

Metastable Vacuum in Spontaneously Broken $N=2$
Supersymmetric Gauge Theory

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Metastable vacuum in spontaneously broken $\mathcal{N} = 2$ supersymmetric gauge theoryMasato Arai,^{1,*} Claus Montonen,^{1,†} Nobuchika Okada,^{2,3,‡} and Shin Sasaki^{1,§}¹*High Energy Physics Division, Department of Physical Sciences, University of Helsinki and Helsinki Institute of Physics, P.O. Box 64, FIN-00014 Helsinki, Finland*²*Department of Physics, University of Maryland, College Park, Maryland 20742, USA*³*Theory Division, KEK, Tsukuba 305-0801, Japan*

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We consider an $\mathcal{N} = 2$ supersymmetric $SU(2) \times U(1)$ gauge theory with $N_f = 2$ massless flavors and a Fayet-Iliopoulos (FI) term. In the presence of the FI term, supersymmetry is spontaneously broken at tree level (on the Coulomb branch), leaving a pseudoflat direction in the classical potential. This vacuum degeneracy is removed once quantum corrections are taken into account. Because of the $SU(2)$ gauge dynamics, the effective potential exhibits a local minimum at the dyon point, where not only supersymmetry but also $U(1)_R$ symmetry is broken, while a supersymmetric vacuum would be realized toward infinity with the runaway behavior of the potential. This local minimum is found to be parametrically long-lived. Interestingly, from a phenomenological point of view, in this metastable vacuum the massive hypermultiplets inherent in the theory play the role of the messenger fields in the gauge mediation scenario, when the standard model gauge group is embedded into their flavor symmetry.

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I. INTRODUCTION

Recently, the possibility that a supersymmetry (SUSY) breaking vacuum is not the global minimum but a local one has been proposed by Intriligator, Seiberg and Shih (ISS) [1]. They have investigated an $\mathcal{N} = 1$ SUSY $SU(N_c)$ gauge theory (SUSY QCD). The number of flavors is taken to be in the range, $N_c + 1 \leq N_f < \frac{3}{2}N_c$, so that this theory is described as the infrared free magnetic dual theory at low energies and can be analyzed perturbatively. The effective theory has the same structure as the O’Raifeartaigh model, and SUSY is broken with the pseudoflat directions parametrized by meson fields in the dual theory. The vacuum degeneracy is removed once one-loop corrections to the Kähler potential are taken into account, and a SUSY breaking minimum shows up at the origin in the moduli space. In addition to this minimum, there exist SUSY vacua in this model, away from the local minimum. It has been shown in Ref. [1] that this false vacuum can be long-lived and thus metastable.

The idea of a metastable SUSY breaking vacuum opens up a lot of theoretical possibilities for SUSY breaking. For such a false vacuum, the conventional argument using the Witten index is not applicable, and it is generally possible for a theory to include a local minimum with broken SUSY even though the Witten index implies the existence of a SUSY vacuum. Similarly, the theorem [2] that a model with spontaneous SUSY breaking should have an R symmetry is not applicable to a model with a SUSY breaking local minimum. This feature is welcome from a

phenomenological point of view, because R symmetry forbids gauginos to obtain masses. The R symmetry should be broken spontaneously or explicitly to realize a phenomenologically viable model. For example, it has been argued [3] that SUSY breaking in a metastable vacuum requires only an approximate R symmetry. Spontaneous R symmetry breaking in gauged O’Raifeartaigh models [3] and modified O’Raifeartaigh models [4,5] with metastable SUSY breaking vacua have been discussed.

Since the paper by ISS, there have been lots of explorations of models with metastable vacua. A metastable SUSY breaking vacuum can simplify the gauge mediation scenario [6] and several simple models have been proposed [7–12]. String theory realizations of the metastable SUSY breaking vacuum have been investigated in Refs. [13–18], where a SUSY breaking scale lower than the string scale can be realized.

In $\mathcal{N} = 1$ SUSY models, a metastable vacuum can be analyzed only in a weak coupling regime in the (effective) theory by perturbative means. Our lack of knowledge about the nonperturbative Kähler potential prevents us from moving away from the weak coupling limit. However, in a class of $\mathcal{N} = 2$ SUSY gauge theories, we can analyze the vacuum structure of a model beyond perturbation theory as first demonstrated by Seiberg and Witten [19,20], using the properties of holomorphy and duality. In Refs. [21,22], $\mathcal{N} = 2$ SUSY gauge theories perturbed by an appropriate superpotential have been studied beyond the perturbative regime. It has been shown that such perturbed $\mathcal{N} = 2$ SUSY gauge theories can have metastable vacua at generic points in the moduli space.

In this paper, we revisit the $\mathcal{N} = 2$ SUSY gauge theory with a Fayet-Iliopoulos (FI) term investigated in Ref. [23]. The model is based on the gauge group $SU(2) \times U(1)$ with

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$N_f = 2$ massless hypermultiplets. At the classical level, this theory has SUSY vacua on the Higgs branch, at the origin of the Coulomb branch. Except for near the origin on the Coulomb branch, the classical potential possesses SUSY breaking minima along a pseudoflat direction on the Coulomb branch. These are far away from the Higgs branch and parametrized by moduli parameters, scalars of vector multiplets. In the quantum theory, these pseudoflat directions are removed and a nontrivial local vacuum may arise while the SUSY vacua on the Higgs branch would remain.

The effective potential along the pseudoflat direction can be analyzed beyond perturbation theory by using the exact results in $\mathcal{N} = 2$ SUSY QCD [19,20]. It is found that the effective potential exhibits a local minimum with broken SUSY at the dyon point through the $SU(2)$ gauge dynamics and also that $U(1)_R$ is dynamically broken there. The global structure of the potential is determined in perturbation theory and the effective potential is found to be of the so-called runaway type, namely, the potential energy decreases toward infinity where the SUSY vacuum would be realized. We discuss the vacuum structure of this model in more detail and give a rough estimate of the decay rate of the local minimum to the runaway vacuum and the SUSY vacua on the Higgs branch. We find that this local minimum is parametrically long-lived and thus metastable. Also, we address phenomenological applications of our model. In fact, in this metastable vacuum, the massive hypermultiplets inherent in the model play the role of messenger fields in the gauge mediation scenario when the flavor symmetry among the hypermultiplets is gauged as the standard model gauge group.

The organization of this paper is as follows. In Sec. II, the model is defined and its classical vacuum structure is studied. In Sec. III, low energy effective Lagrangian is derived using the exact results in SUSY QCD. The effective potential is analyzed in Sec. IV, and we show that the effective potential exhibits a local minimum at the dyon singular point due to nonperturbative $SU(2)$ effects. In Sec. V, we give a rough estimate for the decay rate of the local minimum and show the vacuum can be long-lived. Phenomenological applications of the model are addressed in Sec. VI. The last section is devoted to our conclusion. Detailed derivations of the effective couplings are given in an appendix.

II. VACUUM STRUCTURE OF CLASSICAL THEORY

We first define our classical Lagrangian and analyze its classical vacuum.¹ We describe the classical Lagrangian in terms of $\mathcal{N} = 1$ superfields: Adjoint chiral superfields A_i and vector superfields V_i in the vector multiplet [$i = 1, 2$

denote the index of the $U(1)$ and the $SU(2)$ gauge symmetries, respectively], and two chiral superfields Q_r^I and \tilde{Q}^I_r in the hypermultiplet [$r = 1, 2$ is the flavor index, and $I = 1, 2$ is the $SU(2)$ color index]. The superfield strength is defined by $W_{i\alpha} = -\frac{1}{4}\bar{D}^2(e^{-V_i}D_\alpha e^{V_i})$. The classical Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\text{HM}} + \mathcal{L}_{\text{VM}} + \mathcal{L}_{\text{FI}}, \quad (2.1)$$

$$\begin{aligned} \mathcal{L}_{\text{HM}} = & \int d^4\theta (Q_r^\dagger e^{2V_2+2V_1} Q^r + \tilde{Q}_r e^{-2V_2-2V_1} \tilde{Q}^{\dagger r}) \\ & + \sqrt{2} \left(\int d^2\theta \tilde{Q}_r (A_2 + A_1) Q^r + \text{H.c.} \right), \end{aligned} \quad (2.2)$$

$$\begin{aligned} \mathcal{L}_{\text{VM}} = & \frac{1}{2\pi} \text{Im} \left[\text{tr} \left\{ \tau_{22} \left(\int d^4\theta A_2^\dagger e^{2V_2} A_2 e^{-2V_2} \right. \right. \right. \\ & \left. \left. \left. + \frac{1}{2} \int d^2\theta W_2^2 \right) \right\} \right] \\ & + \frac{1}{4\pi} \text{Im} \left[\tau_{11} \left(\int d^4\theta A_1^\dagger A_1 + \frac{1}{2} \int d^2\theta W_1^2 \right) \right], \end{aligned} \quad (2.3)$$

$$\mathcal{L}_{\text{FI}} = \int d^4\theta \xi V_1, \quad (2.4)$$

where $\tau_{22} = i\frac{4\pi}{g^2} + \frac{\theta}{2\pi}$ and $\tau_{11} = i\frac{4\pi}{e^2}$ are the gauge couplings of the $SU(2)$ and the $U(1)$ gauge interactions, respectively. Here we use the notation, $\text{tr}(T^a T^b) = T(R)\delta^{ab} = \frac{1}{2}\delta^{ab}$ for the $SU(2)$ generators T^a . The common $U(1)$ charge of the hypermultiplets is normalized to one. The last term in Eq. (2.1) is the FI term with a coefficient ξ of mass dimension two. In what follows, we assume that $\xi > 0$. In general, the FI term also appears in the F term, but the $SU(2)_R$ symmetry allows us to take a frame so that it appears only in the D term. Because of this, the $SU(2)_R$ symmetry is explicitly broken down to its subgroup $U(1)_{R'}$. The global symmetry of the theory turns out to be $SU(2)_{\text{Left}} \times SU(2)_{\text{Right}} \times U(1)_{R'} \times U(1)_R$.²

From the above Lagrangian, the classical potential is read off as

$$\begin{aligned} V = & \frac{1}{g^2} \text{tr}[A_2, A_2^\dagger]^2 + \frac{g^2}{2} (q_r^\dagger T^a q^r - \tilde{q}_r T^a \tilde{q}^{\dagger r})^2 \\ & + q_r^\dagger [A_2, A_2^\dagger] q^r - \tilde{q}_r [A_2, A_2^\dagger] \tilde{q}^{\dagger r} + 2g^2 |\tilde{q}_r T^a q^r|^2 \\ & + \frac{e^2}{2} (\xi + q_r^\dagger q^r - \tilde{q}_r \tilde{q}^{\dagger r})^2 + 2e^2 |\tilde{q}_r q^r|^2 \\ & + 2(q_r^\dagger |A_2 + A_1|^2 q^r + \tilde{q}_r |A_2 + A_1|^2 \tilde{q}^{\dagger r}), \end{aligned} \quad (2.5)$$

¹The complete analysis of the classical potential for the one flavor case was originally performed in Ref. [24].

²Without $U(1)$ gauge symmetry (and the FI term), the flavor symmetry $SU(2)_{\text{Left}} \times SU(2)_{\text{Right}}$ is enhanced to $O(4)$ since the representation of Q and \tilde{Q} are in an isomorphic representation of $SU(2)$ gauge group.

where A_2 , A_1 , q_r and \tilde{q}_r are scalar components of the corresponding chiral superfields.

There are supersymmetric vacua in this potential. For example, a solution

$$\begin{aligned} A_1 = A_2 = 0, \quad q_r = 0, \quad \tilde{q}_1 = (v, 0), \\ \tilde{q}_2 = (0, v), \quad \frac{\xi}{2} - |v|^2 = 0, \end{aligned} \quad (2.6)$$

is a possible supersymmetric vacuum.

Let us then investigate the global structure of the vacuum. In order to do that, consider the following field configuration: $q_r = 0$ and $\tilde{q}_1 = (a, 0)$, $\tilde{q}_2 = (0, b)$, $A_2 = \text{diag}(a_2/2, -a_2/2)$ and $A_1 = \text{diag}(a_1, a_1)$. Here, a_1 and a_2 are complex parameters, and a and b are, for simplicity, taken to be real. Then the potential (2.5) is written as

$$\begin{aligned} V = \frac{g^2}{2}(a^2 - b^2)^2 + \frac{e^2}{2}(\xi - a^2 - b^2)^2 \\ + 2a^2 \left| a_1 + \frac{1}{2}a_2 \right|^2 + 2b^2 \left| a_1 - \frac{1}{2}a_2 \right|^2. \end{aligned} \quad (2.7)$$

The behavior of the potential (2.7) is essentially the same as the following scalar potential,

$$V = (\xi - X^2)^2 + X^2 Y^2, \quad (2.8)$$

where X and Y can be regarded as hypermultiplet and vector multiplet directions, respectively. The plot of the potential is depicted in Fig. 1. The potential has a pseudoflat direction along Y with $X = 0$. The vacua on this flat direction are tachyonic along the X direction for $Y < \sqrt{2}\xi$, but there are no longer tachyonic directions for $Y > \sqrt{2}\xi$. In the case (2.7), a pseudoflat direction is, for instance, parametrized by

$$b = 0 \quad \text{and} \quad z = a_1 - \frac{1}{2}a_2, \quad (2.9)$$

for $a_1 + \frac{1}{2}a_2 = 0$, which corresponds to X and Y , respectively. Along this direction, the potential is further mini-

mized with respect to a , whose value at the minimum is given by

$$a = \left(\frac{e^2}{\frac{1}{4}g^2 + e^2 \xi} \right)^{1/2}. \quad (2.10)$$

In this example, the gauge symmetry $SU(2) \times U(1)$ is broken to a linear combination, $U'(1)$. The potential energy at this minimum is given by

$$V = \frac{\xi^2}{2} \frac{e^2 g^2}{4e^2 + g^2}. \quad (2.11)$$

Therefore, the supersymmetry is broken at the minimum along the pseudoflat direction by the nonzero FI parameter.

We are now interested in what happens to this pseudoflat direction in quantum theory. Since a_1 and a_2 have $U(1)_R$ charge $+2$, the $U(1)_R$ symmetry is broken by these nonzero vacuum expectation values (VEVs). We expect that the pseudoflat direction is lifted up, once quantum corrections are taken into account, and some nondegenerate vacua would appear after the effective potential is analyzed. This naive expectation seems natural, if we notice that the above potential energy is described by the bare gauge couplings, which should be replaced by the effective ones (nontrivial functions of the moduli parameters) in the effective theory. In the following sections, we will show that quantum corrections actually remove the vacuum degeneracy and leave two vacua on the Coulomb branch, one of which is a local minimum breaking both SUSY and $U(1)_R$ symmetry and the other is a runaway vacuum.

In addition to this runaway SUSY vacuum in the Coulomb branch, there would be other SUSY vacua. It is known that there are no quantum corrections on the Higgs branch [25], and we thus expect that the classical SUSY vacua (2.6) survive after quantum corrections are taken into account. At these SUSY vacua, the hypermultiplets have very small VEVs because of the theoretical consistency condition $\xi \ll \Lambda^2$ (see the next section). On the Coulomb branch, except for very near the origin, we have a pseudo-

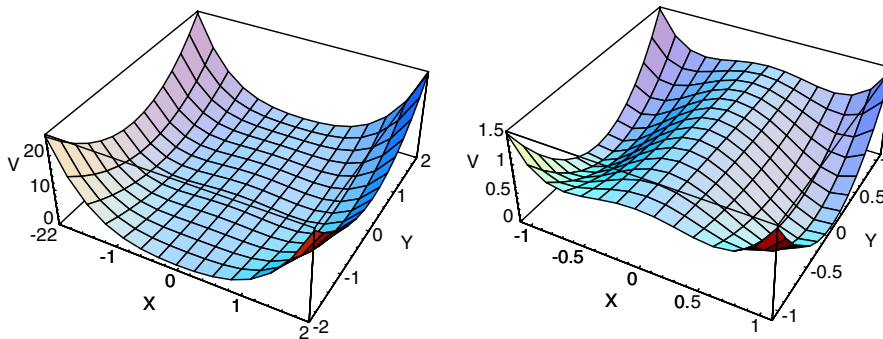


FIG. 1 (color online). Schematic picture of the classical potential. ξ is taken to be 1 in (2.8). For $Y > \sqrt{2}\xi$ (left), the minimum is along $X = 0$. For $Y < \sqrt{2}\xi$ (right), there are tachyonic directions along X .

flat direction as shown above and no tachyonic direction towards the Higgs branch. For this reason, we will analyze the quantum theory along the Coulomb branch and will see how the effective potential can be modified along the pseudoflat directions.

III. QUANTUM THEORY

In this section, we describe the low energy Wilsonian effective Lagrangian of our theory. The detailed derivation of the effective action is found in Ref. [23]. Here, we briefly summarize the results for the convenience of the reader.

In order to derive the exact low energy effective action $\mathcal{L}_{\text{EXACT}}$, which is described by the light fields, the dynamical scale and the coefficient of the FI term ξ , we need to integrate the action to zero momentum. However, this is a highly nontrivial task. Without the FI term, the theory has $\mathcal{N} = 2$ SUSY, which can be utilized to integrate out massive degrees of freedom. In our model, this is not the case because the SUSY is broken at the classical level. In the following discussion, suppose that the coefficient ξ , the order parameter of SUSY breaking, is much smaller than the dynamical scale Λ of the $SU(2)$ gauge interaction. Then we can consider the effective action up to the leading order in ξ . The exact effective Lagrangian, if it could be obtained, can be expanded in the parameter ξ as

$$\mathcal{L}_{\text{EXACT}} = \mathcal{L}_{\text{SUSY}} + \xi \mathcal{L}_1 + \mathcal{O}(\xi^2). \quad (3.1)$$

Here, the first term $\mathcal{L}_{\text{SUSY}}$ is the exact effective Lagrangian containing full SUSY quantum corrections. The second term is the leading term in ξ . Since ξ is a constant and has mass dimension 2, \mathcal{L}_1 should be a gauge-invariant quantity having mass dimension 2. This simple consideration tells us that the second term is nothing but the FI term.³ Analyzing the effective Lagrangian up to the leading order in ξ , we obtain the effective potential to order ξ^2 . The coefficient of ξ^2 in the effective potential includes the full SUSY quantum corrections. Therefore, to achieve our aim, what we need to analyze the effective potential is nothing but the effective Lagrangian $\mathcal{L}_{\text{SUSY}}$.

Except for the FI term, the classical $SU(2) \times U(1)$ gauge theory has a moduli space, which is parametrized by a_2 and a_1 . On this moduli space except at the origin, the gauge symmetry is broken to $U(1)_c \times U(1)$.⁴ Here $U(1)_c$ denotes the gauge symmetry in the Coulomb phase originating from the $SU(2)$ gauge symmetry. Before discussing the effective action of this theory, we should make clear

³The exactness of the FI term is also discussed by using the harmonic superspace formalism in Ref. [26].

⁴In this paper, we study the Coulomb branch, not the mixed branch as in (2.9) and (2.10), on which it is difficult to analyze the effective potential. However, at low energies, the $SU(2)$ gauge coupling is much larger than the $U(1)$ gauge coupling, so that the solution (2.9) and (2.10) is approximately that of the Coulomb branch.

how to treat the $U(1)$ gauge interaction part. In the following analysis, this part is, as usual, discussed as a cutoff theory.⁵ Thus, the Landau pole Λ_L is inevitably introduced in our effective theory, and the defining region of the modulus parameter a_1 is constrained to lie within the region $|a_1| < \Lambda_L$. Because of this constraint, the defining region for the modulus parameter a_2 is found to be also constrained to be in the same region, since the two moduli parameters are related to each other through the hypermultiplets. We take the scale Λ_L to be much larger than the dynamical scale of the $SU(2)$ gauge interaction Λ , so that the $U(1)$ gauge interaction is always weak in the defining region of moduli space. Note that, in our framework, we implicitly assume that the $U(1)$ gauge interaction has no effect on the $SU(2)$ gauge dynamics. This assumption will be justified in the following discussion concerning the monodromy transformation [see Eq. (3.5)].

We first discuss the general formulas for the effective Lagrangian $\mathcal{L}_{\text{SUSY}}$, which consists of two parts described by light vector multiplets and hypermultiplets, $\mathcal{L}_{\text{SUSY}} = \mathcal{L}_{\text{VM}} + \mathcal{L}_{\text{HM}}$. At low energies, the $\mathcal{N} = 2$ effective Lagrangian of the vector multiplet part, \mathcal{L}_{VM} , includes the superfield A_2 of the unbroken Abelian subgroup of $SU(2)$ and the Abelian superfield A_1 . The effective action consistent with $\mathcal{N} = 2$ SUSY and all the symmetries in our theory is given by

$$\mathcal{L}_{\text{VM}} = \frac{1}{4\pi} \text{Im} \left\{ \sum_{i,j=1}^2 \left(\int d^4\theta \frac{\partial F}{\partial A_i} A_i^\dagger + \int d^2\theta \frac{1}{2} \tau_{ij} W_i W_j \right) \right\}, \quad (3.2)$$

where $F(A_2, A_1, \Lambda, \Lambda_L)$ is the prepotential, which is a function of moduli parameters a_2, a_1 , the dynamical scale Λ , and the Landau pole Λ_L . Note that the effective coupling $\tau_{12} (= \tau_{21})$ appears through the quantum corrections. The effective coupling τ_{ij} is defined as

$$\tau_{ij} = \frac{\partial^2 F}{\partial a_i \partial a_j} (i, j = 1, 2). \quad (3.3)$$

The part \mathcal{L}_{HM} is described by a light hypermultiplet with appropriate quantum numbers $(n_e, n_m)_n$, where n_e is the electric charge, n_m is the magnetic charge, and n is the $U(1)$ charge. This part should be added to the effective Lagrangian around a singular point in moduli space, since the hypermultiplet is expected to be light there and enjoys correct degrees of freedom in the effective theory. The explicit description is given by

⁵There is a possibility that a nontrivial fixed point and a strong coupling phase exist in QED [27]. This problem is nontrivial, and is outside our scope (see also Ref. [26] for related discussions).

$$\begin{aligned}
 \mathcal{L}_{\text{HM}} = & \int d^4\theta (M_r^\dagger e^{2n_m V_{2D} + 2n_e V_2 + 2n V_1} M^r \\
 & + \tilde{M}_r e^{-2n_m V_{2D} - 2n_e V_2 - 2n V_1} \tilde{M}^{r\dagger}) \\
 & + \sqrt{2} \left(\int d^2\theta \tilde{M}_r (n_m A_{2D} + n_e A_2 + n A_1) M^r \right. \\
 & \left. + \text{H.c.} \right), \tag{3.4}
 \end{aligned}$$

where M^r and \tilde{M}_r denote light quark, light monopole, or light dyon hypermultiplet, that is, the light BPS states, and V_{2D} is the dual gauge field of $U(1)_c$. Since the $U(1)$ gauge coupling is weak and does not affect the $SU(2)$ gauge dynamics, the flavor symmetry is effectively that of $\mathcal{N} = 2$ SUSY QCD. Recalling that a_1 plays a role of the hypermultiplet mass if it has vacuum expectation value, for vanishing VEV of a_1 , the light BPS states belong to a

spinor representation of $SO(4) \sim SU(2)_- \times SU(2)_+$ [20,28]. A nonzero vacuum expectation value of a_1 breaks the symmetry down to $SU(2)_- \times U(1)_+$. At the quantum level, the global $U(1)_R$ symmetry is anomalous and the resultant anomaly-free symmetry turns out to be $\mathbf{Z}_8 \subset U(1)_R$ [20].

In order to obtain an explicit description of the effective Lagrangian, let us consider the monodromy transformations of our theory. Suppose that the moduli space is parametrized by the vector multiplet scalars a_2 , a_1 and their duals a_{2D} , a_{1D} which are defined as $a_{iD} = \partial F / \partial a_i$ ($i = 1, 2$). These variables are transformed into their linear combinations by the monodromy transformation. In our case, the monodromy transformations form a subgroup of $Sp(4, \mathbf{R})$, which leaves the effective Lagrangian invariant, and the general formula is found to be [29]

$$\begin{pmatrix} a_{2D} \\ a_2 \\ a_{1D} \\ a_1 \end{pmatrix} \rightarrow \begin{pmatrix} \alpha a_{2D} + \beta a_2 + p a_1 \\ \gamma a_{2D} + \delta a_2 + q a_1 \\ a_{1D} + p(\gamma a_{2D} + \delta a_2) - q(\alpha a_{2D} + \beta a_2) - p q a_1 \\ a_1 \end{pmatrix}, \tag{3.5}$$

where

$$\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \in SL(2, \mathbf{Z})$$

and $p, q \in \mathbf{Q}$. Note that this monodromy transformation for the combination (a_{2D}, a_2, a_1) is exactly the same as that for SUSY QCD with massive quark hypermultiplets, if we regard a_1 as the common mass of the hypermultiplets such that $m = \sqrt{2}a_1$. This fact means that the $U(1)$ gauge interaction part only plays the role of the mass term for the $SU(2)$ gauge dynamics. This observation is consistent with our assumptions. On the other hand, the $SU(2)$ dynamics plays an important role for the $U(1)$ gauge interaction part, as can be seen from the transformation law of a_{1D} . This monodromy transformation is also used to derive dual variables associated with the BPS states. As a result, the prepotential of our theory turns out to be essentially the same as the result in Ref. [20] with the additional relation $m = \sqrt{2}A_1$,

$$F(A_2, A_1, \Lambda, \Lambda_L) = F_{SU(2)}^{(SW)}(A_2, m, \Lambda)|_{m=\sqrt{2}A_1} + CA_1^2, \tag{3.6}$$

where the first term on the right-hand side is the prepotential of $\mathcal{N} = 2$ SUSY QCD with hypermultiplets having the same mass m , and C is a free parameter. The freedom of the parameter C is used to determine the scale of the Landau pole relative to the scale of the $SU(2)$ dynamics.

The effective potential can be obtained from the action, after eliminating auxiliary fields⁶

$$\begin{aligned}
 V = & \frac{b_{22}}{2 \det b} \xi^2 + S(a_2, a_1) \{ (|M^r|^2 - |\tilde{M}_r|^2)^2 + 4|M^r \tilde{M}_r|^2 \} \\
 & + 2T(a_2, a_1) (|M^r|^2 + |\tilde{M}_r|^2) \\
 & - U(a_2, a_1) (|M^r|^2 - |\tilde{M}_r|^2), \tag{3.7}
 \end{aligned}$$

where $|M^r|^2 = M^r M_r^\dagger$, $|\tilde{M}_r|^2 = \tilde{M}_r \tilde{M}^{r\dagger}$, $b_{ij} = (1/4\pi) \text{Im} \tau_{ij}$ is the effective coupling and $\det b \equiv b_{22} b_{11} - b_{12}^2$. The functions S , T and U are defined as

$$S(a_2, a_1) = \frac{1}{2b_{22}} + \frac{(b_{12} - nb_{22})^2}{2b_{22} \det b}, \tag{3.8}$$

$$T(a_2, a_1) = |a_2 + na_1|^2, \tag{3.9}$$

$$U(a_2, a_1) = \frac{b_{12} - nb_{22}}{\det b} \xi. \tag{3.10}$$

Solving the stationary conditions with respect to the hypermultiplets, we have the following three solutions:

(1)

$$M = \tilde{M} = 0; \quad V = \frac{b_{22}}{2 \det b} \xi^2, \tag{3.11}$$

⁶We presuppose that the potential is described by the proper variables associated with the light BPS states. For instance, the variable a_2 is understood implicitly as $-a_{2D}$, when we consider the effective potential for the monopole.

(2)

$$|M^r|^2 = -\frac{2T-U}{2S}, \quad \tilde{M} = 0; \quad (3.12)$$

$$V = \frac{b_{22}}{2 \det b} \xi^2 - S|M^r|^4,$$

(3)

$$M = 0, \quad |\tilde{M}^r|^2 = -\frac{2T+U}{2S}; \quad (3.13)$$

$$V = \frac{b_{22}}{2 \det b} \xi^2 - S|\tilde{M}^r|^4.$$

The solution Eq. (3.12) or Eq. (3.13), in which the light hypermultiplet acquires a vacuum expectation value, is energetically favored, because $\det b > 0$ and $S(a_2, a_1) > 0$. Since the hypermultiplet appears in the theory as the light BPS state around the singular point in moduli space, a potential minimum is expected to emerge there. In addition, the points (3.12) and (3.13) are stable in the M, \tilde{M} directions. This is because they are unique solutions and have lower energy than the point (3.11), and the potential at infinity in M, \tilde{M} space is dominated by the M^4, \tilde{M}^4 terms. On the other hand, the solution Eq. (3.11) describes the potential energy away from the singular points, which smoothly connects with the solution Eq. (3.12) or Eq. (3.13).

It was shown that the effective potential is described by the periods a_{2D}, a_2 and the effective gauge coupling b_{ij} . The periods are the same as that of massive SUSY QCD. Although there are some different descriptions of the periods it is convenient for our purpose to write them as integral representations [29] and to write the effective coupling τ_{ij} in terms of the Weierstrass functions.

We first review how to obtain the periods a_{2D} and a_2 . The elliptic curve of $\mathcal{N} = 2$ SUSY QCD with two hypermultiplets having the same mass m was found to be [20]

$$y^2 = x^2(x-u) - \frac{\Lambda^4}{64}(x-u) + \frac{\Lambda^2}{4}m^2x - \frac{\Lambda^4}{32}m^2, \quad (3.14)$$

where $u = \text{Tr}A_2^2$ is identified with the modulus parameter. In this case, the mass formula of the BPS state with the quantum numbers $(n_e, n_m)_n$ is given by $M_{\text{BPS}} = \sqrt{2}|n_m a_{2D} + n_e a_2 + nm/\sqrt{2}|$. If λ is a meromorphic differential on the curve Eq. (3.14) such that

$$\frac{\partial \lambda}{\partial u} = \frac{\sqrt{2}}{8\pi} \frac{dx}{y}, \quad (3.15)$$

the periods are given by the contour integrals

$$a_{2D} = \oint_{\alpha_1} \lambda, \quad a_2 = \oint_{\alpha_2} \lambda, \quad (3.16)$$

where the cycles α_1 and α_2 are defined so as to encircle e_2 and e_3 , and e_1 and e_3 , respectively, which will be given explicitly later on [see Eq. (3.25)]. The meromorphic differential is given by

$$\lambda_{\text{SW}} = -\frac{\sqrt{2}}{4\pi} \frac{y dx}{x^2 - \frac{\Lambda^4}{64}} = -\frac{\sqrt{2}}{4\pi} \frac{dx}{y} \left[x - u + \frac{m^2 \Lambda^2}{4(x + \frac{\Lambda^2}{8})} \right]. \quad (3.17)$$

The differential has a single pole at $x = -\frac{\Lambda^2}{8}$ and the residue is given by

$$\text{Res } \lambda_{\text{SW}} = \frac{1}{2\pi i} (-1) \frac{m}{\sqrt{2}}. \quad (3.18)$$

We calculate the periods by using the Weierstrass normal form for later convenience. In this form, the algebraic curve is rewritten in new variables $x = 4X + \frac{u}{3}$ and $y = 4Y$, such that

$$Y^2 = 4X^3 - g_2 X - g_3 = 4(X - e_1)(X - e_2)(X - e_3), \quad (3.19)$$

$$\sum_{i=1}^3 e_i = 0,$$

where g_2 and g_3 are explicitly written by

$$g_2 = \frac{1}{16} \left(\frac{4}{3} u^2 + \frac{\Lambda^4}{16} - m^2 \Lambda^2 \right), \quad (3.20)$$

$$g_3 = \frac{1}{16} \left(\frac{m^2 \Lambda^4}{32} - \frac{u}{12} m^2 \Lambda^2 - \frac{u \Lambda^4}{96} + \frac{2u^3}{27} \right). \quad (3.21)$$

Converting the Seiberg-Witten differential, Eq. (3.17), into the Weierstrass normal form and substituting it into Eq. (3.16), we obtain the integral representations of the periods as follows (a_{2D} and a_2 are denoted by a_{21} and a_{22} , respectively):

$$a_{2i} = -\frac{\sqrt{2}}{4\pi} \left(-\frac{4}{3} u I_1^{(i)} + 8 I_2^{(i)} + \frac{m^2 \Lambda^2}{8} I_3^{(i)}(c) \right), \quad (3.22)$$

where c is the pole of the differential, given by $c = -\frac{u}{12} - \frac{\Lambda^2}{32}$. The integrals $I_1^{(i)}, I_2^{(i)}$ and $I_3^{(i)}$ are defined as

$$I_1^{(i)} = \frac{1}{2} \oint_{\alpha_i} \frac{dX}{Y}, \quad I_2^{(i)} = \frac{1}{2} \oint_{\alpha_i} \frac{X dX}{Y}, \quad (3.23)$$

$$I_3^{(i)}(c) = \frac{1}{2} \oint_{\alpha_i} \frac{dX}{Y(X-c)}.$$

The roots e_i of the polynomial defining the cubic are chosen so as to lead to the correct asymptotic behavior for large $|u|$,

$$a_{2D}(u) \sim i \frac{2}{2\pi} \sqrt{2u} \log \frac{u}{\Lambda^2}, \quad a_2(u) \sim \frac{\sqrt{2u}}{2}, \quad (3.24)$$

A correct choice is the following:

$$\begin{aligned} e_1 &= \frac{u}{24} - \frac{\Lambda^2}{64} - \frac{1}{8} \sqrt{u + \frac{\Lambda^2}{8} + \Lambda m} \sqrt{u + \frac{\Lambda^2}{8} - \Lambda m}, \\ e_2 &= \frac{u}{24} - \frac{\Lambda^2}{64} + \frac{1}{8} \sqrt{u + \frac{\Lambda^2}{8} + \Lambda m} \sqrt{u + \frac{\Lambda^2}{8} - \Lambda m}, \\ e_3 &= -\frac{u}{12} + \frac{\Lambda^2}{32}. \end{aligned} \quad (3.25)$$

Fixing the contours of the cycles relative to the positions of the poles, which is equivalent to fixing the $U(1)$ charges for the BPS states, the final formulas are given by

$$\begin{aligned} a_{2i} &= -\frac{\sqrt{2}}{4\pi} \left(-\frac{4}{3} u I_1^{(i)} + 8 I_2^{(i)} + \frac{m^2 \Lambda^2}{8} I_3^{(i)} \left(-\frac{u}{12} - \frac{\Lambda^2}{32} \right) \right) \\ &\quad - \frac{m}{\sqrt{2}} \delta_{i2}, \end{aligned} \quad (3.26)$$

with the integrals $I_s^{(1)}$ ($s = 1, 2, 3$) explicitly given by

$$I_1^{(1)} = \int_{e_2}^{e_3} \frac{dX}{Y} = \frac{iK(k')}{\sqrt{e_2 - e_1}}, \quad (3.27)$$

$$I_2^{(1)} = \int_{e_2}^{e_3} \frac{XdX}{Y} = \frac{ie_1}{\sqrt{e_2 - e_1}} K(k') + i\sqrt{e_2 - e_1} E(k'), \quad (3.28)$$

$$\begin{aligned} I_3^{(1)} &= \int_{e_2}^{e_3} \frac{dX}{Y(X-c)} \\ &= \frac{-i}{(e_2 - e_1)^{3/2}} \left\{ \frac{1}{k + \tilde{c}} K(k') + \frac{4k}{1+k} \right. \\ &\quad \left. \times \frac{1}{\tilde{c}^2 - k^2} \Pi_1 \left(\nu, \frac{1-k}{1+k} \right) \right\}, \end{aligned} \quad (3.29)$$

where $k^2 = \frac{e_3 - e_1}{e_2 - e_1}$, $k'^2 = 1 - k^2 = \frac{e_2 - e_3}{e_2 - e_1}$, $\tilde{c} = \frac{c - e_1}{e_2 - e_1}$, and $\nu = -\frac{(k + \tilde{c})^2 (1 - k)^2}{(k - \tilde{c})^2 (1 + k)^2}$. The formulas for $I_s^{(2)}$ are obtained from $I_s^{(1)}$ by exchanging the roots e_1 and e_2 . In Eqs. (3.27), (3.28), and (3.29), K , E , and Π_1 are the complete elliptic integrals [30] given by

$$\begin{aligned} K(k) &= \int_0^1 \frac{dx}{[(1-x^2)(1-k^2x^2)]^{1/2}}, \\ E(k) &= \int_0^1 dx \left(\frac{1-k^2x^2}{1-x^2} \right)^{1/2}, \\ \Pi_1(\nu, k) &= \int_0^1 \frac{dx}{[(1-x^2)(1-k^2x^2)]^{1/2} (1+\nu x^2)}. \end{aligned} \quad (3.30)$$

Next let us consider the effective coupling defined in Eq. (3.3). A detailed derivation of the effective couplings is given in the appendix. The effective couplings τ_{22} and τ_{12} are obtained by

$$\tau_{22} = \frac{\partial a_{2D}}{\partial a_2} = \frac{\omega_1}{\omega_2}, \quad (3.31)$$

$$\tau_{12} = \frac{\partial a_{2D}}{\partial a_1} \Big|_u - \tau_{22} \frac{\partial a_2}{\partial a_1} \Big|_u = -\frac{2z_0}{\omega_2}, \quad (3.32)$$

where ω_i is the period of the Abelian differential,

$$\omega_i = \oint_{\alpha_i} \frac{dX}{Y} = 2I_1^{(i)} \quad (i = 1, 2), \quad (3.33)$$

and z_0 is defined as

$$z_0 = -\frac{1}{\sqrt{e_2 - e_1}} F(\phi, k); \quad \sin^2 \phi = \frac{e_2 - e_1}{c - e_1}. \quad (3.34)$$

Here $F(\phi, k)$ is the incomplete elliptic integral of the first kind given in (A3).

The effective coupling τ_{11} is described in terms of the Weierstrass function. First consider the period a_{1D} by using the Riemann bilinear relation [31],

$$\oint_{\alpha_1} \phi \oint_{\alpha_2} \omega - \oint_{\alpha_1} \omega \oint_{\alpha_2} \phi = 2\pi i \sum_{n=1}^{N_p} \text{Res}_{x_n^+} \phi \int_{x_n^-}^{x_n^+} \omega, \quad (3.35)$$

where ϕ and ω are meromorphic and holomorphic differentials, respectively, N_p is the number of poles ($N_p = 1$ in our case), and x_n^\pm are poles of ϕ on the positive and negative Riemann sheets. Substituting $\phi = \partial \lambda_{SW} / \partial a_1$ and $\omega = \partial \lambda_{SW} / \partial a_2$ into Eq. (3.35), we obtain

$$a_{1D} = -\sum_{n=1}^{N_p} \int_{x_n^-}^{x_n^+} \lambda_{SW} + \tilde{C}, \quad (3.36)$$

where \tilde{C} is a constant independent of a_2 . The effective coupling τ_{11} is obtained by differentiating Eq. (3.36) with respect to a_1 with a_2 fixed. The integral in Eq. (3.36) after the differentiation can be evaluated by the uniformization method discussed in the appendix. After regularizing the integral by using the freedom of the constant \tilde{C} , we finally obtain (see also the appendix for details)

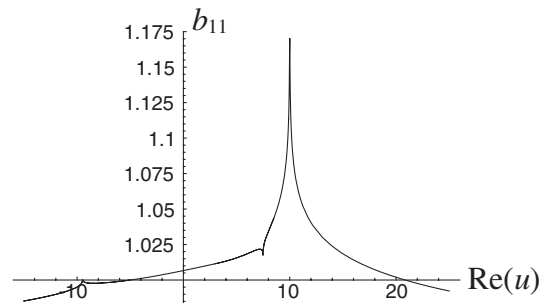
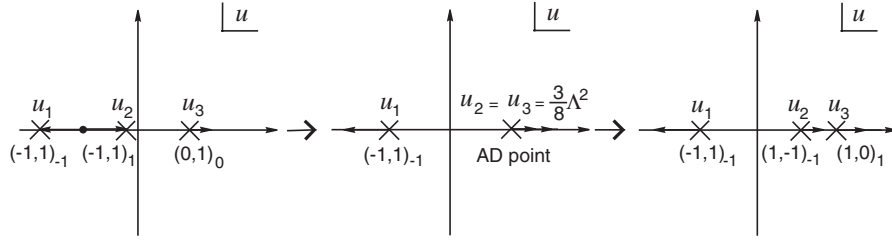


FIG. 2. The effective gauge coupling b_{11} for $a_1 = 3/\sqrt{2}$ along the real u axis.


 FIG. 3. Flow of the singular points as $\text{Re}(a_1)$ increases with $\text{Im}(a_1) = 0$.

$$\tau_{11} = -\frac{1}{\pi i} \left[\log \sigma(2z_0) + \frac{4z_0^2}{\omega_2} I_2^{(1)} \right] + C, \quad (3.37)$$

where σ is the Weierstrass sigma function, and C is the constant in Eq. (3.6).

We now define the Landau pole associated with the $U(1)$ interaction. In the ultraviolet region far away from the origin of the moduli space, the effective coupling is dominated by the $U(1)$ gauge interaction since the $SU(2)$ interaction is asymptotic free and small. As we expect, the gauge coupling b_{11} is found to be a monotonically decreasing function of the large $|a_1|$ with fixed u , and vice versa (see, for example, Fig. 2 in the case of fixed a_1). The Landau pole is defined as $|a_1| = \Lambda_L$ at which $b_{11} = 0$. The large Λ_L required in our assumption is realized by taking an appropriate value for C . In the following analysis, we fix $C = 4\pi i$, which corresponds to $\Lambda_L = 10^{17-18}$ in units of Λ .

IV. NUMERICAL ANALYSIS OF THE EFFECTIVE POTENTIAL

In this section, we examine the effective potential minimum numerically. As explained in the previous section, the minimum is expected to appear at the singular point since it is energetically favored due to the nonzero condensation [see Eq. (3.7)] of the light BPS state such as a quark, monopole or dyon with appropriate quantum number $(n_e, n_m)_n$. Thus, let us first investigate the singular points, and then analyze the effective potential at the singular point.

The singular points on the moduli space is determined by the cubic polynomial [20]. The solutions of the cubic polynomial give the positions of the singular points in the u plane. In the $N_f = 2$ case with the same hypermultiplet masses, the solution is easily obtained as

$$\begin{aligned} u_1 &= -m\Lambda - \frac{\Lambda^2}{8} \Big|_{m=\sqrt{2}a_1}, \\ u_2 &= m\Lambda - \frac{\Lambda^2}{8} \Big|_{m=\sqrt{2}a_1}, \\ u_3 &= m^2 + \frac{\Lambda^2}{8} \Big|_{m=\sqrt{2}a_1}. \end{aligned} \quad (4.1)$$

The flow of the singular points with respect to the real

hypermultiplet mass is sketched in Fig. 3. For $a_1 = 0$, the singular points appear at $u_1 = u_2 = -\Lambda^2/8$ and $u_3 = \Lambda^2/8$. Here, at $u = -\Lambda^2/8$, two singular points coincide. For nonzero $a_1 > 0$,⁷ this singular point splits into two singular points u_1 and u_2 , which correspond to the BPS states with quantum numbers $(-1, 1)_{-1}$ and $(-1, 1)_1$, respectively. As a_1 is increasing, these singular points, u_1 and u_2 , are moving to the left and the right on the real u axis, respectively. The two singular points, u_2 and u_3 , collide and coincide at the so-called Argyres-Douglas (AD) point [32] ($u = \frac{3\Lambda^2}{8}$) for $a_1 = \frac{\Lambda}{2\sqrt{2}}$, where it is believed that the theory becomes superconformal. As a_1 increases further, there appear two singular points u_2 and u_3 again, and the quantum numbers of the corresponding BPS states, $(-1, 1)_1$ at u_2 and $(0, 1)_0$ at u_3 , change into $(1, -1)_{-1}$ and $(1, 0)_1$, respectively. The singular point u_3 is then moving away to the right faster than u_2 .

Now let us examine the effective potential at the singular point. First note that the effective potential is a function of u and a_1 , $V(a_2(u, a_1), a_1)$ [see (3.12) and (3.13)]. Furthermore, (4.1) tells us that the singular point is completely determined by the value of a_1 , and therefore the potential at the singular point is a function of a_1 only. In the following, we investigate the effective potential at some fixed value of a_1 , and see how the minimum appears at the singular point. Then we examine the evolution of the minimum by varying a_1 . In our numerical analysis, we take $\Lambda = 2\sqrt{2}$ and $\xi = 0.1$.

$$(i) \quad 0 \leq \text{Re}(a_1) < \frac{\Lambda}{2\sqrt{2}}, \quad \text{Im}(a_1) = 0$$

The effective potentials for several values of a_1 in the range, $0 < a_1 < \Lambda/2\sqrt{2}$ (corresponding to the left figure in Fig. 3), are depicted in Fig. 4. The potential minima, V_{\min}^1 and V_{\min}^2 , appear at two singular points u_1 and u_2 , respectively, while there is no minimum at the singular point u_3 since the monopole condensation is too small for the potential to have a minimum. In the middle figure the top and the bottom curves show the effective potential without and with the dyon condensations, respectively. The cusps

⁷For $\text{Im}(a_1) = 0$, it is enough to consider only the case $a_1 > 0$, since the result for $a_1 < 0$ can be obtained by exchanging $u_1 \leftrightarrow u_2$, as can be seen from the first two equations in Eq. (4.1).

are smoothed out in the bottom curve, which means that the correct degrees of freedom in the theory are considered. The two minima in Fig. 4 approach each other and their values become smaller as a_1 decreases. Such a behavior can be shown in Figs. 5. The left figure shows the evolution of the potential minima, V_{\min}^1 and V_{\min}^2 at the dyon singular points as a_1 changes on the real u axis. The top and the bottom curves are plots without and with dyon condensates, respectively. From this figure, one sees that the condensation lowers the potential energy. The right figure shows that the evolution of the potential minima along the real a_1 axis. In this plot, the behaviors at the two dyon singular points completely coincide because of the symmetry, $a_1 \rightarrow -a_1$ [see (4.1)]. From the analysis we find that the potential is bounded from below, at least along the real u axis, and it is expected that there is a (local) minimum at $u \rightarrow -\Lambda^2/8$ and $a_1 \rightarrow 0$.

(ii) $\text{Re}(a_1) = 0, \quad \text{Im}(a_1) \neq 0$

Next we examine the effective potential for a complex value of a_1 around $a_1 = 0$. For our purpose, it is sufficient to investigate small values of $\text{Im}(a_1)$ near $\text{Re}(a_1) = 0$ since

we want to know whether the effective potential is bounded from below or not at the point $\text{Re}(a_1) = 0$. Once again let us go back to the flow of the singular points. Figure 6 shows the flow of the singular points u_1 and u_2 for several complex values of a_1 . The left figure shows the flow as $\text{Im}(a_1)$ increases. The singular points u_1 and u_2 are moving in opposite directions along the $u = -\Lambda^2/8 = -1$ axis. The middle (right) figure shows the flow of the singularities for the $\text{Re}(a_1) > 0, \text{Im}(a_1) > 0$ [$\text{Re}(a_1) < 0, \text{Im}(a_1) < 0$] case. The plots of the potential corresponding to these flows are shown in Figs. 7 and 8. The left figure in Fig. 7, corresponding to the left figure in Fig. 6 shows the evolution of the potential minima for two dyons along the $u = -1$ axis. The right figure is the same plot, but along the $\text{Im}(a_1)$ axis. Note that in the latter plot, the evolution of the two dyon minima completely coincides as in the case of the right figure in Fig. 5. These two dyon points roll down to the point $a_1 \rightarrow 0$ ($u \rightarrow -1$), and the potential near $a_1 = 0$ is bounded from below in the pure imaginary direction of a_1 . Figure 8 shows the contour and 3D plots of the effective potential at the u_2 -dyon point as a function of $\text{Re}(a_1) > 0$ and $\text{Im}(a_1) > 0$. The dark (light) color shows lower (higher) value of the effective potential.

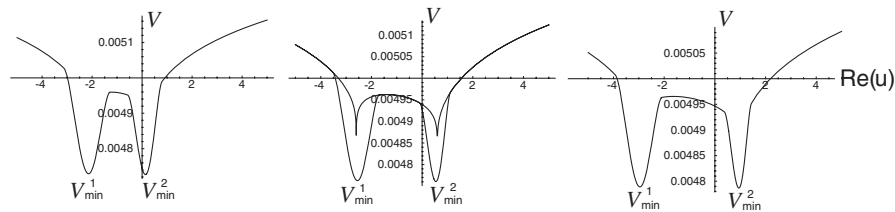


FIG. 4. The effective potential for $a_1 = 0.3$ (left), $a_1 = 0.4$ (middle) and $a_1 = 0.5$ (right) on the real u axis.

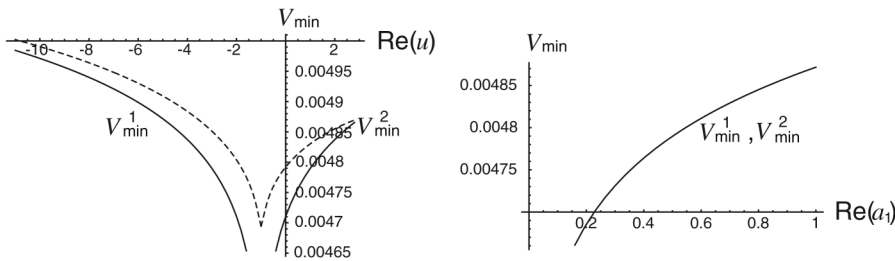


FIG. 5. The evolution of the potential minima V_{\min}^1 and V_{\min}^2 at the singular points u_1 and u_2 , respectively, as a_1 varies on the real u axis (left) and on the real a_1 axis (right). The solid (dashed) curve shows the plots with (without) dyon condensation.

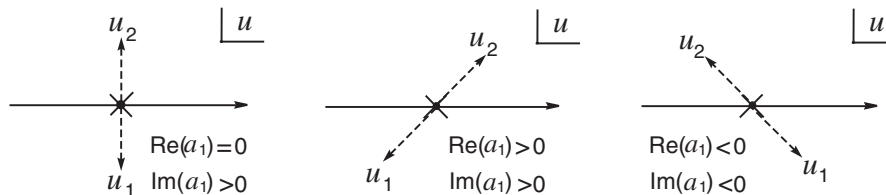


FIG. 6. Flow of the singular points u_1 and u_2 for general values of a_1 .

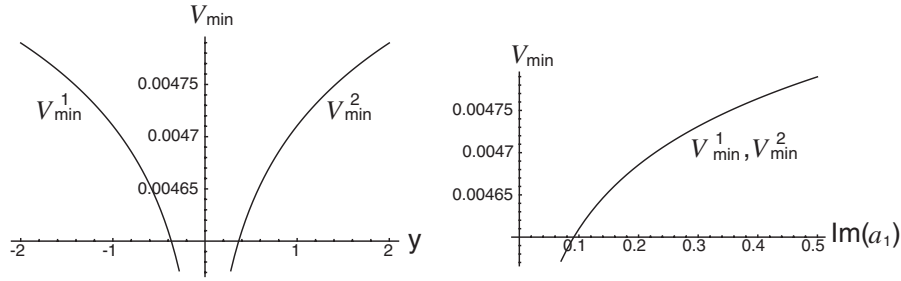


FIG. 7. The evolution of the minima V_{\min}^1 and V_{\min}^2 at u_1 and u_2 , respectively, with varying the pure imaginary part of a_1 along $u = -1 + iy$ axis (left) and the imaginary a_1 axis.

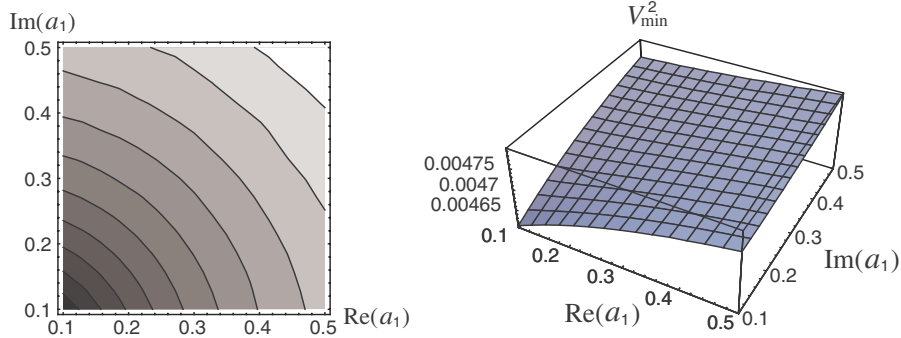


FIG. 8 (color online). The contour and the 3D plots of the effective potential at the singular point u_2 as a function of a_1 .

The effective potential is invariant under $\text{Re}(a_1) \rightarrow -\text{Re}(a_1)$ and/or $\text{Im}(a_1) \rightarrow -\text{Im}(a_1)$, and the plot for other parameter range of a_1 is obtained through this invariance. The plot of the effective potential at u_1 -dyon point is obtained by exchanging $a_1 \rightarrow -a_1$. In conclusion, the point $u \rightarrow -1$ and $a_1 \rightarrow 0$ is expected to be a local vacuum.

However, note that our description is not applicable for very small $|a_1|$, since the condensations of the two dyon states are going to overlap with each other (see Fig. 4). Unfortunately, we have no knowledge about the correct description of the effective theory in this situation. Nevertheless, we conclude that there must appear a local minimum with broken SUSY in the limit $a_1 \rightarrow 0$: In this limit, the effective potential without the dyon condensations is depicted in the left figure in Fig. 5. We find that a potential minimum appears at $u = -\Lambda^2/8 = -1$, and the

value of the effective potential at the cusp is nonzero, $V \simeq 0.0047 > 0$. If we had the correct description of the effective theory for $a_1 = 0$, this cusp might be smoothed out. However, there is no reason for SUSY to be restored at $u = -1$, because the correct effective theory must have no singularity in the Kähler metric. Therefore, there is the promising possibility of the appearance of a local minimum with broken SUSY at $u = -1$ and $a_1 = 0$. Note that in this local minimum the global $\mathbf{Z}_8 \subset U(1)_R$ symmetry is broken down to \mathbf{Z}_4 .

$$(iii) \text{Re}(a_1) > \frac{\Lambda}{2\sqrt{2}}$$

Let us get back to the case of $\text{Im}(a_1) = 0$. For $a_1 > \frac{\Lambda}{2\sqrt{2}}$, the effective potential has two minima, V_{\min}^1 and V_{\min}^3 at two singular points u_1 and u_3 . The dyon condensation is too small for the effective potential to have a minimum at

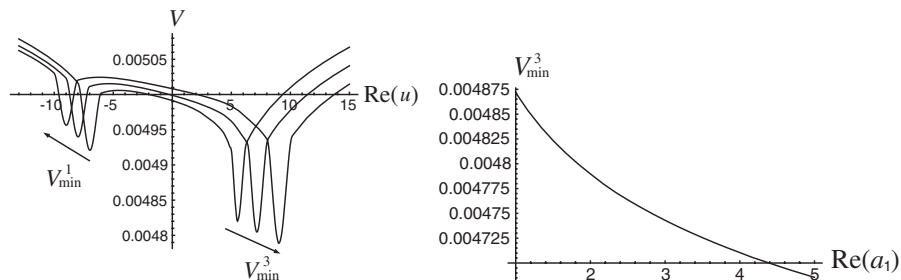


FIG. 9. The evolution of the potential minima at the singular points as a_1 varies on the real u axis (left) and on the real a_1 axis (right).

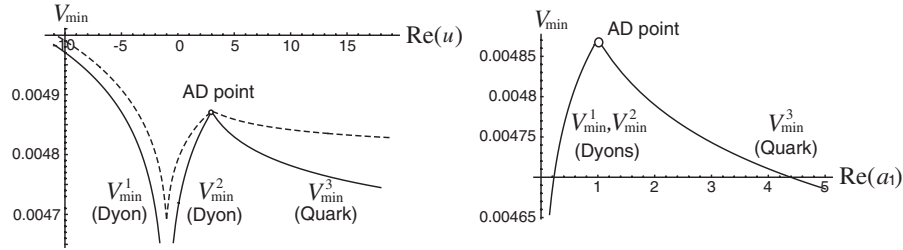


FIG. 10. The effective potential energy at each singular point.

u_2 . The plot of the effective potential is shown in Fig. 9. While the evolution of the potential energy with the singular point u_1 is the same as for $0 < a_1 < \frac{\Lambda}{2\sqrt{2}}$, the potential energy on the quark singular point at u_3 is monotonically decreasing, as a_1 is increasing. Thus, there is a runaway direction along the flow of the quark singular point. We can find the same global structure along the flow of the quark singular point for general complex a_1 values.

The evolutions of the potential energies according to the flows of the singular points along the real u -axis are simultaneously plotted in Fig. 10. The global structure of the effective potential is of the runaway type. However, we found the promising possibility that there exists a local minimum with broken SUSY in the theory. Precisely speaking, since there is no well-defined vacuum in the runaway direction, this minimum with broken SUSY is the unique and promising candidate for the vacuum in the theory. Unfortunately, we have no knowledge of the correct description about the effective theory around the degenerate dyon point.

Finally, we would like to comment on the possible SUSY vacua which are present on the Higgs branch at the classical level. As we have mentioned in Sec. II, SUSY vacua at the point $a_1 = a_2 = 0$ would exist even at the quantum level. At the classical level, the SUSY breaking vacua on the pseudoflat direction roll down to the SUSY vacua on the Higgs branch near the origin $|a_1|, |a_2| \sim \sqrt{\xi}$. In our analysis of the Coulomb branch, the effective potential (3.7) does not give such a picture. Instead, we have found that the effective potential realizes a local minimum at $u \sim \Lambda^2 \gg \xi$, such that the minimum is far away from the Higgs branch. In order to realize the SUSY vacua on the Higgs branch, which would exist even in the quantum level, appropriate hypermultiplets should be introduced in the effective action, while we, unfortunately, do not know the correct treatment of such degrees of freedom.

V. LIFETIME OF THE LOCAL MINIMUM

In this model, we have found two possible kinds of SUSY vacua: A runaway vacuum at infinity in moduli space on the Coulomb branch and the vacua on the Higgs branch. Here we will estimate the decay rate from the local vacuum to these SUSY vacua.

First we estimate it for the runaway vacuum. As we discussed earlier, consistency requires that we restrict the moduli space to a region bounded by the Landau pole. When the boundary of moduli space is located far away from the dynamical scale, the potential energy at the boundary is almost zero, and the true, almost SUSY, vacuum of the theory lies somewhere in this region. If our world is trapped in the local minimum we found in the previous sections, it will eventually decay to this approximately supersymmetric vacuum. The decay rate is expected to be very small, as the potential barrier is very wide.

As analyzed in the previous section, the effective potential can be described as a function of the modulus parameter a_1 . In our calculation, the effective potential is treated in the triangle approximation [33]. Let us take the path in the direction of $\text{Re}(a_1)$: climbing up from the local minimum ($a_1 = 0$) to the AD point ($a_1 = \Lambda/2\sqrt{2}$), then rolling down to Landau pole point ($a_1 = \Lambda_L$). This is similar to the situation in the ISS model.

In the triangle approximation, parameters characterizing the potential are

$$\Delta V_{\pm}, \quad \Delta \Phi_{\pm}, \quad (5.1)$$

where ΔV_{\pm} and $\Delta \Phi_{\pm}$ are the difference of potential height and the distances between local/Landau pole points and potential barrier (see Fig. 11). Following Ref. [33], we define

$$\lambda_{\pm} \equiv \frac{\Delta V_{\pm}}{\Delta \Phi_{\pm}}, \quad c \equiv \frac{\lambda_-}{\lambda_+} = \frac{\Delta V_-}{\Delta V_+} \frac{\Delta \Phi_+}{\Delta \Phi_-}. \quad (5.2)$$

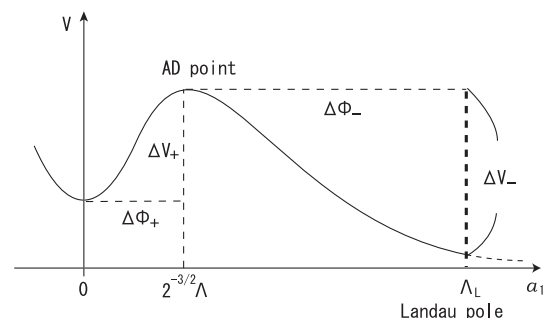


FIG. 11. Schematic picture of the effective potential.

In our case,

$$\Delta\Phi_+ \sim \Lambda, \quad \Delta\Phi_- \sim \Lambda_L, \quad (5.3)$$

and the height of the effective potential is controlled by the SUSY breaking order parameter ξ as

$$V \sim \xi^2. \quad (5.4)$$

Through numerical analysis, the ratio $\Delta V_-/\Delta V_+$ is estimated to be $\mathcal{O}(10)$, so that the condition of Eq. (13) in Ref. [33] can be satisfied,

$$\left(\frac{\Delta V_-}{\Delta V_+}\right)^{1/2} \geq \frac{2\Delta\Phi_-}{\Delta\Phi_- - \Delta\Phi_+} = \frac{2}{1 - \frac{\Delta\Phi_+}{\Delta\Phi_-}} \sim 2. \quad (5.5)$$

Here we have used

$$\frac{\Delta\Phi_+}{\Delta\Phi_-} \sim \frac{\Lambda}{\Lambda_L} \ll 1. \quad (5.6)$$

Since for our choice of parameters, $\Lambda_L = 10^{17-18}$ in units of Λ , we can safely use the formula of the bounce action [33],

$$B = \frac{32\pi^2}{3} \frac{1+c}{(\sqrt{1+c}-1)^4} \frac{\Delta\Phi_+^4}{\Delta V_+}. \quad (5.7)$$

Because the parameter c is very small

$$c = \frac{\lambda_-}{\lambda_+} = \frac{\Delta V_-}{\Delta V_+} \frac{\Delta\Phi_+}{\Delta\Phi_-} \sim 10 \frac{\Lambda}{\Lambda_L} \ll 1, \quad (5.8)$$

we find

$$B \sim \frac{\Lambda_L^4}{\xi^2} \gg \frac{\Lambda^4}{\xi^2} \gg 1. \quad (5.9)$$

Here we used the condition $\Lambda^2 \gg \xi$ for our analysis in the previous section to be theoretically consistent. As a result, the decay rate per unit volume $\Gamma/V \sim e^{-B}$ from the local minimum to the Landau pole point is very small, and the vacuum at the local minimum is very long-lived, i.e. metastable, as expected. Although the final formula seems to indicate that the decay rate becomes zero in the limit $\Lambda_L \rightarrow \infty$, it is, in fact, nonzero due to the barrier penetration from the local minimum to the runaway direction.

Next we estimate the decay rate from the local minimum to the SUSY vacua present on the Higgs branch. The most conservative path to such SUSY vacua from the local one is first climbing up to the origin $a_1 = a_2 = 0$, $q = \tilde{q} = 0$ and then rolling down to the SUSY vacuum $a_1 = a_2 = 0$, $q, \tilde{q} \sim \sqrt{\xi}$. In this situation, the potential parameters are estimated from the numerical analysis to be

$$\begin{aligned} \Delta\Phi_+ &= \Lambda = 2\sqrt{2}, & \Delta\Phi_- &= \sqrt{\xi} = \sqrt{0.1}, \\ \Delta V_+ &= 0.00891, & \Delta V_- &= 0.00421. \end{aligned} \quad (5.10)$$

These parameters do not satisfy the condition (5.5), so that we can not use the formula (13) in Ref. [33]. Instead, we can use another formula (20) in Ref. [33],

$$B = \frac{\pi^2 \lambda_+^2 R_T^3}{96} [-\beta_+^3 + 3c\beta_+^2\beta_- + 3c\beta_+\beta_-^2 - c^2\beta_-^3]. \quad (5.11)$$

Here

$$\beta_{\pm} \equiv \sqrt{\frac{8\Delta\Phi_{\pm}}{\lambda_{\pm}}}, \quad R_T = \frac{1}{2} \left(\frac{\beta_+^2 + c\beta_-^2}{c\beta_- - \beta_+} \right). \quad (5.12)$$

The bounce action is estimated to be $B \sim \mathcal{O}(10^6) \gg 1$ from the numerical values of the potential parameters (5.10). Thus the decay rate from the local minimum to the SUSY vacua can be very small.

VI. APPLICATION TO PHENOMENOLOGY

We have found a metastable vacuum with broken SUSY and also broken $U(1)_R$ symmetry in the previous sections. Here we address the application of our model to phenomenology. Supersymmetric extensions of the standard model have been considered as one of the most promising ways to solve the gauge hierarchy problem in the standard model. Since any supersymmetric partners of the standard model particles have not been observed yet, supersymmetry should be broken at low energies. The origin of supersymmetry breaking and its mediation to the supersymmetric version of the standard model are still prime questions in particle physics. As mentioned several times before, in order to obtain a realistic model, $U(1)_R$ symmetry breaking is necessary as well as breaking of supersymmetry. This is because $U(1)_R$ symmetry forbids gauginos to obtain masses. From this point of view, the metastable vacuum we have found is suitable for phenomenology. Furthermore, the model automatically provides the structure necessary in the gauge mediation scenario.

First, let us give a brief review on the gauge mediation scenario [6]. The basic structure of this scenario is described as the messenger sector superpotential,

$$W = S\tilde{\Phi}\Phi, \quad (6.1)$$

where S is a gauge singlet chiral superfield, and $\tilde{\Phi}$ and Φ are a vectorlike pair of chiral superfields, so-called messenger fields, which are charged under the standard model gauge group. Suppose that both the scalar component and the F component of the singlet superfield S develop VEVs so that SUSY and also $U(1)_R$ symmetry are broken. Through quantum corrections with messenger fields, gauginos and scalar partners of the standard models particles obtain soft SUSY breaking masses,

$$M_{\text{soft}} \sim \frac{\alpha_{\text{SM}} \langle F_S \rangle}{4\pi \langle S \rangle}, \quad (6.2)$$

where $\langle S \rangle$ and $\langle F_S \rangle$ are VEVs of the scalar and the F component of the superfield S , α_{SM} stands for the standard model gauge coupling.

Now we return to our model. At the metastable vacuum, $u \rightarrow -\Lambda^2/8$ and $a_1 \rightarrow 0$, the model possesses flavor symmetry $SU(2)_- \times SU(2)_+$ [20]. The BPS states, dyon hypermultiplet, which describe the low energy effective theory around the metastable vacuum, belong to a doublet under $SU(2)_+$. Note that the model also includes the other massless hypermultiplets (\tilde{D} and D) doublets under $SU(2)_-$ at the singular point, $u = \Lambda^2/8$. However, they can no longer be massless at the other singular point and they exist as massive states in the metastable vacuum. One can understand such a structure in the classical theory. In the classical superpotential in Eq. (2.2),

$$W = \sqrt{2}\tilde{Q}_{I_r}(A_2 + A_1)^I J^r, \quad (6.3)$$

considering that the Cartan part of $A_2 \sim \text{diag}(A_2/2, -A_2/2)$ is left in the low energy effective theory, there are two moduli points, $a_2/2 + a_1 = 0$ and $-a_2/2 + a_1 = 0$, where the hypermultiplets are massless. But at one of the moduli points, some hypermultiplets are massless, but the others are massive and are integrated out. The same structure should be realized at the quantum level. Now the massive hypermultiplet would have the following form of the superpotential,

$$W = \sqrt{2}\tilde{D}_r(n_m A_D + n_e A + n A_1)D^r, \quad (6.4)$$

with certain quantum numbers $(n_e, n_m)_n$ as in Eq. (3.4). Since $\langle A_D \rangle \sim \Lambda$ at the local minimum we have found, the massive hypermultiplets are heavy and integrated out from the low energy effective theory. However, once we take supersymmetry breaking effects into account at the minimum, this superpotential is found to play an important role in phenomenology.

Supersymmetry is broken at the local minimum and so the F component of A_D develops a VEV characterized as $\langle F_{A_D} \rangle \sim \xi$, so that Eq. (6.4) has the same structure as Eq. (6.1) with $A_D = \langle A_D \rangle + \theta^2 \langle F_{A_D} \rangle \sim \Lambda + \theta^2 \xi$. Therefore, when the $SU(2)$ flavor symmetry is weakly gauged as, for example, the $SU(2)$ weak gauge group in the standard model, the hypermultiplets, \tilde{D} and D , play the role of messenger fields and the $SU(2)$ gaugino and all doublet scalars in the supersymmetric standard model obtain masses through the gauge mediation such as

$$M_{\text{soft}} \sim \frac{\alpha_2}{4\pi} \frac{\xi}{\Lambda}, \quad (6.5)$$

where α_2 is the $SU(2)$ weak gauge coupling. Suitable choices of model parameters, supersymmetry breaking order parameter ξ and the messenger scale Λ , lead to phenomenologically favored values for soft supersymmetry breaking masses around 1 TeV.

In order to obtain a more realistic phenomenological model, it is necessary to extend our model so as to provide a larger flavor symmetry. For example, an ideal choice would be an $SU(5)$ flavor symmetry, whose subgroup can be gauged as the standard model gauge group $SU(3)_C \times$

$SU(2)_L \times U(1)_Y \subset SU(5)$ so that all gauginos and scalar partners obtain masses. To implement such a large global symmetry into a supersymmetric gauge theory, a model should be based on a general $SU(N)$ ($N > 2$) gauge group with an appropriate number of flavors. It is nontrivial to construct such a model more suitable for phenomenology, and we leave this issue for future works.

Our SUSY breaking model is based on $\mathcal{N} = 2$ SUSY gauge theories, while the supersymmetric standard model is a chiral theory and should obey $\mathcal{N} = 1$ supersymmetry. It may be somewhat unusual to realize such a setup in four dimensions. As a natural realization, we can consider a $\mathcal{N} = 1$ five-dimensional brane world scenario, where the SUSY breaking sector resides in the bulk while the SUSY standard model sector resides on a ‘‘3-brane.’’ The Lagrangian for the bulk fields is described in terms of $\mathcal{N} = 2$ SUSY theory in four-dimensional point of view, while that for the brane fields obeys only $\mathcal{N} = 1$ supersymmetry. If we could extend our four-dimensional model to a five-dimensional one, such a natural phenomenological model would be realized.

VII. CONCLUSION

We have investigated an $\mathcal{N} = 2$ supersymmetric gauge theory based on the gauge group $SU(2) \times U(1)$ with $N_f = 2$ flavors and the FI term associated with the $U(1)$ gauge group. Thanks to the exact results in $\mathcal{N} = 2$ supersymmetric gauge theories, we can analyze the model beyond perturbation theory with respect to the $SU(2)$ gauge coupling, but as a perturbation with respect to the FI term smaller than the $SU(2)$ dynamical scale. We have found that the effective potential exhibits a local SUSY breaking minimum at the degenerate dyon point due to the strong $SU(2)$ dynamics. On the other hand, away from the origin of the moduli space, the potential energy decreases as we move toward infinity eventually realizing an almost SUSY vacuum. In addition to this runaway vacuum, there are SUSY vacua on the Higgs branch which survive quantum corrections. We have estimated the decay rate of the local minimum in the triangle approximation and found that the false vacuum is parametrically long-lived. In this metastable vacuum, not only SUSY but also the R symmetry are broken. Interestingly, the basic structure of a messenger sector in the gauge mediation scenario is inherent in our model in the metastable vacuum. Once the flavor symmetry among massive hypermultiplets is gauged as the standard model gauge group, they play the role of messenger fields and supersymmetry breaking is transmitted into the SUSY standard model sector through the standard model gauge interactions. In order to obtain a more realistic phenomenological model, it is necessary to enlarge the gauge group so as to include more flavors. It is an interesting question whether such a model still exhibits a metastable vacuum suitable for phenomenology. This direction is worth investigating in the future.

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APPENDIX: DERIVATIONS OF THE EFFECTIVE COUPLINGS IN TERM OF THE WEIERSTRASS FUNCTIONS

In this appendix, we exhibit the derivations of the effective couplings in term of the Weierstrass functions. The derivations are applicable for all the case of flavors ($N_f = 1, 2, 3$), so that we shall write the dynamical scale Λ_{N_f} , corresponding to each flavor case. It is convenient to introduce the uniformization variable z through the map with the Weierstrass \wp function,

$$(\wp(z), \wp'(z)) = (X, Y). \quad (\text{A1})$$

Using this map, the half period $\omega_i/2$ is mapped into the root $e_i = \wp(\omega_i/2)$ ($\omega_3 = \omega_1 + \omega_2$). The inverse map is defined as

$$z_0 = \Psi^{-1}(x_0) = \int_{x_0}^{\infty} \frac{dX}{Y} = -\frac{1}{\sqrt{e_2 - e_1}} F(\phi, k), \quad (\text{A2})$$

where we changed the integration variable X by $t^2 = (e_2 - e_1)/(X - e_1)$, and $F(\phi, k)$ is the incomplete elliptic integral given by

$$F(\phi, k) = \int_0^{\sin\phi} \frac{dt}{[(1-t^2)(1-k^2t^2)]^{1/2}}; \quad (\text{A3})$$

$$\sin^2\phi = \frac{e_2 - e_1}{x_0 - e_1}.$$

We derive the effective couplings, τ_{12} and τ_{11} , by using the map of Eq. (A1). The effective coupling τ_{12} is described by

$$\tau_{12} = \left. \frac{\partial a_{2D}}{\partial a_1} \right|_{a_2} = \left. \frac{\partial a_{2D}}{\partial a_1} \right|_u - \tau_{22} \left. \frac{\partial a_2}{\partial a_1} \right|_u. \quad (\text{A4})$$

The partial derivative of the periods a_{2D} and a_2 with respect to a_1 can be calculated using Eqs. (3.16) and (3.17) as

$$\left. \frac{\partial a_{2i}}{\partial a_1} \right|_u = \oint_{\alpha_i} \frac{\partial \lambda_{SW}}{\partial a_1} \Big|_u$$

$$= Q^{(N_f)}(a_1, \Lambda_{N_f}) \int_{e_j}^{e_3} \frac{dX}{2Y(X-c)} (i \neq j), \quad (\text{A5})$$

where the coefficient $Q^{(N_f)}$ is given by

$$Q^{(N_f)}(a_1, \Lambda_{N_f}) = -\frac{N_f(\sqrt{2}a_1)^{N_f-1} \Lambda_{N_f}^{4-N_f}}{16\pi}. \quad (\text{A6})$$

Using the map of Eq. (A1), the integral can be described as

$$\left. \frac{\partial a_{2i}}{\partial a_1} \right|_u = Q^{(N_f)} \int_{\omega_j}^{\omega_3} \frac{dz}{2(\wp(z) - \wp(z_0))}$$

$$= \frac{Q^{(N_f)}}{2} \frac{1}{\wp'(z_0)} \left(\log \frac{\sigma(z - z_0)}{\sigma(z + z_0)} + 2z\zeta(z_0) \right), \quad (\text{A7})$$

where $\wp(z_0) = c$, $\zeta(z)$ is the Weierstrass zeta function, and we used the definition of the Weierstrass sigma function, $\zeta(z) = \frac{d}{dz} \log \sigma(z)$, and the relation

$$\frac{\wp'(z_0)}{\wp(z) - \wp(z_0)} = \zeta(z - z_0) - \zeta(z + z_0) + 2\zeta(z_0). \quad (\text{A8})$$

Taking into account that Y corresponds to $\wp'(z)$ under the map of Eq. (A1), the pole $\wp'(z_0)$ can be easily obtained as

$$\wp'(z_0)^2 = -\left(\frac{N_f 2^{(N_f-1)/2} \Lambda_{N_f}^{4-N_f}}{32} \right)^2. \quad (\text{A9})$$

Using the pseudoperiodicity of the Weierstrass sigma function,

$$\sigma(z_0 + \omega_i) = -\sigma(z_0) \exp\left(2\zeta\left(\frac{\omega_i}{2}\right) \left(z_0 + \frac{1}{2}\omega_i \right) \right), \quad (\text{A10})$$

we obtain

$$\left. \frac{\partial a_{2i}}{\partial a_1} \right|_u = -\frac{1}{\pi i} \left[\omega_i \zeta(z_0) - 2z_0 \zeta\left(\frac{\omega_i}{2}\right) \right]. \quad (\text{A11})$$

The zeta function at half period can be described by integral representations as

$$\zeta\left(\frac{\omega_i}{2}\right) = -I_2^{(i)}. \quad (\text{A12})$$

Substituting Eq. (A11) into Eq. (A4) and using the Legendre relation

$$\omega_1 \zeta\left(\frac{\omega_2}{2}\right) - \omega_2 \zeta\left(\frac{\omega_1}{2}\right) = i\pi, \quad (\text{A13})$$

we finally obtain

$$\tau_{12} = -\frac{2z_0}{\omega_2}. \quad (\text{A14})$$

Next we derive the effective coupling τ_{11} , which is given by differentiating a_{1D} of Eq. (3.36) with respect to a_1 with a_2 fixed such as

$$\tau_{11} = -\int_{x_n^-}^{x_n^+} \left[\left. \frac{\partial \lambda_{SW}}{\partial u} \right|_{a_1} \left. \frac{\partial u}{\partial a_1} \right|_{a_2} + \left. \frac{\partial \lambda_{SW}}{\partial a_1} \right|_u \right] + \frac{\partial \tilde{C}}{\partial a_1}. \quad (\text{A15})$$

The integral can be evaluated by using the map (A1). Although the integral contains a divergence, it can be

regularized by using the freedom of the integration constant \tilde{C} . Let us demonstrate this regularization by introducing the regularization parameter ϵ as follows.

$$\begin{aligned}
\tau_{11} &= - \int_{x_0^- + \epsilon}^{x_0^+ + \epsilon} \left[\frac{\partial \lambda_{SW}}{\partial u} \Big|_{a_1} \left(- \frac{\partial u}{\partial a_2} \Big|_{a_1} \frac{\partial a_2}{\partial a_1} \Big|_{a_2} \right) + \frac{\partial \lambda_{SW}}{\partial a_1} \Big|_u \right] + \frac{\partial \tilde{C}}{\partial a_1} \\
&= - \int_{-z_0 + \epsilon}^{z_0 + \epsilon} dz \left[- \frac{1}{\pi i \omega_2} \left(\omega_2 \zeta(z_0) - 2z_0 \zeta\left(\frac{\omega_2}{2}\right) \right) + \frac{Q^{(N_f)}}{4(\wp(z) - \wp(z_0))} \right] + \frac{\partial \tilde{C}}{\partial a_1} \\
&= - \frac{1}{\pi i} \left(\log \sigma(2z_0) - \frac{4z_0^2}{\omega_2} \zeta\left(\frac{\omega_2}{2}\right) \right) + \frac{1}{\pi} \log \sigma(\epsilon) + \frac{1}{2} + \frac{\partial \tilde{C}}{\partial a_1}.
\end{aligned} \tag{A16}$$

The divergent part, $\log \sigma(\epsilon)$, can be subtracted by taking the integration constant such that $\tilde{C} = Ca_1 - \frac{a_1}{2} - \frac{a_1}{\pi} \log \sigma(\epsilon)$, and we finally obtain Eq. (3.37) with the relation of Eq. (A12).

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