

## Higgs Boson Mass Bounds in a Type II Seesaw Model with Triplet Scalars

I. Gogoladze – University of Delaware

N. Okada – Theory Division, KEK

Q. Shafi – University of Delaware

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**Higgs boson mass bounds in a type II seesaw model with triplet scalars**Ilia Gogoladze,<sup>1,\*</sup> Nobuchika Okada,<sup>2,+</sup> and Qaisar Shafi<sup>1,‡</sup><sup>1</sup>*Bartol Research Institute, Department of Physics and Astronomy, University of Delaware, Newark, Delaware 19716, USA*<sup>2</sup>*Theory Division, KEK, Tsukuba 305-0801, Japan*

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In a type II seesaw utilized to explain the observed neutrino masses and mixings, one extends the standard model (SM) by introducing scalar fields which transform as a triplet under the electroweak gauge symmetry. New scalar couplings involving the Higgs doublet then appear and, as we show, these have important implications for the Higgs boson mass bounds obtained using vacuum stability and perturbativity arguments. We identify, in particular, regions of the parameter space which permit the SM Higgs boson to be as light as 114.4 GeV, the LEP2 bound. The triplet scalars include doubly charged particles whose masses could, in principle, be in the few hundred GeV range, and so they may be accessible at the LHC.

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The discovery of the standard model (SM) Higgs boson is arguably the single most important mission for the LHC. Under the somewhat radical assumption that the next energy frontier lies at Planck scale ( $M_{\text{pl}}$ ), it has been found that the SM Higgs boson mass lies in the range  $127 \text{ GeV} \leq M_H \leq 170 \text{ GeV}$  [1]. Here the lower bound of 127 GeV on  $M_H$  derives from arguments based on the stability of the SM vacuum (more precisely, that the Higgs quartic coupling does not turn negative at any scale between  $M_Z$  and  $M_{\text{pl}}$ ). Thus, from the point of view of the SM it is perhaps not too surprising that the Higgs boson has not yet been found. The upper bound of about 170 GeV on  $M_H$  comes from the requirement that the Higgs quartic coupling remains perturbative and does not exceed  $\sqrt{4\pi}$ , say, during its evolution between  $M_Z$  and  $M_{\text{pl}}$ .

It has become abundantly clear in recent years that an extension of the SM is needed to explain a number of experimental observations. These include solar and atmospheric neutrino oscillations [2], existence of nonbaryonic dark matter [3], the observed baryon asymmetry of the Universe, etc. Neutrino oscillations, in particular, cannot be understood within the SM, even after including dimension five operators with Planck scale cutoff. These operators yield neutrino masses of order  $10^{-5}$  eV or less, which is far below the 0.05–0.01 eV scale needed to explain the observed atmospheric and solar neutrino oscillations, respectively.

Two attractive seesaw mechanisms exist for explaining the measured neutrino masses (more accurately, mass differences squared). In the so-called type I seesaw [4], new physics is added to the SM by introducing at least two heavy right-handed neutrinos. The seesaw mechanism then ensures that the observed neutrinos acquire masses which are suppressed by the heavy right-handed neutrino mass

scale(s). One expects that the heaviest right-handed neutrino has a mass less than or of order of  $10^{14}$  GeV (this, roughly speaking, comes from the seesaw formula  $m_D^2/M_R$  for the light neutrino mass, where  $m_D$  and  $M_R$  denote the Dirac and right-handed neutrino masses, respectively, and assumes that  $m_D$  is less than or of order of the electroweak scale).

In the type II seesaw [5], the SM is supplemented by an  $SU(2)_L$  triplet scalar field  $\Delta$  which also carries unit hypercharge. There exist renormalizable couplings  $\ell^T \Delta \ell$  which enable the neutrinos to acquire their tiny (observed) masses through the nonzero vacuum expectation value (VEV) of  $\Delta$ .

From our point of view one of the most interesting features in the type II seesaw derives from the fact that the  $SU(2)_L$  triplet  $\Delta$  interacts with the SM Higgs doublet via both cubic and quartic scalar couplings. These, as we will show in this paper, can have far reaching implications for the SM bounds on  $M_H$ , which can be studied by employing the coupled renormalization group equations (RGEs) involving the SM Higgs doublet  $\phi$  and  $\Delta$ . We find, in particular, that for a plausible choice of parameters, the SM Higgs boson mass  $M_H$  can be as low as the LEP2 bound of 114.4 GeV [6].

This is in sharp contrast with the type I seesaw in which the lower bound of 127 GeV for  $M_H$  is increased by the presence of Dirac Yukawa coupling(s) involving right-handed neutrinos [7]. With a Dirac Yukawa coupling equal, say, to unity, the vacuum stability bound is increased to 156 GeV, while the perturbativity bound remains close to 170 GeV. The key difference is that in the type II seesaw a mass for the SM Higgs boson, say in the range of 115–127 GeV, is easily realized, which is not the case for the type I seesaw.

Before moving to the technical part let us note that  $\Delta$  contains doubly charged particles which, if not too heavy, may be produced at the LHC and Tevatron [8]. It is amusing that with the type II seesaw, a “light” SM Higgs is

\*ilia@physics.udel.edu

+okadan@post.kek.jp

‡shafi@bartol.udel.edu

consistent with relatively light  $\Delta$  particles with masses around a few hundred GeV. Of course, it may turn out that the mass scale for  $\Delta$  lies well above the TeV range, in which case we will only find the light Higgs boson at the LHC.

We begin by introducing a Higgs triplet scalar  $\Delta$ , which transforms as  $(\mathbf{3}, 1)$  under the electroweak gauge group  $SU(2)_L \times U(1)_Y$ :

$$\Delta = \frac{\sigma^i}{\sqrt{2}} \Delta_i = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}. \quad (1)$$

The scalar potential involving both the SM Higgs doublet and the Higgs triplet is given by (throughout this paper, we follow the notation of Ref. [9], except that we employ lower case Greek letters for dimensionless couplings)

$$\begin{aligned} V(\Delta, \phi) = & -m_\phi^2(\phi^\dagger \phi) + \frac{\lambda}{2}(\phi^\dagger \phi)^2 + M_\Delta^2 \text{tr}(\Delta^\dagger \Delta) \\ & + \frac{\lambda_1}{2}(\text{tr} \Delta^\dagger \Delta)^2 + \frac{\lambda_2}{2}[(\text{tr} \Delta^\dagger \Delta)^2 - \text{tr}(\Delta^\dagger \Delta \Delta^\dagger \Delta)] \\ & + \lambda_4 \phi^\dagger \phi \text{tr}(\Delta^\dagger \Delta) + \lambda_5 \phi^\dagger [\Delta^\dagger, \Delta] \phi \\ & + \left[ \frac{\Lambda_6}{\sqrt{2}} \phi^T i \sigma_2 \Delta^\dagger \phi + \text{H.c.} \right], \end{aligned} \quad (2)$$

where  $\phi$   $(\mathbf{2}, 1/2)$  is the SM Higgs doublet. Without loss of generality the coupling constants  $\lambda_i$  are taken to be real through a phase rotation of  $\Delta$ . Note that we define a dimensionless parameter  $\lambda_6 \equiv \Lambda_6/M_\Delta$ . The Higgs triplet has a Yukawa coupling with the lepton doublets ( $\ell_L^i$ , with generation index  $i$ ) of the form,

$$\mathcal{L}_\Delta = -\frac{1}{\sqrt{2}}(Y_\Delta)_{ij} \ell_L^{Ti} C i \sigma_2 \Delta \ell_L^j + \text{H.c.}, \quad (3)$$

where  $C$  is the Dirac charge conjugate matrix and  $(Y_\Delta)_{ij}$  denotes elements of the Yukawa matrix.

Assuming the hierarchy  $M_Z \ll M_\Delta$  and integrating out the heavy Higgs triplet, we obtain a low energy effective potential for the SM doublet,

$$V(\phi)_{\text{eff}} = -m_\phi^2(\phi^\dagger \phi) + \frac{1}{2}(\lambda - \lambda_6^2)(\phi^\dagger \phi)^2. \quad (4)$$

Below  $M_\Delta$  the SM Higgs quartic coupling is given by [10]

$$\lambda_{\text{SM}} = \lambda - \lambda_6^2. \quad (5)$$

For a given  $\lambda_6$ , the Higgs quartic coupling is shifted down to the SM coupling by  $\lambda_6^2$  through this matching condition at  $\mu = M_\Delta$ , so that the resulting Higgs boson mass, as we will show, is lowered.

A nonzero VEV ( $v = 246.2$  GeV) for the Higgs doublet induces a tadpole term for  $\Delta$  via the last term in Eq. (2). A nonzero VEV of the Higgs triplet is thereby generated,  $\langle \Delta \rangle \sim \lambda_6 v^2/M_\Delta$ , which then provides the desired neutrino masses using Eq. (3).

Note that the triplet Higgs VEV contributes to the weak boson masses and alters the  $\rho$ -parameter from the SM prediction,  $\rho \approx 1$ , at tree level. The current precision

measurement [11] constrains this deviation to be in the range,  $\Delta\rho = \rho - 1 \approx \langle \Delta \rangle/v \lesssim 0.01$ , so that  $\lambda_6 \lesssim 0.01 M_\Delta/v$ . This constraint is especially relevant when we consider  $M_\Delta \sim \text{TeV}$ , in which case the region  $\lambda_6 \gtrsim 0.1$  is excluded.

We are now ready to analyze the Higgs boson mass bounds from vacuum stability and perturbativity constraints in the presence of the type II seesaw mechanism. There are several new parameters  $\lambda_1, \lambda_2, \lambda_4, \lambda_5, \lambda_6$ , and  $Y_\Delta$ , which potentially affect the RGE running of the Higgs quartic coupling and thus the corresponding Higgs boson mass. We have already noted the important role that  $\lambda_6$  plays via the matching condition in Eq. (5). It turns out that both  $\lambda_4$  and  $\lambda_5$  will also play an important role through their contributions to the renormalization group evolution of the Higgs quartic coupling. They help to lower both the vacuum stability and perturbativity bounds on the SM Higgs mass. In our analysis, we employ two-loop RGEs for the SM couplings and one-loop RGEs for the new couplings associated with the type II seesaw scenario.

For renormalization scale  $\mu < M_\Delta$ , the triplet Higgs is decoupled. For the three SM gauge couplings, we have

$$\frac{dg_i}{d \ln \mu} = \frac{b_i}{16\pi^2} g_i^3 + \frac{g_i^3}{(16\pi^2)^2} \sum_{j=1}^3 b_{ij} g_j^2, \quad (6)$$

where  $g_i$  ( $i = 1, 2, 3$ ) are the SM gauge couplings and

$$b_i = \left( \frac{41}{10}, -\frac{19}{6}, -7 \right), \quad b_{ij} = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{6}{2} & -26 \end{pmatrix}. \quad (7)$$

The top quark pole mass is taken to be the central value  $M_t = 170.9$  GeV [12], with  $(\alpha_1, \alpha_2, \alpha_3) = (0.01681, 0.03354, 0.1176)$  at the  $Z$ -pole ( $M_Z$ ) [11]. For the top Yukawa coupling  $y_t$ , we have [13],

$$\frac{dy_t}{d \ln \mu} = y_t \left( \frac{1}{16\pi^2} \beta_t^{(1)} + \frac{1}{(16\pi^2)^2} \beta_t^{(2)} \right). \quad (8)$$

Here the one-loop contribution is

$$\beta_t^{(1)} = \frac{9}{2} y_t^2 - \left( \frac{17}{20} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2 \right), \quad (9)$$

while the two-loop contribution is given by

$$\begin{aligned} \beta_t^{(2)} = & -12 y_t^4 + \left( \frac{393}{80} g_1^2 + \frac{225}{16} g_2^2 + 36 g_3^2 \right) y_t^2 + \frac{1187}{600} g_1^4 \\ & - \frac{9}{20} g_1^2 g_2^2 + \frac{19}{15} g_1^2 g_3^2 - \frac{23}{4} g_2^4 + 9 g_2^2 g_3^2 - 108 g_3^4 \\ & + \frac{3}{2} \lambda^2 - 6 \lambda y_t^2. \end{aligned} \quad (10)$$

In solving Eq. (8), the initial top Yukawa coupling at  $\mu = M_t$  is determined from the relation between the pole mass and the running Yukawa coupling [14, 15],

$$M_t \approx m_t(M_t) \left( 1 + \frac{4}{3} \frac{\alpha_3(M_t)}{\pi} + 11 \left( \frac{\alpha_3(M_t)}{\pi} \right)^2 - \left( \frac{m_t(M_t)}{2\pi v} \right)^2 \right), \quad (11)$$

with  $y_i(M_t) = \sqrt{2}m_i(M_t)/v$ , where  $v = 246.2$  GeV. Here, the second and third terms in the parenthesis correspond to one- and two-loop QCD corrections, respectively, while the fourth term comes from the electroweak corrections at one-loop level. The numerical values of the third and fourth terms are comparable (signs are opposite). The electroweak corrections at two-loop level and the three-loop QCD corrections [15] are of comparable and sufficiently small magnitude [15] to be safely ignored.

The RGE for the Higgs quartic coupling is given by [13],

$$\frac{d\lambda}{d\ln\mu} = \frac{1}{16\pi^2} \beta_\lambda^{(1)} + \frac{1}{(16\pi^2)^2} \beta_\lambda^{(2)}, \quad (12)$$

with

$$\beta_\lambda^{(1)} = 12\lambda^2 - \left( \frac{9}{5}g_1^2 + 9g_2^2 \right) \lambda + \frac{9}{4} \left( \frac{3}{25}g_1^4 + \frac{2}{5}g_1^2g_2^2 + g_2^4 \right) + 12y_t^2\lambda - 12y_t^4, \quad (13)$$

and

$$\begin{aligned} \beta_\lambda^{(2)} = & -78\lambda^3 + 18 \left( \frac{3}{5}g_1^2 + 3g_2^2 \right) \lambda^2 - \left( \frac{73}{8}g_1^4 - \frac{117}{20}g_1^2g_2^2 \right. \\ & + \frac{2661}{100}g_1^4 \lambda - 3\lambda y_t^4 + \frac{305}{8}g_2^6 - \frac{289}{40}g_1^2g_2^4 - \frac{1677}{200}g_1^4g_2^2 \\ & - \frac{3411}{1000}g_1^6 - 64g_3^2y_t^4 - \frac{16}{5}g_1^2y_t^4 - \frac{9}{28}g_2^4y_t^2 \\ & + 10\lambda \left( \frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2 \right) y_t^2 - \frac{3}{5}g_1^2 \left( \frac{57}{10}g_1^2 - 21g_2^2 \right) y_t^2 \\ & \left. - 72\lambda^2 y_t^2 + 60y_t^6 \right). \end{aligned} \quad (14)$$

The Higgs boson pole mass  $M_H$  is determined by the relation to the running Higgs quartic coupling through the one-loop matching condition [16]

$$\lambda(M_H)v^2 = M_H^2(1 + \Delta_h(M_H)), \quad (15)$$

where

$$\begin{aligned} \Delta_h(M_H) = & \frac{G_F}{\sqrt{2}} \frac{M_Z^2}{16\pi^2} \left[ \frac{M_H^2}{M_Z^2} f_1 \left( \frac{M_H^2}{M_Z^2} \right) + f_0 \left( \frac{M_H^2}{M_Z^2} \right) \right. \\ & \left. + \frac{M_Z^2}{M_H^2} f_{-1} \left( \frac{M_H^2}{M_Z^2} \right) \right]. \end{aligned} \quad (16)$$

The functions are given by

$$\begin{aligned} f_1(\xi) &= \frac{3}{2} \ln \xi - \frac{1}{2} Z \left( \frac{1}{\xi} \right) - Z \left( \frac{c_w^2}{\xi} \right) - \text{In} c_w^2 + \frac{9}{2} \left( \frac{25}{9} - \frac{\pi}{\sqrt{3}} \right), \\ f_0(\xi) &= -6 \ln \frac{M_H^2}{M_Z^2} \left[ 1 + 2c_w^2 - 2 \frac{M_t^2}{M_Z^2} \right] + \frac{3c_w^2 \xi}{\xi - c_w^2} \ln \frac{\xi}{c_w^2} + 2Z \left( \frac{1}{\xi} \right) + 4c_w^2 Z \left( \frac{c_w^2}{\xi} \right) + \left( \frac{3c_w^2}{s_w^2} + 12c_w^2 \right) \text{In} c_w^2 - \frac{15}{2} (1 + 2c_w^2) \\ & - 3 \frac{M_t^2}{M_Z^2} \left[ 2Z \left( \frac{M_t^2}{M_Z^2 \xi} \right) + 4 \ln \frac{M_t^2}{M_Z^2} - 5 \right], \\ f_{-1}(\xi) &= 6 \ln \frac{M_H^2}{M_Z^2} \left[ 1 + 2c_w^4 - 4 \frac{M_t^4}{M_Z^4} \right] - 6Z \left( \frac{1}{\xi} \right) - 12c_w^4 Z \left( \frac{c_w^2}{\xi} \right) - 12c_w^4 \text{In} c_w^2 + 8(1 + 2c_w^4) \\ & + 24 \frac{M_t^4}{M_Z^4} \left[ \ln \frac{M_t^2}{M_Z^2} - 2 + Z \left( \frac{M_t^2}{M_Z^2 \xi} \right) \right], \end{aligned} \quad (17)$$

with  $s_w^2 = \sin^2 \theta_w$ ,  $c_w^2 = \cos^2 \theta_w$  ( $\theta_w$  denotes the weak mixing angle) and

$$Z(z) = \begin{cases} 2A \arctan(1/A) & (z > 1/4) \\ A \ln[(1+A)/(1-A)] & (z < 1/4), \end{cases} \quad (18)$$

with  $A = \sqrt{|1-4z|}$ .

For  $\mu \geq M_\Delta$ , the Higgs triplet contributes to the one-loop RGEs. Consequently, we replace  $b_i$  in Eq. (7) with

$$b_i = \left( \frac{47}{10} - \frac{5}{2} - 7 \right). \quad (19)$$

The RGE for the top Yukawa coupling is unchanged.

The RGEs for the new couplings in Eqs. (2) and (3) have been calculated in Ref. [17] and more recently in Ref. [9]. The RGE of the Higgs quartic coupling acquires a new entry in Eq. (13),

$$\beta_\lambda^{(1)} \rightarrow \beta_\lambda^{(1)} + 6\lambda_4^2 + 4\lambda_5^2. \quad (20)$$

Note that in Eq. (20) the contributions from the couplings  $\lambda_4$  and  $\lambda_5$  are both positive. This feature will be crucial in lowering both the vacuum stability and perturbativity bounds with the type II seesaw.

For the remaining couplings we have [9]

$$\begin{aligned} 16\pi^2 \frac{d\lambda_1}{d\ln\mu} = & - \left( \frac{36}{5}g_1^2 + 24g_2^2 \right) \lambda_1 + \frac{108}{25}g_1^4 + 18g_2^4 \\ & + \frac{72}{5}g_1^2g_2^2 + 14\lambda_1^2 + 4\lambda_1\lambda_2 + 2\lambda_2^2 + 4\lambda_4^2 \\ & + 4\lambda_5^2 + 4 \text{tr}[\mathbf{S}_\Delta] \lambda_1 - 8 \text{tr}[\mathbf{S}_\Delta^2], \end{aligned} \quad (21)$$

$$16\pi^2 \frac{d\lambda_2}{d\ln\mu} = -\left(\frac{36}{5}g_1^2 + 24g_2^2\right)\lambda_2 + 12g_2^4 - \frac{144}{5}g_1^2g_2^2 + 3\lambda_2^2 + 12\lambda_1\lambda_2 - 8\lambda_5^2 + 4\text{tr}[\mathbf{S}_\Delta]\lambda_2 + 8\text{tr}[\mathbf{S}_\Delta^2], \quad (22)$$

$$16\pi^2 \frac{d\lambda_4}{d\ln\mu} = -\left(\frac{9}{2}g_1^2 + \frac{33}{2}g_2^2\right)\lambda_4 + \frac{27}{25}g_1^4 + 6g_2^4 + (8\lambda_1 + 2\lambda_2 + 6\lambda + 4\lambda_4 + 6y_t^2) + 2\text{tr}[\mathbf{S}_\Delta]\lambda_4 + 8\lambda_5^2 - 4\text{tr}[\mathbf{S}_\Delta^2], \quad (23)$$

$$16\pi^2 \frac{d\lambda_5}{d\ln\mu} = -\frac{9}{2}g_1^2\lambda_5 - \frac{33}{2}g_2^2\lambda_5 - \frac{18}{5}g_1^2g_2^2 + (2\lambda_1 - 2\lambda_2 + 2\lambda + 8\lambda_4 + 6y_t^2) + 2\text{tr}[\mathbf{S}_\Delta]\lambda_5 + 4\text{tr}[\mathbf{S}_\Delta^2]. \quad (24)$$

Here,  $\mathbf{S}_\Delta = Y_\Delta^\dagger Y_\Delta$  and its corresponding RGE is given by

$$16\pi^2 \frac{d\mathbf{S}_\Delta}{d\ln\mu} = 6\mathbf{S}_\Delta^2 - 3\left(\frac{3}{5}g_1^2 + 3g_2^2\right)\mathbf{S}_\Delta + 2\text{tr}[\mathbf{S}_\Delta]\mathbf{S}_\Delta. \quad (25)$$

In our analysis, we do not consider the RGE for  $\lambda_6$  because, at one-loop level, it is decoupled from the other RGEs. We assume, though, that like all other couplings, it remains in the perturbative region throughout. However, note that  $\lambda_6$  plays an important role in our analysis through the matching condition for the Higgs quartic coupling between high and low energies. The SM Higgs quartic coupling as the low energy effective coupling is defined in Eq. (5). Therefore, the RGE solution of the SM Higgs quartic coupling is connected to the solution for the Higgs quartic coupling at high energies through the matching condition at  $\mu = M_\Delta$  [10]. For a given  $\lambda_6$ , the Higgs quartic coupling is shifted down to the SM coupling by  $\lambda_6^2$  through this matching condition at  $\mu = M_\Delta$ , so that the resulting Higgs boson mass is lowered.

Next we analyze the RGEs numerically and show how much the vacuum stability and perturbativity bounds on the Higgs boson mass are altered in the presence of the type II seesaw. In addition to the matching condition,  $\lambda_{4,5}$  have important effects on the running of the Higgs quartic coupling since they appear in Eq. (20). Their contributions are positive and these couplings work so as to reduce the Higgs quartic coupling at low energies. Consequently, we can expect that the resultant Higgs boson mass tends to be reduced from the effects of  $\lambda_6$  and  $\lambda_{4,5}$  in the type II seesaw. Although the other parameters  $\lambda_{1,2}$  and  $Y_\Delta$  are not involved in the RGE of Higgs quartic coupling, they, of course, do affect the Higgs boson mass through their RGEs coupled with  $\lambda_{4,5}$ . To keep the discussion as simple as possible we proceed as follows. We first investigate the Higgs boson mass bounds by varying  $\lambda_6$ , while keeping the other (non-SM) parameters fixed. We show that a Higgs

mass as low as the LEP 2 bound can be realized in this case. With the seesaw scale at TeV, however, the lower bound on the Higgs mass is closer to 120 GeV. For our next examples we vary  $\lambda_5$ , keeping the other (non-SM) parameters fixed. This case yields a lower bound of 114.4 GeV for the Higgs boson mass which is compatible with a low (TeV or so) seesaw scale.

Fixing the cutoff scale as  $M_{\text{pl}} = 1.2 \times 10^{19}$  GeV, we define the vacuum stability bound as the lowest Higgs mass given by the running Higgs quartic coupling which satisfies the condition  $\lambda(\mu) \geq 0$  for any scale between  $M_H \leq \mu \leq M_{\text{pl}}$ . On the other hand, the perturbativity bound is defined as the highest Higgs boson mass given by the running Higgs quartic coupling which satisfies the condition  $\lambda(\mu) \leq \sqrt{4\pi}$  for any scale between  $M_H \leq \mu \leq M_{\text{pl}}$ .

In Fig. 1, the running Higgs mass defined as  $\sqrt{\lambda(\mu)}v$  for the vacuum stability bound is depicted for various  $\lambda_6$  and a fixed seesaw scale  $M_\Delta = 10^{12}$  GeV. Here we took simple inputs as  $\lambda_1 = \sqrt{4\pi}$ ,  $\lambda_2 = -1$ ,  $\lambda_4 = \lambda_5 = 0$ , and  $Y_\Delta = 0$ . Figure 2 shows a running Higgs mass for the perturbativity bound for various  $\lambda_6$  with the same inputs for the other parameters as in Fig. 1. We find that as  $\lambda_6$  is increased, the vacuum stability and perturbativity bounds eventually merge. The corresponding Higgs mass coincides, it turns out, with the vacuum stability bound for the SM Higgs boson mass obtained with a cutoff scale  $\Lambda = M_\Delta$ .

In other words, the window for the Higgs boson mass between the vacuum stability and perturbative bounds becomes narrow and is eventually closed as  $\lambda_6$  becomes sufficiently large. Figure 3 shows the window for the Higgs boson pole mass versus  $\lambda_6$  for various  $M_\Delta$ . For a suitable choice of  $\lambda_6$  and  $M_\Delta$ , values for  $M_H$  close to the LEP2 bound are easily realized. Note that with  $M_\Delta$  of order a TeV or so, the lower bound on the Higgs mass is close to 120 GeV, which is still well below the standard value of 127 GeV in the absence of the type II seesaw.

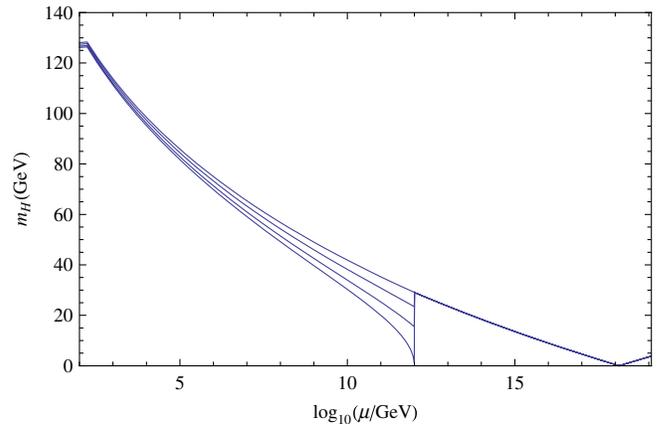


FIG. 1 (color online). Evolution of running Higgs boson mass [ $m_h(\mu) = \sqrt{\lambda(\mu)}v$ ] corresponding to the vacuum stability bound for various  $\lambda_6$  and  $M_\Delta = 10^{12}$  GeV. Each line corresponds to  $\lambda_6 = 0, 0.07, 0.1$ , and  $0.118$ , from top to bottom.

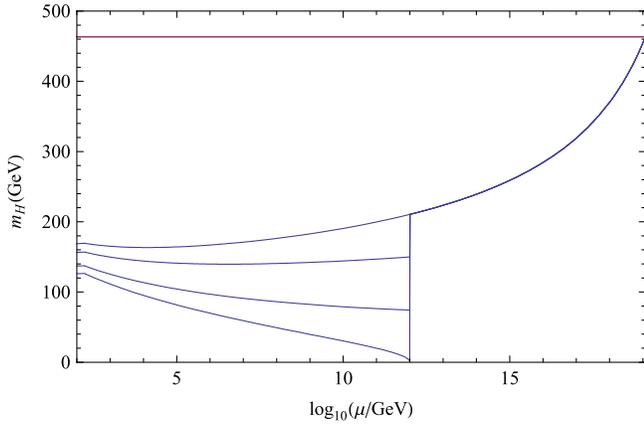


FIG. 2 (color online). Evolution of running Higgs boson mass [ $m_H(\mu) = \sqrt{\lambda(\mu)}v$ ] corresponding to the perturbativity bound for various  $\lambda_6$  and  $M_\Delta = 10^{12}$  GeV. Each line corresponds to  $\lambda_6 = 0, 0.6, 0.8,$  and  $0.855$  from top to bottom. The horizontal line corresponds to  $M_H = (4\pi)^{1/4}v = 464$  GeV.

In Fig. 4, a running Higgs mass defined as  $\sqrt{\lambda(\mu)}v$  for the vacuum stability bound is depicted for various  $\lambda_5$  and a fixed seesaw scale  $M_\Delta = 10^{12}$  GeV. Here we took sample inputs as  $\lambda_1 = \sqrt{4\pi}$ ,  $\lambda_2 = -1$ ,  $\lambda_4 = 0$ ,  $\Lambda_6 = 0$ , and  $Y_\Delta = 0$ . Figure 5 shows a running Higgs mass for the perturbativity bound for various  $\lambda_5$  with the same inputs for the other parameters as in Fig. 4. This time we find that as  $\lambda_5$  is raised, the vacuum stability and perturbativity bounds eventually merge. The corresponding Higgs mass coincides with the vacuum stability bound for the SM Higgs boson with a cutoff scale  $\Lambda = M_\Delta$ . Namely, the window for the Higgs boson mass between the vacuum stability and

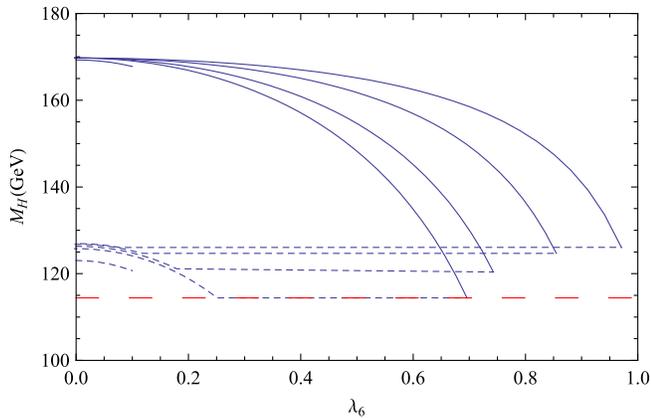


FIG. 3 (color online). The perturbativity (solid line) and vacuum stability (dotted line) bounds on the Higgs boson pole mass  $M_H$  versus  $\lambda_6$  for various  $M_\Delta$ . Each solid and dashed lines correspond to  $M_\Delta = 10^{14}, 10^{12}, 10^9, 1.14 \times 10^7,$  and  $10^3$  GeV, from top to bottom. The results for  $M_\Delta = 1$  TeV are shown only in the region  $\lambda_6 \leq 0.1$  consistent with the  $\rho$ -parameter measurement. The dashed horizontal line denotes the LEP2 bound  $M_H = 114.4$  GeV.

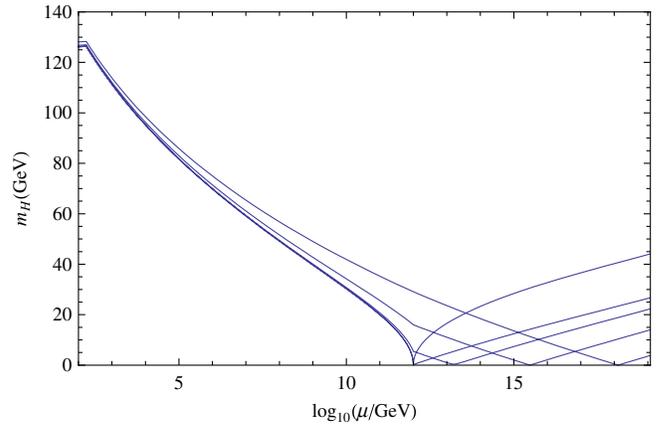


FIG. 4 (color online). Evolution of the running Higgs boson mass corresponding to the vacuum stability bound for various value  $\lambda_5$  and  $M_\Delta = 10^{12}$  GeV. Each line corresponds to  $\lambda_5 = 0.3, 0.22, 0.2, 0.15,$  and  $0$ , from top to bottom at  $\log_{10}(\mu/\text{GeV}) = 19$ .

perturbative bounds becomes narrow and is eventually closed when  $\lambda_5$  becomes sufficiently large. Figure 6 shows the window for the Higgs boson pole mass versus  $\lambda_5$  for various  $M_\Delta$ . Note that with  $M_\Delta = 1$  TeV and  $\lambda_5$  close to 0.1, the vacuum stability bound for  $M_H$  coincides with the LEP2 bound.

Some comments regarding our analysis are in order here. We employ a set of input parameters such that all couplings remain in the perturbative regime for  $M_\Delta \leq \mu \leq M_{\text{pl}}$ . For example, for an input parameter set, it may happen that  $\lambda_{1,2}$  exceed the perturbative regime at a scale  $M_\Delta \leq \mu \leq M_{\text{pl}}$ . We checked that the sample parameter set used in Figs. 1–6 did not cause such theoretical inconsistency. To reproduce the current neutrino oscillation data through the type II seesaw mechanism,  $\lambda_6 Y_\Delta$  should not be

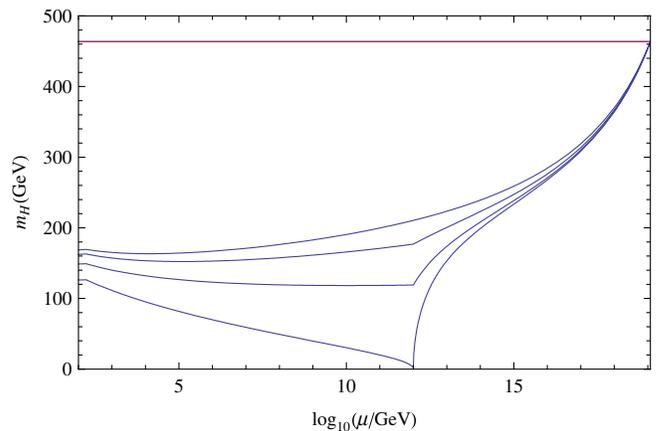


FIG. 5 (color online). Evolution of the running Higgs boson mass corresponding to the perturbativity bound for various  $\lambda_5$  and  $M_\Delta = 10^{12}$  GeV. Each line corresponds to  $\lambda_5 = 0, 1.0, 1.25,$  and  $1.35$ , from top to bottom. The horizontal line corresponds to  $M_H = (4\pi)^{1/4}v = 464$  GeV.

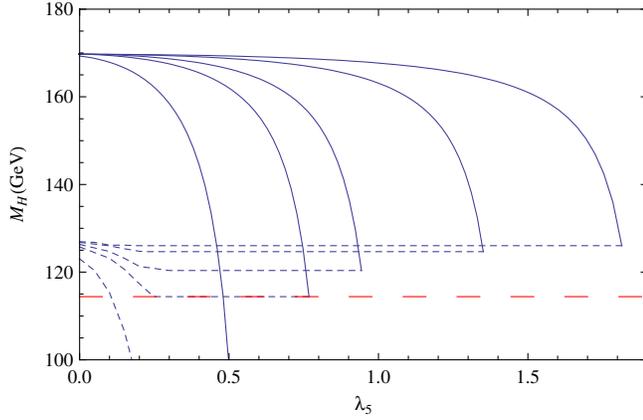


FIG. 6 (color online). The perturbativity (solid line) and vacuum stability (dotted line) bounds versus  $\lambda_5$  for various  $M_\Delta$ . Each solid and dashed line corresponds to  $M_\Delta = 10^{14}$ ,  $10^{12}$ ,  $10^9$ ,  $1.14 \times 10^7$ , and  $10^3$  GeV, from top to bottom. The dashed horizontal line denotes the LEP2 bound  $M_H = 114.4$  GeV.

zero. A tiny  $\lambda_6$  would require a very large  $Y_\Delta$ , which may cause theoretical inconsistency for the RGEs of  $\Lambda_i$  [see Eq. (21)–(24)]. For this reason, we do not strictly set  $\lambda_6 = 0$  and  $Y_\Delta = 0$  in Figs. 1–6. However, we can check that the impact of  $Y_\Delta$  in Fig. 3 is negligible when  $Y_\Delta \leq 0.1$ . In Fig. 6, the effects from  $Y_\Delta$  and  $\lambda_6$  are negligible with  $Y_\Delta, \lambda_6 \leq 0.1$ .

In conclusion, we have considered the potential impact of the type II seesaw on the vacuum stability and perturbativity bounds on the Higgs boson mass of the SM. There are two main effects that we have considered. One is the

tree-level matching condition for the SM Higgs quartic coupling induced by the coupling  $\Lambda_6$  at the seesaw (triplet mass) scale. The implications for the SM Higgs mass are then studied using the appropriate RGEs. In our second set of examples a different coupling, namely  $\lambda_5$ , plays an essential role, with  $\Lambda_6$  essentially subdominant. In both cases we have identified regions of the parameter space for which the lower bound on the SM Higgs boson mass lies well below 127 GeV, the value obtained in the absence of the type II seesaw. Perhaps the most interesting result from our analysis is that with the type II seesaw the SM Higgs boson can have a mass as low as the LEP 2 bound of 114.4 GeV. This can be achieved for plausible values of the parameters and with the mass scale for the triplet in the TeV range. Thus, it is exciting to speculate that the SM Higgs boson, when found at the LHC, may be accompanied by additional scalar fields with some carrying two units of electric charge.

In this paper, we have required absolute stability of the electroweak minimum. If one modifies this into the meta-stability condition, namely, that the electroweak minimum is not the true vacuum but only sufficiently long-lived, the lower bound on the Higgs boson mass in this case can be as low as the LEP 2 bound [18].

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