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Skyrmion Solutions to the Weinberg-Salam Model

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We find a spherically symmetric solution to the gauged $SU(2)_L \otimes SU(2)_R$ chiral model. It corresponds to a new classical solution to the Weinberg-Salam model in the limit of infinite self-coupling and $\sin^2\theta_W = 0$. It has an energy of 11.6 TeV and is classically unstable under small perturbations of the fields. Quantum corrections may stabilize the solution via the introduction of higher-order terms in the effective action. We then investigate the solutions when a particular choice of a correction, the Skyrme term, is added to the Lagrangean. The energies of the (presumably) classically stable solutions are in the teraelectronvolt region.

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As accelerator energies approach the electroweak-symmetry–breaking scale and beyond, it may become feasible to observe nonperturbative effects in the Weinberg-Salam model. In particular, a number of authors[1,2] have investigated the possibility of nontopological soliton solutions to the classical equations of motion.

Concurrently, there has been a recent revival of interest in soliton solutions in the nonlinear chiral model.[3–7] These solitons discovered by Skyrme[3] over twenty years ago are topological in nature, where the topological index is associated with the baryon number.[5]

When the Higgs-boson mass M_H in the Weinberg-Salam model goes to infinity, the model becomes equivalent to a gauged chiral model. Some authors have considered the possibility of solutions to the Weinberg-Salam model (with $M_H \to \infty$) which are analogous to Skyrme's soliton.[8,9] In this Letter we shall exhibit such a solution. The solution is nontopological in nature. This is essentially due to the fact that the boundary conditions of the gauged chiral model differ from those of the ungauged chiral model.

In Skyrme's model a fourth-order term (the "Skyrme term") was added to the standard chiral-model Lagrangean. This term insured the existence of a static solution, since it scaled differently from the usual chiral Lagrangean. For us, no modification of the Weinberg-Salam model is required for this purpose. The gauge-boson kinetic-energy term scales differently from the Higgs-boson Lagrangean, allowing for the possibility of a localized solution. (In the Skyrme model, the Skyrme term may be replaced by an interaction with vector mesons. Static localized solutions to such models have been found by Adkins and Nappi.[10])

Unlike the Skyrme soliton, our solution is unstable under small perturbations of the classical fields. Stability[11] however may be recovered with the inclusion of fourth-order terms like the Skyrme term in the effective Lagrangean. Such terms result naturally as quantum corrections to the model.[12] We will discuss the solutions with the (gauged) Skyrme term added, and conjecture that they correspond to real particles.

We assume the standard Weinberg-Salam model with a single Higgs doublet $\Phi = (\phi_1, \phi_2)$. The Higgs-boson Lagrangean is

$$\mathscr{L}_\Phi = (\mathscr{D}_\mu \Phi)^\dagger (\mathscr{D}^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2, \quad (1)$$

where $\mathscr{D}_\mu$ is the $SU(2)_L \otimes U(1)$ covariant derivative, $\mathscr{D}_\mu = \partial_\mu - (ig/2) A_\mu - (ig'/2) B_\mu$, $A_\mu = \mathbf{A}_\mu \cdot \boldsymbol{\tau}$. It is convenient to define the matrix

$$M = \begin{pmatrix} \phi_2^* & \phi_1 \\ -\phi_1^* & \phi_2 \end{pmatrix}, \quad (2)$$

and then perform the polar decomposition $M = hU$, where $U \in SU(2)$ and h is real. We shall make two assumptions: (i) $g' = 0$ and (ii) h is "frozen" in its vacuum expectation value $\langle h \rangle^2 = \mu^2 / 2\lambda > 0$. Assumption (i) is equivalent to our taking $\sin^2\theta_W = 0$. As was pointed our previously[13] it is necessary for finding a spherically symmetric solution. It is hoped that since $\sin^2\theta_W$ is small, we are not too far from reality. Assumption (i) leads to the global $SU(2)_V$ symmetry $U \to VUV^\dagger$, $A_\mu \to VA_\mu V^\dagger$, $V \in SU(2)$. Assumption (ii) is equivalent to our taking an infinite Higgs-boson mass in the tree-level approximation. Applying (i) and (ii) and adding the Yang-Mills kinetic-energy term we get

$$\mathscr{L}_\Phi+\mathscr{L}_A = \tfrac{1}{2}\langle h\rangle^2 \mathrm{Tr}(D_\mu U)^\dagger (D^\mu U) - \tfrac{1}{8}\mathrm{Tr} F_{\mu\nu}F^{\mu\nu}, \quad (3)$$

where $F_{\mu\nu}=\partial_\mu A_\nu - \partial_\nu A_\mu - (ig/2)[A_\mu, A_\nu]$, and $D_\mu U = \partial_\mu U - (ig/2)A_\mu U$.

Equation (3) is the classical Lagrangean for the gauged $SU(2)_L \otimes SU(2)_R$ chiral model [with $SU(2)_L$ being the gauge group]. Chiral models are known to have an identically conserved current

$$K^\mu = (24\pi^2)^{-1}\epsilon^{\mu\nu\lambda\rho}\mathrm{Tr}\partial_\nu U U^\dagger \partial_\lambda U U^\dagger \partial_\rho U U^\dagger. \quad (4)$$

For us K^μ is not meaningful since it is gauge variant. A gauge-invariant current $\tilde{K}^\mu$ can instead be defined which, however, is not conserved:

$$\tilde{K}^\mu = (24\pi^2)^{-1}\epsilon^{\mu\nu\lambda\rho}\mathrm{Tr}(D_\nu U U^\dagger D_\lambda U U^\dagger D_\rho U U^\dagger + \tfrac{3}{4} g D_\nu U U^\dagger F_{\lambda\rho}), \quad \partial_\mu \tilde{K}^\mu = -(g^2/128\pi^2)\epsilon^{\mu\nu\lambda\rho}\mathrm{Tr}F_{\mu\nu}F_{\lambda\rho}. \quad (5)$$

In the (ungauged) chiral model the charge $q = \int d^3x\, K^0$ is an integer. Finite energy demands that U tends to a constant at spatial infinity. Space is then compactified to a three-sphere which is mapped by U to the SU(2) manifold. The integer q is associated with the homotopy group $\pi_3(SU(2)) = Z$. Now when $SU(2)_L$ is gauged, we only have the condition $D_\mu U \to 0$ at spatial infinity. If we like, we can choose $U \to 1$ at $|\mathbf{x}| \to \infty$ resulting in an integer-valued q, but this procedure has no gauge-invariant meaning. On the other hand, the charge $\tilde{q} = \int d^3x\, \tilde{K}^0$ is gauge invariant, but it has no topological meaning. Only in the case where A_μ is a pure gauge need it be an integer. In what follows we show that there exists a static localized solution to the equations of motion, with $\tilde{q}$ not an integer. One can of course claim that q being an integer means that we have a topological configuration. However, under a gauge transformation G with $G \to 1$ at infinity, q changes by an integer, and therefore no physical significance can be assigned to it.

We specialize to the spherically symmetric configuration. By spherical symmetry we mean that the fields are invariant under simultaneous rotations in Minkowski space and the internal $SU(2)_V$ space:

$$\begin{aligned} &-2i\epsilon_{ijk}x_j\nabla_k U + [\tau_i, U] = 0, \\ &-2i\epsilon_{ijk}x_j\nabla_k A_l + [\tau_i, A_l] - 2i\epsilon_{ilk}A_k = 0. \end{aligned} \quad (6)$$

We work in the gauge $A_0 = 0$. The general solution to (6) is

$$\begin{aligned} &U = \cos\theta + i\boldsymbol{\tau}\cdot\hat{\mathbf{x}}\sin\theta, \\ &-\frac{g}{2}A_i = \frac{a-\frac{1}{2}}{r}(\hat{\mathbf{x}}\times\boldsymbol{\tau})_i + \frac{\beta}{r}\tau_i + \frac{\delta-\beta}{r}(\boldsymbol{\tau}\cdot\hat{\mathbf{x}})\hat{x}_i, \end{aligned} \quad (7)$$

where θ, a, β, and δ are functions of the radial coordinate r and $\hat{x}_i = x_i/r$. After substituting (7) into the expression for the energy we find

$$E = (8\pi\langle h\rangle/g)\int_0^\infty d\rho\{2\rho^{-2}[a^2+\beta^2-\tfrac{1}{4}]^2 + (a'+2\beta\sigma)^2 + (\beta' - 2\sigma a)^2 + \rho^2(\theta'+\sigma)^2 + 2(a+\sin^2\theta - \tfrac{1}{2})^2 + 2(\beta+\sin\theta\cos\theta)^2\}. \quad (8)$$

Here $\rho = g\langle h\rangle r/2$ is a dimensionless variable, the prime denotes differentiation with respect to ρ, and $\sigma \equiv \delta\rho$.

Only two of the four variables θ, a, β, and δ in (8) correspond to dynamical degrees of freedom. This is so since (i) σ appears in (8) with no derivatives and hence is an auxiliary variable; (ii) there is a residual U(1) gauge symmetry,

$$\begin{aligned} &\theta \to \theta - \chi(r), \quad a \to a\cos 2\chi(r) - \beta\sin 2\chi(r), \\ &\beta \to a\sin 2\chi(r) + \beta\cos 2\chi(r), \quad \sigma \to \sigma + \chi'(r). \end{aligned} \quad (9)$$

We shall eliminate the gauge freedom by setting $\beta = 0$. In this gauge the equations of motion are

$$\begin{aligned} &[\rho^2(\theta'+\sigma)]' = 2a\sin 2\theta, \\ &a'' = (4a/\rho^2)(a^2 - \tfrac{1}{4} + \rho^2\sigma^2) + 2a - \cos 2\theta, \\ &\sigma = -\rho^2\theta'/(\rho^2+4a^2). \end{aligned} \quad (10)$$

The boundary conditions at the origin result from the demand that Eqs. (7) be well defined. We have $\theta(0) = \pi n$, $a(0) = \frac{1}{2}$, where n is an integer, which we can set equal to zero. By requiring finite energy we have $\theta(\infty) = \frac{1}{2}\pi m$, $a(\infty) = (-1)^m\frac{1}{2}$, where m is an integer.

Our method for solving Eqs. (10) is first to rewrite them in first-order form. We then use the asymptotic expressions for the solutions, i.e.,

$$\theta \to B\rho, \quad a \to \tfrac{1}{2} + A\rho^2, \quad (11)$$

for $\rho \to 0$

$$\begin{aligned} &\theta \to \tfrac{1}{2}m\pi - D[1+(\sqrt{2}\rho)^{-1}]e^{-\sqrt{2}\rho}, \\ &a \to (-1)^m\tfrac{1}{2} + C[1+(\sqrt{2}\rho)^{-1}]e^{-\sqrt{2}\rho}, \end{aligned} \quad (12)$$

for $p \to \infty$, and integrate both (11) and (12) to some finite $\rho = \rho_0$. By adjusting the parameters A, B, C, and D we then match the values of the functions and their derivatives at ρ_0. One would expect that the minimum-energy solution occurs when $|m| = 1$. However, we were unable to find solutions for $|m|$ odd, because our first-order equations are singular when a goes through a zero. For $m = 2$ we were able to find a solution. Its energy density is localized inside a small fraction of a fermi, and for the total energy we find $E=(8\pi\langle h\rangle/g)(1.79)$; using $M_W=83$ GeV and $g=0.67$, then $E=11.6$ TeV.

Upon substituting Eqs. (7) into the expression for the charge $\tilde{q}$, and applying the gauge condition $\beta=0$, we get $\tilde{q}\cong 0.18$ for the above solution.

In the Skyrme model the topological charge q is identified with fermion number.[5] Similarly, it has been claimed that the charge $\tilde{q}$ is identified with fermion number in the Weinberg-Salam model.[14] The computation is performed by a coupling to fermion fields and application of a derivative expansion. The latter is valid if the inverse fermion mass is smaller than the "size" of the soliton. Thus if (a) there exist sufficiently heavy fermions and (b) our solution corresponds to a state in the quantum theory, it is expected to have very exotic quantum numbers.

Concerning (b), the existence of a state in the quantum theory is in general insured for solutions which are stable under perturbations in the classical fields. This, however, is not the case for our solution. It is not difficult to find a variation $\delta\theta(\rho)$ which lowers the energy of the solution. Hence we do not have a local minimum in the energy.

Even though the solution is classically unstable it may be possible to recover stability in the quantum theory. In the quantum theory higher-order terms appear in the effective action.[12] For the Skyrme model these terms were necessary for the existence of a soliton solution. For us they could possibly cure the classical instability. An example of such a term is

$$(32e^2)^{-1}\,\mathrm{Tr}[U^\dagger D_\mu U, U^\dagger D_\nu U]^2, \qquad (13)$$

which reduces to the Skyrme term in the limit $g\to 0$. By continuous variation of the parameters g and e it is possible to deform our solution (corresponding to the limit $e\to\infty$) to the Skyrme soliton solution $(g\to 0)$.[15] The Skyrme soliton was shown to be classically stable.[16] Thus in traversing a path in the g-e plane from our solution to the Skyrme solution we should encounter a transition from a classically unstable solution to a stable one. So there could exist some range for the parameter e which admits stable solutions. Although it is often easy to find unstable modes of a particular unstable solution, the proof of stability is in general very difficult.[16] After adding the gauged Skyrme term [Eq. (13)] to the Lagrangean, we solve the corresponding equations of motion. For each value of $(g/e)^2\lesssim 0.39$ there are two solutions [for higher $(g/e)^2$ no solutions exist]. The first is classically unstable and reaches the previously discussed solution as $1/e$ goes to zero. The second solution approaches the classically stable solution of the ungauged Skyrme model[7] as $g\to 0$, and is likely to be classically stable too. Energies for both solutions are depicted in Fig. 1. $\tilde{q}$ is given there for selected values of $(g/e)^2$; it approaches unity (i.e., $\tilde{q}\to q$) only as the lower branch of the solution approaches the Skyrme solution,[7] where the gauge degrees of freedom become unimportant.

In the case of unstable or saddle-point solutions, the quantum mechanical relevance of the solutions is an open theoretical question.[17] Already quite a few examples of saddle-point solutions have been discovered in the Weinberg-Salam model.[1,2] One which is thought to have physical significance is the sphaleron solution of Klinkhamer and Manton.[13] It corresponds to an energy maximum along a noncontractible loop (NCL) of field configurations which passes through the vacuum. Its energy $E_0\cong 10$ TeV is the height of the barrier for tunneling between topologically distant vacua. Such processes are known not to conserve baryon number. Tunneling via instantons is negligibly small[18]; however, baryon-number–nonconserving processes may be greatly enhanced if energies of order E_0 are readily available, as in the time of the very early universe.[19]

Unlike in Ref. 13 our solutions are neither associated with baryon-number nonconservation nor with NCL's, since we are at $M_H=\infty$.[20] The first fact results from the vanishing of the integral over the anomaly, and the second one from the absence of a zero for h.

The lower branch of the solutions may correspond to real particles (in analogy with the nucleon) which can be called "weak skyrmions," of mass of the order of teraelectronvolts. If $1/e\simeq 0.2$ as in strong interactions, then $E\simeq 3$ TeV, thus raising the exciting possi-

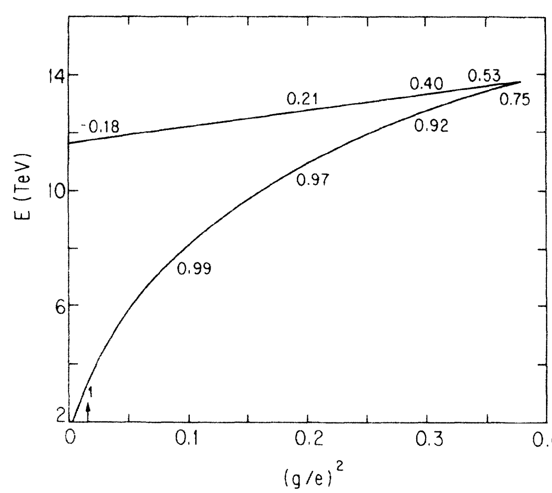

FIG. 1. The energy in teraelectronvolts of the solutions to the equations of motion with the gauged Skyrme term [Eq. (13)] added, as a function of $(g/e)^2$. $g=0.67$ is the standard coupling and e appears in the coefficient of the Skyrme term. The arrow denotes the value of $(g/e)^2$ if $e\simeq 5.4$ as in strong interactions. Values of $\tilde{q}=\int d^3x\,\tilde{K}^0$ [see Eq. (5)] are given near the curves.

bility of directly observing weak skyrmions through the process of longitudinal-W-boson fusion in future colliders.

In a future publication we will discuss in detail our solutions, noncontractible loops in the limit $M_H = \infty$, baryon-number nonconservation, quantization, and phenomonology.

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Note added.—After completion of this work we obtained a report by Ambjørn and Rubakov[21] where static solutions were found to the equations of motion resulting from Eqs. (3) and (13) for various values of $g/e \neq 0$. We agree with their energies, but disagree with their conclusions about stability (contrary to their claim stability was not demonstrated by them for any g/e) and about baryon-number nonconservations (there are none for $M_H = \infty$). Furthermore, because of their coordinate rescaling they could not discuss the solution in the pure Weinberg-Salam model ($g/e = 0$) displayed here.

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