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T. Fukuyama – Ritsumeikan University, Kusatsu
T. Kikuchi – Theory Division, KEK
N. Okada – Theory Division, KEK



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# Solving problems of the 4D minimal SO(10) model in a warped extra dimension

Takeshi Fukuyama,[1,*] Tatsuru Kikuchi,[2,†] and Nobuchika Okada[2,3,‡]

[1]*Department of Physics, Ritsumeikan University, Kusatsu, Shiga, 525-8577, Japan*

[2]*Theory Division, KEK, Oho 1-1, Tsukuba, Ibaraki, 305-0801, Japan*

[3]*Department of Particle and Nuclear Physics, The Graduate University for Advanced Studies, Oho 1-1, Tsukuba, Ibaraki, 305-0801, Japan*

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The minimal renormalizable supersymmetric SO(10) model, an SO(10) framework with only one **10** and one $\overline{\mathbf{126}}$ Higgs multiplets in the Yukawa sector, is attractive because of its high predictive power for the neutrino oscillation data. However, this model suffers from problems related to running of gauge couplings. The gauge coupling unification may be spoiled due to the presence of Higgs multiplets much lighter than the grand unification (GUT) scale. In addition, the gauge couplings blow up around the GUT scale because of the presence of Higgs multiplets of large representations. We consider the minimal SO(10) model in the warped extra dimension and find a possibility to solve these problems.

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## I. INTRODUCTION

A particularly attractive idea for the physics beyond the standard model (SM) is the possibility of grand unified theory (GUT). In the context of GUTs, the diverse set of particle representations and parameters in the SM are unified into a simple and more predictive framework. From this unified picture, one can explain, for example, quantization of electric charges of quarks and leptons. Current experimental data for the standard model gauge coupling constants suggest the successful gauge coupling unification in the minimal supersymmetric standard model (MSSM), and thus strongly support the emergence of a supersymmetric (SUSY) GUT around $M_{\rm GUT} \simeq 2 \times 10^{16}$ GeV.

Among several GUTs, a model based on the gauge group SO(10) is particularly attractive for the following reason. SO(10) is the smallest simple gauge group under which the entire SM matter content of each generation is unified into a single anomaly-free irreducible representation, namely, the spinor **16** representation. In fact, this **16** representation includes right-handed neutrino and SO(10) GUT incorporates the seesaw mechanism [1] that can naturally explain the lightness of the light neutrino masses.

Among several models based on the gauge group SO(10), the so-called minimal SO(10) model has been paid a particular attention, where two Higgs multiplets $\{\mathbf{10} \oplus \overline{\mathbf{126}}\}$ are utilized for the Yukawa couplings with matters $\mathbf{16}_i$ ($i =$ generation) [2]. A remarkable feature of the model is its high predictive power of the neutrino oscillation parameters as well as reproducing charged fermion masses and mixing angles. It has been pointed out that $CP$-phases in the Yukawa sector play an important role to reproduce the neutrino oscillation data [3]. More detailed analysis incorporating the renormalization group (RG) effects in the context of MSSM [4] has explicitly shown that the model is consistent with the neutrino oscillation data.

However, after KamLAND data [5] was released, the results in Ref. [4] were found to be deviated by $3\sigma$ from the observations. Afterward this minimal SO(10) was modified by many authors, using the so-called type-II seesaw mechanism [6] and/or considering a **120** Higgs coupling to the matter in addition to the $\overline{\mathbf{126}}$ Higgs [7]. Based on an elaborate input data scan [8,9] it has been shown that the minimal SO(10) is essentially consistent with low energy data of fermion masses and mixing angles.

On the other hand, it has been long expected to construct a concrete Higgs sector of the minimal SO(10) model. A simplest and renormalizable Higgs superpotential was constructed explicitly and the patterns of the SO(10) gauge symmetry breaking to the standard model one was shown [10–13]. This construction gives some constraints among the vacuum expectation values (VEVs) of several Higgs multiplets, which give rise to a trouble in the gauge coupling unification. The trouble comes from the fact that the observed neutrino oscillation data suggests the right-handed neutrino mass around $10^{12-13}$ GeV, which is far below the GUT scale. This intermediate scale is provided by Higgs field VEV, and several Higgs multiplets are expected to have their masses around the intermediate scale and contribute to the running of the gauge couplings. Therefore, the gauge coupling unification at the GUT scale may be spoiled. This fact has been explicitly shown in Ref. [9], where the gauge couplings are not unified anymore and even the SU(2) gauge coupling blows up below the GUT scale. In order to avoid this trouble and keep the successful gauge coupling unification as usual, it is desirable that all Higgs multiplets have masses around the GUT scale, but some Higgs fields develop VEVs at the intermediate scale. More Higgs multiplets and some parameter

*E-mail: fukuyama@se.ritsumei.ac.jp
†E-mail: tatsuru@post.kek.jp
‡E-mail: okadan@post.kek.jp

 

tuning in the Higgs sector are necessary to realize such a situation.

In addition to the issue of the gauge coupling unification, the minimal SO(10) model potentially suffers from the problem that the gauge coupling blows up around the GUT scale. This is because the model includes many Higgs multiplets of higher dimensional representations. In the field theoretical point of view, this fact implies that the GUT scale is a cutoff scale of the model, and a more fundamental description of the minimal SO(10) model would exist above the GUT scale.

In order to solve these problems related to the gauge coupling running, we can consider two possibilities. One is to replace Higgs fields of large representations into smaller ones and to provide Yukawa couplings in the original minimal SO(10) model as higher dimensional operators [14]. In this way, we can keep the gauge couplings in the perturbative regime until the Planck scale or the string scale. The other possibility is what we explore in this paper: to provide the GUT scale as the cutoff scale of effective field theory in a natural way. For our purpose, we embed the minimal SO(10) model into a warped extra dimension model [15]. In this scenario, the warped metric give rise to an effective cutoff in 4-dimensional effective theory, which is warped down to a low scale from the fundamental mass scale of the original model (a higher dimensional Planck scale). We choose appropriate model parameters so as to realize the effective cutoff scale as the GUT scale. Furthermore, in the context of a warped extra dimension we can propose a simple setup that naturally generates right-handed neutrino masses at intermediate scale even with Higgs field VEVs at the GUT scale. Thus, the gauge coupling unification remains as usual in the MSSM.

This paper is organized as follows: In the next section, we give a brief review of the minimal SUSY SO(10) model, and claim the problems related to the running of the gauge couplings. In Sec. III, we construct a minimal SO(10) model in the contest of the warped extra dimension and propose a simple setup that can solve the problems. The last section is devoted to summary.

## II. MINIMAL SUPERSYMMETRIC SO(10) MODEL

We begin by giving a brief review on the minimal SUSY SO(10) model and show the GUT relation among fermion mass matrices. Even when we concentrate our discussion on the issue how to reproduce the realistic fermion mass matrices in the SO(10) model, there are lots of possibilities for introduction of Higgs multiplets. The minimal supersymmetric SO(10) model is the one where only one **10** and one $\overline{\mathbf{126}}$ Higgs multiplets have Yukawa couplings with **16** matter multiplets such as

$$W = Y_{10}^{ij}\mathbf{16}_i\mathbf{10}_H\mathbf{16}_j + Y_{126}^{ij}\mathbf{16}_i\overline{\mathbf{126}}_H\mathbf{16}_j, \qquad (1)$$

where $\mathbf{16}_i$ is the matter multiplet of the $i$-th generation, $\mathbf{10}_H$ and $\overline{\mathbf{126}}_H$ are the Higgs multiplet of **10** and $\overline{\mathbf{126}}$ representations under SO(10), respectively. Note that, by virtue of the gauge symmetry, the Yukawa couplings, $Y_{10}$ and $Y_{126}$, are complex symmetric $3 \times 3$ matrices. We assume some appropriate Higgs multiplets, whose vacuum expectation values (VEVs) correctly break the SO(10) GUT gauge symmetry into the standard model one at the GUT scale, $M_{\rm GUT} = 2 \times 10^{16}$ GeV. Suppose the Pati-Salam subgroup, $G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$, at the intermediate breaking stage. Under this symmetry, the above Higgs multiplets are decomposed as $\mathbf{10} \rightarrow (\mathbf{6}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{2}, \mathbf{2})$ and $\overline{\mathbf{126}} \rightarrow (\mathbf{6}, \mathbf{1}, \mathbf{1}) + (\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1}) + (\mathbf{10}, \mathbf{1}, \mathbf{3}) + (\mathbf{15}, \mathbf{2}, \mathbf{2})$, while $\mathbf{16} \rightarrow (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$. Breaking down to the standard model gauge group, $SU(4)_c \times SU(2)_R \rightarrow SU(3)_c \times U(1)_Y$, is accomplished by nonzero VEV of the $(\mathbf{10}, \mathbf{1}, \mathbf{3})$ Higgs multiplet. Note that Majorana masses for the right-handed neutrinos are also generated by this VEV through the Yukawa coupling $Y_{126}$ in Eq. (1). In general, the $SU(2)_L$ triplet Higgs in $(\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1}) \subset \overline{\mathbf{126}}$ would obtain the VEV induced through the electroweak symmetry breaking and may play a crucial role of the light Majorana neutrino mass matrix. This model is called the type-II seesaw model, and we include this possibility in the following.

After the symmetry breaking, we find two pair of Higgs doublets in the same representation as the pair in the MSSM. One pair comes from $(\mathbf{1}, \mathbf{2}, \mathbf{2}) \subset \mathbf{10}$ and the other comes from $(\mathbf{15}, \mathbf{2}, \mathbf{2}) \subset \overline{\mathbf{126}}$. Using these two pairs of the Higgs doublets, the Yukawa couplings of Eq. (1) are rewritten as

$$\begin{aligned} W_Y = {} & (U^c)_i(Y_{10}^{ij}H_{10}^u + Y_{126}^{ij}H_{126}^u)Q \\ & + (D^c)_i(Y_{10}^{ij}H_{10}^d + Y_{126}^{ij}H_{126}^d)Q_j \\ & + (N^c)_i(Y_{10}^{ij}H_{10}^u - 3Y_{126}^{ij}H_{126}^u)L_j \\ & + (E^c)_i(Y_{10}^{ij}H_{10}^d - 3Y_{126}^{ij}H_{126}^d)L_j + L_i(Y_{126}^{ij}v_T)L_j \\ & + (N^c)_i(Y_{126}^{ij}v_R)(N^c)_j, \end{aligned} \qquad (2)$$

where $U^c$, $D^c$, $N^c$, and $E^c$ are the right-handed $SU(2)_L$ singlet quark and lepton superfields, $Q$ and $L$ are the left-handed $SU(2)_L$ doublet quark and lepton superfields, $H_{10}^{u,d}$ and $H_{126}^{u,d}$ are up-type and down-type Higgs doublet superfields originated from **10** and $\overline{\mathbf{126}}$, respectively, and the last two terms are the Majorana mass term of the left-handed and the right-handed neutrinos, respectively, developed by the VEV of the $(\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1})$ Higgs ($v_T$) and the $(\mathbf{10}, \mathbf{1}, \mathbf{3})$ Higgs ($v_R$). The factor $-3$ in the lepton sector is the Clebsch-Gordan (CG) coefficient.

In order to keep the successful gauge coupling unification, suppose that one pair of Higgs doublets given by a linear combination $H_{10}^{u,d}$ and $H_{126}^{u,d}$ is light while the other pair is heavy ($\simeq M_{\rm GUT}$). The light Higgs doublets are identified as the MSSM Higgs doublets ($H_u$ and $H_d$) and given by

$$H_u = \tilde{\alpha}_u H_{10}^u + \tilde{\beta}_u H_{126}^u, \qquad H_d = \tilde{\alpha}_d H_{10}^d + \tilde{\beta}_d H_{126}^d, \tag{3}$$

where $\tilde{\alpha}_{u,d}$ and $\tilde{\beta}_{u,d}$ denote elements of the unitary matrix which rotate the flavor basis in the original model into the (SUSY) mass eigenstates. Omitting the heavy Higgs mass eigenstates, the low energy superpotential is described by only the light Higgs doublets $H_u$ and $H_d$ such that

$$\begin{aligned} W_Y = &(U^c)_i(\alpha^u Y_{10}^{ij} + \beta^u Y_{126}^{ij})H_u Q_j \\ &+ (D^c)_i(\alpha^d Y_{10}^{ij} + \beta^d Y_{126}^{ij})H_d Q_j \\ &+ (N^c)_i(\alpha^u Y_{10}^{ij} - 3\beta^u Y_{126}^{ij})H_u L_j \\ &+ (E^c)_i(\alpha^d Y_{10}^{ij} - 3\beta^d Y_{126}^{ij})H_d L_j + L_i(Y_{126}^{ij} v_T)L_j \\ &+ (N^c)_i(Y_{126}^{ij} v_R)(N^c)_j, \end{aligned} \tag{4}$$

where the formulas of the inverse unitary transformation of Eq. (3), $H_{10}^{u,d} = \alpha^{u,d} H_{u,d} + \cdots$ and $H_{126}^{u,d} = \beta^{u,d} H_{u,d} + \cdots$, have been used.

Providing the Higgs VEVs, $H_u = v \sin\beta$ and $H_d = v \cos\beta$ with $v = 174.1$ [GeV], the quark and lepton mass matrices can be read off as

$$\begin{aligned} M_u &= c_{10} M_{10} + c_{126} M_{126}, \qquad M_d = M_{10} + M_{126}, \\ M_D &= c_{10} M_{10} - 3c_{126} M_{126}, \qquad M_e = M_{10} - 3M_{126}, \\ M_T &= c_T M_{126}, \qquad M_R = c_R M_{126}, \end{aligned} \tag{5}$$

where $M_u$, $M_d$, $M_D$, $M_e$, $M_T$, and $M_R$ denote the up-type quark, down-type quark, neutrino Dirac, charged-lepton, left-handed neutrino Majorana, and right-handed neutrino Majorana mass matrices, respectively. Note that all the quark and lepton mass matrices are characterized by only two basic mass matrices, $M_{10}$ and $M_{126}$, and four complex coefficients $c_{10}$, $c_{126}$, $c_T$, and $c_R$, which are defined as $M_{10} = Y_{10}\alpha^d v \cos\beta$, $M_{126} = Y_{126}\beta^d v \cos\beta$, $c_{10} = (\alpha^u/\alpha^d)\tan\beta$, $c_{126} = (\beta^u/\beta^d)\tan\beta$, $c_T = v_T/(\beta^d v \cos\beta)$, and $c_R = v_R/(\beta^d v \cos\beta)$, respectively. These are the mass matrix relations required by the minimal SO(10) model.

Low energy data of six quark masses, three mixing angles, and one phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and three charged-lepton masses are extrapolated to the GUT scale according to the renormalization group equations (RGEs) with given $\tan\beta$, and the data set of quark and lepton mass matrices at the GUT scale is obtained. Using the data, free parameters of fermion mass matrices are determined so as to reproduce the neutrino oscillation data. As usually expected through the seesaw mechanism, the mass scale of the right-handed neutrinos is found to be around $M_R = 10^{12-13}$ GeV.

Note that in the minimal SO(10) model, $Y_{126}$ is related to other fermion mass matrices and determined so as to reproduce the fermion mass matrix data. Accordingly, the Higgs VEV, $v_R$, is determined so as to provide the correct scale for the right-handed neutrino masses, which is found to be $v_R \simeq 10^{14}$ GeV (see, for example, the second paper in Ref. [7] for the explicit presentations of the neutrino Dirac Yukawa matrix and the right-handed neutrino mass matrix). This intermediate scale gives rise to the problem on the gauge coupling unification discussed in the Introduction.

In addition, as discussed by Chang *et al.* in Ref. [14], when we introduce other Higgs multiples to break the GUT gauge symmetry into the SM one, beta function coefficients of RGEs of the gauge coupling become very large and the gauge coupling quickly blows up around the GUT scale. In the field theoretical point of view, this implies that the minimal SO(10) model should be defined as an effective model with the cutoff around the GUT scale. The discrepancy between the GUT scale and the Planck scale or the string scale, that would be a natural cutoff scale of 4-dimensional field theory, can be understood as a conceptual problem of the minimal SO(10) model.

## III. MINIMAL SO(10) MODEL IN A WARPED EXTRA DIMENSION

We consider a SUSY model in the warped five dimensional brane world scenario [15]. The fifth dimension is compactified on the orbifold $S^1/Z_2$ with two branes, ultraviolet (UV) and infrared (IR) branes, sitting on each orbifold fixed point. With an appropriate tuning for cosmological constants in the bulk and on the branes, we obtain the warped metric [15],

$$ds^2 = e^{-2kr_c|y|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 dy^2, \tag{6}$$

for $-\pi \le y \le \pi$, where $k$ is the AdS curvature, and $r_c$ and $y$ are the radius and the angle of $S^1$, respectively.

By the compactification on the orbifold, $N = 1$ SUSY of the five dimensional theory, which corresponds to $N = 2$ SUSY in the four dimensional point of view, is broken down to four dimensional $N = 1$ SUSY. Supersymmetric Lagrangian of this system can be described in terms of the superfield formalism of four dimensional $N = 1$ SUSY theories [16–18]. Now we consider the minimal SUSY SO(10) model in this warped geometry. There are lots of possibilities to construct such a model, where some fields reside in the bulk and some reside on the UV or the IR brane. The most important feature of the warped extra dimension model is that the mass scale of the IR brane is warped down to a low scale by the warp factor [15], $\omega = e^{-kr_c\pi}$, in four dimensional effective theory. For simplicity, we take the cutoff of the original five dimensional theory and the AdS curvature as $M_5 \simeq k \simeq M_P = 2.4 \times 10^{18}$ GeV, the four dimensional (reduced) Planck mass, and so we obtain the effective cutoff scale as $\Lambda_{\rm IR} = \omega M_P$ in effective four dimensional theory. Now let us take the warp factor so as for the GUT scale to be the effective cutoff scale $M_{\rm GUT} = \Lambda_{\rm IR} = \omega M_P$, namely $\omega \simeq 0.01$. As a result, we can realize, as four dimensional effective theory, the minimal SUSY SO(10) model with the effective cutoff at the GUT scale.

Before going to a concrete setup of the minimal SO(10) model in the warped extra dimension, let us see the Lagrangian for the hypermultiplet in the bulk,

$$\mathcal{L} = \int dy \bigg\{ \int d^4\theta r_c e^{-2kr_c|y|}(H^\dagger e^{-V} H + H^c e^V H^{c\dagger}) + \int d^2\theta e^{-3kr_c|y|} H^c \bigg[ \partial_y - \bigg(\frac{3}{2} - c\bigg) kr_c \epsilon(y) - \frac{\chi}{\sqrt{2}} \bigg] H + \text{h.c.} \bigg\}, \tag{7}$$

where $c$ is a dimensionless parameter, $\epsilon(y) = y/|y|$ is the step function, $H$, $H^c$ is the hypermultiplet charged under some gauge group, and

$$V = -\theta\sigma^\mu \bar{\theta} A_\mu - i\bar{\theta}^2 \theta \lambda_1 + i\theta^2 \bar{\theta} \bar{\lambda}_1 + \frac{1}{2}\theta^2 \bar{\theta}^2 D,$$
$$\chi = \frac{1}{\sqrt{2}}(\Sigma + iA_5) + \sqrt{2}\theta\lambda_2 + \theta^2 F, \tag{8}$$

are the vector multiplet and the adjoint chiral multiplets, which form a $N = 2$ SUSY gauge multiplet. $Z_2$ parity for $H$ and $V$ is assigned as even, while odd for $H^c$ and $\chi$.

When the gauge symmetry is broken down, it is generally possible that the adjoint chiral multiplet develops its VEV [19]. Since its $Z_2$ parity is odd, the VEV has to take the form,

$$\langle \Sigma \rangle = 2\alpha k r_c \epsilon(y), \tag{9}$$

where the VEV has been parametrized by a parameter $\alpha$. In this case, the zero mode wave function of $H$ satisfies the following equation of motion:

$$[\partial_y - (\tfrac{3}{2} - c + \alpha) kr_c \epsilon(y)] H = 0 \tag{10}$$

which yields

$$H = \frac{1}{\sqrt{N}} e^{(3/2 - c + \alpha) kr_c |y|} h(x^\mu), \tag{11}$$

where $h(x^\mu)$ is the chiral multiplet in four dimensions. Here, $N$ is a normalization constant by which the kinetic term is canonically normalized,

$$\frac{1}{N} = \frac{(1 - 2c + 2\alpha)k}{e^{(1-2c+2\alpha)kr_c\pi} - 1}. \tag{12}$$

Hence, at $y = \pi$, the wave function becomes

$$H(y = \pi) \simeq \sqrt{(1 - 2c + 2\alpha)k}\,\omega^{-1} h(x^\mu) \tag{13}$$

if $e^{(1/2-c+\alpha)kr_c\pi} \gg 1$, while

$$H(y = \pi) \simeq \sqrt{-(1 - 2c + 2\alpha)k}\,\omega^{-1} e^{(1/2-c+\alpha)kr_c\pi} h(x^\mu) \tag{14}$$

for $e^{(1/2-c+\alpha)kr_c\pi} \ll 1$.

The Lagrangian for chiral multiplets on the IR brane is given by

$$\mathcal{L}_{\text{IR}} = \int d^4\theta \omega^\dagger \omega \Phi^\dagger \Phi + \bigg[ \int d^2\theta \omega^3 W(\Phi) + \text{h.c.} \bigg], \tag{15}$$

where we have omitted the gauge interaction part for simplicity. If it is allowed by the gauge invariance, we can write the interaction term between fields in the bulk and on the IR brane,

$$\mathcal{L}_{\text{int}} = \int d^2\theta \omega^3 \frac{Y}{\sqrt{M_5}} \Phi^2 H(y = \pi) + \text{h.c.}, \tag{16}$$

where $Y$ is a Yukawa coupling constant, and $M_5$ is the five dimensional Planck mass (we take $M_5 \sim M_P$ as mentioned above, for simplicity). Rescaling the brane field $\Phi \to \Phi/\omega$ to get the canonically normalized kinetic term and substituting the zero-mode wave function of the bulk fields, we obtain Yukawa coupling constant in effective four dimensional theory as

$$Y_{4\text{D}} \sim Y \tag{17}$$

if $e^{(1/2-c+\alpha)kr_c\pi} \gg 1$, while

$$Y_{4\text{D}} \sim Y \times e^{(1/2-c+\alpha)kr_c\pi} \ll Y, \tag{18}$$

for $e^{(1/2-c+\alpha)kr_c\pi} \ll 1$. In the latter case, we obtain a suppression factor since $H$ is localized around the UV brane.

Now we give a simple setup of the minimal SO(10) model in the warped extra dimension. We put all **16** matter multiplets on the IR ($y = \pi$) brane, while the Higgs multiplets **10** and $\overline{\mathbf{126}}$ are assumed to live in the bulk. In Eq. (16), replacing the brane field into the matter multiplets and the bulk field into the Higgs multiplets, we obtain Yukawa couplings in the minimal SO(10) model. The Lagrangian for the bulk Higgs multiplets is given in the same form as Eq. (7), where $\chi$ is the SO(10) adjoint chiral multiplet, **45**. As discussed above, since the SO(10) gauge group is broken down to the SM one, some components in $\chi$ which is singlet under the SM gauge group can in general develop VEVs. Here we consider a possibility that the $\text{U}(1)_X$ component in the adjoint $\chi = \mathbf{45}$ under the decomposition $\text{SO}(10) \supset \text{SU}(5) \times \text{U}(1)_X$ has a nonzero VEV,[1]

$$\mathbf{45} = \mathbf{1}_0 \oplus \mathbf{10}_{+4} \oplus \overline{\mathbf{10}}_{-4} \oplus \mathbf{24}_0.$$

The **10** Higgs multiplet and the $\overline{\mathbf{126}}$ Higgs multiplet are

[1]Since $\chi$ has an odd $Z_2$ parity, its nonzero VEV leads to the Fayet-Iliopoulos D-terms localized on both the UV and IR branes [20], which should be canceled to preserve SUSY. For this purpose, we have to introduce new fields on both branes by which the D-terms are compensated. If such fields are in the same representations as matters or Higgs fields like **16** or $\overline{\mathbf{126}}$, we would need to impose some global symmetry to distinguish them.

decomposed under $SU(5) \times U(1)_X$ as

$$\mathbf{10} = \mathbf{5}_{+2} \oplus \bar{\mathbf{5}}_{-2},$$
$$\overline{\mathbf{126}} = \mathbf{1}_{+10} \oplus \mathbf{5}_{+2} \oplus \overline{\mathbf{10}}_{+6} \oplus \mathbf{15}_{-6} \oplus \overline{\mathbf{45}}_{-2} \oplus \mathbf{50}_{+2}.$$

In this decomposition, the coupling between a bulk Higgs multiplet and the $U(1)_X$ component in $\chi$ is proportional to $U(1)_X$ charge,

$$\mathcal{L}_{\rm int} \supset \frac{1}{2} \int d^2\theta \omega^3 Q_X \langle \Sigma_X \rangle H^c H + \text{h.c.}, \quad (19)$$

and thus each component effectively obtains the different bulk mass term,

$$(\tfrac{3}{2} - c) k r_c + \tfrac{1}{2} Q_X \langle \Sigma_X \rangle, \quad (20)$$

where $Q_X$ is the $U(1)_X$ charge of corresponding Higgs multiplet, and $\Sigma_X$ is the scalar component of the $U(1)_X$ gauge multiplet ($\mathbf{1}_0$). Now we obtain different configurations of the wave functions for these Higgs multiplets. Since the $\mathbf{1}_{+10}$ Higgs has a large $U(1)_X$ charge relative to other Higgs multiplets, we can choose parameters $c$ and $\langle \Sigma_X \rangle$ so that Higgs doublets are mostly localized around the IR brane while the $\mathbf{1}_{+10}$ Higgs is localized around the UV brane. For example, the parameter choice, $c = -7/2$ for both $\mathbf{10}$ and $\overline{\mathbf{126}}$ Higgs multiplets and $\langle \Sigma_X \rangle = -k r_c$, can realize this situation.

Using the decomposition of matter multiplets,

$$\mathbf{16}^i = \mathbf{1}^i_{-5} \oplus \bar{\mathbf{5}}^i_{+3} \oplus \mathbf{10}^i_{-1},$$

the Yukawa couplings between matters and the $\overline{\mathbf{126}}$ Higgs multiplet on the IR brane are decomposed into

$$\begin{aligned} W_{Y_{126}} = {} & Y_u^{ij} \mathbf{5}_{+2} \mathbf{10}^i_{-1} \mathbf{10}^j_{-1} + Y_d^{ij} \overline{\mathbf{45}}_{-2} \bar{\mathbf{5}}^i_{+3} \mathbf{10}^j_{-1} \\ & + Y_D^{ij} \mathbf{5}_{+2} \mathbf{1}^i_{-5} \bar{\mathbf{5}}^j_{+3} + Y_e^{ij} \overline{\mathbf{45}}_{-2} \bar{\mathbf{5}}^i_{+3} \mathbf{10}^j_{-1} \\ & + Y_{\nu_L}^{ij} \mathbf{15}_{-6} \bar{\mathbf{5}}^i_{+3} \bar{\mathbf{5}}^j_{+3} + Y_{\nu_R}^{ij} \mathbf{1}_{+10} \mathbf{1}^i_{-5} \mathbf{1}^j_{-5}. \end{aligned} \quad (21)$$

Here, all the Yukawa couplings coincide with the original Yukawa coupling $Y_{126}$ up to appropriate CG coefficients. As discussed above, the $\mathbf{1}_{+10}$ Higgs multiplet giving masses for right-handed neutrinos is localized around the UV brane and, therefore, we obtain a suppression factor as in Eq. (18) for the effective Yukawa coupling between the Higgs and right-handed neutrinos. In effective four dimensional description, the GUT mass matrix relation is partly broken down, and the last term in Eq. (4) is replaced into

$$Y_{126}^{ij} v_R \rightarrow Y_{126}^{ij} (\epsilon v_R), \quad (22)$$

where $\epsilon$ denotes the suppression factor. By choosing appropriate parameters so as to give $\epsilon \simeq 10^{-2}$, we can take $v_R \simeq M_{\rm GUT}$ and keep the successful gauge coupling unification in the MSSM. In fact, the above parameter set, $c = -7/2$ and $\langle \Sigma_X \rangle = -k r_c$, leads to $\epsilon = \omega = M_{\rm GUT}/M_P \simeq 10^{-2}$. The other Higgs multiplets are localized around the IR brane, so that there is no suppression factor for other effective Yukawa couplings.

In our setup, all the matters reside on the brane while the Higgs multiplets reside in the bulk. This setup shares the same advantage as the so-called orbifold GUT [21–23]. We can assign even $Z_2$ parity for MSSM doublet Higgs superfields while odd for triplet Higgs superfields, as a result, the proton decay process through dimension five operators is forbidden.

## IV. CONCLUSION

The minimal renormalizable supersymmetric SO(10) model is a simple framework to reproduce current data for fermion masses and flavor mixings with some predictions. However, this model suffers from some problems related to the running of the gauge couplings. To fit the neutrino oscillation data, the mass scale of right-handed neutrinos lies at the intermediate scale. This implies the presence of some Higgs multiplets lighter than the GUT scale. As a result, the gauge coupling unification in the MSSM may be spoiled. In addition, since Higgs multiplets of large representations are introduced in the model, the gauge couplings blow up around the GUT scale. Thus, the minimal SO(10) model would be an effective theory with a cutoff around the GUT scale, far below the Planck scale.

In order to solve these problems, we have considered the minimal SO(10) model in the warped extra dimension. As a simple setup, we have assumed that matter multiplets reside on the IR brane while the Higgs multiplets reside in the bulk. The warped geometry leads to a low scale effective cutoff in effective four dimensional theory, and we fix it at the GUT scale. Therefore, the four dimensional minimal SO(10) model is realized as the effective theory with the GUT scale cutoff.

After the GUT symmetry breaking, the adjoint scalar in the gauge multiplet in five dimensional SUSY can generally develop a VEV, which plays a role of bulk mass for the bulk Higgs multiplets. This bulk mass is proportional to the charge of each Higgs multiplets and cause the difference between wave functions of each Higgs multiplet. We have found the possibility that the singlet Higgs which provides right-handed neutrino with masses is localized around the UV brane and the geometrical suppression factor emerges in Yukawa couplings of the right-handed neutrinos. As a result, we can set the mass scale of the right-handed neutrinos at the intermediate scale nevertheless the singlet Higgs VEV is around the GUT scale. All Higgs multiplets naturally have masses around the GUT scale and the gauge coupling unification in the MSSM remains the same.

Finally, we give some comments. One can easily extend our setup to put some of matter multiplets in the bulk [24–26]. In this case, we may explain the fermion mass hierarchy in terms of the different overlapping of fermion wave functions between different generations. In this paper, we have assumed that the GUT gauge symmetry is successfully broken down to the SM one. There are several possibilities for the GUT symmetry breaking. It is easy to introduce appropriate Higgs multiplets and superpotential so as to break the GUT symmetry on a brane as in four dimensional SO(10) models. We also can introduce an appropriate boundary condition for bulk gauge multiplets to (explicitly) break the GUT symmetry to a subgroup with rank five in total, as in the orbifold GUTs.

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