might conclude that the lightest KK particle is the first KK top quark with its mass eigenvalue $1/R - m_t$. However, for more precise evaluation for the KK-mode mass spectrum, we need to take into account the one-loop quantum corrections to the first KK top quark mass. As has been calculated in Ref. [19], these corrections are dominated by QCD effects with a logarithmic divergence, which makes the resultant first KK top quark mass sizably larger than $1/R$. On the other hand, quantum corrections to the KK mode gauge boson masses are found to be finite because of the original 5D gauge invariance [19]. Since the first KK $SU(2)_L$ gauge boson X_μ receives such finite corrections proportional to $SU(2)_L$ and $U(1)_Y$ gauge couplings squared, we find that the first KK top quark is heavier than the first KK mode of X_μ . Therefore, X_{DM}^μ is a unique DM candidate.

We have found that a pair of the vector DM (X_{DM}^μ) has no coupling with a single Higgs boson. Here we show that a triple vector coupling among a DM pair and Z -boson is also absent. If it exists, such a triple vector coupling is included in the three-point self-interaction of the 5D $SU(3)$ gauge boson:

$$\mathcal{L}_{5D} \supset -\frac{1}{2} \text{Tr}[F_{MN} F^{MN}] \supset ig \text{Tr}[(\partial_\mu A_\nu - \partial_\nu A_\mu)[A^\mu, A^\nu]]. \quad (9)$$

By using the explicit forms in Eqs. (4) and (5) in terms of Eq. (7), we find

$$\begin{aligned}
 ig \text{Tr}[(\partial_\mu A_\nu - \partial_\nu A_\mu)[A^\mu, A^\nu]] \\
 \supset \frac{ig}{4} \left[-2\partial_\mu \left(W_\nu^3 + \frac{1}{\sqrt{3}} B_\nu \right) (-X_{\text{DM}}^\mu X^\nu + X_{\text{DM}}^\nu X^\mu) \right. \\
 + 2\partial_\mu X_{\text{DM}\nu} \{ -X^\mu (-W^{3\nu} + \sqrt{3} B^\nu) + X^\nu (-W^{3\mu} + \sqrt{3} B^\mu) \} \\
 \left. + 2\partial_\mu X_\nu \{ X_{\text{DM}}^\mu (-W^{3\nu} + \sqrt{3} B^\nu) - X_{\text{DM}}^\nu (-W^{3\mu} + \sqrt{3} B^\mu) \} \right]. \quad (10)
 \end{aligned}$$

Now we see that the triple vector coupling among a DM pair and Z -boson (which is a linear combination of W_μ^3 and B_μ) is absent in the model.

Finally, we discuss the relic density of the vector DM particle. All couplings of the vector DM with the SM particles are derived from the original 5D Lagrangian, $\mathcal{L}_{5D} \supset -\frac{1}{2} \text{Tr}[F_{MN} F^{MN}]$. Because of the vector nature, there are so many interaction terms in the 4D effective Lagrangian, and the complete analysis for the relic density is very tedious. Thus, we here perform a rough estimate of the DM relic density. In 4D effective theory, we can express the kinetic term for the DM $SU(2)_L$ doublet as

$$\mathcal{L}_{4D} = -\frac{1}{2}(D_\mu\chi_\nu - D_\nu\chi_\mu)^\dagger(D^\mu\chi^\nu - D^\nu\chi^\mu), \quad (11)$$

where $\chi_\mu = (X_\mu^1 X_\mu^2)^T$ is the DM $SU(2)_L$ doublet, and D_μ is the covariant derivative, which is the same as the covariant derivative for the SM Higgs doublet. For our estimate of the DM relic density, let us use the four-point interactions among the DM particles and the W -bosons, which can be extracted as

$$\begin{aligned} \mathcal{L}_{4D} &= -\frac{1}{2}(D_\mu\chi_\nu - D_\nu\chi_\mu)^\dagger(D^\mu\chi^\nu - D^\nu\chi^\mu) \\ &\supset -g^2(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma})X_{\text{DM}\mu}X_{\text{DM}\nu}W_\rho^+W_\sigma^-. \end{aligned} \quad (12)$$

Using these interactions, we calculate the DM pair annihilation cross section in the nonrelativistic limit as

$$\sigma v_{\text{rel}} = \frac{5g^4}{48\pi m_{\text{DM}}}, \quad (13)$$

where v_{rel} is a relative velocity of two annihilating DM particles, and m_{DM} is a DM mass. It is well known that the annihilation cross section of $\sigma v_{\text{rel}} = 1$ pb is required to reproduce the observed DM relic abundance [20], which is achieved by the DM mass of $m_{\text{DM}} \simeq 1.5$ TeV. Here, we have used $g \simeq 0.65$ for the $SU(2)_L$ gauge coupling.

In summary, we have proposed a new vector DM scenario in the context of the GHU, where the DM particle is included in the first KK mode of the $SU(2)_L$ doublet vector field. Because of the orbifold boundary condition, the first KK mode of the $SU(2)_L$ doublet vector field is odd under the Z_2 -parity in 4D effective theory and hence the

lightest component is stable. We have found that the real component of the electric-charge neutral component in the $SU(2)_L$ doublet vector, which is the gauge-Higgs partner of the Higgs boson, is the lightest and hence the DM candidate. Since the $SU(2)_L$ doublet vector is incorporated in any GHU models as the gauge-Higgs partner of the Higgs doublet, our proposal is model independent and the electric charge-neutral component of the $SU(2)_L$ doublet vector is a primary DM candidate in the GHU scenario. Based on a simple $SU(3)$ GHU model in the flat 5D Minkowski space, we have shown the basic structure of the DM scenario. The DM mass is determined by the compactified radius with no contribution from the Higgs doublet VEV. Because of the higher-dimensional gauge structure of the GHU model, a coupling among a DM particle pair and a single Z -boson/Higgs boson is absent in the model. Thus, the vector DM particle evades the severe constraint from the current direct DM detection experiments. The observed relic abundance of the DM particle is obtained through its pair annihilation processes into the weak gauge bosons for a TeV-scale DM mass. This DM mass, which is nothing but the compactification scale of the GHU model, is the natural scale for the GHU scenario to provide a solution to the gauge hierarchy problem.

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