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Alexey N. Volkov – University of Virginia

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Numerical Modelling of the Magnus Force and the Aerodynamic Torque on a Spinning Sphere in Transitional Flow

Alexey N. Volkov

Material Science and Engineering Department, University of Virginia, 117 Engineer's way, Charlottesville, VA 22903, USA

Abstract. Three dimensional transitional flow over a spinning sphere is studied numerically by the direct simulation Monte Carlo method. The flow is assumed to be steady-state, gas molecules interact with each other as hard spheres and the specular-diffuse scattering model describes the interaction between molecules and the sphere surface. The translational and rotational velocities of the sphere is assumed to be perpendicular to each other. The drag coefficient, the Magnus force coefficient and the torque coefficient are found as functions of the Mach and Reynolds numbers and the dimensionless rotation parameter for subsonic and supersonic flows. Computational results are compared with the analytical solution for a spinning sphere in free molecular flow and with available semi-empirical data. The "critical" Knudsen number when the Magnus force is equal to zero is found as a function of the Mach number.

Keywords: Spinning sphere, Transitional flow, Free molecular flow, Magnus force, Aerodynamic torque, Parallel DSMC modelling
PACS: 47.45.-n, 47.45.Dt, 47.45.Gx, 47.32.Ef, 05.10.Ln

INTRODUCTION

It is well known that a lateral force affects a spinning body moving in a gas [1]–[6]. This phenomenon is often called the Magnus effect and the lateral force is known as the Magnus force. The Magnus force is of great importance for various applications. For example, it influences trajectories of spinning shells. This effect is also important for geometrically simplest bodies, e.g. for sphere. A model of spherical particle is a widely used model of solid particles in two-phase gas-solid flows. Solid particles can gain high angular velocities due to their rebound from solid surfaces. As a result the Magnus force has a profound effect on the motion of such particles.

In [1] an analytical solution for the lateral force affecting a spinning sphere was found for the continuum flow regime and low velocities corresponding to the Stokes flow. In later papers (see [2] and references there) corrections to this analytical solution were obtained for finite Reynolds numbers. In [3] the lateral force was obtained for a spinning sphere in free molecular flow. Free molecular flow over a general axially symmetric body was considered in [4] and both the aerodynamic force and torque were calculated. It was found in [5, 4] that in free molecular flow the Magnus force has the opposite direction as compared with continuum flow. Recently, in [5, 6] the Magnus force on a sphere in free molecular flow was studied in a more general framework and the last phenomenon was called "the inverse Magnus effect". In particular, it means that the lateral force in transitional flow depends significantly on the Knudsen number and for a some "critical" Knudsen number it is equal to zero. The aim of this work is to study the aerodynamic properties of a spinning sphere in the transitional flow regime and to find the critical Knudsen number.

THE MATHEMATICAL MODEL

A flow of rarefied gas over a homogeneous sphere of radius R is considered. The vector of sphere's translational velocity $\mathbf{V}$, the vector of its rotational velocity $\boldsymbol{\omega}$ and its homogeneous temperature T are assumed to be constant. It is also assumed that (1) the gas is rarefied and monoatomic and its flow can be described by the kinetic model based on the Boltzmann equation; (2) molecules velocity distribution in the free stream is the equilibrium Maxwellian distribution with constant concentration n_∞, velocity $\mathbf{V}_\infty$ and temperature T_∞; (3) the flow is steady-state; (4) gas molecules interact with each other as hard spheres of diameter d and mass m; (5) the model of specular-diffuse scattering [7] describes interactions of molecules with the sphere surface and the relaxation temperature in this model is equal to the temperature T of the sphere; (6) external (gravity) forces affecting gas molecules are negligible.

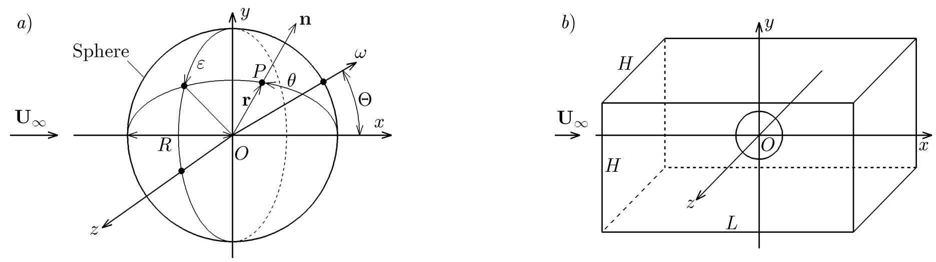

FIGURE 1. Coordinates (*a*) and the computational domain (*b*) used in DSMC simulations of flow over a spinning sphere.

The problem is considered in the frame of reference which is at rest with the center O of the sphere (Fig. 1, *a*). Introduce the right cartesian coordinates $Oxyz$ with the basic vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$, where the axis Ox directs along the vector of macroscopic gas velocity against the sphere $\mathbf{U}_\infty = \mathbf{V}_\infty - \mathbf{V}$ and the axis Oz is perpendicular to the plane of vectors $\mathbf{U}_\infty$ and $\boldsymbol{\omega}$. Position of an arbitrary point P at the sphere surface is defined by the radius vector $\mathbf{r} = R\mathbf{n}$, where $\mathbf{n}$ is the unit vector normal to the surface in the point P pointed outward from the sphere. The spherical angles θ and ε are introduced in order to calculate components of the vector $\mathbf{n}$, so that $\mathbf{n} = \cos\theta\mathbf{i} + \sin\theta\cos\varepsilon\mathbf{j} + \sin\theta\sin\varepsilon\mathbf{k}$. Then the aerodynamic force $\mathbf{F}$ and torque $\mathbf{M}$ affecting the sphere and the heat flux Q at the sphere surface can be found by integrating of the stress vector $\mathbf{p}(\mathbf{n})$ and the heat flux density $q(\mathbf{n})$ over the sphere surface

$$\mathbf{F} = R^2 \int_0^{2\pi}\int_0^{\pi} \mathbf{p}(\mathbf{n}) \sin\theta d\theta d\varepsilon, \qquad \mathbf{M} = R^3 \int_0^{2\pi}\int_0^{\pi} \mathbf{n}\times\mathbf{p}(\mathbf{n}) \sin\theta d\theta d\varepsilon, \qquad Q = R^2 \int_0^{2\pi}\int_0^{\pi} q(\mathbf{n}) \sin\theta d\theta d\varepsilon. \tag{1}$$

The stress vector $\mathbf{p}(\mathbf{n})$ and the heat flux density $q(\mathbf{n})$ can be calculated from the velocity distribution function of gas molecules $f(\mathbf{r},\mathbf{v})$ (here $\mathbf{r}$ and $\mathbf{v}$ are the radius-vector and the velocity vector of a molecule). In order to calculate $\mathbf{p}(\mathbf{n})$ and $q(\mathbf{n})$ it is convenient to introduce in the point P at the sphere surface a local frame of reference which moves with the velocity $\mathbf{V}_\mathrm{w} = \mathbf{V} + R\boldsymbol{\omega}\times\mathbf{n}$ of this point. It this frame of reference a molecule has velocity $\mathbf{v}' = \mathbf{v} - \mathbf{V}_\mathrm{w}$ and

$$\mathbf{p}(\mathbf{n}) = -m\int \mathbf{v}'\mathbf{v}'\cdot\mathbf{n} f'(R\mathbf{n},\mathbf{v}')d\mathbf{v}', \qquad q(\mathbf{n}) = -\frac{m}{2}\int (\mathbf{v}')^2\mathbf{v}'\cdot\mathbf{n} f'(R\mathbf{n},\mathbf{v}')d\mathbf{v}', \tag{2}$$

where $f'(R\mathbf{n},\mathbf{v}') = f(R\mathbf{n},\mathbf{v}'+\mathbf{V}_\mathrm{w})$, $\mathbf{ab}$ denotes the tensor production of vectors $\mathbf{a}$ and $\mathbf{b}$.

The steady flow is calculated as a limit of the time-dependant solution of the Boltzmann equation [7] at time $t\to\infty$

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}} = I_B, \qquad I_B = \frac{d^2}{2}\int\int_0^{2\pi}\int_0^{\pi} (f'f_1' - ff_1)|(\mathbf{v}_1-\mathbf{v})\cdot\mathbf{d}|\sin\vartheta d\vartheta d\varphi d\mathbf{v}_1, \tag{3}$$

where $f = f(\mathbf{v})$, $f_1 = f(\mathbf{v}_1)$, $f' = f(\mathbf{v}+\mathbf{w})$, $f_1' = f(\mathbf{v}_1-\mathbf{w})$, $\mathbf{w} = [(\mathbf{v}_1-\mathbf{v})\cdot\mathbf{d}]\mathbf{d}$, $\mathbf{d} = \sin\vartheta\cos\varphi\mathbf{i} + \cos\vartheta\mathbf{j} + \sin\vartheta\sin\varphi\mathbf{k}$.

Boundary conditions for equation (3) include the specular-diffuse scattering at the sphere surface [7, 8]

$$\text{at } \mathbf{v}'\cdot\mathbf{n} > 0: \quad f'(R\mathbf{n},\mathbf{v}') = (1-\alpha_\tau)f'(\mathbf{n},\mathbf{v}'-2(\mathbf{v}'\cdot\mathbf{n})\mathbf{n}) + \alpha_\tau\frac{2}{\pi C^4}\exp\left(-\frac{(\mathbf{v}')^2}{C^2}\right)\int_{\mathbf{v}''\cdot\mathbf{n}<0} |\mathbf{v}''\cdot\mathbf{n}| f'(R\mathbf{n},\mathbf{v}'')d\mathbf{v}'', \tag{4}$$

and the condition in the free stream, where the velocity distribution is assumed to be Maxwellian

$$f(\mathbf{r},\mathbf{v},t)\to f_\infty(\mathbf{v}) \text{ at } |\mathbf{r}|\to\infty, \qquad f_\infty(\mathbf{v}) = \frac{n_\infty}{(\pi C_\infty)^3}\exp\left(-\frac{(\mathbf{v}-\mathbf{V}_\infty)^2}{C_\infty^2}\right). \tag{5}$$

Here α_τ is the accommodation coefficient, $C = \sqrt{2R_\mu T}$, $C_\infty = \sqrt{2R_\mu T_\infty}$, $R_\mu = k/m$, k is the Boltzmann's constant. The initial condition at $t = 0$ for the steady-state solution of the problem (3)–(5) is arbitrary. In the simplest case the uniform velocity distribution $f_\infty(\mathbf{v})$ can be used as the initial one.

The problem (3)–(5) can be transformed into an appropriate dimensionless form. The dimensionless solution of the problem depends on the velocity coefficient $S_\infty = |\mathbf{U}_\infty|/C_\infty$, the Knudsen number $Kn_\infty = \lambda_\infty/R$ ($\lambda_\infty = 1/(\sqrt{2}\pi d^2 n_\infty)$

is the mean free path of molecules in the undisturbed flow), the ratio of temperatures T/T_∞, the rotation parameter $W = R|\boldsymbol{\omega}|/C_\infty$, the angle Θ between vectors of translational $\mathbf{U}_\infty$ and rotational $\boldsymbol{\omega}$ velocities and the accommodation coefficient α_τ. These six parameters are criteria of similarity for the problem (3)–(5). The Mach number $M_\infty = |\mathbf{U}_\infty|/\sqrt{\gamma R_\mu T_\infty} = \sqrt{\gamma/2}S_\infty$ and the Reynolds number $Re_\infty = 2R\rho_\infty|\mathbf{U}_\infty|/\mu_\infty = 2.51\sqrt{\gamma}M_\infty/Kn_\infty$ ($\gamma = 5/3$ for the monoatomic gas, $\rho_\infty = mn_\infty$, $\mu_\infty = 0.798\lambda_\infty\rho_\infty\sqrt{R_\mu T_\infty}$ is the dynamic viscosity coefficient in the undisturbed flow) can be also used instead of S_∞ and Kn_∞ in the analysis of the dimensionless problem.

FREE MOLECULAR FLOW OVER A SPINNING SPHERE

For the case of free molecular flow over a sphere at $\lambda_\infty/R \gg 1$ the collision integral I_B can be eliminated in equation (3), and an analytical solution for $\mathbf{F}$, $\mathbf{M}$ and Q in the problem (3)-(5) can be obtained. It is well-known [8] that for a steady free molecular flow over a convex body the distribution function of molecules incident to the body surface is equal to the distribution function (5) in the free stream, i.e.

$$\text{at}\ \ \mathbf{v}'\cdot\mathbf{n} < 0: \qquad f'(\mathbf{n},\mathbf{v}') = \frac{n_\infty}{(\pi C_\infty)^3}\exp\left(-\frac{[\mathbf{v}' - (\mathbf{V}_\infty - \mathbf{V}_\mathrm{w}(\mathbf{n}))]^2}{C_\infty^2}\right). \tag{6}$$

Then f' for reflected molecules can be calculated substituting (6) into (4). Substituting (6) and (4) into (2) one can find

$$\mathbf{p}(\mathbf{n}) = \mathbf{p}_n(\mathbf{n}) + \mathbf{p}_\tau(\mathbf{n}), \tag{7}$$

$$\mathbf{p}_n(\mathbf{n}) = p_\infty\left\{(\alpha_\tau - 2)\left[(1+\mathrm{erf}(S_n))\left(S_n^2 + \frac{1}{2}\right) + \frac{S_n}{\sqrt{\pi}}\exp(-S_n^2)\right] - \frac{\alpha_\tau}{2}\chi(S_n)\sqrt{\frac{T}{T_\infty}}\right\}\mathbf{n}, \tag{8}$$

$$\mathbf{p}_\tau(\mathbf{n}) = p_\infty\frac{\alpha_\tau\chi(S_n)}{\sqrt{\pi}}\left[S_\infty(\mathbf{e} - \mathbf{e}\cdot\mathbf{n}\mathbf{n}) - W\mathbf{e}_\omega\times\mathbf{n}\right], \tag{9}$$

$$q(\mathbf{n}) = q_\infty\frac{\alpha_\tau}{\sqrt{\pi}}\left[\left(\frac{S^2}{2} + \frac{5}{4} - \frac{T}{T_\infty}\right)\chi(S_n) - \frac{\exp(-S_n^2)}{4} + \frac{W^2}{2}(\mathbf{n}\times\mathbf{e}_\omega)^2 + S_\infty W\mathbf{e}\cdot(\mathbf{n}\times\mathbf{e}_\omega)\right], \tag{10}$$

where $\mathbf{e}$ and $\mathbf{e}_\omega$ are the unit vectors pointed along vectors $\mathbf{U}_\infty$ and $\boldsymbol{\omega}$, respectively, $S = |S_\infty\mathbf{e} - W\mathbf{e}_\omega\times\mathbf{n}|$, $S_n = -S_\infty\mathbf{e}\cdot\mathbf{n}$, $p_\infty = \rho_\infty C_\infty^2/2$, $q_\infty = \rho_\infty C_\infty^3/2$, $\chi(x) = \exp(-x^2) + \sqrt{\pi}x[1+\mathrm{erf}(x)]$, $\mathrm{erf}(x) = (2/\sqrt{\pi})\int_0^x\exp(-y^2)dy$.

Substituting (7)–(9) into the definition of the aerodynamic force $\mathbf{F}$ in (1) one can write

$$\mathbf{F} = \mathbf{F}_D + \mathbf{F}_L, \qquad \mathbf{F}_D = \frac{1}{2}\rho_\infty\pi R^2 C_D|\mathbf{V}_\infty - \mathbf{V}|(\mathbf{V}_\infty - \mathbf{V}), \qquad \mathbf{F}_L = \frac{1}{2}\rho_\infty\pi R^3 C_L(\mathbf{V}_\infty - \mathbf{V})\times\boldsymbol{\omega}, \tag{11}$$

where $\mathbf{F}_D$ and $\mathbf{F}_L$ are the drag and lateral Magnus forces, C_D and C_L are the drag and Magnus force coefficients

$$C_D = \frac{\exp(-S_\infty^2)}{\sqrt{\pi}S_\infty^3}(2S_\infty^2 + 1) + \frac{\mathrm{erf}(S_\infty)}{S_\infty^4}\left(2S_\infty^4 + 2S_\infty^2 - \frac{1}{2}\right) + \frac{2\alpha_\tau\sqrt{\pi}}{3S_\infty}\sqrt{\frac{T}{T_\infty}}, \qquad C_L = -\frac{4}{3}\alpha_\tau. \tag{12}$$

The drag coefficient C_D in (12) is the same as for the non-rotating sphere [9]. The coefficient C_L was found in [3] for the case $\mathbf{U}_\infty\cdot\boldsymbol{\omega} = 0$ and in [4] for the arbitrary orientation of the rotational velocity. In continuum flow at the Stokes regime when $Re_\infty \ll 1$ and $Re_\omega = R^2\rho_\infty|\boldsymbol{\omega}|/\mu_\infty \ll 1$ the Magnus force coefficient is equal to 2 [1]. Therefore, in free molecular flow the Magnus force has the opposite direction as compared with continuum Stokes flow.

The aerodynamic torque can be found in the similar manner substituting (7)–(9) into the torque definition in (1):

$$\mathbf{M} = -\frac{1}{2}\rho_\infty\pi R^5|\boldsymbol{\omega}|^2(C_m\mathbf{e}_\omega + C_{m\perp}\mathbf{e}_{\omega\perp}), \quad \mathbf{e}_{\omega\perp} = \frac{\mathbf{e} - (\mathbf{e}\cdot\mathbf{e}_\omega)\mathbf{e}_\omega}{|\mathbf{e} - (\mathbf{e}\cdot\mathbf{e}_\omega)\mathbf{e}_\omega|}, \tag{13}$$

where C_m and $C_{m\perp}$ are the coefficients of the torque components which are parallel and perpendicular to the vector $\boldsymbol{\omega}$

$$C_m = \frac{\alpha_\tau}{\sqrt{\pi}W}\left[I_1 + I_2 + (I_2 - 3I_1)(\mathbf{e}\cdot\mathbf{e}_\omega)^2\right], \qquad C_{m\perp} = \frac{\alpha_\tau}{\sqrt{\pi}W}(I_2 - 3I_1)(\mathbf{e}\cdot\mathbf{e}_\omega)|\mathbf{e} - (\mathbf{e}\cdot\mathbf{e}_\omega)\mathbf{e}_\omega|, \tag{14}$$

$$I_1 = \frac{\sqrt{\pi}}{2}\frac{\mathrm{erf}(S_\infty)}{S_\infty^3}\left(S_\infty^4 + \frac{1}{4}\right) + \frac{\exp(-S_\infty^2)}{2S_\infty^2}\left(S_\infty^2 - \frac{1}{2}\right), \qquad I_2 = \sqrt{\pi}\frac{\mathrm{erf}(S_\infty)}{S_\infty}\left(S_\infty^2 + \frac{1}{2}\right) + \exp(-S_\infty^2). \tag{15}$$

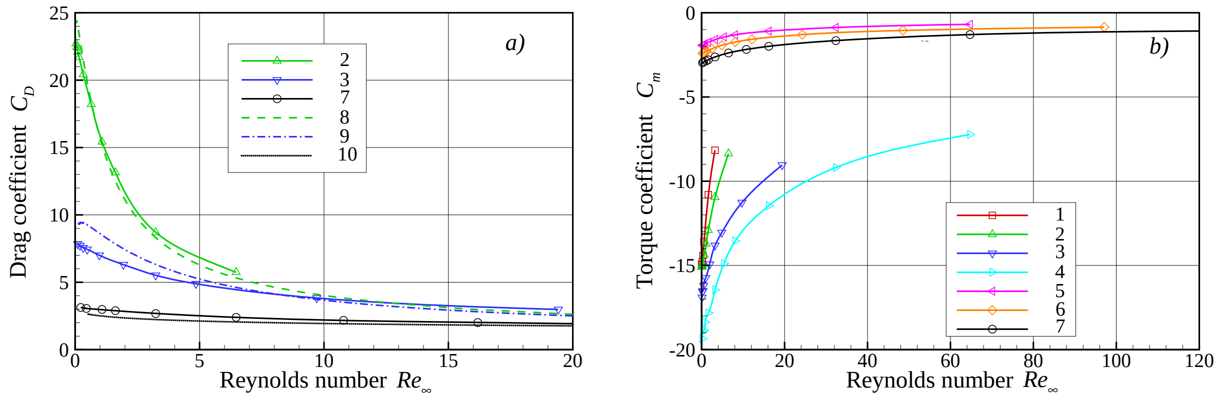

FIGURE 2. The drag coefficient C_D (*a*) and the torque coefficient C_m (*b*) versus the Reynolds number Re_∞. Curve 1, $M_\infty = 0.1$; 2 and 8, 0.2; 3 and 9, 0.6; 4 and 5, 1; 6, 1.5; 7 and 10, 2. Curves 1–4, $W = 0.1$; 5–7, 1. Curves 8–10, C_D from the Henderson's relations [10].

The torque vector $\mathbf{M}$ lies in the plane of vectors $\mathbf{U}_\infty$ and $\boldsymbol{\omega}$, but $\mathbf{M}$ points along $\boldsymbol{\omega}$ only if vectors $\mathbf{U}_\infty$ and $\boldsymbol{\omega}$ either parallel or perpendicular to each other. Formulae (13)–(15) were obtained in another mathematical form in [4].

The heat flux at the surface of a spinning sphere can be found substituting (10) into the definition of Q in (1):

$$Q = 4\rho_\infty c_p \pi R^2 St|\mathbf{V}_\infty - \mathbf{V}|(T_{r0} - T_\infty), \tag{16}$$

where $c_p = (5/2)R_\mu$, St is the Stanton number which does not depend on sphere rotation, T_{r0} is the adiabatic temperature (it is the uniform body temperature for which $Q = 0$) which depends on the rotation parameter W

$$St = \frac{\alpha_\tau}{5\sqrt{\pi}}\frac{I_2}{S_\infty}, \qquad \frac{T_{r0}}{T_\infty} = \frac{T_{r00}}{T_\infty} + W^2\frac{J_2}{I_2}, \qquad \frac{T_{r00}}{T_\infty} = \frac{J_1}{I_2}, \tag{17}$$

$$J_1 = \frac{\sqrt{\pi}}{2}\frac{\operatorname{erf}(S_\infty)}{S_\infty}\left(S_\infty^4 + 3S_\infty^2 + \frac{3}{4}\right) + \left(\frac{S_\infty^2}{2} + \frac{5}{4}\right)\exp(-S_\infty^2), \qquad J_2 = \frac{I_2}{2} - \frac{1}{4}\left[(3I_1 - I_2)(\mathbf{e}\cdot\mathbf{e}_\omega)^2 + I_2 - I_1\right]. \tag{18}$$

Here T_{r00} is the adiabatic temperature of the non-rotating sphere [8, 9]. To the best of author's knowledge, relations (16)–(18) for the heat flux Q at a spinning sphere in free molecular flow were not published earlier.

COMPUTATIONAL RESULTS FOR TRANSITIONAL FLOW

Computational results for transitional flow at $\lambda_\infty \sim R$ are obtained in the case when the relative velocity $\mathbf{U}_\infty$ is perpendicular to the rotational velocity $\boldsymbol{\omega}$ of the sphere ($\Theta = \pi/2$), $T/T_\infty = 1$ and $\alpha_\tau = 1$. Then the force $\mathbf{F}$ and the torque $\mathbf{M}$ can be also represented in the form (11) and (13), where coefficients C_D, C_L and C_m are functions of M_∞, Re_∞ and W. Calculations were carried out for subsonic ($0.1 \leqslant M_\infty \leqslant 1$, $W = 0.1$) and supersonic ($1 \leqslant M_\infty \leqslant 2$, $W = 1$) flows with the help of parallel algorithms of the direct simulation Monte Carlo method based on the NTC scheme [7]. Sphere was placed in the center of the rectangular computational domain (Fig. 1, *b*) of sizes $L \times H \times H$, where $L/R = 40$, $H/R = 20$ for subsonic and $L/R = H/R = 10$ for supersonic flows. Free molecular flows over a spinning sphere were calculated in order to verify the code. The maximal difference between numerical values of coefficients C_D, C_L, C_m and $C_Q = St(T_{r0}/T_\infty - 1)$ and their values computed from analytical relations (11)–(18) was less than 0.5%.

Values of aerodynamic coefficients C_D, C_m and C_L of the spinning sphere obtained with help of DSMC simulation in the transitional flow regime are shown in Fig. 2 and 3. It was found that the rotation parameter W almost does not influence the drag coefficient C_D for considered ranges of Kn_∞, M_∞ and W (the maximal difference between C_D for the rotating and non-rotating sphere is obtained to be less than 5%). In Fig. 2, *a* computed values of C_D (curves 2, 3 and 7) are compared with values C_D^H predicted by the Henderson's semi-empirical relations [10] (curves 8, 9 and 10). The maximal difference between C_D and C_D^H is observed at very low Reynolds numbers which correspond to the free molecular flow regime. Apparently, this difference is observed because C_D^H tends at $Kn_\infty \to \infty$ to the approximate Epstein formula for the sphere drag in free molecular flow at $\alpha_\tau = 0.89$ while C_D tends to formula (12) at $\alpha_\tau = 1$. For

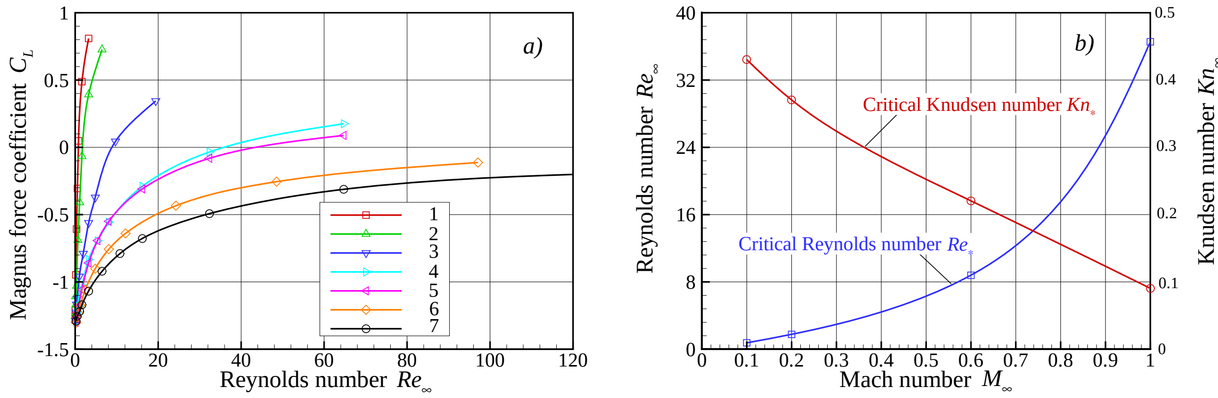

FIGURE 3. The Magnus force coefficient C_L versus the Reynolds number Re_∞ (a) and the critical Reynolds Re_* and Knudsen Kn_* numbers (b). Curve 1, $M_\infty = 0.1$; 2, 0.2; 3, 0.6; 4 and 5, 1; 6, 1.5; 7, 2. Curves 1–4, $W = 0.1$, 5–7, $W = 1$.

transitional flow at $Kn_\infty \leqslant 1$ the difference between C_D and C_D^H is less than 8% while the difference between C_D^H and the reliable experimental data by Bailey and Hyatt [11] and Zarin [12] lies in the range from 4% to 16%.

According to (14) the torque coefficient C_m in free molecular flow is inversely proportional to the rotation parameter W. The same dependance takes place in continuum Stokes flow over a spinning sphere at $\mathbf{U}_\infty = 0$ and $Re_\omega \ll 1$ where $C_m = 16/Re_\omega$, see [13]. Computational results for C_m shown in Fig. 2, b demonstrate that in transitional flows C_m is also approximately inversely proportional to W. In particular, C_m for $W = 1$ (curves 5–7) is of order of magnitude less than C_m for $W = 0.1$ (curves 1–4). With increasing of the Reynolds number Re_∞ the torque coefficient C_m tends to a some value which is a function of M_∞ and W while in transitional flow C_m depends essentially on Re_∞. For the best of author's knowledge, this effect of the sphere's translational velocity on its aerodynamic torque has not been studied yet.

Computed values of the Magnus force coefficient C_L are shown in Fig. 3, a. One can see that C_L tends from the negative value $-4/3$ in free molecular flow to the positive value with increasing Re_∞ (or decreasing Kn_∞) but the "critical" Reynolds Re_* and Knudsen Kn_* numbers (Fig. 3, b, "critical" value corresponds to zero value of C_L, e.g. $C_L(M_\infty, Re_*) = 0$) depend significantly on M_∞. The critical Knudsen number almost linearly decreases as M_∞ increases, so that C_L can be positive at $M_\infty > 1$ only in the near-continuum and continuum flow regimes. The influence of the rotation parameter W on C_L is relatively weak in the considered range of W. Comparison of curves 4 and 5 corresponding to $W = 0.1$ and $W = 1$, respectively, at $M_\infty = 1$ shows that C_L slightly decreases if W increases.

It is interesting, of course, to correlate the change in C_L and changes in the flow field around a sphere and the stress distribution on its surface. However, this question is beyond the scope of the short paper. It is significant that both normal $\mathbf{p}_n$ and tangential $\mathbf{p}_\tau$ stresses at the sphere surface determine the Magnus force in transitional flow. Fields of z-components of vectors $\mathbf{p}_n$ and $\mathbf{p}_\tau$ at the sphere surface at $M_\infty = 2$ and $W = 1$ in free molecular flow and for $Kn_\infty = 0.05$ are shown in Fig. 4 and 5, respectively. One can see that stresses at the back part of the sphere surface ($\theta < 90^\circ$) are negligibly small as compared with their values at the frontal part of the surface ($\theta > 90^\circ$). The Magnus force coefficient can be represented in the form $C_L = C_{L(n)} + C_{L(\tau)}$ where $C_{L(n)}$ and $C_{L(\tau)}$ are contributions of the normal and tangential stresses. Fields in Fig. 4 are calculated using formulae (8) and (9). In this case the distribution of p_{nz} is symmetrical with respect to the plane $z = 0$, therefore $C_{L(n)} = 0$. The field of p_{nz} in transitional flow (Fig. 5, a) is no longer symmetrical, $C_{L(n)} > 0$ and it increases as Kn_∞ decreases. The maximal absolute value of the $p_{\tau z}$ in transitional flow (Fig. 5, b) is much less as compared with the case of free molecular flow. It means that $C_{L(\tau)}$ in transitional flow remains negative but its absolute value decreases as Kn_∞ decreases. Therefore the combined contribution of normal and tangential stresses changed its sign at a some critical Knudsen number Kn_*.

CONCLUSION

The Magnus force and torque coefficients for a spinning sphere were calculated for wide ranges of governing parameters corresponding to free molecular, transitional and near-continuum flows. It was found that the Magnus force changes its direction at a some critical Knudsen number which decreases with increase of the Mach number. The

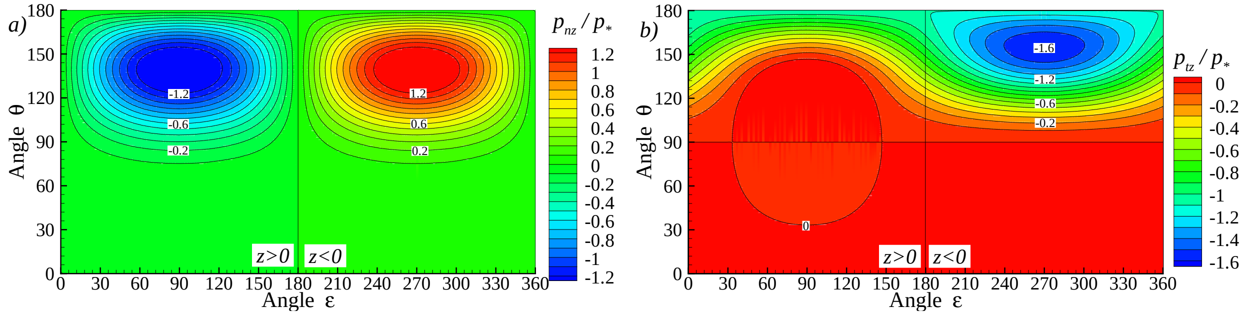

FIGURE 4. Contours of constant stress components p_{nz} (a) and $p_{\tau z}$ (b) at the sphere surface in free molecular flow. $M_\infty = 2$, $W = 1$. $p_* = \rho_\infty U_\infty^2/2 = p_\infty S_\infty^2$.

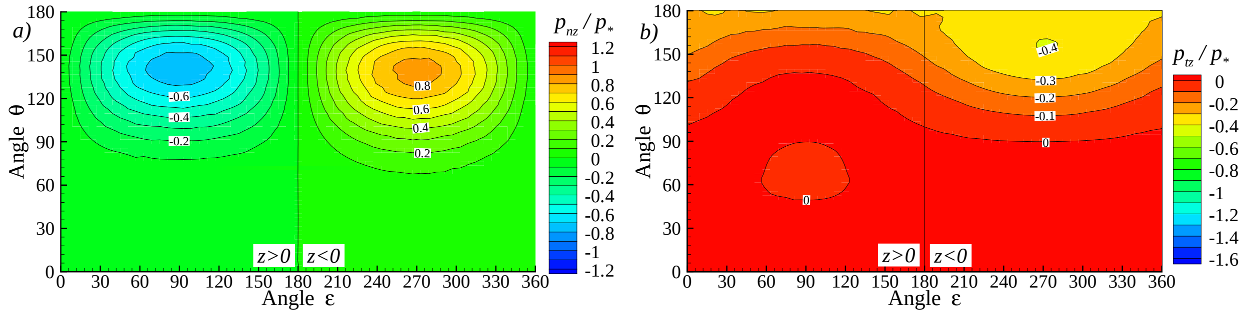

FIGURE 5. Contours of constant stress components p_{nz} (a) and $p_{\tau z}$ (b) at the sphere surface in transitional flow. $Kn_\infty = 0.05$, $M_\infty = 2$, $W = 1$. $p_* = \rho_\infty U_\infty^2/2 = p_\infty S_\infty^2$.

change in the direction of the Magnus force is caused by the redistribution of both normal and tangential stresses at the sphere surface. The increase in the rotational velocity results in a weak decrease of the Magnus force coefficient. The torque coefficient essentially depends on the translational velocity of the sphere. The obtained results can be used for more accurate prediction of interaction between spinning spherical particles and a carrying gas in gas-solid flows.

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