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General Action Principle for Supersymmetric Particles

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A new Lagrangean formulation for supersymmetric particles is given. With the inclusion of a Wess-Zumino term one can obtain any irreducible representation of supersymmetry upon quantization. The quantization is carried out in a gauge-invariant manner. A possible generalization of this Lagrangean to superstrings is given.

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Much attention has been given recently to superstrings as possible unified theories containing gravitation.¹ A covariant action² has been given for the superstring; however, to carry out the quantization one must resort to a light-cone gauge fixing. It is not known whether there is an alternative formulation which can give a Lorentz-covariant quantization. Another question arises from the presence of a certain Wess-Zumino term which must be included in the total action of Ref. 2. The coefficient of this term is not arbitrary and its meaning has not been completely understood.

Instead of tackling these questions for superstrings, I examine analogous questions for superparticles. A new classical action for supersymmetric particles is given. Canonical quantization is carried out in a gauge-invariant manner. The Lagrangean contains a Wess-Zumino term, whose coefficient is associated with the superspin of the particle. By suitable choice of this coefficient, along with the mass, any irreducible representation of supersymmetry is obtained in the

quantum theory.

In the work of Balachandran *et al.*³ a Lagrangean formulation was given for relativistic spinning particles. Upon quantization it was found that all unitary irreducible representations of the covering group of the connected Poincaré group P_{\downarrow}^+ were obtainable. My approach will be to give a supersymmetric extension of the Lagrangean of Ref. 3, in such a way that upon quantization we recover all irreducible representations of the super Poincaré group.

The configuration space for the Lagrangean of Ref. 3 was taken to be $P_{\downarrow}^+ = \{(x, \Lambda)\}$. There, $x = (x^m; m = 0, 1, 2, 3)$ corresponded to the space-time coordinates of the particle, while $\Lambda = [\Lambda^m_n]$ was an element of the connected component of the Lorentz group. Here the configuration space is the super Poincaré group $SP_{\downarrow}^+ = \{(x, \theta, \bar{\theta}, M)\}$, where x is as before, $\theta = (\theta^\alpha; \alpha = 1, 2)$ and $\bar{\theta} = (\bar{\theta}^{\dot{\alpha}}; \dot{\alpha} = 1, 2)$ are Grassmann coordinates, and $M = [M^\alpha_\beta]$ is an element of $SL(2, C)$. We will assume that M is unaffected by global supersymmetry transformations:

$$(x^m, \bar{\theta}, \theta, M) \rightarrow (x^m + i\theta\sigma^m\bar{\zeta} - i\zeta\sigma^m\bar{\theta}, \theta + \zeta, \bar{\theta} + \bar{\zeta}, M),$$

where ζ and $\bar{\zeta}$ are infinitesimal Grassmann parameters. Through M , the physical momenta P_m and the "superspin" S_{mn} can be defined,

$$P = P_m \sigma^m = M\tilde{P}M^\dagger, \quad S = S_{mn} \sigma^{mn} = M\tilde{S}M^{-1}, \quad (1)$$

where $\tilde{P} = \tilde{P}_m \sigma^m$ and $\tilde{S} = \tilde{S}_{mn} \sigma^{mn}$ are constant matrices. The σ^{mn} 's are $SL(2, C)$ generators which are written in terms of the σ^m and $\bar{\sigma}^m$ matrices. Here and throughout this Letter the conventions of Wess and Bagger⁴ are used.

For massive particles \tilde{P} and \tilde{S} can be chosen to be the momenta and superspin in the particle rest frame, e.g.,

$$\tilde{P} = m \sigma^0, \quad \tilde{S} = 2\lambda \sigma^{12}. \quad (2)$$

With this choice we have the following identities:

$$P_m P^m = -\det P = -m^2, \quad (3)$$

$$\frac{1}{2} S_{mn} S^{mn} = -\frac{1}{4} [\text{Tr} S^2 + \text{Tr} (S^\dagger)^2] = \lambda^2, \quad (4)$$

$$S_{mn} P^m = \frac{1}{4} \text{Tr} \bar{\sigma}_n (PS^\dagger + SP) = 0. \quad (5)$$

For massless particles we may replace (2) by

$$\tilde{P} = \omega (\sigma^0 + \sigma^3), \quad \tilde{S} = 2\lambda \sigma^{12}. \quad (6)$$

With this choice we get $P^m P_m = 0$ as well as (4) and (5).

For the Lagrangean we first consider

$$L_0 = P_m V^m, \quad V^m = \dot{x}^m + i\theta\sigma^m\dot{\bar{\theta}} - i\dot{\theta}\sigma^m\bar{\theta}, \quad (7)$$

where the dot indicates a derivative with respect to proper time τ . For the case $m = 0$, (7) is similar to Lagrangeans which have previously been studied.^{5,6} However, unlike in those treatments, we need not add Lagrange-multiplier terms since variations of P_m are already constrained by (1).

The action $\int d\tau L_0$ is invariant under global supersymmetry transformations, as well as the usual reparametrizations $\tau \rightarrow f(\tau)$. Additional local symmetries are present for L_0 . L_0 is unchanged under the

right action of three-parameter subgroup of $SL(2, C)$

$$\delta_\xi M = M[\epsilon_{mnr} \tilde{P}^m \xi^n(\tau) \sigma^{rs}]. \quad (8)$$

(ξ^n are three infinitesimal parameters since $\delta_\xi M = 0$ when $\xi^n = \mu \tilde{P}^n$.) For the case of massless particles only there is a local supersymmetry. It is due to the fact that when $\det P = 0$, there are nonzero solutions y^α to the equation $P_{\alpha\dot{\alpha}} y^\alpha = 0$. When \tilde{P} has the form given in (6), the supersymmetry transformation is⁷

$$\begin{aligned} \delta_\eta \theta^\alpha &= (M^{-1})^\alpha_\eta \eta(\tau), & \delta_\eta \bar{\theta}^{\dot{\alpha}} &= (M^\dagger)^{\dot{\alpha}}_\eta \bar{\eta}(\tau), \\ \delta_\eta x^m &= i \delta_\eta \theta \sigma^m \bar{\theta} - i \theta \sigma^m \delta_\eta \bar{\theta}, \end{aligned} \quad (9)$$

where η and $\bar{\eta}$ are infinitesimal Grassmann parameters.

The classical equations of motion resulting from the variations δx^m , $\delta \theta^\alpha$, and $\delta \bar{\theta}^{\dot{\alpha}}$ in L_0 are

$$\dot{P} = 0, \quad \dot{\theta}^\alpha P_{\alpha\dot{\alpha}} = 0, \quad P_{\alpha\dot{\alpha}} \dot{\bar{\theta}}^{\dot{\alpha}} = 0. \quad (10)$$

For the massive case, the latter two equations imply $\dot{\theta}^\alpha = \dot{\bar{\theta}}^{\dot{\alpha}} = 0$. The most general variation of M is of the form $\delta M = \epsilon_{mn} \sigma^{mn} M$, which leads to $\delta L_0 = 2\epsilon_{mn} P^m V^n$. With use of (10) the resulting equations of motion may be written

$$j^{mn}_{(0)} = 0, \quad (11)$$

$$\int_{R^1} d\tau L_{WZ} = -\frac{1}{16\lambda^2} \int_{G^2} \text{Tr}(S dS \wedge dS + S^\dagger dS^\dagger \wedge dS^\dagger),$$

where $R^1 = \partial G^2$. Such contributions are commonly called Wess-Zumino terms.⁹

Next we carry out the canonical quantization. Because there exist constraints on the phase-space variables we will rely on Dirac's quantization procedure.¹⁰ We define π_m , χ_α , and $\bar{\chi}_{\dot{\alpha}}$ to be the momenta conjugate to x^m , θ^α , and $\bar{\theta}^{\dot{\alpha}}$. The Poisson brackets (PB's) are

$$\begin{aligned} \{x^n, \pi_m\} &= \eta^n_m, & \{\theta^\alpha, \chi_\beta\} &= -\delta^\alpha_\beta, \\ \{\bar{\theta}^{\dot{\alpha}}, \bar{\chi}_{\dot{\beta}}\} &= -\delta^{\dot{\alpha}}_{\dot{\beta}}. \end{aligned} \quad (15)$$

Further we need the six-momenta F^{mn} conjugate to the "angles" $\alpha_{mn} = -\alpha_{nm}$ which parametrize M . Actually, it is more convenient to work with the variables³ $R^{mn} = -N^{mn}_{rs} F^{rs}$, where N is a nonsingular matrix with

$$M^{-1} \frac{\partial M}{\partial \alpha_{mn}} = (N^{-1})^{mn}_{rs} \sigma^{rs}. \quad (16)$$

R^{mn} generate right-handed $SL(2, C)$ transformations on M ,

$$\begin{aligned} \{R^{mn}, M\} &= M \sigma^{mn}, \\ \{R^{rs}, R^{mn}\} &= -\eta^{sm} R^{rn} + \eta^{sn} R^{rm} + \eta^{rm} R^{sn} - \eta^{rn} R^{sm}. \end{aligned} \quad (17)$$

Since both terms in L are first order in τ derivatives

where $J^{mn}_{(0)} = x^m P^n - x^n P^m - \epsilon^{mnr} P_r \theta \sigma_s \bar{\theta}$. For massive particles, the last term in $J^{mn}_{(0)}$ is separately conserved.

I shall show that upon quantization L_0 yields the fundamental representation Ω_0 of $N=1$ supersymmetry. Higher representations Ω_s , $s = \frac{1}{2}, 1, \frac{3}{2}, \dots$, are obtained from the Lagrangean

$$\begin{aligned} L &= L_0 + L_{WZ}, \\ L_{WZ} &= \lambda(\text{Tr} \sigma^{12} M^{-1} \dot{M} - \text{Tr} \bar{\sigma}^{12} \dot{M}^\dagger M^{\dagger-1}). \end{aligned} \quad (12)$$

Upon the setting of θ and $\bar{\theta}$ equal to zero, L becomes equivalent to the Lagrangean of Ref. 3.

Under $\delta_\xi M = M[\xi_{12}(\tau) \sigma^{12} + \xi_{03}(\tau) \sigma^{03}]$, L_{WZ} changes by a total time derivative. However, the total action $\int d\tau L$ is invariant only under

$$\delta_\xi M = M \xi_{12}(\tau) \sigma^{12}. \quad (13)$$

The term L_{WZ} does not alter the equations of motion (10). The only effect that L_{WZ} has on Eq. (11) is that the conserved angular momentum gets redefined: $J^{mn}_{(0)} \rightarrow J^{mn}_{(\lambda)} = J^{mn}_{(0)} + S^{mn}$.

In the work of Balachandran, Lizzi, and Sparano⁸ it was shown that $\int d\tau L_{WZ}$ could be written in terms of a closed but not exact two-form on the coset space $SL(2, C)/U(1) \times R^1$ spanned by S_{mn} . In the notation of Ref. 4,

$$\int_{R^1} d\tau L_{WZ} = -\frac{1}{16\lambda^2} \int_{G^2} \text{Tr}(S dS \wedge dS + S^\dagger dS^\dagger \wedge dS^\dagger), \quad (14)$$

all fourteen momenta are constrained,

$$\begin{aligned} \phi_n &= P_n - \pi_n \approx 0, \\ t^{mn} &= R^{mn} - \lambda(\eta^m_1 \eta^n_2 - \eta^m_2 \eta^n_1) \approx 0, \\ \psi_\alpha &= \chi_\alpha + iP_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \approx 0, & \bar{\psi}_{\dot{\alpha}} &= \bar{\chi}_{\dot{\alpha}} + i\theta^\alpha P_{\alpha\dot{\alpha}} \approx 0, \end{aligned} \quad (18)$$

and the Hamiltonian is just a linear combination of the constraints, $H = \alpha_{mn} t^{mn} + \beta^n \phi_n + \mu^\alpha \psi_\alpha + \bar{\mu}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}}$. Here α_{mn} , β^n , μ^α , and $\bar{\mu}_{\dot{\alpha}}$ are Lagrange multipliers, the latter two being odd Grassmann. The condition that the constraints hold for all τ imposes no further (secondary) constraints on phase space.

It is useful to define variables which have (weakly) zero PB's with all constraints. Such variables are

$$\pi^m, \quad Q_\alpha = \chi_\alpha - i\pi_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}}, \quad \bar{Q}_{\dot{\alpha}} = \bar{\chi}_{\dot{\alpha}} - i\theta^\alpha \pi_{\alpha\dot{\alpha}}, \quad (19)$$

$$j^{mn} = x^m \pi^n - x^n \pi^m - \theta \sigma^{mn} \chi - \bar{\chi} \bar{\sigma}^{mn} \bar{\theta} + L^{mn}.$$

Here $\pi_{\alpha\dot{\alpha}} = (\sigma^m)_{\alpha\dot{\alpha}} \pi_m$ and L^{mn} generate left-handed $SL(2, C)$ transformations on M .

$$L^{mn} = \Lambda^{mn}_{rs} R^{rs} = S^{mn} + \Lambda^{mn}_{rs} t^{rs}, \quad (20)$$

where $\Lambda^{mn}_{rs} \sigma^{rs} = M^{-1} \sigma^{mn} M$. On the constrained phase space j^{mn} is identical to $J^{mn}_{(\lambda)}$. Up to linear

combinations of the constraints, the PB's of π^m , Q_α , $\bar{Q}_{\dot{\alpha}}$, and j^{mn} form the usual $N=1$ supersymmetry algebra,

$$\begin{aligned} \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} &= 2i\pi_{\alpha\dot{\alpha}}, & \{j^{mn}, \pi^r\} &= \pi^n\eta^{mr} - \pi^m\eta^{nr}, \\ \{j^{mn}, Q_\alpha\} &= (\sigma^{mn})_\alpha{}^\beta Q_\beta, \\ \{j^{mn}, \bar{Q}_{\dot{\alpha}}\} &= \bar{Q}_{\dot{\beta}}(\bar{\sigma}^{mn})^{\dot{\beta}}{}_{\dot{\alpha}}, \\ \{j^{rs}, j^{mn}\} &= \eta^{rn}j^{ms} - \eta^{sm}j^{rn} - \eta^{sn}j^{mr} + \eta^{rm}j^{sn}. \end{aligned} \quad (21)$$

Because L possesses gauge symmetries, we expect that certain linear combinations of the constraints (18) are first class and are the generators of the symmetries. Since the variables (19) have zero PB's with all constraints, they have zero PB's with the first-class constraints and, hence, are gauge invariant. Further, since (19) have zero PB's with all second-class constraints, the Poisson bracket relations (21) are identical to the corresponding Dirac brackets.

The present approach to quantization of the theory will be to quantize only with respect to the gauge-invariant variables. It remains to show that (19) form a complete set of such variables. For this we consider the following cases separately.

$m \neq 0$, $\lambda = 0$.—Here our action is just $\int d\tau L_0$, which is invariant under $\tau \rightarrow f(\tau)$ and $\delta_\xi M = M\xi_{ij}(\tau)\sigma^i\sigma^j$ ($i, j = 1-3$). The associated four first-class constraints are $C^m\phi_m \approx 0$ and $t_{ij} \approx 0$, where $C^m = \text{Tr}M^{-1} \times \bar{\sigma}^0 M^{-1} \sigma^m$. Thus, among the fourteen variables which parametrize the constrained phase space a total of ten can be gauge invariant. The latter are precisely (19). The reason that the set (19) consists of ten variables (and not fourteen) is that when we impose the constraints and set $\lambda = 0$ we get four conditions:

$$\pi^m\pi_m = -m^2, \quad W^m\pi^n - W^n\pi^m = 0, \quad (22)$$

where $W^m = \frac{1}{2}\epsilon^{mnr}s\pi_n j_{rs} - \frac{1}{4}\bar{Q}\bar{\sigma}^m Q$ is the supersymmetric generalization of the Pauli-Lubanski vector.

The quantum theory is obtained by replacement of the (antisymmetric) symmetric Dirac brackets by $-i$ times the (commutator) anticommutator. Upon imposing the conditions (22) on the quantum mechanical operators π^m , j^{mn} , Q_α , and $\bar{Q}_{\dot{\alpha}}$, we immediately obtain the (massive) fundamental representation Ω_0 . In the rest frame ($\pi = \pi_m\sigma^m = m\sigma^0$), $Q_\alpha/(2m)^{1/2}$ and $\bar{Q}_{\dot{\alpha}}/(2m)^{1/2}$ become fermionic creation and annihilation operators which act on a system of four states. Since $W^i = 0$, the rest-frame angular momentum is $J_i = \frac{1}{2}\epsilon_{ijk}j^{jk} = \frac{1}{4}m\bar{Q}\bar{\sigma}^i Q$, yielding two scalars and one spin- $\frac{1}{2}$ particle.

$m \neq 0$, $\lambda \neq 0$.—For this system there are two gauge symmetries: reparametrizations and (13). Now twelve independent gauge-invariant variables may be defined on the constrained phase space. Once again they are just (19). When $\lambda \neq 0$ there exist only two conditions

on the variables (19); i.e.,

$$\pi^m\pi_m = -m^2, \quad (W^m\pi^n - W^n\pi^m)^2 = 2m^4\lambda^2. \quad (23)$$

The second equation in (23) follows from identities (3)–(5). Equations (23) correspond to Casimir invariants for the super Poincaré group.¹¹ If we now choose $\lambda^2 = s(s+1)$, where s is a half-integer, we obtain the Ω_s representation of supersymmetry in the quantum theory. [Quantization is not possible if $\lambda^2 \neq s(s+1)$.]

$m = 0$, $\lambda = 0$.—There are six gauge symmetries: (8), local supersymmetry (9), and reparametrizations $\tau \rightarrow f(\tau)$. The first-class constraints which generate the local supersymmetries are $(M^{-1})_2^\alpha\psi_\alpha \approx 0$ and $\bar{\psi}_{\dot{\alpha}}(M^{+-1})^{\dot{\alpha}}_2 \approx 0$. A total of eight gauge-invariant variables exist on the constrained phase space and this corresponds to the number of independent variables in (19). Here we have

$$\pi_m\pi^m = 0, \quad W^m = \frac{1}{8}\bar{Q}\bar{\sigma}^m Q. \quad (24)$$

In addition, we have the identity $\frac{1}{8}\bar{Q}\bar{\sigma}^m Q = H\pi^m$, H being the helicity. Since $\pi_m W^m$ is identically zero, (24) imposes four conditions on variables (19). The remaining two conditions are easily seen by going to the frame $\pi = (\omega, 0, 0, \omega)$. There $Q_2 = \bar{Q}_2 = 0$.

Upon quantization we get the fundamental representation for massless particles. In the frame $\pi = (\omega, 0, 0, \omega)$, $Q_1/2\sqrt{\omega}$ and $\bar{Q}_1/2\sqrt{\omega}$ are lowering and raising operators acting on a two-state system. Since $H = (1/8\omega)\bar{Q}\sigma^3 Q$, the helicity eigenvalues are 0 and $-\frac{1}{2}$.

$m = 0$, $\lambda \neq 0$.—Now there are four gauge symmetries: $\tau \rightarrow f(\tau)$, (9), and (13), leaving ten gauge-invariant variables on the constrained phase space. There are also ten variables in (19), since we now have

$$\pi_m\pi^m = 0, \quad W_m W^m = 0, \quad (25)$$

as well as the above two conditions on Q and \bar{Q} . Again we have the identity $W_m\pi^m = 0$, which along with (25) implies that π^m and W^m are parallel; $W^m = H\pi^m$. To determine the helicity H we can go to the frame $\pi = (\omega, 0, 0, \omega)$ and $S = 2\lambda\sigma^{12}$. We find $H = -\lambda + (1/8\omega)\bar{Q}\sigma^3 Q$. Quantization is possible when $\lambda = \pm\frac{1}{2}, \pm 1, \dots$. The resulting helicity eigenvalues are $-\lambda$ and $-\lambda - \frac{1}{2}$, corresponding to the Ω_λ representation for massless particles.

Exotic representations of the super Poincaré group are also obtainable from (12). Supersymmetric tachyons result from the choice $\tilde{P} = \kappa\sigma^3$. Another exotic representation comes from $\tilde{P} = 0$.

To obtain the representations of $N > 1$ supersymmetry we simply replace $(\theta^\alpha, \bar{\theta}^{\dot{\alpha}})$ by $(\theta_A^\alpha, \bar{\theta}_A^{\dot{\alpha}})$, $A = 1, 2, \dots, N$. So V^m in (7) becomes $\dot{x}^m + i\theta_A \times \sigma^m \bar{\theta}_A - i\dot{\theta}_A \sigma^m \bar{\theta}_A$.

A possible generalization to strings is as follows: The configuration-space variables $(x^m, \theta_A^\alpha, \bar{\theta}_A^\alpha, M)$ are now functions of two parameters τ^a ($a=0,1$). Now instead of defining the Lorentz vector P_m as in Eq. (1), we define an antisymmetric tensor Σ_{mn} :

$$\Sigma_{mn} \sigma^{mn} = M \tilde{\Sigma} M^{-1}, \quad (26)$$

where $\tilde{\Sigma} = \tilde{\Sigma}_{mn} \sigma^{mn}$ is a constant matrix. The generalization of L_0 is

$$\mathcal{L}_0 = \Sigma_{mn} \epsilon^{ab} V_a^m V_b^n, \quad (27)$$

$V_a^m = \partial_a x^m + i \theta_A \sigma^m \partial_a \bar{\theta}_A - i \partial_a \theta_A \sigma^m \bar{\theta}_A$, where $\partial_a = \partial/\partial \tau^a$. The action $\int d^2 \tau \mathcal{L}_0$ is invariant under global supersymmetry and reparametrizations $\tau^a \rightarrow f^a(\tau^0, \tau^1)$. The bosonic version of (27) has been examined in Ref. 8. It was found to contain the Nambu-Goto action and Schild's null string as special cases.

The Lagrangean (27) and possible Wess-Zumino terms are currently under investigation.

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