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Can classical wormholes stabilize the brane–anti-brane system?

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Abstract

We investigate the static solutions of Callan and Maldecena and Gibbons to lowest order Dirac–Born–Infeld theory. Among them are charged wormhole solutions connecting branes to anti-branes. It is seen that there are no such solutions when the separation between the brane and anti-brane is smaller than some minimum value. The minimum distance coincides with the energy minimum, and depends monotonically on the charge. Making the charge sufficiently large, such that the minimum separation is much bigger than $\sqrt{\alpha'}$, may suppress known quantum processes leading to decay of the brane–anti-brane system. For this to be possible the zeroth order wormhole solutions should be reasonable approximations of solutions in the full D-brane theory. With this in mind we address the question of whether the zeroth order solutions are stable under inclusion of higher order corrections to the Dirac–Born–Infeld action.

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1. Introduction

The Born–Infeld nonlinear description of electrodynamics [1] and its subsequent generalization to membranes à la Dirac [2] is of current interest due to its role as an effective theory for Dp -branes. The associated Dirac–Born–Infeld (DBI) action appears at lowest order in the derivative expansion for the effective Dp -brane action [3,4]. The original Born–Infeld theory has a charged static solution, which was generalized by Callan and Maldecena [5] and Gibbons [6] to families of static solutions on the brane. The families are associated with orbits of the $SO(1, 1)$ group. The Lagrangian is invariant under $SO(1, 1)$ and can be used to label the orbits. One such orbit contains the solution of Born and Infeld.

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Another is a BPS solution representing a fundamental string attached to the brane. Finally, there is a third family of solutions corresponding to wormholes which connect the brane to an anti-brane a finite distance away. Here we show that there are no charged wormhole solutions having a separation between the p -brane and anti- p -brane smaller than some minimum value. The minimum separation distance goes like $|Q|^{1/(p-1)}$, Q being the $U(1)$ charge. At minimum separation, the self-energy is also a minimum, where there appears a cusp singularity in the plot of the energy versus separation distance. In the quantum analysis of the brane–anti-brane system an instability is known to occur at distance scales of order $\sqrt{\alpha'}$ due to excitation of tachyonic modes [7]. Then for sufficiently large charge, i.e.,

$$|Q|^{1/(p-1)} \gg \sqrt{\alpha'}, \quad (1.1)$$

such quantum processes may be suppressed, and it is possible that charged wormholes can stabilize the brane–anti-brane system.

For the above scenario to be correct, however, classical stability of the wormhole solutions should be checked. This means (a) enlarging to time dependent solutions, and investigating whether solutions are stable with respect to perturbations about the static solution. It also means (b) checking whether the solutions to the zeroth order effective theory are a reasonable approximation of solutions to the full effective Dp -brane action. Here we shall only consider (b). One signal that solutions may be unstable in the sense (b) is the presence of singularities in the field strength, where the derivative expansion cannot be trusted. Such a singularity is present for the original Born–Infeld solution, in that case at a single point, and for the entire orbit of solutions connected to the Born–Infeld solution. Despite the singularity, these solutions are associated with a finite self-energy at zeroth order. For the BPS case the singularity occurs an infinite distance away from the brane, and appears harmless. The wormhole-type solutions were originally constructed by joining together two local solutions, obtained in the static gauge, at the minimum circumference of the wormhole [5,6]. A singularity in the field strength occurs along the throat—precisely at the minimum circumference. The singularity in the field strength is a coordinate singularity, and thus can be removed by going to another gauge. Nevertheless, it is a signal that higher order corrections may not be negligible.

To check stability in the sense (b), we will rely on recent computations of the derivative corrections to Born–Infeld theory. The first order corrections to the action were obtained separately by Wyllard [8] and Das, Mukhi and Suryanarayana [9]. (Higher order corrections seem currently out of reach.) Using their results we carried out a stability check previously for the case of the original Born–Infeld solution [10]. There we argued that the original Born–Infeld solution is unstable under inclusion of these first order corrections. More specifically, we numerically obtained corrections to the zeroth order Born–Infeld solution, but found that they give an infinitely large correction to the Lagrangian. We give a simpler proof of the result here. Because the Lagrangian is $SO(1, 1)$ invariant the result applies to the entire orbit of solutions connected to the Born–Infeld solution. Concerning the BPS solution, it is known that such a solution is stable *to all orders* in the derivative expansion [11]. We verify that this is consistent with the first order results of [8,9]. We find that the stability analysis for the wormhole solutions leads to the same results obtained for the Born–Infeld solution. Namely, corrections to the zeroth order solution lead to an

infinitely large correction of the Lagrangian. In this case, we need to rely on numerical computation for the result.

In Section 2 we give analytic expressions for the three families of zeroth order solutions along with their self-energies. Here we show that the charged wormhole solutions have a minimum length. The question of stability of the zeroth order solutions is addressed in Section 3. In Appendix A we write down the wormhole solution in another gauge, where the field strength is singularity-free. In fact, it is a constant in that gauge. In Appendix B we use the results of [8,9] to obtain the first order corrections to the field equations for the BPS case, and show that the answer agrees with [11].

2. Zeroth order solutions

2.1. Dirac–Born–Infeld theory

We consider the p -dimensional brane embedded in a ten-dimensional space–time with flat metric $[\eta_{AB}] = \text{diag}(-1, 1, \dots, 1)$, $A, B, \dots = 0, 1, \dots, 9$. We denote the brane coordinates by X^A . They are functions of $p + 1$ parameters ξ^μ , $\mu, \nu, \dots = 0, 1, \dots, p$. Additional degrees of freedom on the brane are $U(1)$ potentials $\mathcal{A}_\mu(\xi)$. The DBI action is written in terms of the $(p + 1) \times (p + 1)$ matrix

$$h_{\mu\nu} = \eta_{AB} \partial_\mu X^A(\xi) \partial_\nu X^B(\xi) + 2\pi\alpha' F_{\mu\nu}(\xi), \tag{2.1}$$

where $\partial_\mu = \frac{\partial}{\partial \xi^\mu}$. The first term is the induced metric on the brane, while $F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$ is the $U(1)$ field strength. We assume the two-form contribution is absent $B_{AB} = 0$. The DBI action is [1,2]

$$\mathcal{S}_{\text{DBI}}^{(0)} = \frac{T_p}{g_s} \int d^{p+1} \xi \mathcal{L}_{\text{DBI}}^{(0)}, \quad \mathcal{L}_{\text{DBI}}^{(0)} = 1 - \sqrt{-\det[h_{\mu\nu}]}, \tag{2.2}$$

where T_p is the tension, which is expressible in terms of α' according to

$$T_p = (4\pi^2 \alpha')^{-(p+1)/2}, \tag{2.3}$$

and g_s is the string coupling. $\mathcal{S}_{\text{DBI}}^{(0)}$ is invariant under diffeomorphisms on the brane $\xi^\mu \rightarrow \xi'^\mu(\xi)$, $U(1)$ gauge transformations $\mathcal{A}_\mu(\xi) \rightarrow \mathcal{A}_\mu(\xi) + \partial_\mu \Lambda(\xi)$, as well as ten-dimensional Poincaré transformations. Variations in $X^A(\xi)$ and $\mathcal{A}_\mu(\xi)$ lead to the equations of motion

$$\begin{aligned} \partial_\mu \{ \sqrt{-\det h} (h^{\mu\nu} + h^{v\mu}) \eta_{AB} \partial_\nu X^B \} &= 0, \\ \partial_\mu \{ \sqrt{-\det h} (h^{\mu\nu} - h^{v\mu}) \} &= 0, \end{aligned} \tag{2.4}$$

respectively, where $h^{\mu\nu} h_{\nu\rho} = \delta_\rho^\mu$.

The known families of spherically symmetric static solutions [5,6] can be classified in terms of $SO(1, 1)$ orbits (we do this below), and they describe different topologies embedded in the flat ten-dimensional background. For one family of solutions, containing the original Born–Infeld solution, a time slice is \mathbb{R}^p minus a point. We could therefore describe it with the introduction of a delta function source to the right-hand side of (2.4).

Another family corresponds to a brane and anti-brane connected by a wormhole. In that case one can patch together local solutions to (2.4). The families of solutions are written in terms of two integration constants Q and C , Q being the electric charge. Locally, all solutions can be expressed in the static gauge, where one identifies ξ^μ with the first $p + 1$ brane coordinates X^μ , $\mu = 0, 1, \dots, p$. The remaining X^α , $\alpha = p + 1, p + 2, \dots, 9$, then denote normal coordinates, and (2.1) becomes

$$h_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu X^\alpha \partial_\nu X^\alpha + 2\pi\alpha' F_{\mu\nu}. \tag{2.5}$$

The static solutions of [5,6] are for a radial electric field with a single transverse mode excited. Choose the nonvanishing degrees of freedom to be $\mathcal{A}_0(r)$ and $X_{p+1}(r)$, where r is the radial coordinate on the brane. Since the metric is diagonal, the resulting matrix h is diagonal except for the 2×2 submatrix with corresponding indices μ and ν equal to 0 and r . That 2×2 submatrix and its inverse are given by

$$\begin{pmatrix} -1 & -f(r) \\ f(r) & 1 + g(r)^2 \end{pmatrix} \quad \text{and} \quad \frac{1}{1 + g(r)^2 - f(r)^2} \begin{pmatrix} -1 - g(r)^2 & -f(r) \\ f(r) & 1 \end{pmatrix}, \tag{2.6}$$

respectively, where $f(r) = 2\pi\alpha' \partial_r \mathcal{A}_0(r)$ and $g(r) = \partial_r X_{p+1}(r)$. Substituting into the equations of motion (2.4) gives

$$\partial_r \left\{ \frac{r^{p-1} f(r)}{\sqrt{1 + g(r)^2 - f(r)^2}} \right\} = \partial_r \left\{ \frac{r^{p-1} g(r)}{\sqrt{1 + g(r)^2 - f(r)^2}} \right\} = 0. \tag{2.7}$$

The solutions for $f(r)$ and $g(r)$ are

$$\frac{f(r)}{Q} = \frac{g(r)}{C} = \frac{1}{\sqrt{Q^2 - C^2 + r^{2p-2}}}. \tag{2.8}$$

The integration constants Q and C have units of $[\text{length}]^{p-1}$.

For the configurations (2.8) the Lagrangian and equations of motion are invariant under the $SO(1, 1)$ transformation

$$\begin{pmatrix} f(r) \\ g(r) \end{pmatrix} \rightarrow \begin{pmatrix} \cosh \nu & \sinh \nu \\ \sinh \nu & \cosh \nu \end{pmatrix} \begin{pmatrix} f(r) \\ g(r) \end{pmatrix}. \tag{2.9}$$

The integration constants transform in the same way

$$\begin{pmatrix} Q \\ C \end{pmatrix} \rightarrow \begin{pmatrix} \cosh \nu & \sinh \nu \\ \sinh \nu & \cosh \nu \end{pmatrix} \begin{pmatrix} Q \\ C \end{pmatrix}. \tag{2.10}$$

There are then three kinds of orbits: (i) $|Q| > |C|$, (ii) $|Q| = |C|$ and (iii) $|Q| < |C|$. (i) is connected to the original Born–Infeld solution, (ii) is the BPS solution and (iii) is associated with wormhole solutions. The orbits can be classified by their corresponding value for the spatial integral of the Lagrangian density $\mathcal{L}_{\text{DBI}}^{(0)}$

$$L_{\text{DBI}}^{(0)}(r) = -\Omega_{p-1} \left\{ \int_r^\infty dr \frac{r^{2p-2}}{\sqrt{r^{2p-2} + Q^2 - C^2}} - \int_0^\infty dr r^{p-1} \right\}, \tag{2.11}$$

where Ω_{p-1} is the volume of a unit $p - 1$ sphere. We removed a hole of radius r around the origin in the integration for the first integral, in order to accommodate the different cases, as their domains differ. The second integral is the vacuum subtraction. The first integral can be expressed in terms of the one for $X_{p+1}(r)$, yielding

$$L_{\text{DBI}}^{(0)}(r) = \frac{\Omega_{p-1}}{p} \left\{ r \sqrt{r^{2p-2} + Q^2 - C^2} + (C^2 - Q^2) \frac{X_{p+1}(r)}{C} \right\}. \tag{2.12}$$

For cases (i), (ii) and (iii) we find that the result (after setting r to its appropriate value) is positive, zero and negative, respectively.

2.2. Analytic solutions

The right-hand side of (2.8) can be integrated to obtain analytic expressions for the potential $\mathcal{A}_0(r)$ and the transverse displacement $X_{p+1}(r)$. For this expand in powers of $\frac{r^{2p-2}}{Q^2 - C^2}$ and integrate term by term. The result for the indefinite integral can be expressed in terms of a hypergeometric

$$\frac{r}{\sqrt{Q^2 - C^2}} F\left(\frac{1}{2}, \frac{1}{2p-2}; \frac{2p-1}{2p-2}; -\frac{r^{2p-2}}{Q^2 - C^2}\right), \tag{2.13}$$

for $|\frac{r^{2p-2}}{Q^2 - C^2}| < 1$. Now set the limits of integration to be r and ∞ , with the assumption that the potentials vanish at the latter. To evaluate (2.13) for these limits we analytic continue $F(a, b; c; z)$ using [12]

$$F(a, b; c; z) = \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)} (-z)^{-a} F\left(a, 1-c+a; 1-b+a; \frac{1}{z}\right) + \frac{\Gamma(c)\Gamma(a-b)}{\Gamma(a)\Gamma(c-b)} (-z)^{-b} F\left(b, 1-c+b; 1-a+b; \frac{1}{z}\right), \tag{2.14}$$

where $|\arg(-z)| < \pi$. So (2.13) can be rewritten as

$$\frac{1}{(2-p)r^{p-2}} F\left(\frac{1}{2}, \frac{p-2}{2p-2}; \frac{3p-4}{2p-2}; -\frac{Q^2 - C^2}{r^{2p-2}}\right) + \frac{(Q^2 - C^2)^{\frac{2-p}{2p-2}}}{\sqrt{\pi}} \Gamma\left(\frac{2p-1}{2p-2}\right) \Gamma\left(\frac{p-2}{2p-2}\right),$$

since $F(a, 0; c; z) = 1$. For $p > 2$, only the last term survives when evaluating at ∞ , which then get subtracted out after evaluating between r and ∞ . For the transverse displacement X_{p+1} one gets

$$X_{p+1}(r) = \frac{-C}{(p-2)r^{p-2}} F\left(\frac{1}{2}, \frac{p-2}{2p-2}; \frac{3p-4}{2p-2}; -\frac{Q^2 - C^2}{r^{2p-2}}\right), \quad \text{for } p > 2. \tag{2.15}$$

Next we examine the result for the three different orbits.

Case (i) $|Q| > |C|$. The limit $C \rightarrow 0$ where the transverse mode is not excited gives the original Born–Infeld solution [1], while $|Q| > |C| > 0$ yields a deformation of the Born–Infeld solution where a spike protrudes from the brane. In Fig. 1 we plot the function

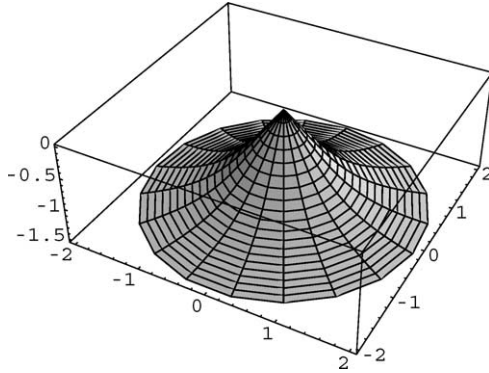


Fig. 1. $p = 3, |Q| = 1, |C| = 0.8$.

$X_{p+1}(r)$ for $p = 3$ on a two-dimensional spatial slice. From (2.15) (and (2.14)) the maximum size of the spike is the absolute value of

$$X_{p+1}(0) = \frac{C\Gamma(\frac{3p-4}{2p-2})\Gamma(\frac{1}{2p-2})}{(2-p)\sqrt{\pi}}(Q^2 - C^2)^{\frac{2-p}{2p-2}}, \quad \text{for } p > 2. \tag{2.16}$$

For the integral of the Lagrangian density (cf. (2.12)) one gets

$$L_{\text{DBI}}^{(0)}(0) = \frac{\Omega_{p-1}\Gamma(\frac{3p-4}{2p-2})\Gamma(\frac{1}{2p-2})}{p(p-2)\sqrt{\pi}}(Q^2 - C^2)^{\frac{p}{2p-2}} > 0, \quad \text{for } p > 2. \tag{2.17}$$

Case (ii) $|Q| = |C|$. One arrives at the BPS solution for this case, where (2.8) reduce to the Coulomb solutions $f(r) = g(r) = \frac{Q}{r^{p-1}}$. Since $F(a, b; c; 0) = 1$, (2.15) reduces to

$$X_{p+1}(r) = \frac{-Q}{(p-2)r^{p-2}}, \quad \text{for } p > 2 \tag{2.18}$$

and the spike becomes infinitely long (see Fig. 2), representing a fundamental string attached to the brane [5]. In this case the integral of the Lagrangian density $L_{\text{DBI}}^{(0)}(0)$ goes to zero. These solutions are BPS because they preserve half of the supersymmetries of the ground state solution. Supersymmetries are present when the matrix

$$\partial_\mu X_{p+1}(\xi)[\Gamma^\mu, \Gamma^{p+1}] + 2\pi\alpha' F_{\mu\nu}(\xi)[\Gamma^\mu, \Gamma^\nu] \tag{2.19}$$

is degenerate. Γ^A are Γ matrices for the ten-dimensional background space, $[\Gamma^A, \Gamma^B] = 2\eta^{AB}$. To see that this holds when $|Q| = |C|$, one observes that (2.19) is proportional to $(Q\Gamma^0 + C\Gamma^{p+1})\Gamma^r$, whose square is $(C^2 - Q^2)\mathbb{1}$. (2.19) is then nilpotent when $|Q| = |C|$.

Case (iii) $|Q| < |C|$. Here one gets a finite diameter tube with a minimum radius $r_0 = (C^2 - Q^2)^{\frac{1}{2p-2}}$, as illustrated in Fig. 3. Both g and f , and consequently also the electric field, are singular at $r = r_0$. Nevertheless, \mathcal{A}_0 and X_{p+1} are not. From the latter the tube has a finite length. After expressing C in terms of r_0 and Q , it is

$$X_{p+1}(r_0) = \frac{\sqrt{\pi(r_0^{2p-2} + Q^2)}\Gamma(\frac{2p-1}{2p-2})}{(2-p)r_0^{p-2}\Gamma(\frac{p}{2p-2})}, \quad \text{for } p > 2. \tag{2.20}$$

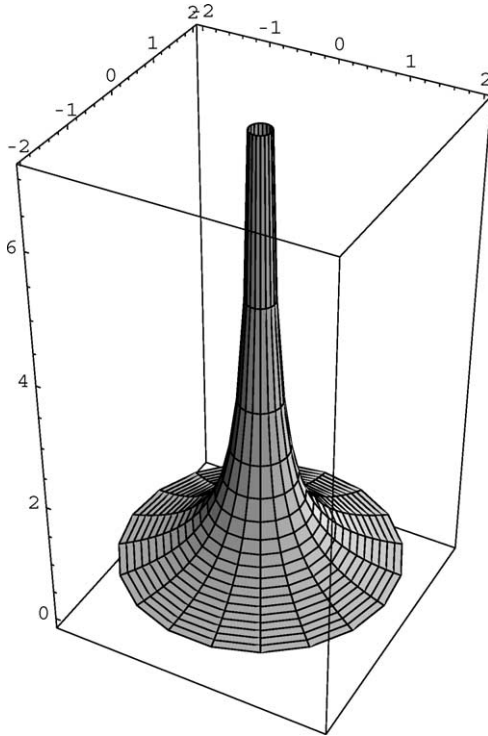


Fig. 2. $p = 3, |Q| = |C| = 1$.

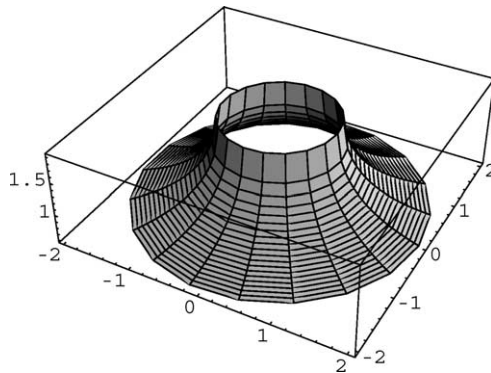


Fig. 3. $p = 3, |Q| = 1, |C| = 1.2$.

The domain of integration for the integral of the Lagrangian density (cf. (2.12)) now goes from r_0 to ∞ . One gets

$$L_{\text{DBI}}^{(0)}(r_0) = -\frac{\Omega_{p-1} \sqrt{\pi} \Gamma(\frac{2p-1}{2p-2})}{p(p-2) \Gamma(\frac{p}{2p-2})} r_0^p < 0, \quad \text{for } p > 2. \quad (2.21)$$

The static gauge breaks down at $r = r_0$, and so the above solution is only local. A global solution was proposed by gluing this one to the analogous solution on an anti-brane [5,6]. The global solution then represents a wormhole connecting the brane with an anti-brane a distance of $2|X_{p+1}(r_0)|$ away, with a throat of minimum radius r_0 . The gluing of the two local solutions to form a wormhole occurs at $r = r_0$, precisely where there is a singularity in the electric field, which might be a matter of concern. On the other hand, the electric field singularity is a coordinate singularity, which is easily seen by transforming to another gauge. Take for example the gauge where the r -coordinate of the static gauge is replaced by $z = X_{p+1}$. In the new gauge the solution is described by the inverse, call it $R(z)$, of the function $X_{p+1}(r)$. Now the electrostatic field is in the z -direction, and there are coordinate singularities at the location of the brane (and anti-brane). We denote the electric field ($\times 2\pi\alpha'$) in the new gauge by $E(z)$. It can be computed locally by performing a coordinate transformation from the static gauge,

$$E(z) = \frac{\partial R}{\partial z} f(R(z)) = \frac{1}{g(R(z))} f(R(z)). \tag{2.22}$$

Substituting in the solution for f and g given in (2.8) gives a constant electric field

$$E(z) = Q/C. \tag{2.23}$$

So in this coordinate frame there are no singularities in the electric field (for $C \neq 0$). In Appendix A we write down the action in this gauge and show that (2.23) solves the corresponding equations of motion.

From (2.20) it follows that there is a minimum separation distance between the brane and anti-brane for a fixed Q (and $p > 2$). It is equal to

$$\min 2|X_{p+1}(r_0)| = \frac{2\sqrt{\pi(p-1)} \Gamma(\frac{2p-1}{2p-2})}{(p-2)^{\frac{3p-4}{2p-2}} \Gamma(\frac{p}{2p-2})} |Q|^{\frac{1}{p-1}}, \tag{2.24}$$

and occurs when C and Q are constrained by

$$C^2 = (p-1)Q^2. \tag{2.25}$$

In Fig. 4 we plot the separation distance versus throat size for a fixed Q when $p = 3$.

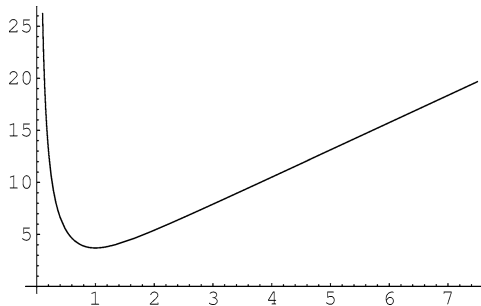


Fig. 4. $p = 3, Q = 1, 2|X_{p+1}(r_0)|$ vs. r_0 .

2.3. Self-energy

Concerning the energy, one can apply the canonical formalism starting from the Lagrangian in (2.2). For this again assume the static gauge and hence (2.5). The Hamiltonian density is

$$\mathcal{H}_{\text{DBI}}^{(0)} = P^\alpha \dot{X}_\alpha + \Pi^\mu \dot{\mathcal{A}}_\mu - \mathcal{L}_{\text{DBI}}^{(0)}, \tag{2.26}$$

where the dot denotes a time derivative and

$$\begin{aligned} P^\alpha &= -\frac{1}{2} \sqrt{-\det h} (h^{0\mu} + h^{\mu 0}) \partial_\mu X^\alpha, \\ \Pi^\mu &= -\pi \alpha' \sqrt{-\det h} (h^{\mu 0} - h^{0\mu}), \end{aligned} \tag{2.27}$$

are the momenta conjugate to X_α and \mathcal{A}_μ , respectively. As usual, the momentum conjugate to \mathcal{A}_0 is constrained to be zero. After integrating by parts

$$\mathcal{H}_{\text{DBI}}^{(0)} = P^\alpha \dot{X}_\alpha + \Pi^i F_{0i} + \sqrt{-\det h} - 1 - \partial_i \Pi^i \mathcal{A}_0, \tag{2.28}$$

where $i = 1, 2, \dots, p$. The coefficient of \mathcal{A}_0 gives the Gauss law constraint. The remaining terms are equal to

$$-\sqrt{-\det h} h^{00} - 1. \tag{2.29}$$

Although the Lagrangian density is invariant under the $SO(1, 1)$ symmetry (2.9), the Hamiltonian density is not. Then unlike the integral of the Lagrangian density, the integral of the energy density will not be constant along the orbits of $SO(1, 1)$.

The integral of (2.29) gives the self-energy of the DBI solutions (2.8). After removing a hole around the origin of radius r in the integration domain one gets

$$\begin{aligned} \mathcal{U}(r) &= \frac{T_p \Omega_{p-1}}{g_s} \mathcal{E}(r), \\ \mathcal{E}(r) &= \int_r^\infty dr \frac{(r^{2p-2} + Q^2)}{\sqrt{r^{2p-2} + Q^2 - C^2}} - \int_0^\infty dr r^{p-1}, \end{aligned} \tag{2.30}$$

where the factor T_p/g_s comes from (2.2). In the second integral we subtract off the total vacuum energy of the brane. Note that we must restrict the lower limit in the first integral to be greater than or equal to r_0 for the case $|C| > |Q|$. The result can be expressed in terms of $X_{p+1}(r)$:

$$\mathcal{E}(r) = -[(p-1)Q^2 + C^2] \frac{X_{p+1}(r)}{pC} - \frac{r}{p} \sqrt{r^{2p-2} + Q^2 - C^2}. \tag{2.31}$$

This gives a positive answer for the self-energy since $X_{p+1}(r)/C$ is negative for the solutions, while the second term vanishes after evaluating at the minimum value of r (0 for $|Q| \geq |C|$ and r_0 for $|C| > |Q|$).

For case (i) $|Q| > |C|$, one gets the total self-energy of the solution by setting r in (2.31) to zero, which yields

$$\mathcal{U}(0) = \frac{T_p \Omega_{p-1}}{g_s} \frac{\Gamma(\frac{3p-4}{2p-2}) \Gamma(\frac{1}{2p-2})}{p(p-2) \sqrt{\pi}} \frac{(p-1)Q^2 + C^2}{(Q^2 - C^2)^{\frac{p-2}{2p-2}}}, \quad \text{for } p > 2. \tag{2.32}$$

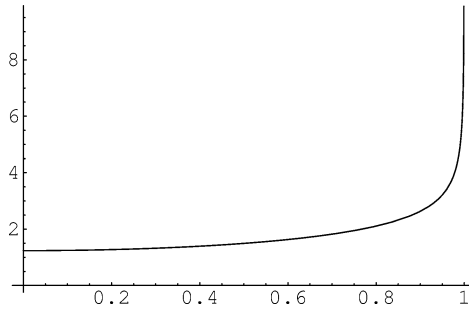


Fig. 5. $p = 3, Q = 1, \mathcal{E}$ vs. $C (C < Q)$.

For a fixed Q it goes monotonically from the Born–Infeld value

$$\mathcal{U}_{\text{Born–Infeld}} = \frac{T_p \Omega_{p-1}}{g_s} \frac{p-1}{p(p-2)\sqrt{\pi}} \Gamma\left(\frac{3p-4}{2p-2}\right) \Gamma\left(\frac{1}{2p-2}\right) |Q|^{\frac{p}{p-1}}, \quad \text{for } p > 2 \tag{2.33}$$

corresponding to $C = 0$, to infinity in the BPS limit, corresponding to $|C| \rightarrow |Q|$. In Fig. 5 we plot \mathcal{E} vs. C for $Q = 1$ and $p = 3$.

For case (ii) $|Q| = |C|$ the total self-energy is infinite. At large distances $|X_{p+1}|$, the energy per unit length of the infinite string solution goes to a constant. From (2.31) it follows that

$$\frac{d\mathcal{U}}{d|X_{p+1}|} \rightarrow \frac{T_p \Omega_{p-1}}{g_s} |Q|, \quad \text{as } |X_{p+1}| \rightarrow \infty. \tag{2.34}$$

For case (iii) $|Q| < |C|$, one gets the total self-energy by setting r in (2.31) equal to r_0 :

$$\mathcal{U}(r_0) = \frac{T_p \Omega_{p-1}}{g_s} \frac{\sqrt{\pi}}{p-2} \frac{\Gamma(\frac{2p-1}{2p-2})}{\Gamma(\frac{p}{2p-2})} \left(\frac{Q^2}{r_0^{p-2}} + \frac{r_0^p}{p} \right), \quad \text{for } p > 2. \tag{2.35}$$

For a fixed Q , the minimum energy configuration occurs for $r_0^{2p-2} = (p-2)Q^2$, corresponding to the minimum separation distance (2.24) between the brane and anti-brane. The minimum value for $\mathcal{U}(r_0)$ is

$$\mathcal{U}_{\min} = \frac{T_p \Omega_{p-1}}{g_s} \frac{2\sqrt{\pi}}{p(p-2)} \frac{(p-1)}{2^{p-2}} \frac{\Gamma(\frac{2p-1}{2p-2})}{\Gamma(\frac{p}{2p-2})} |Q|^{\frac{p}{p-1}}, \quad \text{for } p > 2. \tag{2.36}$$

If $Q = 0$ the minimum energy configuration occurs when the brane and anti-brane coincide. For $Q \neq 0$ and a separation distance greater than the minimum value (2.24), there are two possible solutions with different throat sizes. The one with a smaller throat is energetically favored. Call $\mathcal{U}_0(X_{p+1})$ and $\mathcal{U}_1(X_{p+1})$ the energy of the thin and fat wormholes, respectively. For a large separation distance,

$$\begin{aligned} \mathcal{U}_0(X_{p+1}) &\rightarrow \frac{T_p \Omega_{p-1}}{g_s} |Q X_{p+1}|, \\ \mathcal{U}_1(X_{p+1}) &\rightarrow \frac{T_p \Omega_{p-1}}{p g_s} \left\{ \frac{\sqrt{\pi}}{p-2} \frac{\Gamma(\frac{2p-1}{2p-2})}{\Gamma(\frac{p}{2p-2})} \right\}^{1-p} |X_{p+1}|^p. \end{aligned} \tag{2.37}$$

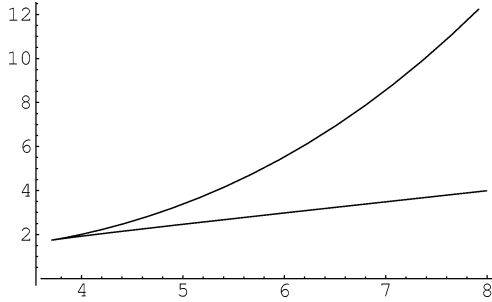


Fig. 6. $p = 3, Q = 1, \mathcal{E}$ vs. $2|X_{p+1}(r_0)|$.

Upon plotting the energy versus the separation distance one gets a double-valued function, with a cusp at the minimum separation, as is illustrated in Fig. 6 for $Q = 1$ and $p = 3$.

The minimum energy solution for fixed Q in case (i) $|Q| > |C|$ was the original Born–Infeld solution, while in case (iii) $|Q| < |C|$ it corresponded to (2.36). In both cases the energy goes like $|Q|^{p/p-1}$. Assuming charge conservation, such solutions are energetically unstable under fission into far separated solutions with total charge equal to Q . It was however pointed out in [6] that fission may not be realized at the classical level for singular field configurations, and the above solutions are of this type. Assuming fission does occur, either classically or quantum mechanically the minimum energy configuration should be an ensemble of far separated wormholes in case (iii) or Born–Infeld solutions in case (i) with the fundamental charge. In such a case it then appears difficult to satisfy the condition (1.1) for quantum stability. For example, when $p = 3$ and taking the electron charge as fundamental, Q would be equal to $2\pi\alpha'$ times the square root of the fine structure constant. Finally, in comparing (i) with (iii), the ratio $\mathcal{U}_{\min}/\mathcal{U}_{\text{Born–Infeld}}$ is less than one for $p \geq 4$, while it is greater than one for $p = 3$. Thus for $p \geq 4$, it is energetically more favorable for wormholes to develop between a charged brane and oppositely charged anti-brane than for Born–Infeld configurations to develop on the brane and anti-brane. The opposite is true for $p = 3$.

2.4. Thermodynamic considerations

Here we make a side remark concerning the thermodynamics of wormholes. Once again, for case (iii) when the energy is greater than the minimum, two types of wormholes with different thickness may be present. Say they are in a heat bath with temperature T and call ρ_0 and ρ_1 the density of the thin and fat wormholes, respectively. If one assumes they are in dissipative and thermal equilibrium, then the ratio of their densities at a temperature T is given by

$$\frac{\rho_1}{\rho_0} = \exp\left(\frac{\mathcal{U}_0(X_{p+1}) - \mathcal{U}_1(X_{p+1})}{k_B T}\right). \tag{2.38}$$

3. Inclusion of first order corrections

Here we examine what happens to the zeroth order classical solutions upon including the first order derivative corrections in the action. We already checked in [10] that the zeroth order Born–Infeld solution ($C = 0$) does not survive upon the inclusion of such corrections. More specifically, we numerically found a classical solution to the corrected field equations, but it was associated with an infinite value for the Lagrangian. Because as with zeroth order, the Lagrangian is $SO(1, 1)$ invariant, the infinite value for the Lagrangian follows for the entire orbit of solutions connected to the $C = 0$ solution; i.e., case (i). On the other hand, the case (ii) BPS solution ($|Q| = |C|$) is unchanged upon inclusion of the first order corrections, and just like at zeroth order, the Lagrangian vanishes. In fact the BPS solution is known to survive to all orders in the derivative expansion [11]. We shall verify that this result is consistent with the explicit expression for the first order terms obtained in [8,9]. The stability analysis for the wormhole solutions case (iii) leads to the same results we obtain for case (i). Namely, corrections to the zeroth order solution lead to an infinitely large correction of the Lagrangian.

The first order corrections were initially computed in [8,9] for the space-filling D9-brane. A dimensional reduction could then be performed to get the corrections to the DBI action (2.2) for an arbitrary Dp -brane. We first briefly recall the results of the dimensional reduction procedure at zeroth order [4]. One starts with the Born–Infeld (BI) action $\mathcal{S}_{\text{BI}}^{(0)}$ for the space-filling D9-brane. It is written in terms of a 10×10 matrix \tilde{h} with elements

$$\tilde{h}_{AB} = \eta_{AB} + 2\pi\alpha' F_{AB}, \quad A, B, \dots = 0, 1, \dots, 9, \quad (3.1)$$

where $F_{AB} = \partial_A \mathcal{A}_B - \partial_B \mathcal{A}_A$ is the ten-dimensional field strength and we again assume a flat background metric η_{AB} . \tilde{h} in (3.1) can be obtained from h in (2.1) by assuming the static gauge, which here means $X^A = \xi^A$, for all A . $\mathcal{S}_{\text{BI}}^{(0)}$ is given by

$$\mathcal{S}_{\text{BI}}^{(0)} = \frac{T_9}{g_s} \int d^{10}\xi \mathcal{L}_{\text{BI}}^{(0)}, \quad \mathcal{L}_{\text{BI}}^{(0)} = 1 - \sqrt{-\det[\tilde{h}_{AB}]}, \quad (3.2)$$

and from (2.3), $T_9 = 1/(4\pi^2\alpha')^5$. In dimensional reduction to the Dp -brane, $9 - p$ of the nine spatial directions are ‘T-dualized’. Choose the T-dual directions to be $A = \alpha = p + 1, p + 2, \dots, 9$. One of the consequence of this procedure, is that the gauge potentials \mathcal{A}_α in the T-dual directions get replaced with the transverse modes X_α of the Dp -brane according to

$$2\pi\alpha' \mathcal{A}_\alpha \rightarrow X_\alpha. \quad (3.3)$$

The fundamental degrees of freedom are then X_α and the remaining $p + 1$ gauge potentials \mathcal{A}_μ , $\mu = 0, 1, \dots, p$, which are functions of the $p + 1$ coordinates of the brane ξ^μ . Then the nonvanishing matrix elements of \tilde{h} are $\tilde{h}_{\mu\nu}$ and they are identical to $h_{\mu\nu}$ of the DBI action written in the static gauge and given in (2.5). Finally after performing integrations in the T-dual directions (3.2) gets replaced by

$$\mathcal{S}_{\text{BI}}^{(0)} = \frac{T_p}{g_s} \int d^{p+1}\xi (1 - \sqrt{-\det[\tilde{h}_{\mu\nu}]}, \quad (3.4)$$

where the Dp -brane tension is again given by (2.3), and one recovers (2.2) in the static gauge. So instead of working with $h_{\mu\nu}$ as we did in the previous section we could have started with the 10×10 matrix \tilde{h} . Then for the static spherically symmetric solutions of the previous section where just the $p + 1$ transverse mode is excited,

$$2\pi\alpha' F_{0i} = f\hat{r}_i, \quad 2\pi\alpha' F_{p+1i} = g\hat{r}_i, \quad i = 1, 2, \dots, p,$$

where \hat{r} is the unit vector in the radial direction and spherical symmetry means f and g are only functions of the radial variable r . The 10×10 matrix \tilde{h} takes the form

$$\tilde{h} = \begin{pmatrix} -1 & -f\hat{r} & & & \\ f\hat{r} & \mathbb{1}_{p \times p} & g\hat{r} & & \\ & -g\hat{r} & 1 & & \\ & & & \mathbb{1}_{(8-p) \times (8-p)} & \end{pmatrix}. \tag{3.5}$$

The first order corrections $\mathcal{S}_{\text{BI}}^{(1)}$ to the action $\mathcal{S}_{\text{BI}}^{(0)}$ of the space-filling D9-brane obtained in [8,9] involve first and second derivatives of the field strength F_{AB} . They are contained in the rank-4 tensor

$$S_{ABCD} = 2\pi\alpha' \partial_A \partial_B F_{CD} + (2\pi\alpha')^2 \tilde{h}^{EG} (\partial_A F_{CE} \partial_B F_{DG} - \partial_A F_{DE} \partial_B F_{CG}), \tag{3.6}$$

which is anti-symmetric in the last two indices. Here $\tilde{h}^{AB} \tilde{h}_{BC} = \delta_C^A$. The total action is

$$\begin{aligned} \mathcal{S}_{\text{BI}}^{(0)} + \mathcal{S}_{\text{BI}}^{(1)} &= \frac{T_9}{g_s} \int d^{10}x \left\{ 1 - \sqrt{-\det[\tilde{h}_{AB}]} \left(1 + \frac{\kappa}{4} \Delta \right) \right\}, \\ \Delta &= \tilde{h}^{AB} \tilde{h}^{CD} \tilde{h}^{EG} \tilde{h}^{IJ} (S_{BCEG} S_{DAIJ} - 2S_{GIBC} S_{JEDA}), \end{aligned} \tag{3.7}$$

where $\kappa = \frac{(2\pi\alpha')^2}{48}$. We again specialize to the case where a single transverse mode (the $(p + 1)$ th mode) is excited on a $p \leq 8$ brane, and consider static spherically symmetric fields. So the ansatz for \tilde{h} is again (3.5). Its determinant and inverse are given by

$$\det \tilde{h} = -1 + f^2 - g^2 \tag{3.8}$$

and

$$\tilde{h}^{-1} = \frac{1}{\det \tilde{h}} \begin{pmatrix} 1 + g^2 & \hat{r} f & -fg & & \\ -\hat{r} f & -\mathbb{1} + (f^2 - g^2)P & \hat{r} g & & \\ -fg & -\hat{r} g & f^2 - 1 & & \\ & & & \det \tilde{h} \mathbb{1}_{(8-p) \times (8-p)} & \end{pmatrix}, \tag{3.9}$$

respectively. P is the projection matrix $P_{ij} = \delta_{ij} - \hat{r}_i \hat{r}_j$, satisfying $P_{ij} \hat{r}_j = 0$ and $P_{ij} P_{jk} = P_{ik}$. Some work shows that the nonvanishing components of S_{ABCD} are

$$\begin{aligned} S_{ijk0} &= -S_{ij0k} = \det \tilde{h} H_f' \hat{r}_i \hat{r}_j \hat{r}_k - \frac{1}{r} \left(H_f + \frac{f}{r} \right) (P_{ik} \hat{r}_j + P_{jk} \hat{r}_i) + \left(\frac{f}{r} \right)' P_{ij} \hat{r}_k, \\ S_{ijkp+1} &= -S_{ijp+1k} = \det \tilde{h} H_g' \hat{r}_i \hat{r}_j \hat{r}_k - \frac{1}{r} \left(H_g + \frac{g}{r} \right) (P_{ik} \hat{r}_j + P_{jk} \hat{r}_i) + \left(\frac{g}{r} \right)' P_{ij} \hat{r}_k, \\ S_{ijkl} &= \frac{(\ln \det \tilde{h})'}{2r} (P_{ik} \hat{r}_j \hat{r}_\ell + P_{j\ell} \hat{r}_i \hat{r}_k - P_{i\ell} \hat{r}_j \hat{r}_k - P_{jk} \hat{r}_i \hat{r}_\ell) \\ &\quad + \frac{1 + (\det \tilde{h})^{-1}}{r^2} (P_{ik} P_{j\ell} - P_{i\ell} P_{jk}), \end{aligned} \tag{3.10}$$

where

$$H_f = \frac{f'}{\det \tilde{h}}, \quad H_g = \frac{g'}{\det \tilde{h}}, \quad (3.11)$$

the prime here denoting derivatives in r . In addition we define

$$H_k = f H_g - g H_f.$$

Substituting into the formula in (3.7) for Δ gives

$$\begin{aligned} \mathcal{E} &= -\frac{1}{4}(\det \tilde{h})^2 \Delta \\ &= H_f'^2 - H_g'^2 + H_k'^2 + \frac{p-1}{r^2} \left\{ (2 + (\det \tilde{h})^2)(H_f'^2 - H_g'^2 + H_k'^2) \right. \\ &\quad \left. + \frac{1}{2}(\ln \det \tilde{h})'^2 + \frac{2}{r}(\ln \det \tilde{h})' - \frac{1}{r}(\det \tilde{h})' \right. \\ &\quad \left. + \frac{1}{r^2}(1 + \det \tilde{h})(p + 1 + (p-2)\det \tilde{h}) \right\}. \end{aligned} \quad (3.12)$$

So for the above ansatz the correction to the zeroth order Lagrangian density $\mathcal{L}_{\text{DBI}}^{(0)}$ in (2.2) is

$$\mathcal{L}_{\text{DBI}}^{(1)} = \frac{\kappa \mathcal{E}}{(-\det \tilde{h})^{3/2}}. \quad (3.13)$$

In obtaining the equations of motion one must again write $f(r)$ and $g(r)$ in terms of potentials and extremize with respect to the latter. As the general system is quite involved, below we shall restrict to functions $f(r)$ and $g(r)$ which are related by a constant factor

$$\frac{f(r)}{Q} = \frac{g(r)}{C}, \quad (3.14)$$

as what occurred for the zeroth order solutions (2.8). This set of configurations respects the $SO(1, 1)$ symmetry (2.9) and (2.10). Using

$$\frac{H_f(r)}{Q} = \frac{H_g(r)}{C}, \quad H_k(r) = 0 \quad (3.15)$$

the Lagrangian density simplifies, and it is $SO(1, 1)$ invariant. Once again there are three distinct orbits: (i) $|Q| > |C|$, (ii) $|Q| = |C|$ and (iii) $|Q| < |C|$, and one expects that these orbits are classified by the corresponding value for the spatial integral of the Lagrangian density, which is now $\mathcal{L}_{\text{DBI}}^{(0)} + \mathcal{L}_{\text{DBI}}^{(1)}$. To compute the latter we only have to examine one point on each of the orbits, which we do below.

(i) $|Q| > |C|$. A convenient point on this orbit is the purely electrostatic case, where the transverse mode is suppressed: $f(r) = 2\pi\alpha' \mathcal{A}'_0(r)$ and $g(r) = 0$. Substituting this into

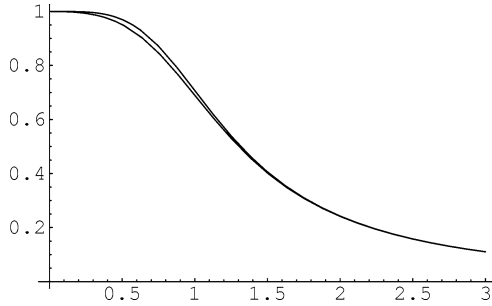


Fig. 7. f_0 and $f_0 + f_1$ vs. r for $p = 3, Q = 1, C = 0$.

$\mathcal{L}_{\text{DBI}}^{(0)} + \mathcal{L}_{\text{DBI}}^{(1)}$, and varying with respect to $\mathcal{A}_0(r)$ gives the corrected Born–Infeld equation

$$\begin{aligned} & \frac{\sqrt{-\det \tilde{h}}}{\kappa} (r^{p-1} f - Q \sqrt{-\det \tilde{h}}) \\ &= \left[\frac{2r^{p-1} H'_f}{(-\det \tilde{h})^{3/2}} \right]'' - \frac{3r^{p-1} f H_f'^2}{(-\det \tilde{h})^{3/2}} - \frac{2(p-1)(3 + f^4)(r^{p-3} H_f)'}{(-\det \tilde{h})^{3/2}} \\ &+ \frac{(p-1)r^{p-5} f}{(-\det \tilde{h})^{3/2}} \{ f^4 (p-2 - r^2 H_f^2) + f^2 (-2p-3 + 4r^2 H_f^2) \\ &+ 3(-2p+6 + 3r^2 H_f^2) \}. \end{aligned} \tag{3.16}$$

To get back the zeroth order equations set the left-hand side equal to zero. So the right-hand side represents the derivative corrections. The zeroth order Born–Infeld solution satisfies (3.16) as $r \rightarrow \infty$, so the corrections are negligible in this region. In [10], starting with the zeroth order Born–Infeld solution at $r \rightarrow \infty$, we used (3.16) to numerically integrate to finite r . We found the resulting corrections to $f(r)$ at finite r to be small, and just like at zeroth order, f tends to 1 as $r \rightarrow 0$. Call $f_0(r)$ the zeroth solution, and $f_1(r)$ the correction it receives at first order. In Fig. 7 we plot $f_0(r)$ and $f_0(r) + f_1(r)$ when $Q = 1, C = 0$ and $p = 3$. In Fig. 7 we set $\kappa = 1$ which is equivalent to choosing the scale for r . If the zeroth order Born–Infeld solution gives a reasonable approximation to a classical solution in the full effective theory, and one can apply the derivative expansion to get the latter, then higher order corrections should be small. In particular, we expect only a small change in the value of the Lagrangian at the next order. If one assumes this to be the case one can apply a Taylor expansion about the zeroth order solution

$$\begin{aligned} & \int d^p \xi [\mathcal{L}_{\text{DBI}}^{(0)} + \mathcal{L}_{\text{DBI}}^{(1)}](f_0 + f_1) \\ & \approx \int d^p \xi [\mathcal{L}_{\text{DBI}}^{(0)}](f_0) + \int d^p \xi [\mathcal{L}_{\text{DBI}}^{(1)}](f_0) + \int d^p \xi \left. \frac{\delta \mathcal{L}_{\text{DBI}}^{(0)}}{\delta f} \right|_{f=f_0} f_1. \end{aligned} \tag{3.17}$$

The last term vanishes by the zeroth order field equations, and so the first order correction to the Lagrangian is $\int d^p \xi [\mathcal{L}_{\text{DBI}}^{(1)}](f_0)$. However, it is easy to check that $[\mathcal{L}_{\text{DBI}}^{(1)}](f_0)$

diverges near the origin as $1/r^{3p+1}$. So the first order correction is not small; rather, $\int d^p \xi [\mathcal{L}_{\text{DBI}}^{(1)}](f_0)$ is infinite! This agrees with the result found numerically in [10], and indicates that the Born–Infeld solution, and indeed all case (i) solutions, are unstable under inclusion of first order derivative corrections.

(ii) $|Q| = |C|$. This is the BPS case $g(r) = f(r)$. Here $\mathcal{L}_{\text{DBI}}^{(1)} = 0$, and so just like at zeroth order the Lagrangian vanishes. To find equations of motion we must first vary f and g (or more precisely X_{p+1} and \mathcal{A}_0) separately and then impose the BPS condition. We do this in Appendix B. (Actually, there we do not impose the restriction of spherical symmetry.) The result is simply

$$\nabla^2 \{1 + 2\kappa(\nabla^2)^2\} \mathcal{A}_0 = 0, \tag{3.18}$$

with the same equation for X_{p+1} . For the case of spherically symmetric solutions we can use $\nabla^2 = \frac{1}{r^{p-1}} \partial_r r^{p-1} \partial_r$. Eq. (3.18) says that the zeroth order solution is also valid at first order. This agrees with [11], where it was shown that the BPS solution is valid to all orders. The result (3.18) thus provides a check of the computations in [8,9].

(iii) $|Q| < |C|$. A convenient point is the purely transverse case. Here the electric field vanishes: $f(r) = 0$ and $g(r) = X'_{p+1}(r)$. Substituting this in (3.13) and varying $X_{p+1}(r)$ gives

$$\begin{aligned} & \frac{\sqrt{-\det \tilde{h}}}{\kappa} (r^{p-1} g - C \sqrt{-\det \tilde{h}}) \\ &= \left[\frac{2r^{p-1} H'_g}{(-\det \tilde{h})^{3/2}} \right]'' + \frac{3r^{p-1} g H_g'^2}{(-\det \tilde{h})^{3/2}} - \frac{2(p-1)(3+g^4)(r^{p-3} H_g)'}{(-\det \tilde{h})^{3/2}} \\ &+ \frac{(p-1)r^{p-5} g}{(-\det \tilde{h})^{3/2}} \{g^4(p-2+r^2 H_g^2) + g^2(2p+3+4r^2 H_g^2) \\ &+ 3(-2p+6-3r^2 H_g^2)\}. \end{aligned} \tag{3.19}$$

Again to get back the zeroth order equations set the left-hand side equal to zero, and so the right-hand side represents the derivative corrections. (3.19) is also obtained by making the transformation $f(r) \rightarrow ig(r)$ and $Q \rightarrow iC$ in (3.16), so solving for real $g(r)$ in (3.19) is equivalent to solving for imaginary $f(r)$ in (3.16). As with case (i), starting with the zeroth order solution at $r \rightarrow \infty$, we can use (3.19) to numerically integrate to finite r . We find that just like at zeroth order, g becomes singular at some finite r , which appears to be slightly greater than r_0 . We can then conclude that the corrections cause the wormhole to become wider. Call $g_0(r)$ the zeroth order solution, and $g_1(r)$ the correction it receives at first order. In Fig. 8 we plot $g_0(r)$ and $g_0(r) + g_1(r)$ when $C = 1, Q = 0$ and $p = 3$. The lower curve is g_0 . Again we set $\kappa = 1$. Fig. 8 shows that the correction g_1 to the solution is small away from the wormhole throat. On the other hand, the corresponding correction to the Lagrangian density appears not to be small, as is indicated in Fig. 9 where we numerically compare $[\mathcal{L}_{\text{DBI}}^{(0)} + \mathcal{L}_{\text{DBI}}^{(1)}](g_0 + g_1)$ and $[\mathcal{L}_{\text{DBI}}^{(0)}](g_0)$. The lower curve in Fig. 9 is $[\mathcal{L}_{\text{DBI}}^{(0)} + \mathcal{L}_{\text{DBI}}^{(1)}](g_0 + g_1)$. We then expect large corrections to its integral. In fact, we find that the numerical integration of $[\mathcal{L}_{\text{DBI}}^{(0)} + \mathcal{L}_{\text{DBI}}^{(1)}](g_0 + g_1)$ fails to give a convergent result. So just as in case (i), the integral of the Lagrangian density appears

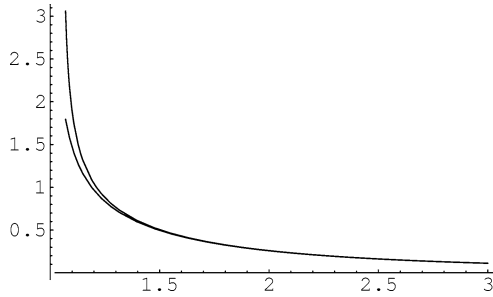


Fig. 8. g_0 and $g_0 + g_1$ vs. r for $p = 3, C = 1, Q = 0$.

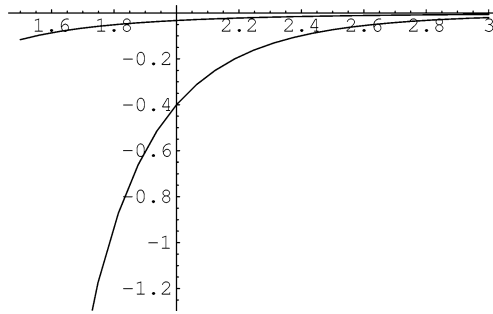


Fig. 9. $[\mathcal{L}_{\text{DBI}}^{(0)}](g_0)$ and $[\mathcal{L}_{\text{DBI}}^{(0)} + \mathcal{L}_{\text{DBI}}^{(1)}](g_0 + g_1)$ vs. r for $p = 3, C = 1, Q = 0$.

to be ill-defined, indicating that the case (iii) solutions are also unstable under inclusion of first order derivative corrections.

4. Conclusion

The preliminary indications here are that the classical wormhole solution may not be a reasonable approximation to a solution in the full D -brane theory. If so it therefore cannot prevent the decay of the brane–anti-brane system. More generally, it appears that the only solution that survives higher order derivative corrections may be the BPS solution. On the other hand, a more extensive analysis may be possible. For example, it would be interesting to drop the restriction to static solutions, or to excite additional degrees of freedom such as magnetic fields. Perhaps time dependent configurations can survive first order derivative corrections, or perhaps the zeroth order static solutions evolve to time dependent ones after including the higher order. To check this would require combining two separate stability analyses, which we referred to as (a) and (b) in the introduction. A final but unpleasant (from a computational point of view) possibility is that solutions are recovered only after going beyond the first order. Moreover, all orders may be required, meaning the solutions may be nonperturbative.

Acknowledgement

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Appendix A. Alternative gauge

Here we reconstruct the zeroth order wormhole solutions starting with the action written in an alternative gauge. This gauge is obtained by replacing the r -coordinate of the static gauge by $z = X_{p+1}$ gauge. It has the advantage that it removes the coordinate singularity appearing at the midway point of the wormhole, and shows that there is a smooth solution connecting the brane to anti-brane. Coordinate singularities reappear, however, at the location of the brane and anti-brane. We show that the electric field is well behaved in this gauge, and in fact is a constant.

Consider the domain $S^{p-1} \times \mathbb{R}^2$, with local coordinate patches used to parametrize the Dp -brane. Denote by z one of the coordinates of \mathbb{R}^2 , while the other corresponds to time t . We look for spherically symmetric static solutions with

$$X_0 = t, \quad X_1^2 + X_2^2 + \dots + X_p^2 = R(z)^2, \quad X_{p+1} = z. \quad (\text{A.1})$$

So for example for the D3-brane we can write

$$\begin{aligned} X_1 &= R(z) \sin \theta \cos \phi, \\ X_2 &= R(z) \sin \theta \sin \phi, \\ X_3 &= R(z) \cos \theta, \end{aligned} \quad (\text{A.2})$$

using standard spherical coordinates θ and ϕ ,

$$\begin{aligned} h_{tt} &= -1, & h_{\theta\theta} &= R(z)^2, \\ h_{\phi\phi} &= R(z)^2 \sin^2 \theta, & h_{zz} &= R'(z)^2 + 1, \end{aligned} \quad (\text{A.3})$$

where here the prime denotes a derivative in z . Now introduce a z -dependent electrostatic field in the z -direction, leading to the off-diagonal components

$$h_{tz} = -h_{zt} = -E(z). \quad (\text{A.4})$$

In terms of the electrostatic potential \mathcal{A}_0 which we now write as a function of z , $E(z) = 2\pi\alpha' \partial_z \mathcal{A}_0(z)$. After performing the angular integrations, the DBI Lagrangian $L_{\text{DBI}}^{(0)}$ (ignoring the vacuum term) will be proportional to $R(z)^2 \sqrt{R'(z)^2 - E(z)^2 + 1}$. Generalizing to arbitrary p ,

$$L_{\text{DBI}}^{(0)} \propto R(z)^{p-1} \sqrt{R'(z)^2 - E(z)^2 + 1}. \quad (\text{A.5})$$

Variations in the electrostatic potential and $R(z)$ give

$$\left(\frac{R(z)^{p-1} E(z)}{\sqrt{R'(z)^2 - E(z)^2 + 1}} \right)' = 0, \quad (\text{A.6})$$

$$R''(z)R(z) = (p-1)(R'(z)^2 - E(z)^2 + 1), \quad (\text{A.7})$$

respectively. From the first equation

$$\left| \frac{E(z)}{Q} \right| = \sqrt{\frac{R'(z)^2 + 1}{R(z)^{2p-2} + Q^2}}, \tag{A.8}$$

where Q is an integration constant, and substituting into the second equation

$$R''(z) = (p - 1)R(z)^{2p-3} \frac{R'(z)^2 + 1}{R(z)^{2p-2} + Q^2}. \tag{A.9}$$

After integrating once

$$R'(z) = \frac{1}{C} \sqrt{R(z)^{2p-2} - C^2 + Q^2}, \tag{A.10}$$

where C is an integration constant. Since $R'(z)$ corresponds to $1/g(r)$, the result agrees with (2.8). For the wormhole solution, $R(z)$ is nonsingular everywhere except at the location of the brane and anti-brane. At the midway-point on the wormhole, R is a well-defined function of z , and is a minimum since

$$R''(z_{\text{mid}}) = \frac{p-1}{r_0} \left(1 - \frac{Q^2}{C^2} \right), \quad R(z_{\text{mid}}) = r_0 = (C^2 - Q^2)^{1/(2p-2)}, \tag{A.11}$$

and $|Q| < |C|$ for wormhole solutions. So now coordinate singularities appear at the brane and anti-brane, rather than at the midway-point on the wormhole. By substituting (A.10) into (A.8) one gets that the electric field $E(z)$ in the z -direction is a constant

$$|E(z)| = \left| \frac{Q}{C} \right|, \tag{A.12}$$

which agrees with (2.23). It goes to one in the BPS limit, and $1/\sqrt{p-1}$ for the minimum energy wormhole. We conclude that in this coordinate frame there are no singularities in the electric field (for $C \neq 0$).

Appendix B. First order BPS equations

Here we derive the first order BPS equation (3.18). Unlike in Section 3, we make no restriction to spherical symmetry. Our starting point is then not (3.5), but

$$\tilde{h} = \begin{pmatrix} -1 & -\vec{f} & & & \\ \vec{f} & \mathbb{1}_{p \times p} & \vec{g} & & \\ & -\vec{g} & 1 & & \\ & & & \mathbb{1}_{(8-p) \times (8-p)} & \end{pmatrix}, \tag{B.1}$$

where $\vec{f} = 2\pi\alpha' \vec{\nabla} A_0$ and $\vec{g} = \vec{\nabla} X_{p+1}$ are vector fields on the Dp -brane. The general BPS condition is $\vec{f} = \vec{g}$. Upon imposing this condition on \tilde{h}^{-1} one gets

$$\tilde{h}^{-1}|_{\text{BPS}} = \begin{pmatrix} -1 - \vec{f}^2 & -\vec{f} & \vec{f}^2 & & \\ \vec{f} & \mathbb{1}_{p \times p} & -\vec{f} & & \\ \vec{f}^2 & \vec{f} & -\vec{f}^2 + 1 & & \\ & & & \mathbb{1}_{(8-p) \times (8-p)} & \end{pmatrix}, \tag{B.2}$$

while the only nonvanishing components of S_{ABCD} are

$$S_{ijk0}|_{\text{BPS}} = S_{ijkp+1}|_{\text{BPS}} = -S_{ij0k}|_{\text{BPS}} = -S_{ijp+1k}|_{\text{BPS}} = \partial_i \partial_j f_k. \quad (\text{B.3})$$

Since the BPS action vanishes, we must impose $\vec{f} = \vec{g}$ after performing the variations in the action to find the BPS field equations, i.e., we must perform separate variations of \mathcal{A}_0 and X_{p+1} . By varying \mathcal{A}_0 and then setting $\vec{f} = \vec{g}$,

$$\begin{aligned} \delta \tilde{h}^{00}|_{\text{BPS}} &= -2(1 + \vec{f}^2) \vec{f} \cdot \delta \vec{f}, \\ \delta \tilde{h}^{p+1p+1}|_{\text{BPS}} &= -2\vec{f}^2 \vec{f} \cdot \delta \vec{f}, \\ \delta \tilde{h}^{0p+1}|_{\text{BPS}} &= (1 + 2\vec{f}^2) \vec{f} \cdot \delta \vec{f} = \delta \tilde{h}^{p+10}|_{\text{BPS}}, \\ \delta \tilde{h}^{i0}|_{\text{BPS}} &= (1 + \vec{f}^2) \delta f_i + \vec{f} \cdot \delta \vec{f} f_i = -\delta \tilde{h}^{0i}|_{\text{BPS}}, \\ \delta \tilde{h}^{ip+1}|_{\text{BPS}} &= -\vec{f}^2 \delta f_i - \vec{f} \cdot \delta \vec{f} f_i = -\delta \tilde{h}^{p+1i}|_{\text{BPS}}, \\ \delta \tilde{h}^{ij}|_{\text{BPS}} &= -f_i \delta f_j + f_j \delta f_i, \end{aligned} \quad (\text{B.4})$$

while the nonvanishing components of δS_{ABCD} are

$$\begin{aligned} \delta S_{ijk0}|_{\text{BPS}} &= -\delta S_{ij0k}|_{\text{BPS}} = \partial_i \partial_j \delta f_k + \delta S_{ijkp+1}|_{\text{BPS}}, \\ \delta S_{ijkp+1}|_{\text{BPS}} &= -\delta S_{ijp+1k}|_{\text{BPS}} = \partial_i f_\ell \partial_j f_k \delta f_\ell - \tilde{h}^{0\ell} \partial_i \delta f_k \partial_j f_\ell + (i \rightleftharpoons j), \\ \delta S_{ijk\ell}|_{\text{BPS}} &= \partial_j f_\ell \partial_i \delta f_k + \partial_i f_k \partial_j \delta f_\ell - (i \rightleftharpoons j). \end{aligned} \quad (\text{B.5})$$

In evaluating $\delta \Delta|_{\text{BPS}}$ we can use $(\tilde{h}^{AB} S_{ijAB})|_{\text{BPS}} = 0$. Then

$$\begin{aligned} \delta \Delta|_{\text{BPS}} &= -4\tilde{h}^{CD} S_{jDA} \{ \delta \tilde{h}^{AB} S_{ijBC} + \tilde{h}^{AB} \delta S_{ijBC} + \tilde{h}^{AB} \delta \tilde{h}^{i\ell} S_{\ell jBC} \} |_{\text{BPS}} \\ &= -8\partial_i \partial_j f_k \partial_i \delta f_k. \end{aligned} \quad (\text{B.6})$$

Now substitute $\vec{f} = 2\pi\alpha' \vec{\nabla} \mathcal{A}_0$ to obtain (3.18) from the variation of \mathcal{A}_0 in $\mathcal{L}_{\text{DBI}}^{(0)} + \mathcal{L}_{\text{DBI}}^{(1)}$. One gets the same results from variations of X_{p+1} .

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