Kinetic Model of a Collisional Admixture in Dusty Gas and its Application to Calculating Flow Past Bodies

Alexey N. Volkov – Baltic State Technical University
Yury M. Tsirkunov – Baltic State Technical University

Citation of published version:

Volkov, A. and Tsirkunov, Y. (2000): Kinetic Model of a Collisional Admixture in Dusty Gas and its Application to Calculating Flow Past Bodies. *Fluid Dynamics*, 35(3). DOI: 10.1007/BF02697751

Fluid Dynamics, Vol. 35, No. 3, 2000

KINETIC MODEL OF A COLLISIONAL ADMIXTURE IN DUSTY GAS AND ITS APPLICATION TO CALCULATING FLOW PAST BODIES

A. N. Volkov and Yu. M. Tsirkunov

UDC 532.529.5:533.7

Using the methods of statistical physics, the basic kinetic equation describing the dynamics of a polydisperse admixture of solid particles in a dilute dusty-gas flow is derived. Particle rotation, inelastic collisions, and interaction with the carrier gas are taken into account. The basic kinetic equation is used to obtain a Boltzmann-type equation for the one-particle distribution function, for which the boundary conditions for the problem of dusty-gas flow past a body are formulated. On the basis of the kinetic model developed, using direct statistical modeling, the flow patterns and the fields of the dispersed-phase macroparameters in a uniform crosswise dusty-gas flow past a cylinder are obtained for various free-stream particle sizes and concentrations.

In dusty-gas flows past bodies, for monodisperse particles several qualitatively different kinds of dispersed-phase flow patterns may be observed [1]. The realization of one or other kind of flow depends on the free-stream particle concentration, the particle inertia (Stokes number), and the mechanism of particle interaction with the body surface. All kinds of particle flow, except one, can be described within the framework of the collisionless particle medium model. However, in many problems interparticle collisions play a decisive role in forming both the dispersed-phase and carrier-gas flow patterns. For example, in a coarse-particle dusty-gas flow past a blunt body the effect of collisions between the incident particles and those rebounding from the body on the dispersed-phase flow field is appreciable for very small free-stream particle volume fractions ($\alpha_{p\infty} \sim 10^{-6}$). This is approximately ten times smaller than the particle concentration at which the dispersed-phase effect on the carrier-gas flow becomes important [1]. This type of flow is of great interest from the standpoint of the shielding effect, i.e. the "scattering" of part of the kinetic energy of the incident particles due to collisions with the rebounding and chaotically moving particles, which results in a reduced rate of abrasive erosion of the body [2, 3].

Despite the significant progress in the kinetic theory of gas-particle mixtures achieved during the last two decades (for example, see monograph [4] and the accompanying references), so far there have been only a few papers published in which the dusty-gas flow past bodies is modeled with account for interparticle collisions [3, 5, 6]. In historically the first study [5], the collisional dispersed-phase equations are introduced phenomenologically, i. e.: the "gas" of particles with its own pressure is described by modified Euler equations with source terms which take into account the interphase exchange and the passage of particles from the collisionless to the collisional (chaotically moving) fraction. Similarly structured continuum equations of the dispersed phase, obtained using a Boltzmann-type kinetic equation, are employed in [6]. In both studies, it is assumed that the characteristic free path between interparticle collisions λ_* is much smaller than the characteristic length scale of the flow L_* (i.e. in the particle "gas", the Knudsen number is small $\mathrm{Kn}_* = \lambda_*/L_* \ll 1$) and the particle velocity distribution is locally equilibrium (Maxwellian).

However, in a dilute dusty-gas flow past a body neither the first nor the second of these assumptions is valid [7]. Firstly, in the free stream the admixture flow is usually collisionless ($\mathrm{Kn}_* \gg 1$), while near the body there may exist a region in which $\mathrm{Kn}_* \sim 1$ and, for fairly high particle concentrations, even a region of continuum flow ($\mathrm{Kn}_* \ll 1$). Secondly, for inelastic interparticle collisions, in an isolated system the kinetic energy of particle chaotic motion can only decrease, and there is no equilibrium Maxwell-type particle velocity distribution [7].

In [3], for calculating the admixture motion a direct statistical modeling method based on a kinetic model of the particle medium is used, which makes it possible to avoid the above-mentioned shortcomings of studies [5, 6]. However, the results obtained in [3] are concerned only with the effect of interparticle collisions on the surface erosion while the dispersed-phase flow pattern is not considered. In addition, in [3] particle rotation, which plays an important role both in particle-particle and particle-wall collisions and in the particle motion through the gas flow, is neglected.

The goals of this study are as follows: (i) using the methods of statistical physics to obtain a basic kinetic equation for a dilute system of polydisperse particles traveling in a carrier gas and inelastically colliding with each other, and (ii) formulating and numerically solving by a direct statistical modeling method the problem of uniform dusty-gas flow past a body (cylinder), when the gas phase is treated as a continuum and the admixture as a discrete system of particles.

Sankt-Peterburg. Translated from Izvestiya Rossiiskoi Akademii Nauk, Mekhanika Zhidkosti i Gaza, No. 3, pp. 81–97, May–June, 2000. Original article submitted April 7, 1999.

1. DYNAMICS OF ADMIXTURE PARTICLES

We assume that the admixture particles are spherical and, in general, have different diameters. As usual for coarse aerosol mechanics, we also assume that near an individual particle the carrier gas flow can be considered to be continuous and that the memory and nonstationary effects inducing the Basset, Archimedes, and virtual mass forces can be neglected [8].

We will consider the motion of a system of N particles in the carrier gas. The state of the i-th particle is characterized by a point $\mathbf{x}_i$ of phase space which includes the particle radius-vector $\mathbf{r}_i$, the particle translational ($\mathbf{v}_i$) and rotational ($\boldsymbol{\omega}_i$) velocities and the radius r_i. We assume that there is no interphase mass transfer or particle fragmentation in collisions and hence that the particle radius remains unchanged. On these assumptions, the evolution of the system is described by the equations:

$$\frac{d\mathbf{x}_i}{dt}=\sum_{\substack{j=1\\ j\neq i}}^{N}\mathbf{F}_{ij}(\mathbf{X})+\mathbf{F}_i'(\mathbf{X},\mathbf{Q}'),\qquad \mathbf{X}=(\mathbf{x}_1,\ldots,\mathbf{x}_N),\qquad i=1,\ldots,N$$

$$\mathbf{x}_i=\begin{pmatrix}\mathbf{r}_i\\ \mathbf{v}_i\\ \boldsymbol{\omega}_i\\ T_i\\ r_i\end{pmatrix},\qquad \mathbf{F}_{ij}=\begin{pmatrix}0\\ \mathbf{f}_{ij}/m_i\\ \mathbf{I}_{ij}/I_i\\ q_{ij}/c_i\\ 0\end{pmatrix},\qquad \mathbf{F}_i'=\begin{pmatrix}\mathbf{v}_i\\ \mathbf{f}_i'/m_i\\ \mathbf{I}_i'/I_i\\ q_i'/c_i\\ 0\end{pmatrix},\qquad \mathbf{Q}'=\begin{pmatrix}T'\\ \rho'\\ \mathbf{v}'\end{pmatrix} \tag{1.1}$$

Here, t is time, $m_i=(4/3)\pi r_i^3\rho_p^0$ and $I_i=(2/5)m_i r_i^2$ are the particle mass and moment of inertia, $c_i=m_i c_p^0$ is the particle heat capacity, ρ_p^0 and c_p^0 are the particle material density and specific heat, and T', ρ', and $\mathbf{v}'$ are the carrier-gas temperature, density, and velocity. The force $\mathbf{f}_{ij}$, the torque $\mathbf{I}_{ij}$ and the heat flux q_{ij} are due to the mechanical action of the j-th on the i-th particle in a collision. The hydrodynamic force $\mathbf{f}_i'$, the torque $\mathbf{I}_i$, and the heat flux q_i' are due to the interaction between the i-th particle and the carrier gas.

In the motion of the particle system, the change in the state of one particle may depend on the state of the other particles. This is because of interparticle collisions and hydrodynamic interaction. The latter is usually taken into account for concentrated systems of Stokes particles [9] and neglected for dilute systems. However, as shown experimentally [10], the hydrodynamic particle-particle interaction may also be appreciable in dilute coarse-particle systems for strong velocity phase disequilibrium, when the Reynolds number of the flow past a particle is higher than the least critical value (200–300) and the wakes behind the particles, which consist of long-lived nonuniformities (separated ring vortices), extend over distances of several tens or even hundreds of particle diameter. Let R_*' and Θ_*' be the linear and time scales of the hydrodynamic singularities (for instance, wakes) associated with the motion of the individual particles, $T_*=L_*/v_*$ be the characteristic flow time, and v_* the characteristic flow velocity relative to the body. If it is possible to introduce physically infinitesimal scales of time t_Φ' and length l_Φ' which satisfy the inequalities [8]

$$R_*' \ll l_\Phi' \ll L_*,\qquad \Theta_*' \ll t_\Phi' \ll T_*$$

the actual fields $\mathbf{Q}'$ can be represented as a superposition ($\mathbf{Q}'=\mathbf{Q}+\Delta\mathbf{Q}$) of the flow field $\mathbf{Q}(\mathbf{r}, t)$ averaged over l_Φ and t_Φ' and a correction $\Delta\mathbf{Q}$ describing the microstructure of the carrier-gas flow due to the presence of the particles. Similarly, the term $\mathbf{F}_i'$ on the right side of (1.1) can be formally represented as follows [11]:

$$\mathbf{F}_i'=\mathbf{F}_i(\mathbf{x}_i, \mathbf{Q}(\mathbf{r}_i, t), \alpha_p)+\Delta\mathbf{F}_i(\mathbf{X}, \mathbf{Q}, \Delta\mathbf{Q}),\qquad \mathbf{Q}=\begin{pmatrix}T\\ \rho\\ \mathbf{v}\end{pmatrix} \tag{1.2}$$

Here, the term $\mathbf{F}_i$ describes the interphase interaction between the particle and the averaged carrier-gas flow field $\mathbf{Q}$ in the locally uniform approximation (with correction for the flow constriction at finite particle volume fractions α_p), and the term $\Delta\mathbf{F}_i$ describes the random interaction between the particle and the flow field fluctuations. For modeling $\Delta\mathbf{F}_i$, the diffusion approximation [11] is often used.

Let c_* be the scale of the particle velocity relative to the other particles and w_* the scale of the particle velocity relative to the carrier gas. The processes occurring in the dispersed-phase motion have the following scales: r_* and $\Theta_*=r_*/c_*$ are the radius and time interval of action of the forces resulting from mechanical contact between particles; l_* and $t_*=l_*/c_*$ are the distance between particles and the time which they take to travel this distance, R_* and $\theta_*=R_*/w_*$ are the length and the time of particle hydrodynamic relaxation in the averaged carrier-gas flow field; λ_* and $\tau_*=\lambda_*/c_*$ are the free path and the time interval between two collisions of an individual particle (these are the scales of collisional relaxation in the particle medium); and τ_* is the characteristic time of particle diffusion due to interaction with the carrier-gas parameter fluctuations.

The kinetic model of the dispersed phase considered below is based on the following relations between the scales introduced:

$$L_* \sim R_*, \quad r_* \ll l_*, \quad l_* \ll \lambda_*, \quad l_* \ll L_* \tag{1.3}$$

$$R_*, \ \lambda_* \ll c_* \tau_*' \tag{1.4}$$

Essentially, the first of relations (1.3) gives the definition of a coarse admixture in terms of the characteristic linear scales of the problem. The second and third relations denote the diluteness of the admixture, which, in particular, makes it possible to neglect the dependence of $\mathbf{F}_i$ on α_p. The fourth relation corresponds to the situation in which, in a given domain of physical space, the number of particles is large enough to describe the whole system using the methods of statistical mechanics. Inequality (1.4) makes it possible to neglect the particle interaction with small-scale carrier-gas fluctuations (for example, the wakes of other particles). Thus, in this study the hydrodynamic interaction of the particles is not taken into account.

For a coarse dilute admixture, the following estimates can be easily obtained [1]:

$$R_* \sim r_*(\rho_p^0/\rho_*), \qquad l_* \sim r_*/\alpha_*^{1/3}, \qquad \lambda_* \sim r_*/\alpha_* \tag{1.5}$$

Here, α_* is a characteristic value of the particle volume fraction. It follows from (1.5) that, for example, in the flow past a body with a characteristic dimension $L_* \sim 1$ m, at a solid-to-gaseous phase ratio $\rho_p^0/\rho_* \sim 10^4$, conditions (1.3) are satisfied for $r_* \sim 100$ μm and $\alpha_p \le 10^{-4}$. When $\alpha \sim 10^{-4}$, we have $\lambda_*/L_* \sim 1$, which corresponds to transitional admixture flow in terms of the Knudsen number of the particle "gas". We note that, usually, if inequalities (1.3) are satisfied, the relations $c_* \sim w_* \sim v_*$ hold and, hence, similar inequalities are also valid for the corresponding time scales.

The assumptions (1.3) make it possible to consider the effects described by the terms $\mathbf{F}_{ij}$ as short-range effects (the distances $|\mathbf{r}_j - \mathbf{r}_i|$ on which $\mathbf{F}_{ij}$ are nonzero are much smaller than any other characteristic scale of the problem, in particular, R_*, the characteristic linear scale of the "external" phase interaction forces $\mathbf{F}_{ij}$) and to take into account only pair collisions. These assumptions are similar to those made for the Boltzmann equation in kinetic theory.

On the assumptions formulated, from (1.1) and (1.2) we obtain:

$$\frac{dx_i}{dt} = \sum_{\substack{j=1 \\ j \ne i}}^{N} \mathbf{F}_{ij}(\mathbf{x}_i, \mathbf{x}_j) + \mathbf{F}_i(\mathbf{x}_i, \mathbf{Q}(\mathbf{r}_i, t)), \quad i = 1, \ldots, N \tag{1.6}$$

2. KINETIC MODEL OF THE DISPERSED PHASE

As in the kinetic theory of a rarefied gas, if conditions (1.3) are satisfied it is possible to use a reduced, kinetic description of the dispersed phase.

The solution of the system (1.6) with the initial condition $\mathbf{X} = \mathbf{X}_0$ at $t = 0$ is equivalent to the solution of the transport equation

$$\frac{\partial D_N}{\partial t} + \sum_{i=1}^{N} \frac{\partial}{\partial \mathbf{x}_i} \cdot (\mathbf{F}_i D_N) = -\sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \ne i}}^{N} \frac{\partial}{\partial \mathbf{x}_i} \cdot (\mathbf{F}_{ij} D_N) \tag{2.1}$$

for the dynamic distribution function $D_N(\mathbf{X}, t, \mathbf{X}_0)$ with the initial condition $D_N(\mathbf{X}, 0, \mathbf{X}_0) = \delta_{11N}(\mathbf{X} - \mathbf{X}_0)$, where $\delta_M(\mathbf{x})$ is a M-dimensional Dirac delta-function [12]. Here,

$$\mathbf{x} \in \mathbf{R}^M, \qquad \int \delta_M(\mathbf{x})\, d\mathbf{x} = 1$$

The solution of Eq. (2.1) can be represented in terms of the solution $\mathbf{X}(t) = \mathbf{S}(\mathbf{X}_0, t)$ of the system (1.6) as follows:

$$D_N(\mathbf{X}, t, \mathbf{X}_0) = \delta_{11N}(\mathbf{X} - \mathbf{S}(\mathbf{X}_0, t))$$

In contrast to (1.6), Eq. (2.1) is convenient for passing to the kinetic description of the dispersed phase.

By virtue of the inequalities (1.3), we can neglect the collision duration compared with the characteristic time scale of any other process in the gas-particle mixture and assume that, during collision, the position of the colliding particles does not change. The superscripts "minus" and "plus" denote the parameters of the i-th and j-th particles before and after collision. Then, the relations between the parameters $\mathbf{y}_k = (\mathbf{v}_k, \omega_k, T_k)$ $(k = i, j)$ before and after collision can be represented in the form:

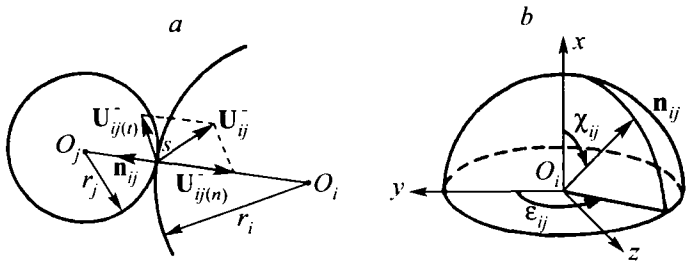

Fig. 1. Diagram of a collision between the i-th and j-th disperse particles (a) and the coordinate system in the collision integral of Eq. (2.4) (b).

$$\mathrm{y}_k^+=\mathrm{y}_k^+(\mathrm{y}_i^-,\ \mathrm{y}_j^-,\ r_i,\ r_j,\ \mathrm{n}_{ij}),\qquad k=i,\ j \tag{2.2}$$

These relations have physical meaning only when $\mathbf{g}_{ij}^- \cdot \mathbf{n}_{ij} \le 0$, where $\mathbf{g}_{ij}=\mathbf{v}_j-\mathbf{v}_i$, and $\mathbf{n}_{ij}$ is a unit vector specifying the relative location of the particle centers at the collision instant (Fig. 1). In what follows, we will assume that Eq. (2.2) describes a one-to-one transformation, so that its Jacobian is nonzero:

$$J_1=|[\partial(\mathrm{y}_i^+,\ \mathrm{y}_j^+)]/[\partial(\mathrm{y}_i^-,\ \mathrm{y}_j^-)]|\neq 0$$

and, hence, Eq. (2.2) can be solved for the particle parameters before the collision

$$\mathrm{y}_k^-=\mathrm{y}_k^-(\mathrm{y}_i^+,\ \mathrm{y}_j^+,\ r_i,\ r_j,\ \mathrm{n}_{ij}),\qquad k=i,\ j,\qquad \mathrm{g}_{ij}^+\cdot\mathrm{n}_{ij}\ge 0 \tag{2.3}$$

For conditions (1.3), as in rarefied-gas theory [12, 13], in the particle medium we can introduce (i) physically infinitesimal length $l_\Phi=r_*/\alpha_*^{1/2}$ and time $t_\Phi=l_\Phi/c_*$ scales which, for $\alpha_* \ll 1$, satisfy the inequalities

$$l_* \ll l_\Phi \ll \lambda_*,\ L_*,\qquad t_* \ll t_\Phi \ll \tau_*,\ T_*$$

and (ii) a smoothing operator P_N corresponding to these scales:

$$P_N G(\mathrm{X},\ t)=\frac{1}{t_\Phi l_\Phi^{3N}}\int_{t-t_\Phi/2}^{t+t_\Phi/2}\int G(\mathrm{r}_1',\ \mathrm{z}_1,\ldots,\ \mathrm{r}_N',\ \mathrm{z}_N,\ t')\,d\mathrm{r}_1'\ldots d\mathrm{r}_N'\,dt'$$

Here, $G(\mathbf{X},\ t)$ is a certain function of the phase coordinates and time, $\mathbf{z}_i=(\mathbf{y}_i,\ r_i)$, and the integration with respect to $\mathbf{r}_i'$ is performed over a symmetric neighborhood of the point $\mathbf{r}_i$, which has the shape of a parallelepiped with the edge length l_Φ.

We apply the operator P_N to Eq. (2.1). From the averaged equation obtained, for a rarefied particle "gas" (formally, as $\alpha_* \to 0$), using the multiple-scale method [14], we can derive a kinetic-type equation for the smoothed distribution function $D_N^{(c)}=P_N D_N$ [12]. By introducing an ensemble of macroscopically identical systems with the initial-state distribution density $F_N(\mathbf{X}_0,\ 0)$ and averaging the resulting equation over the initial states, we obtain the so-called basic kinetic equation

$$\frac{\partial F_N}{\partial t}+\sum_{i=1}^{N}\frac{\partial}{\partial \mathrm{x}_i}(\mathrm{F}_i F_N)=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\delta_3(\mathrm{r}_j-\mathrm{r}_i)I_{ij} \tag{2.4}$$

$$I_{ij}=d_{ij}^2\int_{g_{ij}\cdot n_{ij}\le 0}\left(\frac{F_N(\mathrm{X}_{ij}^-,\ t)}{J}-F_N(\mathrm{X},\ t)\right)|\mathrm{g}_{ij}\cdot\mathrm{n}_{ij}|\sin\chi_{ij}\,d\chi_{ij}\,d\varepsilon_{ij}$$

$$J=\left|\frac{\mathrm{g}_{ij}\cdot\mathrm{n}_{ij}}{\mathrm{g}_{ij}^-\cdot\mathrm{n}_{ij}}J_1\right|,\qquad \mathrm{X}_{ij}^-=(\mathrm{x}_1,\ldots,\ \mathrm{x}_{i-1},\ \mathrm{x}_i^-,\ \mathrm{x}_{i+1},\ldots,\ \mathrm{x}_{j-1},\ \mathrm{x}_j^-,\ \mathrm{x}_{j+1},\ldots,\ \mathrm{x}_N)$$

which describes the evolution of a statistical N-particle distribution function

$$F_N(\mathrm{X},\ t)=\int D_N^{(c)}(\mathrm{X},\ t,\ \mathrm{X}_0)F_N(\mathrm{X}_0,\ 0)\,d\mathrm{X}_0$$

in $(11\times N)$-dimensional phase space. In Eq. (2.4), χ_{ij} and ε_{ij} are the angles of an arbitrary spherical coordinate system, which specify the direction of $\mathbf{n}_{ij}$:

$$\mathrm{n}_{ij}=\cos\chi_{ij}\mathbf{i}+\sin\chi_{ij}\cos\varepsilon_{ij}\mathbf{j}+\sin\chi_{ij}\sin\varepsilon_{ij}\mathbf{k} \tag{2.5}$$

Here, $(\mathbf{i},\ \mathbf{j},\ \mathbf{k})$ is the basis of the Cartesian coordinate system $(x,\ y,\ z)$ (Fig. 1), $d_{ij}=r_i+r_j$, and $\mathbf{x}_k^-=(\mathbf{r}_k,\ \mathbf{y}_k^-(\mathbf{y}_i,\ \mathbf{y}_j,\ r_i,\ r_j,\ \mathbf{n}_{ij}),\ r_i)$, $k=i,\ j$. The multiplier $1/J$ in the collisional operator on the right side of (2.4) takes into account the phase volume "compression", which takes place if the collision is not absolutely elastic.

All the admixture macroparameters $\langle\Phi\rangle_1$ and $\langle\Psi\rangle_2$ which are of interest from the mechanical viewpoint can be expressed in terms of functions of the parameters of one $(\Phi(\mathbf{x}_i))$ or two $(\Psi(\mathbf{x}_i,\ \mathbf{x}_j))$ particles and the one- or two-particle distribution functions (F_1 and F_2):

$$\langle\Phi\rangle_1(\mathrm{r},\ t)=\int\sum_{i=1}^{N}\delta_3(\mathrm{r}_i-\mathrm{r})\Phi(\mathrm{x}_i)F_N(\mathrm{X},\ t)\,d\mathrm{X}=N\int\Phi(\mathrm{r},\ \mathrm{z}_1)F_1(\mathrm{r},\ \mathrm{z}_1,\ t)\,d\mathrm{z}_1 \tag{2.6}$$

$$\begin{aligned}\langle\Psi\rangle_2(\mathrm{r},\ t)=&\int\sum_{i=1}^{N-1}\sum_{i=j+1}^{N}\delta_3(\mathrm{r}_i-\mathrm{r})\delta_3(\mathrm{r}_j-\mathrm{r})\Psi(\mathrm{x}_i,\ \mathrm{x}_j)F_N(\mathrm{X},\ t)\,d\mathrm{X}=\\&\frac{N(N-1)}{2}\int\Psi(\mathrm{r},\ \mathrm{z}_1,\ \mathrm{r},\ \mathrm{z}_2)F_2(\mathrm{r},\ \mathrm{z}_1,\ \mathrm{r},\ \mathrm{z}_2,\ t)\,d\mathrm{z}_1\,d\mathrm{z}_2\end{aligned} \tag{2.7}$$

For instance, the number (n_p) and volume (α_p) concentrations of the admixture, its macroscopic velocity $\mathbf{u}_p$, the mean-square particle chaotic motion velocity c_p^2, and the interparticle collision frequency in unit volume ν_p can be calculated as follows:

$$n_p=\langle 1\rangle_1,\quad \alpha_p=\frac{4}{3}\pi\langle r_i^3\rangle_1,\quad \mathbf{u}_p=\frac{\langle r_i^3\mathbf{v}_i\rangle_1}{\langle r_i^3\rangle_1},\quad c_p^2=\frac{\langle r_i^3(\mathbf{v}_i-\mathbf{u}_p)^2\rangle_1}{\langle r_i^3\rangle_1},\quad \nu_p=\langle\pi d_{ij}^2 g_{ij}\rangle_2$$

Let the system of particles considered be located in a volume V of physical space. In the thermodynamic limit, as the number of particles N and the volume V tend to infinity at a constant mean particle concentration, i. e

$$N,\ V\to\infty\qquad (N/V=\mathrm{const}) \tag{2.8}$$

the use of the "molecular chaos" hypothesis

$$F_2(\mathrm{x}_1,\ \mathrm{x}_2,\ t)=F_1(\mathrm{x}_1,\ t)F_1(\mathrm{x}_2,\ t) \tag{2.9}$$

makes it possible, using (2.4), to obtain the Boltzmann-type equation for the distribution function $f_1=f(\mathbf{x}_1,\ t)=NF_1(\mathbf{x}_1,\ t)$:

$$\begin{aligned}&\frac{\partial f_1}{\partial t}+\frac{\partial}{\partial\mathbf{r}_1}\cdot(\mathbf{v}_1f_1)+\frac{\partial}{\partial\mathbf{v}_1}\left(\frac{\mathbf{f}_1}{m_1}f_1\right)+\frac{\partial}{\partial\boldsymbol{\omega}_1}\left(\frac{\mathbf{I}_1}{I_1}f_1\right)+\frac{\partial}{\partial T_1}(q_1f_1)=\\&\int_0^{\infty}d_{12}^2\int\int_{\mathbf{g}_{12}\cdot\mathbf{n}_{12}\le 0}\left(\frac{f_1^-f_2^-}{J}-f_1f_2\right)|\mathbf{g}_{12}\cdot\mathbf{n}_{12}|\sin\chi_{12}\,d\chi_{12}\,d\varepsilon_{12}\,dy_2\,dr_2\end{aligned} \tag{2.10}$$

Here, $f_2=f(\mathbf{x}_2,\ t)$, $f_1^-=f(\mathbf{x}_1^-,\ t)$, and $f_2^-=f(\mathbf{x}_2^-,\ t)$. A similar kinetic equation with a quite different collision integral was obtained phenomenologically in [15] for a dilute medium of liquid droplets.

Let D be the calculation domain, ∂D be its boundary, $\mathbf{r}_1\in\partial D$, $\mathbf{n}=\mathbf{n}(\mathbf{r}_1)$ be the inward unit normal to ∂D, and $\mathbf{v}_1$ the velocity of the particles entering the calculation domain at the boundary point $\mathbf{r}_1$, i. e. let the condition $\mathbf{v}_1\cdot\mathbf{n}>0$ be satisfied. Then, at each point on the boundary ∂D either an explicit expression for the distribution function f_1 at $\mathbf{v}_1\cdot\mathbf{n}>0$ or a relation between f_1 at $\mathbf{v}_1\cdot\mathbf{n}>0$ and $f(\mathbf{r}_1,\ \mathbf{y}_1',\ r_1,\ t)$ at $\mathbf{v}_1'\cdot\mathbf{n}<0$ must be specified. It is considered [16] that this boundary condition formulation ensures the correctness of the initial-boundary-value problem for an equation of the type (2.10).

The diagram of a calculation domain typical of plane aerodynamical problems is shown in Fig. 2. The calculation domain $ABCD$ is bounded by the entrance boundary AD, the symmetry plane AB, the surface of the body immersed in the flow BC, and the exit boundary CD. At AD and CD, the distribution function over the velocities $\mathbf{v}_i$, ω_i, and sizes r_i of the

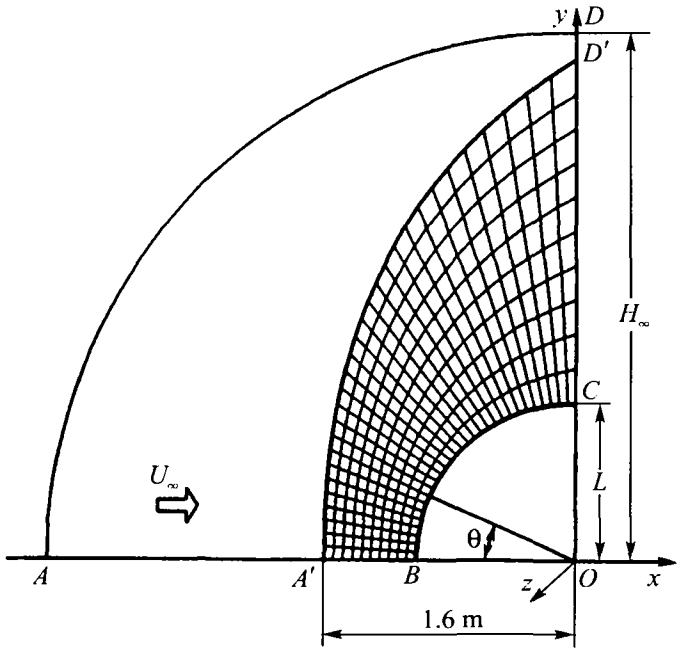

Fig. 2. Calculation domain in the problem of gas-particle flow past a cylinder.

particles whose translational velocity is directed into the domain *ABCD* must be specified. We assume that the boundary *AD* lies in the undisturbed-flow region, where the particles are at rest relative to the gas and equipropably distributed over the mixture volume, while their velocities are statistically uncorrelated. Then the following boundary condition must be specified on *AD*:

$$f(\mathrm{x}_1, t)=n_{p\infty}\delta_3(\mathrm{v}_1-\mathrm{v}_\infty)\delta_3(\omega_1)\delta_1(T_1-T_\infty)f_\infty^0(r_1) \tag{2.11}$$

Here, $n_{p\infty}$ and $\mathbf{v}_\infty$ are the particle number concentration and velocity vector, and $f_\infty^0(r_i)$ is the particle size distribution function in the undisturbed flow ($\int_0^\infty f_\infty^0(r_i)\,dr_i=1$).

Assuming that the exit boundary *CD* is located fairly far downstream, so that on this boundary $|\mathbf{u}_p\cdot\mathbf{n}| \gg c_p$, we find that the particle fraction entering the calculation domain through this boundary is very small and it can be assumed that approximately: $f(\mathbf{x}_1, t)=0$.

It seems likely that, as the exit boundary is displaced downstream, this condition is satisfied more and more accurately because the particle chaotic motion decays due to interaction with the carrier gas.

In the symmetry plane *AB*, the following symmetry condition is specified:

$$f(\mathrm{x}_1, t)=f(\mathrm{r}_1, \mathrm{y}_1', r_1, t)$$
$$\mathrm{v}_1'=\mathrm{v}_1-2\mathrm{n}(\mathrm{v}_1\cdot\mathrm{n}), \qquad \omega_1'=\omega_1-2\mathrm{n}\times(\omega_1\times\mathrm{n}) \tag{2.12}$$

Here, $\mathbf{v}_1'\cdot\mathbf{n} < 0$.

The most complex problem is that of formulating the boundary conditions on the body surface *BC*, which is impermeable to the particles and on which, as a rule, it is required to model an inelastic rebound. Neglecting the particle-surface interaction time and the change in the particle center position during the collision, we can write the boundary condition on the surface at rest in the following form (similar to that in rarefied-gas kinetic theory [16]):

$$|\mathrm{v}_1\cdot\mathrm{n}|f(\mathrm{x}_1, t)=\int\limits_{\mathrm{v}_1'\cdot\mathrm{n}<0}|\mathrm{v}_1'\cdot\mathrm{n}|W_w(\mathrm{y}_1'\to\mathrm{y}_1|r_1, \mathrm{n})f(\mathrm{r}_1, \mathrm{y}_1', r_1, t)\,d\mathrm{y}_1' \tag{2.13}$$

Here, $W_w(\mathbf{y}_1'\to\mathbf{y}_1|r_1, \mathbf{n})$ is the conditional distribution density of the particle parameters $\mathbf{y}_1$ after reflection from the surface for known pre-collision parameters $\mathbf{y}_1'$.

3. MODEL OF A TWO-PARTICLE COLLISION

Our particle collision model is based on the momentum and angular momentum balance equations for a pair of i-th and j-th nondeformable colliding particles:

$$m_i\mathbf{v}_i^- + m_j\mathbf{v}_j^- = m_i\mathbf{v}_i^+ + m_j\mathbf{v}_j^+, \qquad I_k(\boldsymbol{\omega}_k^+ - \boldsymbol{\omega}_k^-) = m_k\mathbf{e}_k \times (\mathbf{v}_k^+ - \mathbf{v}_k^-)$$
$$\mathbf{e}_j = -r_j\mathbf{n}_{ij}, \qquad \mathbf{e}_i = r_i\mathbf{n}_{ij}, \qquad k = i, j \tag{3.1}$$

System (3.1) contains three vector equations for four unknown vector quantities $\mathbf{v}_k^+$ and $\boldsymbol{\omega}_k^+$, $k=i, j$. To make (3.1) close, it is necessary to introduce additional hypotheses for the forces acting at the point of particle contact. Usually [17], these hypotheses are formulated in the form of a relation between the components of the relative velocity of the particle surfaces $\mathbf{U}_{ij}$ at the point of particle contact s (Fig. 1) before and after the collision:

$$\mathbf{U}_{ij} = \mathbf{g}_{ij} - \mathbf{w}_{ij} \times \mathbf{n}_{ij}, \qquad \mathbf{w}_{ij} = r_i\boldsymbol{\omega}_i + r_j\boldsymbol{\omega}_j \tag{3.2}$$

It is often assumed [17] that the relation between $\mathbf{U}_{ij}^-$ and $\mathbf{U}_{ij}^+$ has the following form:

$$\mathbf{U}_{ij}^+ = -a_{pn}\mathbf{U}_{ij(n)}^- + a_{pt}\mathbf{U}_{ij(t)}^- \tag{3.3}$$

Here, a_{pn} and a_{pt} are the restitution coefficients of the normal and tangential components of the relative velocity $\mathbf{U}_{ij}$; here and in what follows, the subscripts (n) and (t) denote the arbitrary-vector component directed along the normal $\mathbf{n}_{ij}$ and that lying in the tangential plane located at right angle to $\mathbf{n}_{ij}$, respectively (for any vector, $\mathbf{a}=\mathbf{a}_{(n)} + \mathbf{a}_{(t)}$ and $\mathbf{a}_{(n)}=(\mathbf{a}\cdot\mathbf{n}_{ij})\mathbf{n}_{ij}$). It is assumed that the coefficients a_{pn} and a_{pt} take into account the losses of particle kinetic energy due to inelastic deformations during collision and due to particle surface roughness, respectively ($0 \le a_{pn}$, $|a_{pt}| \le 1$).

If the values of a_{pn} and a_{pt} are given, the system (3.1)–(3.3) will be closed and can be solved for the parameters of the two particles after the collision. Using the notation

$$\mathbf{G}_{ij} = \mu_i\mathbf{v}_i + \mu_j\mathbf{v}_j, \quad \mathbf{W}_{ij} = \zeta_j\boldsymbol{\omega}_j - \zeta_i\boldsymbol{\omega}_i, \quad \mu_k = \frac{m_k}{m_i + m_j}, \quad \zeta_k = I_k/r_k$$

from (3.1)–(3.3) we can obtain the following relations:

$$\mathbf{G}_{ij}^+ = \mathbf{G}_{ij}^-, \quad \mathbf{g}_{ij}^+ = \mathbf{g}_{ij}^- - (1 + a_{pn})\mathbf{U}_{ij(n)}^- - (2/7)(1 - a_{pt})\mathbf{U}_{ij(t)}^-$$
$$\mathbf{W}_{ij}^+ = \mathbf{W}_{ij}^-, \qquad \mathbf{w}_{ij}^+ = \mathbf{w}_{ij}^- - (5/7)(1 - a_{pt})(\mathbf{U}_{ij}^- \times \mathbf{n}_{ij}) \tag{3.4}$$

Then, the required parameters $\mathbf{v}_k^+$ and $\boldsymbol{\omega}_k^+$ can be found from the formulas

$$\mathbf{v}_i^+ = \mathbf{G}_{ij}^+ - \mu_j\mathbf{g}_{ij}^+, \qquad \mathbf{v}_j^+ = \mathbf{G}_{ij}^+ + \mu_i\mathbf{g}_{ij}^+$$
$$\boldsymbol{\omega}_i^+ = \mu_j\frac{r_j}{r_i}\left(\frac{\mathbf{w}_{ij}^+}{r_j} - \frac{\mathbf{W}_{ij}^+}{\zeta_j}\right), \qquad \boldsymbol{\omega}_j^+ = \mu_i\frac{r_i}{r_j}\left(\frac{\mathbf{w}_{ij}^+}{r_i} + \frac{\mathbf{W}_{ij}^+}{\zeta_i}\right) \tag{3.5}$$

For determining the temperature variation of each particle, we assume that the kinetic energy loss is redistributed between the particles in the form of heat fluxes proportional to the particle masses. Accordingly, we have:

$$T_k^+ = T_k^- + \frac{\mu_k}{c_k}\Delta E_{ij}, \qquad \Delta E_{ij} = \frac{m_i m_j}{m_i + m_j}(B_{ij}^- - B_{ij}^+)$$
$$B_{ij} = \mathbf{g}_{ij}^2/2 + \mathbf{w}_{ij}^2/5, \qquad k = i, j \tag{3.6}$$

The theoretical or experimental determination of the coefficients a_{pn} and a_{pt} is very difficult and, so far, no information on the dependence of a_{pn} and a_{pt} on the velocities of the colliding particles and the other governing parameters of collision dynamics has been published. The use of physically probable assumptions concerning the variation of a_{pn} and a_{pt} with the relative velocity of the colliding particles [5] leads to an increase in the number of free parameters of the model, which makes its numerical study much more complex. This is why in this paper the restitution coefficients are assumed to be constant (a_{pn}, a_{pt}=const). In this case, $J_1 = -a_{pn}a_{pt}^2$ and $J = a_{pn}^2 a_{pt}^2$.

4. MODEL OF PARTICLE REFLECTION FROM A SOLID SURFACE

Neglecting the body surface roughness which can result in random scattering of the particles, we assume that during a collision the particle parameters change in a determinate way. We then have:

$$W_w(\mathbf{y}_i' \to \mathbf{y}_i | r_i, \mathbf{n}) = \delta_0(\mathbf{y}_i - \mathbf{y}_w^+(\mathbf{y}_i', r_i, \mathbf{n}))$$

Here, $\mathbf{y}_i^+ = \mathbf{y}_w^+(\mathbf{y}_i^-, r_i, \mathbf{n})$ is the law of post-collision variation of the particle parameters $\mathbf{y}_i$ (in this section, the minus and plus superscripts signify the parameters of an arbitrary i-th particle before and after collision with the surface).

The subscripts (n) and (t) denote the particle velocity component directed along the normal $\mathbf{n}$ and that lying in the plane tangential to the surface. Then, the velocity vector of a nonrotating particle after collision $\mathbf{v}_i^+$ can be expressed in terms of the velocity vector before collision $\mathbf{v}_i^-$ and the restitution coefficients of the normal and tangential particle velocities a_{wn} and a_{wt} as follows:

$$\mathbf{v}_{i(n)}^+ = -a_{wn}\mathbf{v}_{i(n)}^-, \qquad \mathbf{v}_{i(t)}^+ = a_{wt}\mathbf{v}_{i(t)}^-$$

In this study, for determining the restitution coefficients we used the relations $a_{wn} = a_{wn}(\varphi, |\mathbf{v}_i^-|)$ and $a_{wt} = a_{wt}(\varphi)$ (here, φ is the impact angle, i.e. the angle between the vectors $\mathbf{v}_i^-$ and $\mathbf{n}$) obtained in [18,19] by processing the experimental data on the flow of an air — electrocorundum particle mixture past surfaces made of a St3 mild steel. These relations are valid on the velocity range: 50 m/s $\le |\mathbf{v}_i^-| \le$ 500 m/s.

A particle impacting on the body surface may have an arbitrary orientation of the translational $\mathbf{v}_i^-$ and angular $\boldsymbol{\omega}_i^-$ velocity vectors. It seems probable that the force distribution over the particle-wall contact spot is symmetric about the plane $(\mathbf{n}, \mathbf{U}_i^-)$, which contains the unit vector of the normal $\mathbf{n}$ and the particle bottom point velocity vector $\mathbf{U}_i^- = \mathbf{v}_i^- - r_i\boldsymbol{\omega}_i^- \times \mathbf{n}$. This means that the main vector of the particle-wall interaction forces has no component directed at right angles to the plane $(\mathbf{n}, \mathbf{U}_i^-)$ and the main torque of the interaction forces (relative to the particle center) has no component lying in this plane. We introduce the unit vectors of the local coordinate system

$$\boldsymbol{\varkappa} = (\mathbf{n} \times \mathbf{U}_i^-)/(|\mathbf{n} \times \mathbf{U}_i^-|), \qquad \boldsymbol{\tau} = \boldsymbol{\varkappa} \times \mathbf{n}$$

and denote the $\mathbf{n}$, τ, and $\boldsymbol{\varkappa}$ vector components by corresponding subscripts. Assuming also that all the thermal energy released during impact is absorbed by the body surface, we obtain:

$$v_{i\varkappa}^+ = v_{i\varkappa}^-, \qquad \omega_{in}^+ = \omega_{in}^-, \qquad \omega_{i\tau}^+ = \omega_{i\tau}^-, \qquad T_i^+ = T_i^- \tag{4.1}$$

During particle impact with the surface, only the velocity components v_{in}, $v_{i\tau}$, and $\omega_{i\varkappa}$ change. Their values after impact are given by the following formulas [19]:

$$v_{in}^+ = -a_{wn}v_{in}^-$$

$$v_{i\tau}^+ = \begin{cases} a_{wt}v_{i\tau}^- - (1 - a_{wt})r_i\omega_{i\varkappa}^-, & \varphi > \varphi_* \\ a_{wt}v_{i\tau}^- - (2/7)r_i\omega_{i\varkappa}^-, & \varphi \le \varphi_* \end{cases} \tag{4.2}$$

$$\omega_{i\varkappa}^+ = \begin{cases} \dfrac{5a_{wt} - 3}{2}\omega_{i\varkappa}^- + \dfrac{5}{2r_i}(a_{wt} - 1)v_{i\tau}^-, & \varphi > \varphi_* \\ \dfrac{2}{7}\omega_{i\varkappa}^- - \dfrac{a_{wt}}{r_i}v_{i\tau}^-, & \varphi \le \varphi_* \end{cases}$$

Here, φ is the angle of incidence in the plane $(\mathbf{n}, \tau)$, and φ_* is the root of the equation $q_{wt}(\varphi_*) = 5/7$ (for St3 steel, $\varphi_* = 79.5°$).

5. INTERACTION BETWEEN THE PARTICLES AND THE CARRIER GAS

In accordance with our assumptions, for determining the force $\mathbf{f}_i$ and the torque $\mathbf{I}_i$ exerted on an individual particle by the carrier gas we use the known results for uniform unbounded steady-state flow past a sphere. In calculating $\mathbf{f}_i$ we take

into account only the two most important components, i. e. the aerodynamic drag $\mathbf{f}_{Di}$ and the Magnus force $\mathbf{f}_{Mi}$. Accordingly, $\mathbf{f}_i=\mathbf{f}_{Di}+\mathbf{f}_{Mi}$. The force $\mathbf{f}_{Di}$ can be represented in the form [8]:

$$f_{Di}=0.5C_D(\mathrm{Re}_i)\pi r_i^2\rho|v-v_i|(v-v_i), \qquad \mathrm{Re}_i=\frac{2r_i\rho|v-v_i|}{\mu}$$

Here, μ is the carrier-gas viscosity. For calculating the drag coefficient C_D for incompressible flow past a spherical particle we use the system of relations [20], which describe the sphere drag "standard curve" $C_D(\mathrm{Re}_i)$ in an incompressible gas to within 1–2%.

The lift Magnus force $\mathbf{f}_{Mi}$ is generated by the motion of a rotating particle [8]:

$$f_{Mi}=\frac{4}{3}C_\omega(\mathrm{Re}_i, \mathrm{Re}_{\omega i})\pi r_i^3\rho(\omega-\omega_i)\times(v-v_i)$$

$$\mathrm{Re}_{\omega i}=\frac{4r_i^2\rho|\omega-\omega_i|}{\mu}, \qquad \omega=0.5\,\mathrm{curl}\,v$$

In the limiting case $\mathrm{Re}_i, \mathrm{Re}_{\omega i} \ll 1$, a theoretical solution [21] gives the value $C_\omega=3/4$. According to the experimental results [22], for $\mathrm{Re}_{\omega i} \le 10^3$ the coefficient C_ω varies only slightly as compared with the limiting value. Accordingly, below, in calculating the Magnus force we assume that $C_\omega=3/4$.

The damping moment of the forces $\mathbf{I}_i$ acting on a rotating particle is usually written in the form [8]:

$$I_i=0.5C_L(\mathrm{Re}_{\omega i})\rho r_i^5|\omega-\omega_i|(\omega-\omega_i)$$

For $\mathrm{Re}_{\omega i} \ll 1$ we have $C_L=64\pi/\mathrm{Re}_{\omega i}$. At moderate and high Reynolds numbers $\mathrm{Re}_{\omega i} \le 1.6\cdot10^5$, for calculating C_L we use the formulas from [23] obtained using an approximation of the theoretical, numerical, and experimental data of different authors.

The heat flux across the particle surface is usually written in the form [24]:

$$q_i=2\mathrm{Nu}_p(\mathrm{Re}_i, M_i)\pi r_i\lambda(T-T_i), \qquad M_i=(|v-v_i|)/\sqrt{\gamma RT}$$

Here, $\mathrm{Nu}_p=2r_i\xi/\lambda$, $\lambda=c_p\mu/\mathrm{Pr}$, $\gamma=c_p/c_v$, ξ is the coefficient of heat transfer across the interphase surface, c_p and c_v are the perfect-gas specific heats at constant pressure and volume, and Pr is the Prandtl number.

In this study, for determining Nu_p in the Stokes flow past a particle ($M_i \ll \mathrm{Re}_i \ll 1$) we used the theoretical result $\mathrm{Nu}_p=2$ and, for large Re_i, the semi-empirical Cavano-Dreik formula [24].

6. NUMERICAL STUDY OF THE ADMIXTURE FLOW IN A UNIFORM CROSSWISE GAS-PARTICLE STREAM PAST A CYLINDER

In this study, for collisionless, transitional, and near-continuum particle "gas" flow regimes the macroparameters, e.g. the functionals of the solution $f(\mathbf{x}_1, t)$ of Eq. (2.10), were calculated using a direct statistical modeling method similar to that used in rarefied-gas calculations. The most effective numerical algorithms of this method use Bird's scheme [25] and are based on directly simulating the random motion and collisions of a finite system of N particles. From the theoretical viewpoint, this random process of variation of the parameters $(\mathbf{X}, t)$ is described by an integral analog of Eq. (2.4) [26, 27]. In the calculations, the similarity with respect to the Knudsen number between the "gas" of model particles and a real admixture can always be ensured for a not too large number of particles N.

The results described below were obtained using an approximate direct statistical modeling method based on discretizing the calculation domain in the cells and splitting the particle motion process into two stages: (i) the stage of interparticle collisions and (ii) the stage of collisionless motion in the carrier gas. The collisions were calculated using a majorant frequency scheme [7].

The calculation domain $A'BCD'$ and the grid for modeling the collisional-admixture motion are shown in Fig. 2. The admixture parameter distributions on $A'D'$ were determined from the trajectory calculations [28] of the particle collisionless motion from the undisturbed flow (line AD) to the boundary $A'D'$.

The carrier gas (air, $\gamma=1.4$, Pr=0.77, R=287 J/(kg·mole)) was assumed to be incompressible and the flow to be potential. In the calculations, the parameter values were as follows: cylinder radius L=1 m, cylinder material St3 steel, free-stream velocity v_∞=100 m/s, density ρ=1.197 kg/m^3, temperature T_∞=273 K, and viscosity $\mu_\infty=1.71\cdot10^{-5}$ kg/(m·s). These correspond to the Mach number $M_\infty=v_\infty/(\gamma RT_\infty)^{1/2}=0.3$ and the Reynolds number $\mathrm{Re}_\infty=v_\infty\rho L/\mu_\infty=7\cdot10^6$.

We considered a flow of gas carrying an admixture of electrocorundum particles with density ρ_p^0=3950 kg/m^3 and specific heat c_p^0=1000 J/(kg·K). In the interparticle collision model the restitution coefficients were a_{pn} and $a_{p\tau}$. The free-stream particle volume fraction was varied on the range $\alpha_{p\infty}=10^{-6}$–10^{-4} and the particle radius on the range r_p=5–250 μm.

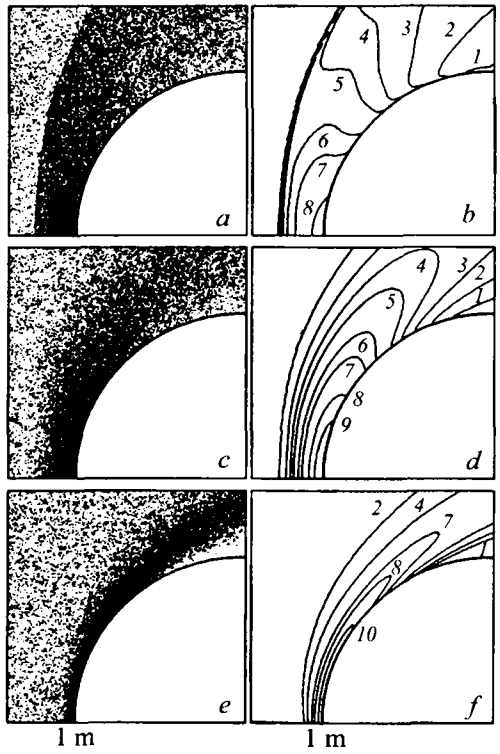

Fig. 3

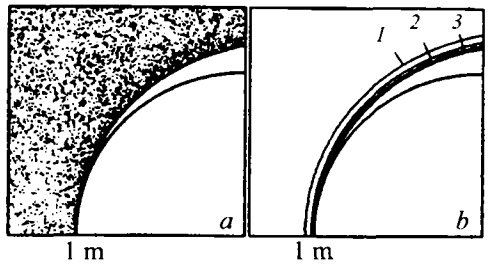

Fig. 4

Fig. 3. Distribution patterns of model particles of radius r_p=100 μm (*a*, *c*, *e*) and the corresponding isolines of the relative admixture concentration $\alpha_p/\alpha_{p\infty}$ (*b*, *d*, *f*: *1* — $\alpha_p/\alpha_{p\infty}$=1, *2* — 1.5, *3* — 2, *4* — 2.5, *5* — 3, *6* — 4, *7* — 5, *8* — 6, *9* — 8, *10* — 12, *11* — 20); *a*, *b* — $\alpha_{p\infty}=10^{-5}$; *c*, *d* — $\alpha_{p\infty}=3\cdot10^{-5}$: *1* — $\alpha_p/\alpha_{p\infty}$=1.5; *2* — 2; *3* — 4.

Fig. 4. Distribution patterns of model particles of radius r_p=10 mm (*a*) and the corresponding isolines of the relative admixture concentration $\alpha_p/\alpha_{p\infty}$ (*b*); $\alpha_{p\infty}=3\cdot10^{-5}$: *1* — $\alpha_p/\alpha_{p\infty}$=1.5; *2* — 2; *3* — 4.

Typical flow patterns for "coarse" (r_p=100 μm) and "fine" (r_p=10 μm) particles are presented in Figs. 3 and 4, respectively. Figure 5 shows the distributions of the relative particle concentration $\alpha_p/\alpha_{p\infty}$ along the critical streamline and the cylinder surface in the flow plane, while Fig. 6 shows the distributions of local values of the Knudsen number Kn_p in the particle "gas" (in Figs. 5 and 6, *r* is the polar radius)

$$\mathrm{Kn}_p=\lambda_p/L, \qquad \lambda_p=\frac{n_p}{2\nu_p}\sqrt{v_p^2}, \qquad v_p^2=\langle r_i^3 v_i^2\rangle_1/\langle r_i^3\rangle_1$$

For collisionless particle motion, the distribution of $\alpha_p/\alpha_{p\infty}$ (broken curves in Fig. 5) does not depend on $\alpha_{p\infty}$. In Fig. 5*a*, *c*, peaks (I) and (II) correspond to the location in the flow plane of the envelopes of the trajectories of the particles moving without collisions after primary and secondary reflection from the surface [28]. On these envelopes the distribution of $\alpha_p/\alpha_{p\infty}$ has a discontinuity of the second kind [29]. These results may be regarded as corresponding to the limiting case $\alpha_{p\infty} \to 0$.

At r_p=100 μm, the influence of interparticle collisions becomes noticeable for $\alpha_{p\infty}=3\cdot10^{-6}$ (curves *1* in Fig. 5*a*, *b* and Fig. 6). As is clear from Fig. 6, in this case $\mathrm{Kn}_p > 1$ over the entire flow region. With increase in the particle volume fraction, as might be expected, the role of interparticle collisions increases. For $\alpha_{p\infty}=10^{-5}$ (Fig. 3*a*, *b* and curves *2* in Fig. 5*a*, *b* and Fig. 6) peaks (I) and (II) become smaller, whereas the relative admixture concentration increases near the stagnation point. In Fig. 3*a*, the envelope of trajectories of the particles after primary rebound from the cylinder surface is still well pronounced, whereas the envelope of particle trajectories after secondary rebound is completely "diffused". When $\alpha_{p\infty}=3\cdot10^{-5}$ (Fig. 3*c*, *d* and curves *3* in Fig. 5*a*, *b* and Fig. 6), the concentration increases monotonously toward the cylinder surface. As follows from Fig. 6 (curves *2* and *3*), in these two cases in the near-wall layer an admixture flow regime

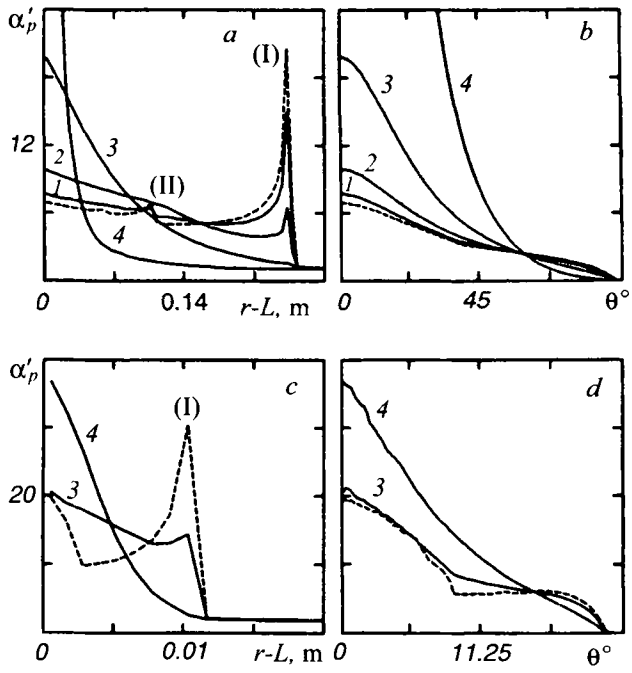

Fig. 5. Distributions of relative admixture volume concentration $\alpha_p/\alpha_{p\infty}$ along the critical streamline (*a*, *c*) and along the cylinder surface (*b*, *d*):*a*, *b* — r_p=100 μm; *c*, *d* — r_p=10 μm. The broken curves correspond to the collisionless admixture; *1* — $\alpha_{p\infty}=3\cdot10^{-6}$; *2* — 10^{-5}; *3* — $3\cdot10^{-5}$; *4* — 10^{-4}.

transitional in Kn_p is realized. For $\alpha_{p\infty}=10^{-4}$ (Fig. 3*e*, *f* and curves *4* in Fig. 5*a*, *b* and Fig. 6), the near-wall layer in which interparticle collisions are important is much thinner. In this case, the particle concentration α_p on the body surface is hundreds of times greater than $\alpha_{p\infty}$ and, near the surface in the stagnation point neighborhood, the admixture flow regime is continuous ($Kn_p < 10^{-2}$). However, with increase in the distance from the surface the value of Kn_p grows rapidly: at $r - L\approx0.05$ m the admixture flow is already transitional and, at $r - L\approx0.25$ m, the particle motion is almost collisionless. In the near-wall layer, the Knudsen number Kn_p also increases rapidly downstream along the cylinder surface and, for $\theta \geq 60°$, the admixture motion is collisionless.

Thus, for given initial parameters of the problem, as the admixture concentration $\alpha_{p\infty}$ varies from $3\cdot10^{-6}$ to 10^{-4}, three qualitatively different particle "gas" flow regimes, namely, collisionless, transitional, and continuous, are realized near the surface. As our results show, with increase in $\alpha_{p\infty}$, starting from $\alpha_{p\infty}\approx3\cdot10^{-6}$, the collisions between the impinging and rebounding particles noticeably affect the admixture concentration field, which confirms the a priori estimates of [1].

For essentially smaller particles (r_p=10 μm), the global admixture flow pattern is qualitatively different from that for particles with r_p=100 μm (see Figs. 3 and 4). As follows from the calculations, for r_p=10 μm even when $\alpha_{p\infty}=3\cdot10^{-5}$ the admixture density field (Fig. 4*b*) almost coincides with that for collisionless flow. This is because the role of interparticle collisions is important only in the near-wall layer which, for r_p=10 μm, is very thin, approximately 15 times thinner than for r_p=100 mm. At the same time, the transverse structures of the near-wall layers for r_p=100 μm and 10 μm are qualitatively identical (see curves *4* in Fig. 5*a*, *c*). It is quite possible that, in both cases, with increase in $\alpha_{p\infty}$ the near-wall layer can be regarded as an infinitely thin "sheet" [30] with a finite dispersed-phase surface density.

Figure 7 shows the distributions of the dispersed particles over translational v_x and rotational ω_z velocities at the stagnation point for various free-stream admixture concentrations. Formally, the distribution function $f_\beta(\mathbf{r}, \beta, t)$ of a certain parameter β can be represented as a ratio of functionals of the type (2.6):

$$f_\beta(\boldsymbol{r}, \beta, t)=\frac{N}{n(\mathrm{r}, t)}\int\delta_1(\beta - \beta_1)F_1(\mathrm{r}, \mathrm{z}_1, t)\,d\mathrm{z}_1, \qquad \beta=v_x, \omega_z \tag{6.1}$$

In Fig. 7, the distribution functions are plotted to a scale uniquely determined by the normalization condition $\int f_\beta\, d\beta = 1$ and the units of measurement of the quantities v_x and ω_z.

For collisionless admixture motion, the particle trajectories may intersect. As a result, in the collisionless region the distribution functions (6.1) take the form of linear combinations of one-dimensional δ-functions. The interparticle collisions

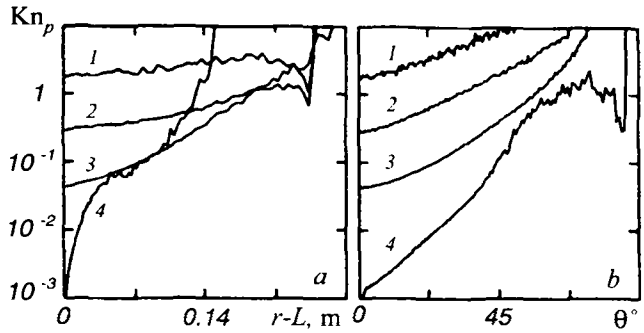

Fig. 6. Distribution of knudsen number Kn_p in the particle "gas" along the critical streamline (*a*) and along the cylinder surface (*b*). r_p=100 μm; *1* — $\alpha_{p\infty}=3\cdot10^{-6}$, *2* — $3\cdot10^{-5}$, *3* — $3\cdot10^{-5}$, *4* — 10^{-4}.

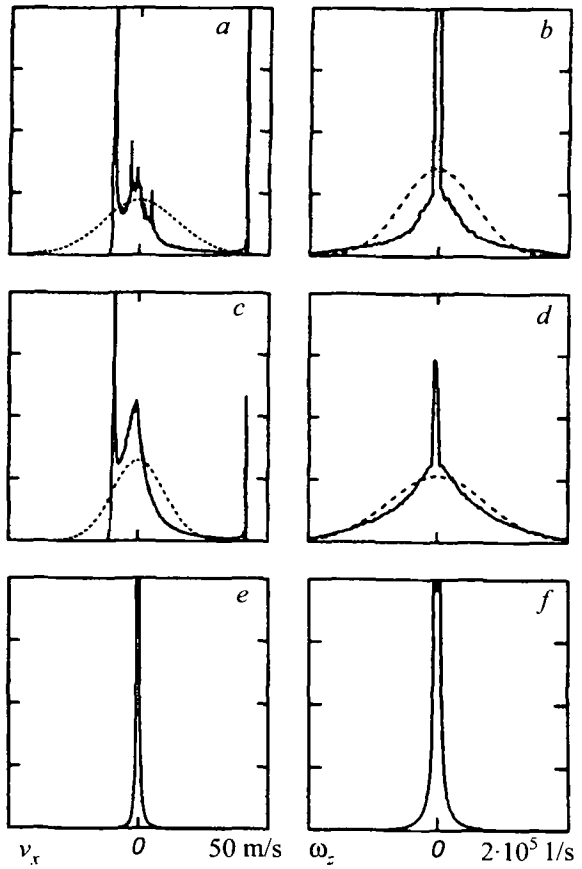

Fig. 7. Distribution functions of the component v_x (*a*, *c*, *e*) of the translational velocity and the component ω_z (*b*, *d*, *f*) of the rotational velocity of the particles at the stagnation point. r_p=100 μm; *a*, *b* — $\alpha_{p\infty}=10^{-5}$; *c*, *d* — $3\cdot10^{-5}$; *e*, *f* — 10^{-4}. The broken curves correspond to the Maxwellian distribution.

result in distribution function "diffusion". However, on the range of $\alpha_{p\infty}$ considered an appreciable fraction of the particles does not collide with other particles. The "peaks" on the graphs of the distribution functions correspond to these regularly moving particles. It is clear from Fig. 7 that when $\alpha_{p\infty} \le 3\cdot10^{-5}$ the fraction of regularly moving particles in the stagnation point neighborhood is fairly large. In this case, the distribution functions differ considerably from the Maxwell distributions [16] (broken curves) with the same means and dispersions.

Summary. It is shown numerically that in dusty-gas flow past a blunt body interparticle collisions play an important role in forming the admixture flow pattern even at very small free-stream particle concentrations. For coarse and fine particles, the global flow patterns are qualitatively different. For coarse particles, the flow pattern varies sharply with the free-stream particle concentration $\alpha_{p\infty}$. As $\alpha_{p\infty}$ increases, interparticle collisions lead to the formation of a near-wall layer of high particle concentration, which is absent from the collisionless admixture model. At the same time, for fine particles the flow pattern varies only slightly with increase in $\alpha_{p\infty}$ and almost everywhere, except for a very thin layer in which interparticle collisions are important, coincides with that for the collisionless admixture.

The work received financial support from the Russian Foundation for Basic Research (projects 96-01-01467 and 99-01-00674).

REFERENCES

1. Yu. M. Tsirkunov, "Admixture flow modeling in problems of two-phase aerodynamics," *Modeling in Mechanics*, 7, No. 2, ITPM SO RAS, Novosibirsk (1993), p. 151–193.
2. S. K. Matveev, "Mathematical description of a gas-particle flow past bodies with account for the reflected particle effect," In: *Motion of a Compressible Fluid and Non-Homogeneous Media* [in Russian], Izd. LGU, Leningrad (1982), p. 189–201.
3. A. Kitron, T. Elpirin, and A. Tamir, "Monte-Carlo analysis of wall erosion and direct contact heat transfer by impinging two-phase jet," *J. Thermophys. Heat Transfer*, **3**, No. 2, 112 (1989).
4. V. A. Tsibarov, *Kinetic Method in Gas-Particle Mixture Theory* [in Russian], Izd. S-Pb University, Sankt-Peterburg (1997).
5. S. K. Matveev, "Model of a particle gas with account for inelastic collisions," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 6, 12 (1983).
6. Yu. E. Gorbachev and V. Yu. Kruglov, "Calculation of the parameters of a two-phase mixture flow past a sphere with account for interparticle collisions," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 4, 93 (1989).
7. A. Volkov and Yu. Tsirkunov, "Direct simulation Monte-Carlo modeling of two-phase gas-solid particle flows with inelastic particle-particle collisions," In: *Proc. 3d ECCOMAS Computational Fluid Dyn. Conf. Paris, 1996*, Wiley, Urichester (1996), p. 662–668.
8. R. I. Nigmatulin, *Fundamentals of the Mechanics of Heterogeneous Media* [in Russian], Nauka, Moscow (1978).
9. J. Happel and H. Brenner, *Low Reynolds Number Hydrodynamics*, Prentice-Hall, Englewood Ceiffs (1965).
10. E. Yokuda and C. T. Crowe, "Effect of Reynolds number and spacing on the dispersion of particles in self-induced turbulence," *In: Turbulence Modification in Multiphase Flows*, **110**, ASME, New York (1991), p. 7–13.
11. V. P. Myasnikov, "Statistical model of the mechanical behavior of disperse systems," In: *Mechanics of Multicomponent Media in Engineering Processes* [in Russian], Nauka, Moscow (1978), p. 70–101.
12. Yu. L. Klimontovich, "Dissipative equations for multi-particle distribution functions," *Uspekhi Fiz. Nauk*, **139**, No. 4, 689 (1983).
13. V. Ya. Rudyak, "Basic kinetic equation of a rarefied gas," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 6, 154 (1989).
14. J. E. McCune, G. Sandri, and E. A. Frieman, "A new approach to nonequilibrium statistical mechanics of gases," In: *Rarefied Gas Dynamics, V.1*, Acad. Press, New York (1963).
15. I. M. Vasenin, V. A. Arkhipov, V. G. Butov, et al., *Gasdynamics of Two-Phase Nozzle Flows* [in Russian], Izd. Tomsk Univ., Tomsk (1986).
16. M. N. Kogan, *Rarefied Gas Dynamics* [in Russian], Nauka, Moscow (1967).
17. G. L. Babukha and A. A. Shraiber, *Interaction of Polydisperse Particles in Two-Phase Flows* [in Russian], Naukova Dumka, Kiev (1972).
18. V. A. Lashkov, "On the experimental determination of particle velocity restitution coefficients for surface impact in a gas-particle flow," *Inzh. Fiz. Zh.*, **60**, No. 2, 197 (1991).
19. Yu. M. Tsirkunov, S. V. Panfilov, and M. B. Klychnikov, "Semi-empirical model of impact interaction between a dispersed particle and a surface in gas-particle flow," *Inzh. Fiz. Zh.*, **67**, No. 5–6, 379 (1994).
20. S. A. Morsi and A. J. Alexander, "An investigation of particle trajectories in two-phase flow systems," *J. Fluid Mech.*, **55**, Pt. 2, 193 (1972).
21. S. I. Rubinow and J. B. Keller, "The transverse force on a spinning sphere moving in viscous fluid," *J. Fluid Mech.*, **11**, Pt. 3, 447 (1961).
22. V. A. Naumov, A. D. Solomenko, and V. P. Yatsenko, "Influence of the Magnus force on the motion of a rigid spherical body at high angular velocity," *Inzh. Fiz. Zh.*, **65**, No. 3, 287 (1993).
23. S. C. R. Dennis, S. N. Singh, and D. B. Ingham, "The steady flow due to a rotating sphere at low and moderate Reynolds numbers," *J. Fluid. Mech.*, **101**, Pt. 2, 257 (1980).
24. L. E. Sternin, B. N. Maslov, A. A. Shraiber, and A. A. Podvysotskii, *Two-Phase Mono- and Poly-Disperse Gas-Particle Flows* [in Russian], Mashinostroenie, Moscow (1980).
25. G. A. Bird, *Molecular Gas Dynamics and the Direct Simulation of Gas Flows*, Clarendon Press, Oxford (1994).
26. M. S. Ivanov and S. V. Rogazinskii, "Comparative analysis of algorithms of the direct statistical modeling method in rarefied- gas dynamics," *Zh. Vych. Mat. Mekh.*, **28**, No. 7, 1058 (1988).
27. M. S. Ivanov and S. V. Rogazinskii, *Efficient Schemes for Statistical Modeling of Spatially Nonuniform Rarefied-Gas Flows* [in Russian], Preprint No. 29–88, ITPM, Novosibirsk (1988).
28. Yu. M. Tsirkunov, A. N. Volkov, and S. V. Panfilov, "Motion of solid admixture particles and surface erosion in a dilute gas-particle flow past bodies," *In: Proc. 13-th Session Intern. School on Models in Continuum Mech. St. Petersburg, 1995* [in Russian], Izd. SPb. Univ., St. Petersburg (1996), p. 109–116.
29. A. N. Osiptsov and E. G. Shapiro, "Dusty-gas flow past a sphere at high supersonic velocity," *In: Investigation of Gasdynamics and Heat Transfer in Complex Homogeneous and Multiphase Flows* [in Russian], Izd. MSU, Moscow (1990), p. 89–105.
30. A. N. Kraiko and S. M. Sulaimanova, "Two-fluid flows of gas-particle mixtures with "sheets" and "filaments" formed in the flow past impenetrable surfaces," *Prikl. Mat. Mekh.* **47**, No. 4, 619 (1983).