# Supersymmetric Radius Stabilization in Warped Extra Dimensions

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# Supersymmetric radius stabilization in warped extra dimensions

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We propose a simple model of extra-dimensional radius stabilization in a supersymmetric Randall-Sundrum model. In our model, we introduce only a bulk hypermultiplet and source terms (tadpole terms) on each boundary brane. With an appropriate choice of model parameters, we find that the radius can be stabilized by supersymmetric vacuum conditions. Since the radion mass can be much larger than the gravitino mass and even the original supersymmetry breaking scale, radius stability is ensured even in the presence of supersymmetry breaking. We find a parameter region in which unwanted scalar masses induced by quantum corrections through the bulk hypermultiplet and a bulk gravity multiplet are suppressed and the anomaly mediation contribution dominates.

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## I. INTRODUCTION

Motivated by an alternative solution to the hierarchy problem, much attention has been recently paid to the brane world scenario [1–3]. In this scenario, the hierarchy between the weak scale and the Planck scale is geometrically obtained by the presence of large extra spatial dimensions [1] or warped extra spatial dimensions [2,3] without supersymmetry (SUSY).

There is another motivation to consider the brane world scenario in the context of SUSY breaking mediation in supergravity (SUGRA) as first discussed in [4]. In 4D SUGRA, once SUSY is broken in the hidden sector, SUSY breaking effects can be mediated to the visible sector automatically through the Planck-suppressed SUGRA contact interactions,

$$\int d^4\theta c_{ij}\frac{Z^\dagger Z Q_i^\dagger Q_j}{M_4^2}\to c_{ij}m_{3/2}^2\tilde{Q}_i^\dagger\tilde{Q}_j, \tag{1}$$

where we obtain soft scalar masses of the order of the gravitino mass $m_{3/2}$ for the scalar partners. Here $Z$ is a SUSY breaking chiral superfield with $F_Z\neq 0$, $Q_i$ is the minimal SUSY standard model (MSSM) chiral superfields of $i$th flavor, $\tilde{Q}_i$ is its scalar component, $c_{ij}$ are flavor-dependent constants, and $M_4$ is a 4D Planck scale. Although the soft SUSY breaking masses are severely constrained to be almost flavor diagonal by experiments, there is no symmetry reason for $c_{ij}=\delta_{ij}$ in 4D SUGRA. Therefore the 4D SUGRA model suffers from the so-called SUSY flavor changing neutral current (FCNC) problem. Recently, it was proposed that direct contact terms such as Eq. (1) between the visible and hidden sectors are naturally suppressed if the two sectors are separated from each other along the direction of extra spatial dimensions [4,5]. This is because the higher dimensional locality forbids direct contact term. This scenario is called the "sequestering scenario." In this setup, soft SUSY breaking terms in the visible sector are generated through a superconformal anomaly (anomaly mediation) and the resultant mass spectrum is found to be flavor blind; there is no SUSY FCNC problem [4,6].[1] Thus it is well motivated to consider the SUSY brane world scenario.

In the brane world scenario, there is an important issue called "radius stabilization." In order for the scenario to be phenomenologically viable, the compactification radius should be stabilized. However, in the normal SUSY brane world scenario, the "radion," a scalar field parametrizing the compactification radius, is found to be a modulus field if SUSY is manifest, and the radius is undetermined. Although a nontrivial radion potential emerges once SUSY is broken, such a potential usually destabilizes the radius. While some fields introduced in the bulk may work to stabilize the radius, these new fields might generate new flavor violating soft SUSY breaking terms in the visible sector larger than the anomaly mediation contributions. For the above discussions, see [8] and [12], for example. Unfortunately, this situation seems to be generic in the SUSY brane world scenario. Therefore, when we construct a realistic SUSY brane world model, we have to consider SUSY breaking, its mediation mechanism, and the radius stabilization all together from the beginning. This makes a model construction very hard. We need a simple model which can stabilize the radius independently of the SUSY breaking and its mediation mechanism.

In this paper, we propose a simple model of extra-dimensional radius stabilization in a SUSY Randall-Sundrum model. We introduce only a bulk hypermultiplet and source terms (tadpole terms) on each boundary branes. With appropriate values of the source terms and the mass of a bulk hypermultiplet, we can find a classical SUSY configuration

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[1] If the visible sector is the MSSM, the sleptons are found to be tachyonic. There are many proposals for nonminimal models providing a realistic mass spectrum [7].

 

connecting two branes, and the radius is completely determined by a SUSY vacuum condition. The radion mass can be much larger than the gravitino mass and even the original SUSY breaking scale. We will show that the radion potential does not receive SUSY breaking effects so much, and the radius stability is ensured even with SUSY breaking. Unwanted soft scalar masses induced by quantum corrections through the bulk hypermultiplet and the bulk gravity multiplet are estimated. We find a parameter region in which they are suppressed and the anomaly mediation contribution dominates. Based on our model, we can discuss the radius stabilization problem independently of SUSY breaking and its mediation mechanism, and the original picture of the sequestering scenario can work. This is a remarkable advantage for model buildings in the SUSY brane world scenario.

For related works, see [5,9–13], for example. We give some brief comments on the relations between our model and the models of Goldberger and Wise [9], Arkani-Hamed *et al.* [10], and Goh, Luty, and Ng [13]. Our model may be understood as a SUSY version of [9] in some sense. In both models, a classical configuration of the bulk scalar field (hypermultiplet in our case) connecting two boundary branes stabilizes the radius by adjusting parameters on the boundaries and in the bulk. The radius stability is ensured by SUSY in our model. As will be seen, our model is similar to the model in Ref. [10]. While in [10] radius stabilization is discussed in the global SUSY theory with a massive hypermultiplet in the bulk in a flat space-time background, our model is based on 5D SUGRA with a Randall-Sundrum background. Even if taking a flat limit, our model does not reduce to the model in Ref. [10], since the hypermultiplet in our model becomes massless in this limit. Our model is also similar to the model in Ref [13]. Radius stabilization is realized with SUSY breaking in their model, while in our model it is realized in a supersymmetric way.

This paper is organized as follows. In the next section, we introduce our model and discuss how the radius is stabilized. Then, the radion mass is calculated in Sec. III and it turns out to be very heavy. In Sec. IV, it is shown that the radius stability is ensured even if we take SUSY breaking effects into account. In Sec. V, we estimate unwanted scalar masses induced by quantum corrections through the bulk hypermultiplet and the bulk gravity multiplet. We find a parameter region in order for our model to be phenomenologically viable. Section VI is devoted to summary.

## II. SIMPLE MODEL OF RADIUS STABILIZATION

The starting point of our discussion is the following Lagrangian[2] in a five-dimensional Randall-Sundrum background [2,3], in which the fifth dimension is compactified on an orbifold $S^1/Z_2$,

$$\mathcal{L}_5 = \int d^4\theta \frac{T+T^\dagger}{2} e^{-(T+T^\dagger)\sigma}[-6M_5^3+|H|^2+|H^c|^2]|\phi|^2 + \left[\int d^2\theta \phi^3 e^{-3T\sigma} H\left\{\left[-\partial_y+\left(\frac{3}{2}+c\right)T\sigma'\right]H^c + W_b(y)\right\} + \text{H.c.}\right], \tag{2}$$

where the five-dimensional spacetime metric is given by

$$ds^2 = e^{-2r\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - r^2 dy^2 \quad (\mu,\nu=0,1,2,3), \tag{3}$$

where $r$ is the radius of the fifth dimension, $0 \leqslant y \leqslant \pi$ is the angle on $S^1$, and $\sigma(y)=k|y|$, with $k$ being an $\text{AdS}_5$ curvature scale. The prime denotes differentiation with respect to $y$, $T$ is a radion chiral multiplet whose real part of the scalar component gives the radius $r$, $\phi = 1+\theta^2 F_\phi$ is a compensating multiplet, $H$ and $H^c$ are hypermultiplet components in terms of superfield notation in $N=1$ SUSY in four dimensions [15,16], and the $Z_2$ parity for $H$ and $H^c$ is defined as even and odd, respectively. $W_b \equiv J_0\delta(y) - J_\pi\delta(y-\pi)$, where $J_{0,\pi}$ are constant source terms on each boundary brane at $y=0,\pi$. Rescaling

$$(H,H^c) \to \frac{1}{\omega}(H,H^c), \quad \omega \equiv \phi e^{-T\sigma}, \tag{4}$$

we obtain a more convenient form such as

$$\mathcal{L}_5 \to \int d^4\theta\left[-3M_5^3(T+T^\dagger)|\omega|^2 + \frac{T+T^\dagger}{2}(|H|^2+|H^c|^2)\right] + \left[\int d^2\theta\, \omega H\left\{-\partial_y H^c + \left(c+\frac{1}{2}\right)T\sigma' H^c + \omega W_b\right\} + \text{H.c.}\right]. \tag{5}$$

Supersymmetric configurations are easily obtained from the F-flatness conditions[3]

$$0 = -\partial_y H^c + \left(c+\frac{1}{2}\right)T\sigma' H^c + e^{-T\sigma}W_b, \tag{6}$$

$$0 = \partial_y H + \left(c-\frac{1}{2}\right)T\sigma' H. \tag{7}$$

[2]This Lagrangian is the one originated from linearized supergravity (see, for example, Ref. [14]). Considering that nonlinear terms in a full five-dimensional supergravity are suppressed by the Planck scale $M_5$ and, as will be seen later, we can take the parameters in our model being much smaller than the Planck scale $M_5$, we can expect their effects negligible. Therefore, the Lagrangian is a good starting point of our arguments.

[3]It is well known that SUSY vacua in global SUSY theory are also SUSY vacua in supergravity if the vaccuum expectation value (VEV) of the superpotential vanishes at the minimum [17]. This fact can be shown in an elegant way by use of a superconformal framework.

It is useful to parametrize the $Z_2$-odd field as $H^c(y)=\varepsilon(y)\tilde{H}^c(y)$ with a step function $\varepsilon(y)=-1,+1$ for $y<0,0<y$ and a regular function $\tilde{H}^c(y)$. Except the boundary points $y=0,\pi$, the solutions can be easily found as

$$H(y)=C_H e^{(1/2-c)T\sigma},\quad \tilde{H}^c(y)=C_{\tilde{H}^c}e^{(c+1/2)T\sigma}, \tag{8}$$

with integration constants $C_H$ and $C_{\tilde{H}^c}$. The source terms on each boundaries lead to the boundary conditions for $\tilde{H}^c$ such as

$$\tilde{H}^c(0)=\frac{J_0}{2},\quad \tilde{H}^c(\pi)=\frac{J_\pi}{2}e^{-Tk\pi}. \tag{9}$$

As a result, we obtain a SUSY vacuum condition of the form

$$J_0-J_\pi e^{-(3/2+c)Tk\pi}=0. \tag{10}$$

Thus, the radius is determined with appropriate values of $J_{0,\pi}$ and the bulk hypermultiplet mass $c$. This is the main point of this paper.

## III. FOUR-DIMENSIONAL EFFECTIVE ACTION AND RADION MASS

It is convenient to describe our model in the form of 4D effective theory with only a light hypermultiplet. Substituting the light mode wave functions for the hypermultiplet,

$$H(x,y)=h(x)e^{(1/2-c)T\sigma}, \tag{11}$$

$$H^c(x,y)=h^c(x)\varepsilon(y)e^{(c+1/2)T\sigma}, \tag{12}$$

into Eq. (5) and performing $y$ integration, we obtain the effective Kähler potential part

$$\int d^4\theta\mathcal{K}_{\rm eff}=\int d^4\theta[f(T,T^\dagger)|\phi|^2+K(T,T^\dagger)|h|^2 +K^c(T,T^\dagger)|h^c|^2], \tag{13}$$

where

$$f(T,T^\dagger)=-\frac{3M_5^3}{k}[1-e^{-(T+T^\dagger)k\pi}],$$

$$K(T,T^\dagger)=\frac{e^{(1/2-c)(T+T^\dagger)k\pi}-1}{(1-2c)k},$$

$$K^c(T,T^\dagger)=\frac{e^{(1/2+c)(T+T^\dagger)k\pi}-1}{(1+2c)k}, \tag{14}$$

and the effective superpotential part

$$\int d^2\theta\phi^2W(h,T)=\int d^2\theta\phi^2h[J_0-J_\pi e^{-(c+3/2)Tk\pi}]. \tag{15}$$

The SUSY vacuum condition $\partial W/\partial h=0$ leads to the same condition as Eq. (10) as it should. It is somewhat complicated but straightforward to calculate the scalar potential and find the potential minimum at $T=T_0$ satisfying Eq. (10), $h=0$, and arbitrary $h^c$. Since the effective superpotential is independent of $h^c$, $h^c$ is left undetermined. At the point $h=0$, the radion potential is found to be

$$V_{\rm radion}=K(T,T^\dagger)^{-1}\left|\frac{\partial W(h,T)}{\partial h}\right|^2 =\frac{(1-2c)k}{e^{(1/2-c)(T+T^\dagger)k\pi}-1}|J_0-J_\pi e^{-(c+3/2)Tk\pi}|^2. \tag{16}$$

One can explicitly see that the potential minimum is given by the SUSY condition (10). Note that $T\to\infty$ also gives the potential minimum. However, this originates from the singularity of the Kähler potential $K(T,T^\dagger)$, and thus this vacuum is not well defined. It is interesting to take the flat limit $k\to0$. The scalar potential reduces to a runaway potential $V_4\sim1/(T+T^\dagger)$; namely, the radius is not stabilized. This means that a warped background metric is crucial for radius stabilization.

Now we calculate the radion mass. Considering canonical normalization of the radion kinetic term, we can estimate a radion mass such as

$$m^2_{\rm radion}\sim\left(\frac{\partial^2f(T,T^\dagger)}{\partial T^\dagger\partial T}\right)^{-1}\frac{\partial^2V_{\rm radion}}{\partial T^\dagger\partial T}\Bigg|_{T=T_0} =\frac{(1-2c)}{e^{(1/2-c)(T+T^\dagger)k\pi}-1}\times\left(\frac{\left(\frac{3}{2}+c\right)^2|J_\pi|^2}{3M_5^3}\right)k^2e^{-(1/2+c)(T+T^\dagger)k\pi}\Big|_{T=T_0} >0. \tag{17}$$

Note that the radion mass squared is always positive irrespective of the value of $c$. This means that the radius is stabilized and the configuration under consideration is stable. As an example, if we take[4] $c=\frac{1}{2}$, $e^{-T_0k\pi}\sim10^{-2}$, $J_\pi\sim(0.1\times M_5)^{3/2}$, and $k\sim0.1\times M_5$, we obtain the radion mass

$$m^2_{\rm radion}\sim(10^{-5}\times M_4)^2\gg m^2_{3/2},F_{\rm hidden}, \tag{18}$$

which is much larger than the gravitino mass ($\sim$10 TeV) in the anomaly mediation scenario and the original SUSY breaking $F$-term scale $F_{\rm hidden}\sim m_{3/2}M_4$ in the hidden sector. This fact implies that SUSY breaking effects have little affect on the radion potential and the radius is not destabilized even in the presence of SUSY breaking. In the next section, we will check this expectation in more detail.

[4]In our model, the gauge hierarchy problem is solved by SUSY, so that it does not need to take $e^{-Tk\pi}\simeq10^{-16}$ as in the original Randall-Sundrum model.

### IV. STABILITY OF RADIUS UNDER SUSY BREAKING EFFECTS

Suppose that the hidden sector fields and visible sector fields reside on the branes at the boundaries $y=0$ and $y=\pi$, respectively. In this setup, the hidden sector fields couple only to the compensating multiplet and the other fields can be regarded as the visible sector fields. Once SUSY is broken in the hidden sector, the SUSY breaking effects emerge in the visible sector only through the nonvanishing $F_\phi$, and we can treat the compensating multiplet as a spurion. In order to prove the stability of the radius in the presence of the SUSY breaking effects, we have to solve equations of motion for $H,H^c$ with nonvanishing $F_\phi$. However, it is hard to solve these complicated equations. Instead of solving them, we prove the radius stability in the effective 4D theory as an approximation since the effect of small $F_\phi$ is important only for light fields.

With the compensating multiplet $\phi=1+\theta^2F_\phi$ as the spurion, the Lagrangian for the auxiliary fields can be read off from Eqs. (13) and (15) such as

$$\begin{aligned}\mathcal{L}_{\rm aux}=&F_T^\dagger[(f_{TT^\dagger}+K^c_{TT^\dagger}|h^c|^2+K_{TT^\dagger}|h|^2)F_T+(K_T^ch^c)^\dagger F^c\\&+(K_Th)^\dagger F+W_T^\dagger+f_{T^\dagger}F_\phi]+F^{c\dagger}[(K_T^ch^c)F_T+K^cF^c]\\&+F^\dagger[(K_Th)F_T+KF+W_h^\dagger]+FW_h+F_TW_T+2(F_\phi W\\&+\mathrm{H.c.})+F_\phi^\dagger f_TF_T+|F_\phi|^2f,\end{aligned}\tag{19}$$

where $f_T$ stands for $\partial f/\partial T$, etc. In the following, we estimate the deviation from the SUSY case in the first order of $F_\phi$. In this approximation, all the $F$ terms and shifts of the field VEVs, $\delta h$ and $\delta T=T-T_0$, around the SUSY vacuum are regarded as $\mathcal{O}(F_\phi)$ variables. Thus we can approximate the equations of motion for auxiliary fields as

$$\begin{aligned}0=&(f_{TT^\dagger}+K^c_{TT^\dagger}|h^c|^2+K_{TT^\dagger}|h|^2)F_T+(K_T^ch^c)^\dagger F^c+(K_Th)^\dagger F\\&+W_T^\dagger+f_{T^\dagger}F_\phi\\\sim&(f_{TT^\dagger}+K^c_{TT^\dagger}|h^c|^2)F_T+(K_T^ch^c)^\dagger F^c+\delta h^\dagger W_{hT}^\dagger+f_{T^\dagger}F_\phi,\end{aligned}\tag{20}$$

$$0=(K_T^ch^c)F_T+K^cF^c,\tag{21}$$

$$0=(K_T\delta h)F_T+KF+W_h^\dagger\sim KF+W_{hT}^\dagger\delta T^\dagger.\tag{22}$$

The solutions $F_T$ and $F$ are given by

$$F_T\sim-\frac{1}{C_T}(\delta h^\dagger W_{hT}^\dagger+f_{T^\dagger}F_\phi)|_{T=T_0,h=0},\tag{23}$$

$$F\sim-\frac{1}{K}W_{hT}^\dagger\delta T^\dagger|_{T=T_0,h=0},\tag{24}$$

where

$$C_T=f_{TT^\dagger}+\left(K^c_{TT^\dagger}-\frac{|K_T^c|^2}{K^c}\right)|h^c|^2.\tag{25}$$

Up to second order, the scalar potential is given by

$$\begin{aligned}\Delta V=&-FW_h-F_TW_T-2(F_\phi W+\mathrm{H.c.})-F_\phi^\dagger f_TF_T-|F_\phi|^2f\\&\sim\frac{1}{K}|W_{hT}\delta T|^2+\frac{1}{C_T}|\delta h^\dagger W_T^\dagger-f_{T^\dagger}F_\phi|^2-|F_\phi|^2f,\end{aligned}\tag{26}$$

and minimization conditions $\partial\Delta V/\partial\delta T=0$ and $\partial\Delta V/\partial\delta h=0$ lead to

$$\delta T\sim0,\tag{27}$$

$$\delta h\sim-\frac{f_T(T_0)}{W_{hT}(T_0)}F_\phi^\dagger\sim\frac{6}{2c+3}\frac{M_5^3F_\phi^\dagger}{J_0k}e^{-(T_0+T_0^\dagger)k\pi}.\tag{28}$$

Therefore, with appropriate values of parameters, the deviations are small enough for our treatment to be consistent. $h^c$ still remains undetermined. Numerical calculations show that the above results give good approximations. Now we have proved the radius stability even with the presence of SUSY breaking.

### V. SCALAR MASSES INDUCED BY BULK FIELDS

We have introduced the hypermultiplet in the bulk for radius stabilization. In general, there is a possibility that the flavor-dependent soft SUSY breaking terms being phenomenologically dangerous are induced through the bulk hypermultiplet, since the ($Z_2$-even) hypermultiplet can directly couple to both the hidden and visible sector fields.

Let us consider the effective Kähler potentials on the hidden brane at $y=0$ and the visible brane at $y=\pi$ such that [in the original basis of Eq. (2)]

$$\mathcal{L}_{\rm hidden}=\int_0^\pi dy\int d^4\theta e^{-(T_0+T_0^\dagger)\sigma}\left[Z^\dagger Z+\frac{H_0^\dagger H_0Z^\dagger Z}{M_5^3}\right]\delta(y),\tag{29}$$

$$\begin{aligned}\mathcal{L}_{\rm visible}=&\int_0^\pi dy\int d^4\theta e^{-(T_0+T_0^\dagger)\sigma}\Bigg[Q_i^\dagger e^{-V}Q_i\\&+c_{ij}\frac{H_0^\dagger H_0Q_i^\dagger Q_j}{M_5^3}\Bigg]\delta(y-\pi).\end{aligned}\tag{30}$$

Here we have assumed minimal Kähler potentials for the first term in each set of brackets for simplicity, $F_\phi=0$ is taken as an approximation suitable for the following discussion, $V$ is a vector superfield in the visible sector, $c_{ij}$ are flavor-dependent constants, and $H_0$ is the massless mode of $H$ given by

$$H_0(x,y)=\frac{1}{N_0}e^{(3/2-c)T_0k|y|}h_0(x),\tag{31}$$

with the normalization constant

$$|N_0|^2 = \frac{e^{(1/2-c)(T_0+T_0^\dagger)k\pi}-1}{(1-2c)k}. \tag{32}$$

We take into account contributions only from the massless mode, since contributions from massive modes are expected to be exponentially suppressed by the Yukawa potential. In terms of canonically normalized $H_0$, $Z$, and $Q_i$, the contact interactions in Eqs. (29) and (30) are rewritten as

$$\mathcal{L}_{\mathrm{hidden}} \supset \frac{1}{|N_0|^2}\int d^4\theta \frac{h_0^\dagger h_0 Z^\dagger Z}{M_5^3}, \tag{33}$$

$$\mathcal{L}_{\mathrm{visible}} \supset c_{ij}\frac{e^{(3/2-c)(T_0+T_0^\dagger)k\pi}}{|N_0|^2}\int d^4\theta \frac{h_0^\dagger h_0 Q_i^\dagger Q_j}{M_5^3}. \tag{34}$$

The scalar masses induced by one-loop corrections through the bulk hypermultiplet are roughly estimated as

$$\Delta\tilde{m}_{ij}^2 \sim \frac{1}{16\pi^2}c_{ij}\frac{e^{(3/2-c)(T_0+T_0^\dagger)k\pi}}{|N_0|^4 M_5^6}|F_Z|^2 V_{\mathrm{eff}}^{-2}$$
$$\sim \frac{1}{16\pi^2}c_{ij}m_{3/2}^2\left(\frac{k}{M_4}\right)^2\left(\frac{1-2c}{e^{(1/2-c)(T_0+T_0^\dagger)k\pi}-1}\right)^2 \times e^{(3/2-c)(T_0+T_0^\dagger)k\pi}. \tag{35}$$

Here we have used the relation $M_4^2 \sim M_5^3/k$ between the 4D Planck and 5D Planck scales, $V_{\mathrm{eff}} \sim 1/k$ is the effective volume of the fifth dimension in the warped background metric by which the loop integral is expected to be cut off physically, and $1/16\pi^2$ is a one-loop suppression factor. For $c > \frac{3}{2}$ or $c < -\frac{1}{2}$, $\Delta\tilde{m}_{ij}^2$ is strongly suppressed. This is because, in the case with $c > \frac{3}{2}$, $H_0$ is localized around the hidden brane and the overlapping with the visible brane is exponentially suppressed. On the other hand, a zero mode $H_0$ is localized around the visible brane and the overlapping with the hidden brane is exponentially suppressed for $c < -\frac{1}{2}$. Even for $c \sim 3/2, -1/2$, the contribution can be suppressed if $k \ll M_4$ or equivalently $k \ll M_5$ through the relation between Planck scales, $M_4^2 \sim M_5^3/k$. This is a natural situation when the bulk gravity is weak enough to be consistent with the classical treatment. For example, if we take $k \sim 0.1 M_5$, then $M_4^2 \simeq 10^3 k^2$, and we find $\Delta\tilde{m}_{ij}^2/\tilde{m}_{\mathrm{AMSB}}^2 \sim 10^{-3} \ll 1$ to be consistent with current experimental results, where $\tilde{m}_{\mathrm{AMSB}}^2 \sim (1/16\pi)^2 m_{3/2}^2$ is the anomaly mediation contribution.

The gravity multiplet always exists in the bulk. Let us consider scalar masses induced through the bulk gravity multiplet loop corrections. This contribution is expected to be flavor blind since the fundamental interactions between fields on the branes and the bulk gravity multiplet are controlled by the SUGRA symmetry. In the flat background case, this contribution is directly calculated in [18] and the result is given by[5]

$$\Delta m_{\mathrm{5D\ flat}}^2 \sim -\frac{1}{16\pi^2}m_{3/2}^2\frac{1}{(M_4 r_0)^2}. \tag{36}$$

Unfortunately, this is the negative contribution and should be suppressed compared with anomaly mediation contributions to avoid tachyonic scalar fields. Although corrections through the gravity multiplet in the warped case have not yet been explicitly calculated, we guess the result from analogy to the flat case. As discussed in [18], the result of Eq. (36) can be obtained from the result of gravitino loop corrections in 4D SUGRA. It is known that in 4D SUGRA the scalar mass squared induced by gravitino one-loop corrections diverges quadratically. The result is given by

$$\Delta m_{\mathrm{4D\ flat}}^2 \sim \frac{1}{16\pi^2}m_{3/2}^2\frac{\Lambda^2}{M_4^2}, \tag{37}$$

where $\Lambda$ is a cutoff scale. The above result in the 5D SUGRA case can be obtained by replacing the cutoff scale $\Lambda$ with the inverse of the extra-dimensional volume $1/r_0$. Recalling the Planck scale matching relations

$$M_4^2 = M_5^3 r_0 \ \ (\text{flat case}), \tag{38}$$

$$M_4^2 \sim \frac{M_5^3}{k} \ \ (\text{warped case}), \tag{39}$$

we naively expect that the scalar mass squared in the 5D warped case is obtained by replacing $1/r_0$ with $k$ such as

$$\Delta\tilde{m}_{\mathrm{5D\ warped}}^2 \sim -\frac{1}{16\pi^2}m_{3/2}^2\left(\frac{k}{M_4}\right)^2. \tag{40}$$

As mentioned before, this negative contribution should be smaller than the anomaly mediation contributions. For example, if we take $k \sim 0.1 M_5$, then $M_4^2 \simeq 10^3 k^2$, and we find $\Delta\tilde{m}_{\mathrm{5D\ warped}}^2/\tilde{m}_{\mathrm{AMSB}}^2 \sim 10^{-3} \ll 1$.

## VI. SUMMARY

We have proposed a simple model of extra-dimensional radius stabilization in the supersymmetric Randall-Sundrum model. With only a bulk hypermultiplet and the source terms on each boundary branes, radius stabilization has been succeeded through SUSY vacuum conditions. The radion mass is found to be large enough; this radius stabilization is ensured even if SUSY breaking effects are taken into account. Our model gives a remarkable advantage for model building in the SUSY brane world, since the radius can be stabilized

[5]Another interesting contribution induced by corrections through the bulk gravity multiplet loop has been calculated in [19,20]. This is found to be proportional to $1/(M_4 r_0)^3$ and is subdominant.

independently of the SUSY breaking and its mediation mechanism. Our model may be applicable to many models. We find a reasonable parameter region where unwanted contributions to the scalar mass squared through bulk multiplets are suppressed enough.

As a bonus of our model, if the bulk hypermultiplet $H$ is identified with the right-handed neutrino and has couplings among Higgs doublets and the left-handed lepton doublet on the visible brane at $y=\pi$, we can naturally obtain a tiny neutrino mass through the mechanism proposed by Grossman and Neubert [21] with the hypermultiplet mass $c>\frac{3}{2}$. In order to obtain a realistic neutrino mass matrix, at least one extra hypermultiplet has to be introduced [21]. Such an extension is straightforward.

Finally, we comment on the dynamical origin of the source terms $J_{0,\pi}$. The source terms on each brane have the same form as the Polonyi model [22]. By introducing some strong coupling gauge theories with some superfields on each brane, we can easily construct a model where the source terms on each brane are dynamically generated through the strong gauge dynamics by the same manner as in [23,24]. Here we give a rough picture of such models. Introduce a SUSY $SU(2)$ gauge theory with four doublets ($V_i$) on a brane ($y=0$ or $\pi$) and consider a superpotential

$$W=\frac{1}{\sqrt{M_5}}[V_iV_j]H\bigg|_{y=0,\pi}. \tag{41}$$

At low energies, the meson composite superfield $[V_iV_j]$ develops a nonzero VEV, $\langle[V_iV_j]\rangle=\Lambda^2$, through the quantum modulus deformation [25], where $\Lambda$ is the dynamical scale of the $SU(2)$ gauge theory. As a result, we obtain $J_{0,\pi}\sim\Lambda^2/\sqrt{M_5}$.


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