OPTIMUM WING SHAPING AND GUST LOAD ALLEVIATION OF HIGHLY FLEXIBLE AIRCRAFT WITH FINITE ACTUATIONS

by

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A THESIS

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ABSTRACT

The idea of improved flight performance is a constant goal within the aircraft design industry. In order to have aircraft which can fly further and for longer durations improved aerodynamic efficiency is required. Traditionally this is achieved through the use of discrete control elements such as flaps and slats. These mechanisms have a useful purpose in instances such as take off and landing, but are not often useful in other flight conditions because they tend to generate large amounts of drag. Recent research has shown that the potential for a continuously deformable wing is desired to effectively improve flight performance at any given flight condition. One example of this technology is NASA’s Variable Camber Continuous Trailing Edge Flap (VCCTEF) which creates a trailing edge for an aircraft wing which can change the camber of individual sections without creating any discontinuities which generate drag. This application deals with small scale deformations (camber change) which can be improved to dealing with large scale deformations (bending, torsion, etc.) through the use of flexible structures and actuator systems. The first step in utilizing these large scale deformations to improve flight performance is to determine what wing geometries produce the most efficient performance. One method of determining this is to utilize an aeroelastic optimization process to define the wing geometry.

Exploration of this optimization requires a definition of improved flight performance. The work expressed within this project used a reduction in drag as a measure of improved flight performance. This was chosen because if one considers an electric aircraft its range and endurance can be improved by reducing the drag experienced by the aircraft. The optimization was further improved when additional objectives were considered. The control
cost required for these geometries gave insight into how much energy is required to gain the energy savings by increasing efficiency. Additionally some wing geometries were shown to produce better results at reducing the effects of wind gusts. After these optimizations were defined, an additional optimization was constructed to determine the best placement and number of actuators used to generate these wing geometries. Moving forward, the optimization will be applied over a range of velocities which will be used to develop a linear parameter varying controller. This controller will be designed to seamlessly transition between the optimum wing geometries at varying flight conditions.
DEDICATION

To my family and friends
**LIST OF SYMBOLS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>Centrifugal acceleration, (m/s^2)</td>
</tr>
<tr>
<td>(B)</td>
<td>Body-fixed frame</td>
</tr>
<tr>
<td>(b_c)</td>
<td>Semi-chord of airfoil, (m)</td>
</tr>
<tr>
<td>(\mathbf{B}^F, \mathbf{B}^M, \mathbf{N}^g)</td>
<td>Influence matrices for aerodynamic force, moment, and gravity force</td>
</tr>
<tr>
<td>(\mathbf{B}_F, \mathbf{B}_B)</td>
<td>Components of influence matrix for (\mathbf{u})</td>
</tr>
<tr>
<td>(\mathbf{B}_u, \mathbf{B}_u^m)</td>
<td>Influence matrices in control loads</td>
</tr>
<tr>
<td>(\mathbf{C}<em>{FF}, \mathbf{C}</em>{FB}, \mathbf{C}<em>{BF}, \mathbf{C}</em>{BB})</td>
<td>Components of generalized damping matrix</td>
</tr>
<tr>
<td>(\mathbf{C}^{GB})</td>
<td>Rotation matrix from body frame to global frame</td>
</tr>
<tr>
<td>(C_1, C_2, C_3, C_4)</td>
<td>Optimization Constraints</td>
</tr>
<tr>
<td>(d)</td>
<td>Distance of midchord in front of beam reference axis, (m)</td>
</tr>
<tr>
<td>(E_{max}, R_{max})</td>
<td>Maximum endurance and range of aircraft</td>
</tr>
<tr>
<td>(\mathbf{F}^a, \mathbf{M}^e)</td>
<td>Aerodynamic force and moment on wing sections</td>
</tr>
<tr>
<td>(\mathbf{F}^{pt}_u, \mathbf{M}^{pt}_u)</td>
<td>Complete points loads due to (\mathbf{u})</td>
</tr>
<tr>
<td>(\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3)</td>
<td>Matrices for inflow states differential equitation</td>
</tr>
<tr>
<td>(G)</td>
<td>Global or inertial frame</td>
</tr>
<tr>
<td>(g)</td>
<td>Gravitational acceleration vector, (m/s^2)</td>
</tr>
<tr>
<td>(J)</td>
<td>Jacobian matrices relating independent and dependent variables</td>
</tr>
<tr>
<td>(\mathbf{K}_{FF})</td>
<td>Generalized stiffness matrix</td>
</tr>
<tr>
<td>(L, D, W)</td>
<td>Total lift, drag, and weight of aircraft, (N)</td>
</tr>
<tr>
<td>(l_{mc}, m_{mc}, d_{mc})</td>
<td>Aerodynamic lift, moment, and drag in local aerodynamic frame about midchord</td>
</tr>
<tr>
<td>(\mathbf{M}<em>{FF}, \mathbf{M}</em>{FB}, \mathbf{M}<em>{BF}, \mathbf{M}</em>{BB})</td>
<td>Components of generalized inertia matrix</td>
</tr>
<tr>
<td>(M_y^g)</td>
<td>Gust-induced aerodynamic bending moment, (N \cdot m)</td>
</tr>
</tbody>
</table>
\( P_B \)  
Inertial rigid-body position of aircraft, \( m \)

\( Q \)  
Tuning matrix for control cost

\( q \)  
Trim or design variables

\( R \)  
Radius of turning path, \( m \)

\( R_F, R_B \)  
Flexible and rigid-body components of generalized load vector

\( R^u_F, R^s_B \)  
Generalized loads due to \( u \)

\( r_F, r_B \)  
Residuals of equilibrium equation

\( T \)  
Thrust force vector, \( N \)

\( U_c \)  
Control cost

\( U_\infty \)  
Flight speed, \( m/s \)

\( u \)  
Distributed wing shaping control force vector

\( w \)  
Wing node-fixed local frame

\( w_g \)  
Gust velocity, \( m/s \)

\( \dot{y}, \dot{z}, \ddot{z}, \dot{\alpha}, \ddot{\alpha} \)  
Airfoil motion variables in local aerodynamic frame

\( \alpha_B \)  
Aircraft pitching angle, rad

\( \alpha_g \)  
Gust-induced angle of attack, rad

\( \beta \)  
Rigid-body velocity of aircraft, \( m/s \)

\( \varepsilon \)  
Complete strain vector of aircraft

\( \varepsilon^0 \)  
Initial strain of aircraft

\( \varepsilon_e (\varepsilon_x, \kappa_x, \kappa_y, \kappa_z) \)  
Elemental strain vector and its components

\( \zeta \)  
Quaternion

\( \eta \)  
Magnitude of mode shapes

\( \lambda \)  
Inflow states for unsteady aerodynamics

\( \lambda_0 \)  
Induced velocity due to wake, \( m/s \)

\( \xi_1, \xi_2 \)  
Tuning parameters in multi-objective optimizations

\( \rho_\infty \)  
Air density, \( kg/m^3 \)

\( \Phi \)  
Linear mode shape of aircraft
\( \varphi_B \)  \hspace{2cm} \text{Aircraft bank angle, } \text{rad}
ACKNOWLEDGMENTS

I want to begin by thanking my advisor, Dr. Weihua Su. From the time I began working with him as an undergraduate student through the completion of my Master’s program, he has been a helpful and encouraging mentor. He has shown me the hard work involved in performing academic research while also letting me understand the sense of accomplishment that comes from completing a difficult problem. I would also like to acknowledge the other members of my committee, Dr. Jinwei Shen and Dr. David MacPhee for their help through this process.

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1 INTRODUCTION

1.1 Background and Motivation

In mankind’s quest for flight, engineers and adventurers have turned to birds for inspiration. These creatures soar through the sky for long periods of time with limited effort. As technology improves, the need for aircraft that can fly for long periods of time and over longer distances while limiting the fuel required has risen. The designs for these aircraft have settled into a pattern of large wingspan and high aspect ratio wings. This improves the aerodynamic efficiency of the aircraft by increasing the lift to drag ratio which improves the flight performance. Additionally, these long slender wings are being constructed using composite materials and other design techniques which reduces the weight of the aircraft to further improve its performance. The drawback of these design is wings which are more flexible and subject to aeroelastic phenomenon such as flutter.

It can be considered, that the flexible nature of the aircraft wings could be used as a benefit for further improvement of flight performance. Nature has shown that there is not one wing configuration which is superior in all flight conditions. Birds bend and flap their wings in different patterns depending upon the actions they are performing. A bird will change the shape of its wings if it is landing, taking off, chasing prey, or riding wind currents to sustain long flight times. Similar systems could be built within an aircraft to actuate the shape of the aircraft wings to tailor the wing geometry to the current flight conditions to optimize performance. It is natural to assume that the optimum wing shape is different for an aircraft at cruise than when it is taking off or landing. This concept is already used as modern aircraft deploy flaps and slats during takeoff and landing to increase lift at low speeds. These concepts are not effective when an aircraft is in cruising
flight as discrete surfaces such as flaps being deployed greatly increase the drag experienced by the wing. This then leads to the idea that there is a need for a fully deformable aircraft wing which could be used to improve flight performance throughout the entire flight envelope of an aircraft.

The improvement of aircraft operation efficiency needs to be considered over the whole flight plan, instead of a single point in the flight envelope, since the flight condition varies in a flight mission. Therefore, it is natural to employ morphing wing designs so that the aircraft can be made adaptive to different flight conditions and missions. At the advent of recent development in advanced composites as well as sensor and actuator technologies, in-flight adaptive wing/aircraft morphing is now becoming a tangible goal. Traditionally, the discrete control surfaces were used to re-distribute the aerodynamic loads along the wing span during the flight, so as to tailor the aircraft performance. However, the deflection of discrete control surfaces may increase the aerodynamic drag. An effective alternative is to introduce conformal wing/airfoil shape changes for the aerodynamic load control. In addition, the flexibility associated to the morphing wing structures may be pro-actively utilized to improve the aircraft performance. The active aeroelastic tailoring techniques would allow aircraft designers to take advantage of the wing flexibility to create the desired wing load distribution according to the mission requirement, so as to improve overall aircraft operating efficiency and performance, without using the traditional discrete control surfaces. The utilization of these concepts is predicated upon the optimum shape being known and a control system which is able to produce this wing shape.

1.2 Literature Survey of Current Work

Research into aircraft performance is not a new subject and has been studied at various levels for some time. A brief overview of some related works completed recently is presented here.
1.2.1 Aerodynamic Optimization

The question of determining the optimum wing shape has been studied in depth. Recently, Chen et al. [1] studied the effects of various trim conditions on the aerodynamic shape optimization of the common research model wing-body-tail configuration. Using a free form distribution for the wing geometry coupled with a RANS solver for the aerodynamics, they studied the impact of a trim constraint on the optimization process. Through a series of optimizations utilizing the trim conditions at varying points in the design process, they concluded that considering the trim during optimization yields the best performance. In a similar study, Lyu and Martins [2] performed an aerodynamic optimization of the trailing edge of a wing. Their optimization showed that drag reductions could be seen with shape optimization of either the entire wing or just the trailing edge. Taking the optimization a step further requires the development of realistic system capable of producing the optimum shape that is suitable for a given flight condition. This concept was highlighted in Nguyen et al. [3], where the design of the Variable Camber Continuous Trailing Edge Flap (VCCTEF) is introduced. In addition, an optimization is performed to determine the deflection angles required throughout the trailing edge to improve the flight performance.

1.2.2 Morphing Technologies

Many wing morphing technologies have been developed over the years as the materials and fabrication methodologies have advanced. In Nguyen et al. [4] the principles of aerodynamic shape optimization and morphing wing structures were explored. The optimization process led to the development of the VCCTEF, which was a novel concept for improving aircraft performance by drag reduction. A further study of the VCCTEF wing model was conducted by Nguyen and Ting [5], where they performed a flutter analysis of the mission-adaptive wing. The methodology included a vortex-lattice aerodynamic model coupled with a finite element structural dynamic model. Urnes et al. [6] provided an updated review of the development, design, and testing of the VCCTEF.
project. Under the support of the U.S. Air Force Research Laboratory, FlexSys, Inc. developed the Mission Adaptive Compliant Wing (MAC-Wing) to test and evaluate its performance. The adaptive trailing edge flap technology was combined with a natural laminar flow airfoil and tested on the Scaled Composites White Knight aircraft. The testing suggested fuel saving, weight reduction, and improved control authority [7, 8]. In an effort to move from an adaptable trailing edge to a completely adaptable wing structure, the Cellular Composite Active Twist Wing was designed and tested in Cramer et al. [9], showing promising results. A scaled airplane model was built, which incorporated active twist wings and was compared to a similar rigid model with traditional control surfaces in wind tunnel tests. The active twist wings showed similar capabilities for symmetric and asymmetric movements as well as added benefits in stall mitigation. An overview of the process used to design the composite lattice-based cellular structures for active wing shaping was presented in Jenett et al. [10], in which they presented a detailed approach for designing a low density and highly compliant structure.

1.2.3 Control Reduction

The addition of a morphing aircraft wing would require the addition of various actuators and sensors within the wing structure to adjust the wing geometry to the desired shape. If the goal of the geometry changes is to morph the entire wing structure it is natural to assume that a fully distributed control scheme is required so that any portion of the wing structure can be bent or twisted to the appropriate position. This however would be very impractical from a design stand point as it would require the installations of more actuators than there is room for and would require a much higher control cost to run this large network of actuators. This leads to research in the reduction of the number of the actuator/sensors applied as well as the concept of optimal placement of the actuators/sensors. With the idea that determining the location of actuators and sensors on actively controlled structures is a very important issue, Bruant et al. [11] developed a formulation to predict the optimum location based upon minimizing the energy required
and maximizing the output of the system. The methodology presented can be used effectively to determine the location of the sensor or actuator, but is however limited in determining the required number of actuators required for a system. In Aldraihem et al [12] the optimum location and size of piezoelectric actuators/sensors was explored using a modal cost and controllability index. This approach gives insight into effectiveness individual actuators/sensors have on the system as well as providing a penalty term to the cost function to reduce the number of impractical designs from the optimization process.

1.3 Thesis Outline

As optimization processes and morphing technology have improved, there is a need for a complete system, in which a controller will actuate the wing members to the desired optimum shape throughout the entire flight envelope and perform the required maneuver and vibration control during the flight. Most current optimization schemes utilize a CFD aerodynamic model coupled with discrete structural points as design variables. These methods produce promising results, but when considered over an entire flight plan could be a very time consuming process. Additionally, these methods generally consider the planform shape of the wing rather than the wing bending and torsions associated with highly flexible, large aspect ratio wing members. Recent developments of morphing technologies such as the Cellular Composite Active Twist Wing take advantage of the flexible nature of high aspect ratio wings. Therefore, it is natural to develop an optimization scheme that mainly considers the bending and torsion of the high aspect ratio wings. This concept was utilized in Su et al. [13], which utilized a modal based optimization approach in determining the best feasible wing shape (wing bending and torsion deformations) of a highly flexible aircraft at any given flight scenario. In this paper, this process will be used going forward to develop a wing shape control algorithm with defined distributed control loads. The optimization process will generate the specific wing shape needed to guarantee the optimum performance and ride quality over the entire flight envelope of an aircraft.
Once the optimization is defined, additional objective functions will be explored. The optimization will attempt to limit the control cost in an effort to make the system more efficient. Gust load alleviation will also be considered. The root bending moment caused by a wind gust will be minimized to improve the ride quality of the aircraft. Both of these parameters will also be explored in the form of multi-objective functions to better understand the trade-offs between various objective. Additionally a study which aims at reducing the required number of actuators for the geometry changes will be explored.
In order to explore the best possible wing geometries for a specified aircraft, the analytical tools and optimization processes must first be defined. The optimization process will focus on improving flight performance through use of a modal based optimization scheme which takes advantage of a flexible aircrafts inherent mode shapes. This methodology will be explored in greater detail in the following sections. The flight performance will be evaluated using an aeroelastic formulation capable of analyzing a highly flexible aircraft. The formulation is described below.

2.1 Aeroelastic Formulation

The solution of the coupled aeroelastic and flight dynamic equations will be obtained through the use of a strain-based geometrically nonlinear beam formulation described by Su et al in [14, 15, 16, 17]. A brief overview of the methodology is outlined below.

2.1.1 System Frames

The formulation begins by defining a global inertial frame, \( G \), which is shown in Figure 2.1, within this frame a body frame, \( B \), which defines the vehicle orientation and position is also defined. Within \( B \), \( B_x(t) \) points to the right wing, \( B_y(t) \) points towards the front of the aircraft, and \( B_z(t) \) is the cross product of \( B_x(t) \) and \( B_y(t) \) and thus points vertically up from the aircraft. With this frame defined the position and orientation of the
Figure 2.1: Global and body frames defining the rigid-body motion of the aircraft

Figure 2.2: Flexible lifting-surfaces frames within the body frame
aircraft can be defined along with their respective time derivatives as

\[ b = \begin{pmatrix} p_B \\ \theta_B \end{pmatrix} \]

\[ \dot{b} = \beta = \begin{pmatrix} \dot{p}_B \\ \dot{\theta}_B \end{pmatrix} = \begin{pmatrix} v_B \\ \omega_B \end{pmatrix} \]

\[ \ddot{b} = \ddot{\beta} = \begin{pmatrix} \ddot{p}_B \\ \ddot{\theta}_B \end{pmatrix} = \begin{pmatrix} \ddot{v}_B \\ \ddot{\omega}_B \end{pmatrix} \tag{2.1} \]

Where \( p_B \) and \( \theta_B \) are the body position and orientation of the aircraft. These are both resolved within the body frame \( B \). It is of note that the origin of this body frame is arbitrary and does not need coincide with the center of gravity of the vehicle.

For highly flexible vehicle the geometry of the wings may be utilized to model the wing sections as beams. Within the body frame, a localized beam, frame, \( w \), is built within each element of the wing along the beam reference line shown in Figure 2.2. This frame allows for the definition of the nodal position and orientation of the flexible members. The \( w \) frame is defined by \( w(s) \) which points along the beam reference line, \( w(s) \) points towards the leading edge, and \( w(s) \) is the normal to the wing surface. The curvilinear coordinate \( s \) provides the nodal location within the body frame.

### 2.1.2 Elements with Constant Strain

As seen in Su and Cesnik [15] a nonlinear beam element can be used to model the elastic deformation of slender beams. Strain degrees or curvatures of the beam reference line are considered as the independent variables in the solution. The strain-based formulation allows simple shape functions such as constant functions to be defined for each element. This allows the strain vector of an element to be described as

\[ \varepsilon^T_e = \begin{pmatrix} \varepsilon_x & \kappa_x & \kappa_y & \kappa_z \end{pmatrix} \tag{2.2} \]
where $\varepsilon_x$ is the extensional strain, $\kappa_x$ is the twist of the beam reference line, $\kappa_y$ is the bending about the local $w_y$ axis, and $\kappa_z$ is the bending about the local $w_z$ axis. From this the total strain vector for the entire aircraft can be obtained by arranging the strain vectors of each element into the global strain vector as shown here

$$\varepsilon^T = \{\varepsilon_{T1}^T \varepsilon_{T2}^T \varepsilon_{T3}^T \ldots \varepsilon_{TN}^T\}$$

(2.3)

where $\varepsilon_{ei}$ denotes the strain of the $i$th element. This constant strain distribution over each element allows for complex geometrically nonlinear deformations to be represented.

### 2.1.3 Equations of Motion

The principle of virtual work or Hamilton’s principle is used to derive the equation of motion for the system. The total virtual work done on a beam is found by integrating the products of all internal and external forces and the corresponding virtual displacements over the volume, given as

$$\delta W = \int_V \delta u^T(x, y, z) f(x, y, z) dV$$

(2.4)

Where $f$ represents the general forces acting on the volume. This includes internal elastic forces, gravity forces, inertial forces, external distributed forces and moments, point moments and forces among others. As outlined in Su and Cesnik [15] the derivation of the equations of motions results in the following

$$M_{FF}(\varepsilon)\ddot{\varepsilon} + M_{FB}(\varepsilon)\dot{\beta} + C_{FF}(\dot{\varepsilon}, \varepsilon, \beta)\dot{\varepsilon} + C_{FB}(\dot{\varepsilon}, \varepsilon, \beta)\dot{\beta} + K_{FF}\varepsilon = R_F(\ddot{\varepsilon}, \dot{\varepsilon}, \varepsilon, \dot{\beta}, \beta, \lambda, \zeta, T, u)$$

$$M_{BF}(\varepsilon)\ddot{\varepsilon} + M_{BB}(\varepsilon)\dot{\beta} + C_{BF}(\dot{\varepsilon}, \varepsilon, \beta)\dot{\varepsilon} + C_{BB}(\dot{\varepsilon}, \varepsilon, \beta)\dot{\beta} = R_B(\ddot{\varepsilon}, \dot{\varepsilon}, \dot{\beta}, \beta, \lambda, \zeta, T, u)$$

(2.5)
The generalized force vector is

\[
\begin{bmatrix}
R_F \\
R_B
\end{bmatrix} = \begin{bmatrix}
K_{FF} \varepsilon_0 \\
0
\end{bmatrix} + \begin{bmatrix}
J^T_{\theta e} \\
J^T_{\theta b}
\end{bmatrix} B^F F^a + \begin{bmatrix}
J^T_{\theta e} \\
J^T_{\theta b}
\end{bmatrix} B^M M^a \\
+ \begin{bmatrix}
J^T_{h e} \\
J^T_{h b}
\end{bmatrix} N^g g + \begin{bmatrix}
J^T_{p e} \\
J^T_{p b}
\end{bmatrix} T + \begin{bmatrix}
\bar{B}_F \\
\bar{B}_B
\end{bmatrix} u
\]  

(2.6)

Where the first term represents any pre-strain associated within the aircraft, the second and third terms involve the aerodynamic forces and moments respectively, the fourth term is the gravity force, and the fifth term is the thrust force associated with any engines mounted to the vehicle. The final term will be discussed in greater detail in section 2.2, as it relates to the required control force to maintain a specific wing geometry. One may also define the rigid body propagation equations as seen in equation 2.7. This allows the flight dynamics equations to be more easily solved.

\[
\dot{\zeta} = -\frac{1}{2} \Omega_{\zeta}(\beta) \zeta
\]

\[
\dot{P}_B = \begin{bmatrix}
C^{GB}(\zeta) & 0
\end{bmatrix} \beta
\]  

(2.7)

### 2.1.4 Unsteady Aerodynamics

The aerodynamic loads expressed in equation 2.6 are calculated based upon the 2-D finite-state inflow theory[18]. At a given station along the wing, the aerodynamic lift, moment and drag are expressed as

\[
l_{mc} = \pi \rho_{\infty} b_c^2 \left(-\ddot{z} + \dot{y}\dot{\alpha} - d\ddot{\alpha}\right) + 2\pi \rho_{\infty} b_c \dot{y}^2 \left[-\frac{\ddot{z}}{\dot{y}} + \left(\frac{1}{2} b_c - d\right) \frac{\dot{\alpha}}{\dot{y}} - \frac{\lambda_0}{\dot{y}}\right]
\]

\[
m_{mc} = \pi \rho_{\infty} b_c^2 \left(-\frac{1}{8} b_c^2 \ddot{\alpha} - \dot{y}\dot{z} - d\ddot{\alpha} - \dot{y} \lambda_0\right)
\]

\[
d_{mc} = -2\pi \rho_{\infty} b_c \left(\dot{z}^2 + d^2 \dot{\alpha}^2 + \lambda_0^2 + 2d \dot{z} \dot{\alpha} + 2\dot{z} \lambda_0 + 2d \dot{\alpha} \lambda_0\right)
\]  

(2.8)
where $b_c$ is the semi-chord, and $d$ is the distance of the mid-chord in front of the reference axis. The quantity $-\dot{z}/\dot{y}$ is the angle of attack that consists of contribution from both the pitching angle and the unsteady plunging motion of the airfoil. The different velocity components are shown in Figure 2.3. Form equation 2.8 that only the induced drag is considered in this current study. The inflow parameter $\lambda_0$ accounts for induced flow due to free vorticity, which is the summation of the inflow states $\lambda$ as described in [18] and given by

$$\dot{\lambda} = F_1 \begin{bmatrix} \dot{\varepsilon} \\ \dot{\beta} \end{bmatrix} + F_2 \begin{bmatrix} \dot{\varepsilon} \\ \beta \end{bmatrix} + F_3 \lambda$$

(2.9)

### 2.2 Fully Distributed Control Scheme

In order to actuate the wing to a given geometry some control scheme is desired. Given that the wing shape will be actuated across the entire span, a scheme which includes full actuation potential is required. This section will describe the formulation of a fully distributed control scheme.
2.2.1 Elemental Forces

As a baseline, a distributed control scheme is developed by assuming every element along the main wing can be actuated. Figure 2.4 shows a generic wing element with applied point force \((u_1)\) and force couples \((ru_2, ru_3\) and \(ru_4)\) on both ends for actuation. The combined loads may independently actuate the extensive, torsional, out-of-plane bending, and in-plane bending deformations of the element. These elemental loads are written as

\[
(F^{pt}_u)_e = \begin{bmatrix} -u_1 & 0 & 0 & 0 & 0 & u_1 & 0 & 0 \end{bmatrix}^T
\]

\[
(M^{pt}_u)_e = \begin{bmatrix} -ru_2 & -ru_3 & -ru_4 & 0 & 0 & ru_2 & ru_3 & ru_4 \end{bmatrix}^T
\]

(2.10)

where the coefficient \(r\) represents the arms of force couples \(u_1, u_2,\) and \(u_3\). Without loss of generality, \(r\) is set to be 1 throughout the studies. Note that there are three nodes defined on each beam element [15]. As no loads are applied at the mid-node of the element for
shaping actuation, the middle three entries of both load vectors \((F^pt_u)_e\) and \((M^pt_u)_e\) are all zeros. Eq. (2.10) is further written into the matrix form of

\[
(F^pt_u)_e = \begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}^T \begin{bmatrix}
u_1 \\
u_2 \\
u_3 \\
u_4 
\end{bmatrix} = (B^f_u)_e u_e
\]

\[
(M^pt_u)_e = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 
\end{bmatrix}^T \begin{bmatrix}
u_1 \\
u_2 \\
u_3 \\
u_4 
\end{bmatrix} = (B^m_u)_e u_e
\]

### 2.2.2 Fully Distributed Control Matrix

Accordingly, the complete control loads are obtained by properly sizing and assembling the elemental matrices in Eq. (2.11), leading to

\[
F^pt_u = B^f_u u \\
M^pt_u = B^m_u u
\]

where \(F^pt_u\) and \(M^pt_u\), as nodal loads along the wing span, can be eventually transformed into the generalized control load by using the Jacobians [15], resulting in

\[
\begin{bmatrix}
R^u_F \\
R^u_B
\end{bmatrix} = \begin{bmatrix}
J^T_{pe} \\
J^T_{pb}
\end{bmatrix} B^f_u + \begin{bmatrix}
J^T_{θε} \\
J^T_{θb}
\end{bmatrix} B^m_u u = \begin{bmatrix}
\bar{B}_F \\
\bar{B}_B
\end{bmatrix} u
\]

which participated in Eq. (2.6) as part of the generalized load for full flexible aircraft.
2.3 Optimization Formulation

With the aeroelastic formulation now fully defined, the optimization problem can be addressed. Typically optimization problems have three parts, the variables which can be updated and changed to find the optimum solution, the objective function which is what is generally minimized, and the constraints which keep the proposed solutions within a feasible realm. This section will highlight the specifics of all of these parts for our specific problem. First a few details regarding the trim of the aircraft will be addressed.

2.3.1 Updated Trim

Whatever wing geometry is chosen to be the optimum condition, must still satisfy the basic trim conditions of the aircraft. Since this optimization is focusing on novel wing shaping concepts, the traditional control surfaces usually used to trim an aircraft will be removed. This means that the new trim variables for this approach include

$$q_{trim} = \begin{bmatrix} \alpha_B & \phi_B & T & u \end{bmatrix}^T$$

(2.14)

Where $u$ which represents the control input described previously will replace the traditional variables such as the elevator angle. When considering the steady level flight case which is what is explored in this paper, the equations of motion (Eq. 2.5) can be simplified to determine the new trim equations. If the transient terms, damping terms, and unsteady aerodynamic terms are removed and the rigid body propagation equations are applied the equations of motion can be rewritten to form our new trim equations

$$K_{FF}\varepsilon - R_F(\alpha_B, \phi_B, T, u, \varepsilon) = 0$$

$$R_B(\alpha_B, \phi_B, T, u, \varepsilon) = 0$$

(2.15)

These equations are a function of the traditional trim variables, $\alpha_B$, $\phi_B$, and $T$ as well as the control force $u$. Also these equation require the strain associated with the current aircraft geometry to be known. This is how one can prescribe a specific geometry to
determine the optimum shape

2.3.2 Modal Description

As previously stated, the trim condition requires a known strain vector which represents the current geometry of the aircraft wings. This could be used as the design space for the optimization process, but the number of variables required is quite large. In an effort to reduce this design space a modal approach is presented to reduce the number of variable required to prescribe a wing geometry. In Su et al [13] a modal approach is described where the wing geometry is represented by a series of truncated modes expressed as

$$\bar{\varepsilon}(s, t) = \sum_{i=1}^{N} \Phi_i(s) \eta_i(t)$$

(2.16)

where $\Phi_i$ are the linear mode shapes of the flexible aircraft and $\eta_i$ the corresponding magnitude of the modes. This approach allows one to use a finite number of flexible modes to search for the optimum wing shape for minimum drag, while maintaining the trim and elastic equilibrium of the aircraft. Replace $\varepsilon$ in Eq. (2.15) by $\bar{\varepsilon}$, one can further write the residual equations as

$$r_F = K_{FF} \bar{\varepsilon} - R_F(\alpha_B, \varphi_B, T, u, \eta_1, \eta_2, \cdots, \eta_N)$$

(2.17)

$$r_B = R_B(\alpha_B, \varphi_B, T, u, \eta_1, \eta_2, \cdots, \eta_N)$$

where the control force $u$ is explicitly solved by enforcing $r_F = 0$ during each iteration of the optimization process. In combination with Eq. (2.6), the control force is then given by

$$u = B^{-1}_F \left( K_{FF} \bar{\varepsilon} - K_{FF} \varepsilon^0 - J^T_{pe} B^a F^a - J^T_{\theta e} B^M M^a - J^T_{he} N^g g - J^T_{pe} T \right)$$

(2.18)

This is the full distributed control load along the wing span to actuate and maintain the desired wing geometry from the optimization solution. A methodology for reducing this scheme will be presented in the numerical studies to follow.
2.3.3 Objective Function

With the aircraft already described using the flexible mode shapes it is natural to utilize the modal magnitudes as design variables in the optimization process. This allows the design space for the optimization to be greatly reduced so that instead of exploring say a large series of defined points on a wing the entire wing shape will be represented by a series of truncated modes as described in 2.16. These along with the traditional trim variables will be used in the optimization to explore the entire subset of possible wing configurations. The set of design variables then becomes

\[ x = \{\alpha_B, \varphi_B, T, \eta_1, \eta_2, \ldots, \eta_N\}^T \]  (2.19)

With the goal of the optimization process being improved flight performance, it is natural to consider a reduction in drag and increase in flight performance. This can be seen from the aircraft range equation where typically one can improve range and endurance by maximizing the lift over drag ratio. However if an electric aircraft is considered, the required lift becomes a constant value so the range and endurance are improved by reduction in drag. Utilizing the aeroelastic formulation described previously, the various aircraft geometries will be investigated to determine the wing shape which maintains the lowest drag. This allows the objective function of the optimization to be described as

\[ \min_{\mathbf{q}} D = D(\mathbf{q}) \]  (2.20)

As the desired results of the optimization change and advance, the optimization can be updated and expanded to include other considerations to improve the flight performance. This will be explored further in the following sections.

2.3.4 Constraints

The final portion of the optimization problem is to define constraints on the model to prevent solutions which are not physically possible. For this optimization problem the
first constraint is that the aircraft must maintain its trim condition which expressed as

\[ C_1 : r_B = 0 \]  \hspace{1cm} (2.21)

In addition, some other variables should be constrained within their search limits, such as the bending curvatures of each wing element or the angle of attack and the required thrust. These are represented as

\[
C_2 : \begin{cases} 
\max |\kappa_x| \leq \kappa_{x\text{lim}} \\
\max |\kappa_y| \leq \kappa_{y\text{lim}} \\
\max |\kappa_z| \leq \kappa_{z\text{lim}} 
\end{cases}
\]  \hspace{1cm} (2.22)

\[
C_3 : \begin{cases} 
|\alpha_B| \leq \alpha_{\text{lim}} \\
0 \leq T \leq T_{\text{lim}} 
\end{cases}
\]  \hspace{1cm} (2.23)

When a maneuver is considered the bank angle can also be constrained as

\[ C_4 : 0 \leq \varphi_B \leq \varphi_{\text{lim}} \]  \hspace{1cm} (2.24)

As the optimization evolves and improves, additional constraints can be considered to further refine the optimization. This will be explored in the following sections.

### 2.3.5 Baseline Optimization Results

In this section, a highly flexible aircraft model is considered for the numerical study. By following Su et al. [13], the vehicle’s geometrical and physical properties are shown in Figure 2.5 and Table 2.1. The aircraft has a wingspan of 32 m and a total mass of 54.5 kg. A list of linear flexible modes, which will be used for finding the optimum wing shaping in the current studies, was also provided in Su et al. [13].

Following the approach proposed in Su et al. [13], the initial condition for design optimization is determined at first. This is achieved by trimming a conventional aircraft with control surfaces, refer to Figure 2.5, for a steady level flight at 20,000 m with a
constant speed of 25 m/s. The resulting trimmed wing shape is shown in Figure 2.6 and trim variables are used as the baseline to perform an optimization for the minimum drag as described by Eq. (2.20), without applying any additional constraints. This process is then repeated, using the previous interim optimum solution as the initial condition for a new optimization process, until the optimum solution converges within a tolerance. During the optimization, only the first seven symmetric modes are included, which was determined to be the converged solution to the optimization in Su et al. [13]. Some details regarding the modes utilized are listed in Table 2.2. The converged solution is compared to the baseline in Table 2.3. The resulting optimum geometry yields a decrease in drag from 59.28 N to 51.39 N, which represents a 13.3% reduction. The table also highlights the magnitude of each mode showing that the first mode is dominant and the third mode is also somewhat significant. The higher order modes are largely unused. This is seen in the resulting optimum wing shape shown in Figure 2.7. The optimum wing shape is mostly flat, which is consistent with the results reported in Su et al. [13]. Since this shape differs so much from the baseline, which is a clear “U” shape as shown in Figure 2.6, it is intuitive that a significant amount of control effort would be required to maintain the shape. More detailed
Table 2.1: Properties of the baseline highly flexible aircraft

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wings</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Span</td>
<td>16</td>
<td>m</td>
</tr>
<tr>
<td>Chord</td>
<td>1</td>
<td>m</td>
</tr>
<tr>
<td>Incidence angle</td>
<td>2</td>
<td>deg</td>
</tr>
<tr>
<td>Sweep angle</td>
<td>0</td>
<td>deg</td>
</tr>
<tr>
<td>Dihedral angle</td>
<td>0</td>
<td>deg</td>
</tr>
<tr>
<td>Beam reference axis (from LE)</td>
<td>50</td>
<td>% chord</td>
</tr>
<tr>
<td>Cross-sectional c.g. (from LE)</td>
<td>50</td>
<td>% chord</td>
</tr>
<tr>
<td>Mass per span</td>
<td>0.75</td>
<td>kg·m</td>
</tr>
<tr>
<td>Rotational moment of inertia</td>
<td>0.1</td>
<td>kg·m</td>
</tr>
<tr>
<td>Torsional rigidity</td>
<td>$1.00 \times 10^4$</td>
<td>N·m$^2$</td>
</tr>
<tr>
<td>Flat bending rigidity</td>
<td>$2.00 \times 10^4$</td>
<td>N·m$^2$</td>
</tr>
<tr>
<td>Edge bending rigidity</td>
<td>$4.00 \times 10^6$</td>
<td>N·m$^2$</td>
</tr>
<tr>
<td><strong>Tails</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Span of horizontal tail</td>
<td>2.5</td>
<td>m</td>
</tr>
<tr>
<td>Span of vertical tail</td>
<td>1.6</td>
<td>m</td>
</tr>
<tr>
<td>Chord of tails</td>
<td>0.5</td>
<td>m</td>
</tr>
<tr>
<td>Incidence of horizontal tail</td>
<td>-3</td>
<td>deg</td>
</tr>
<tr>
<td>Incidence of vertical tail</td>
<td>0</td>
<td>deg</td>
</tr>
<tr>
<td>Sweep of vertical tail</td>
<td>10</td>
<td>deg</td>
</tr>
<tr>
<td>Sweep of horizontal tail</td>
<td>0</td>
<td>deg</td>
</tr>
<tr>
<td>Dihedral of horizontal tail</td>
<td>0</td>
<td>deg</td>
</tr>
<tr>
<td>Beam reference axis (from LE)</td>
<td>50</td>
<td>% chord</td>
</tr>
<tr>
<td>Cross-sectional c.g. (from LE)</td>
<td>50</td>
<td>% chord</td>
</tr>
<tr>
<td>Mass per span</td>
<td>0.8</td>
<td>kg·m</td>
</tr>
<tr>
<td>Rotational moment of inertia</td>
<td>0.01</td>
<td>kg·m</td>
</tr>
<tr>
<td>Torsional rigidity</td>
<td>$1.00 \times 10^4$</td>
<td>N·m$^2$</td>
</tr>
<tr>
<td>Flat bending rigidity</td>
<td>$2.00 \times 10^4$</td>
<td>N·m$^2$</td>
</tr>
<tr>
<td>Edge bending rigidity</td>
<td>$4.00 \times 10^6$</td>
<td>N·m$^2$</td>
</tr>
<tr>
<td><strong>Complete aircraft</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>54.5</td>
<td>kg</td>
</tr>
</tbody>
</table>

discussions and quantitative presentations about the control cost will be provided in the following studies where a balance between flight performance and control cost is explored.
Table 2.2: Description of symmetric structural modes

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Description</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>First symmetric flat bending</td>
<td>0.4244</td>
</tr>
<tr>
<td>3</td>
<td>Second symmetric flat bending</td>
<td>2.431</td>
</tr>
<tr>
<td>5</td>
<td>First symmetric torsion</td>
<td>5.039</td>
</tr>
<tr>
<td>7</td>
<td>First symmetric edge bending</td>
<td>5.915</td>
</tr>
<tr>
<td>8</td>
<td>Third symmetric flat bending</td>
<td>6.698</td>
</tr>
<tr>
<td>10</td>
<td>Fourth symmetric flat bending</td>
<td>13.47</td>
</tr>
<tr>
<td>12</td>
<td>Second symmetric torsion</td>
<td>14.97</td>
</tr>
</tbody>
</table>

Figure 2.6: Baseline trimmed geometry for steady level flight at 25 m/s
Figure 2.7: Optimum geometry for steady level flight at 25 m/s

Table 2.3: Minimum drag optimization results with only trim constraint

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Optimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of Attack, deg</td>
<td>1.26</td>
<td>2.6204</td>
</tr>
<tr>
<td>Drag, N</td>
<td>59.28</td>
<td>51.3937</td>
</tr>
<tr>
<td>Mode 1</td>
<td>1.5654</td>
<td>0.1329</td>
</tr>
<tr>
<td>Mode 3</td>
<td>-0.0164</td>
<td>-0.0347</td>
</tr>
<tr>
<td>Mode 5</td>
<td>0.0071</td>
<td>-0.0027</td>
</tr>
<tr>
<td>Mode 7</td>
<td>0.0004</td>
<td>-0.0004</td>
</tr>
<tr>
<td>Mode 8</td>
<td>0.0005</td>
<td>-0.0013</td>
</tr>
<tr>
<td>Mode 10</td>
<td>-0.0002</td>
<td>-0.0002</td>
</tr>
<tr>
<td>Mode 12</td>
<td>-0.0014</td>
<td>-0.0013</td>
</tr>
</tbody>
</table>
With an optimization process defined and tested, the process of improving it and expanding it can begin. Within a design for an aircraft their may not be one single target which should be focused on in an optimization and often their are multiple factors which are considered. This leads to the use of multi-objective optimizations which can produce results showing how an improvement in one area may cause a regression in another. However, sometimes two targets may prove to be independent of one another which can be best highlighted by a similar multi-objective study. The following sections will discuss the various other flight performance characteristics which will be considered. Studies will also be shown which show how these new factors relate to the drag reduction objective presented in the baseline optimization.

3.1 Control Cost

It was seen in the baseline minimum drag optimization that the desired wing geometry would likely require a great deal of control force input to maintain such an aggressive shape change. The following section will highlight how the corresponding control cost should be minimized to avoid the situation where an excessive control effort outweighs the benefits gained from the minimum drag. First, the control cost is defined as

\[ U_c(q) = u^T(q)Qu(q) \]  

(3.1)

where \( Q \) is a user-defined weighting matrix to tune the control cost. For the numerical studies presented here, the values of the weighting matrix are all set to one. Other values could be used to place a higher cost on specific actuators, but that is not explored in the
3.1.1 Control Cost vs Drag Optimization

In order to understand the trade-off involved in improving the performance and reducing the control cost, a multi-objective optimization is performed. The control cost was defined previously in Eq. (3.1). An objective function is then defined to consider the trade-off between the minimum drag and control cost as

$$
\min_{\xi_1} f = \xi_1 D + (1 - \xi_1)U_c
$$

(3.2)

where $0 \leq \xi_1 \leq 1$. Varying the parameter $\xi_1$ results in varying the trade between minimum drag and minimum control cost. For example, when $\xi_1 = 1$ the objective function becomes entirely minimum drag, while $\xi_1 = 0$ becomes entirely minimum control cost. For this study, the tuning parameter is varied at an increment of 0.1. Additional cases are added for $\xi_1 = 0.92, 0.95, \text{ and } 0.99$, in order to better understand the sharp change in geometry.
Figure 3.2: Optimum geometries with trade-off between minimum drag and control cost
Figure 3.3: Bending moment for each element of the wing for various values of $\xi_1$ from 0.9 to 1. The results are presented in Table 3.1. As the parameter $\xi_1$ increases from 0 to 1, the minimum drag decreases with an increased control cost. This trend highlights the trade-off required between the minimum drag and the control cost. This trend can be seen graphically in Figure 3.1. The optimum geometries of each case are presented in Figure 3.2. The shape does not change dramatically between $\xi_1 = 0$ and $\xi_1 = 0.9$. After this point, the tip deflection starts to decrease more drastically. The shape transition from a fairly deep “U” to the flat shape as seen in the minimum drag study in the previous section. From $\xi_1 = 0$ and $\xi_1 = 1$ the shape undergoes a very smooth transition, which is a positive result meaning a marginal change in wing geometry can result in fairly significant reductions in control cost or drag depending on which direction the shape goes. The out-of-plane bending moment required to actuate each element of the wing structure for a few highlighted values of $\xi_1$ are shown in Figure 3.3. This figure shows the required control effort at each wing segment to achieve the appropriate geometry resulting from the corresponding optimization. It can be seen that as the parameter $\xi_1$ changes from 0
(minimum control cost) to 1 (minimum drag) the required bending moment in each element drastically increases, especially within the elements closer to the wing root. This observation agrees well with the shapes seen in Figure 3.2, the shape becomes much flatter compared to the trim state of the aircraft, which indicates that a large bending moment is needed near the wing root to flatten the shape and thus reduce the drag.

Table 3.1: Minimum drag vs control cost

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>Drag, N</th>
<th>Control Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>53.99375</td>
<td>3.84E+04</td>
</tr>
<tr>
<td>0.1</td>
<td>53.85298</td>
<td>4.02E+04</td>
</tr>
<tr>
<td>0.2</td>
<td>53.82706</td>
<td>4.04E+04</td>
</tr>
<tr>
<td>0.3</td>
<td>53.79521</td>
<td>4.11E+04</td>
</tr>
<tr>
<td>0.4</td>
<td>53.7630</td>
<td>4.22E+04</td>
</tr>
<tr>
<td>0.5</td>
<td>53.70141</td>
<td>4.53E+04</td>
</tr>
<tr>
<td>0.6</td>
<td>53.57382</td>
<td>5.49E+04</td>
</tr>
<tr>
<td>0.7</td>
<td>53.4550</td>
<td>6.97E+04</td>
</tr>
<tr>
<td>0.8</td>
<td>53.26017</td>
<td>1.06E+05</td>
</tr>
<tr>
<td>0.9</td>
<td>52.75547</td>
<td>2.92E+05</td>
</tr>
<tr>
<td>0.92</td>
<td>52.5576</td>
<td>4.12E+05</td>
</tr>
<tr>
<td>0.95</td>
<td>52.2053</td>
<td>7.32E+05</td>
</tr>
<tr>
<td>0.99</td>
<td>51.4596</td>
<td>2.63E+06</td>
</tr>
<tr>
<td>1</td>
<td>51.3411</td>
<td>3.40E+06</td>
</tr>
</tbody>
</table>

3.1.2 Control Cost Constraint

A constraint on the control cost can be used to ensure the optimum shape does not require excessive energy, while achieving drag reduction. This can be formulated as a constrained optimization problem as follows,

$$\min_{\bm{q}} D = D(\bm{q})$$

subject to

$$C_1 : \bm{r}_B = \bm{0}$$

$$C_5 : U_c \leq U_{clim}$$

(3.3)

where $U_{clim}$ is a user-defined value to constrain the control cost. In this study, the limit is set at $U_{clim} = 7 \times 10^4$. This value corresponds to a control cost near the middle of the
previous study. The resulting optimum solution is compared with the baseline in Table 3.2. It can be seen that the drag reduces from 59.28 N to 53.53 N, which is a 9.7% reduction. The resulting optimum geometry can be seen in Figure 3.4. The shape is consistent with the ones from the previous study as it falls somewhere between $\xi_1 = 0.6$ and $\xi_1 = 0.9$, just as the control cost falls between those same weighting parameters.

3.2 Gust Load Alleviation

Highly flexible aircraft with slender wings are often susceptible to the perturbation of wind gust. To account for this situation and ensure aircraft flight safety, another design objective function is defined so as to minimize the wing aerodynamic bending moment induced by gust perturbation. In doing so, a discrete gust model, shown in Figure 3.5, is used to calculate the aerodynamic moment generated at the wing root. The gust width is
25 times the chord length of the main wing. The gust velocity can be expressed as [19]

\[ w_g = \frac{w_0}{2} \left( 1 - \cos \frac{2\pi x}{25c} \right) \]  

(3.4)

where \( w_0 \) is the nominal maximum gust speed and \( c \) the average chord length of the main wing. To further simplify the problem, a method similar to Tang and Dowell [20] is used here, where the gust strength at a given time is assumed as constant along the wing span. This gives an effective angle of attack induced by the wind gust as

\[ \alpha_g = \frac{w_g}{U_\infty} \]  

(3.5)

with the maximum being \( (\alpha_g)_{\text{max}} = w_0/U_\infty \). This additional angle of attack will be added to Eq. (2.8) to calculate the gust-induced aerodynamic loads. With the actual wing shape frozen, i.e. same as with no gust, the gust-induced flatwise bending moment \( M_y^g \) can be assessed at the wing root following Eq. (2.6), which is then to be minimized to alleviate the gust perturbation.

### 3.2.1 Gust Load vs Drag Optimization

Basically, the concept of gust alleviation discussed here is a passive way to improve the ride quality of the aircraft under gust perturbation. The wind gust perturbation model
Figure 3.5: A 1-cosine vertical gust velocity profile with unit peak velocity

Figure 3.6: Trade-off between minimum drag and gust-induced bending moment
and the approach to estimate the gust-induced bending moment were given in Sec. 3.2. The gust velocity, \(w_0\), is set at 2.19 m/s, which gives an induced angle of attack of 5° for the flight speed of 25 m/s. Some aircraft missions may require this to be considered in addition to the flight performance requirements, such as drag reduction. In doing so, a different optimization is performed in order to better understand the potential trade-off between the flight efficiency and ride quality by minimizing the drag and the gust-induced root bending moments. In this study, the optimization problem can be expressed as

\[
\min_{q} f = \xi_2 D + (1 - \xi_2) M_y^g
\]

(3.6)

where \(0 \leq \xi_2 \leq 1\). The control constraint, \(C_5\), was again set at \(U_{\text{clim}} = 7 \times 10^4\) to ensure the control cost is not too large. The parameter \(\xi_2\) is varied at an increment of 0.1. When \(\xi_2 = 0\) the optimization is entirely to find the minimum root moment, and when \(\xi_2 = 1\) the optimization is entirely to find the minimum drag. The results of this study can be seen in Table 3.3 and graphically in Figure 3.6, which very clearly show the required trade-off between flight performance and safety. As a smaller drag is achieved, the wing root aerodynamic bending moment increases and vice versa. The optimum wing geometries associated with each value of the weighting parameter \(\xi_2\) are shown in Figure 3.7. It can be
Figure 3.7: Optimum geometries with trade-off between minimum drag and control cost
seen that the shape does not vary greatly as the weighting parameter changes. The fairly
strict control cost constraint, $C_5$, likely plays a large role in this. The control cost constraint
requires each case to still consider the required input to achieve either drag reduction or
gust load alleviation. The shapes are similar to that produced in section 3.1.2. This also
explains why the drag values only vary slightly going from around 53 N to just under 56 N.

### 3.2.2 Aircraft Endurance and Range

For a battery-powered, propeller-driven aircraft, the maximum endurance and range
are given by

$$E_{\text{max}} = \frac{\bar{\eta} U_\infty}{\sqrt{2}} \frac{\sqrt{L^2}}{D} \frac{\sqrt{\rho_\infty S}}{\sqrt{W^2}} (\bar{C}_0 - \bar{C}_1)$$

$$R_{\text{max}} = \frac{\bar{\eta} U_\infty}{D} \frac{L \bar{C}_0 - \bar{C}_1}{W}$$

where flight speed $U_\infty$ is constant. Lift $L$ equals weight $W$, both of which are unchanged
for a battery-powered aircraft. $\rho_\infty$ is the air density, $S$ is the lifting surface area, and $\bar{\eta}$ is
the propulsion efficiency. $\bar{C}_0$ and $\bar{C}_1$ are initial and final battery capacities. It is clear that both endurance and range are inversely proportional to the drag $D$, with a constant lift at level flight. With this relationship, one can further convert Figure 3.6 into the trade-off between gust-induced root bending moment and flight endurance or range, see Figure 3.8. Since aircraft power capacity and propulsion efficiency are not involved in the current study, Figure 3.8 is plotted with non-dimensional data, that is, all data are normalized with respect to the case when $\xi_2 = 0$. From Figure 3.8, it is easy to observe the gain on maximum endurance or range, at the cost of increased root-bending moment.

3.3 Drag, Control Cost, and Gust Optimizations

3.3.1 Definition

In order to understand the trade-offs between all three quantities, drag $D$, gust-induced root bending moment $M_y^g$, and control cost $U_c$, a study is performed to balance the effects of all three. The root bending moment and the drag can be considered as additional constraints when performing an optimization in which the control cost is to be minimized. The modified optimization problem is expressed as

$$\min_{\mathbf{q}} U_c = U_c(\mathbf{q}) \tag{3.8}$$

subject to the following constraints

$$C_6: D \leq D_{\text{lim}}$$

$$C_7: M_y^g \leq M_y^{g\text{lim}} \tag{3.9}$$

3.3.2 Results

In the study, one may vary the constraints of both drag and gust-induced root bending moment within a given range. With that, the minimum control cost is searched with different combinations of drag and bending moment constraints. Presented here are the results of a study in which the drag constraint is varied from 52 N to 62 N at an
increment of 0.5 N, and the root bending moment constraint is varied from 2700 N-m to 3100 N-m at every 25 N-m. This study provides an aircraft designer a better understanding of the control cost required to fly an aircraft at a certain drag and root moment experienced due to gust. For illustration, a grid of drag and root moment constraints are applied to generate the plot seen in Figure 3.9. It can be seen that there is a region, where the target drag and gust-induced moment are both small, therefore no feasible optimum solutions of wing geometry, and thus the control cost, exist. This is seen on the left corner of the figure. There is a clear border where the control cost is relative high in order to limit the gust moment and achieve the required wing geometry. However, the control cost is reduced when the constraints are relaxed, namely the points on the right side of the figure. Figure 3.10 helps further illustrate the region where no solution is possible.
Figure 3.9: Minimum control cost with varying drag and root moment constraints
Figure 3.10: Minimum control cost with varying drag and root moment constraints
4 OPTIMIZATION IMPROVEMENTS

4.1 Actuation Reduction Scheme

As previously described, the optimization assumes a fully distributed control scheme. This idea would be both impractical and inefficient to apply to an aircraft design. A reduction process must be undertaken to determine the number and placement of actuators needed to most effectively generate the desired wing geometries. A reduction process is outlined in the following section.

The original model contains 20 possible wing elements to be used for control. Each element contains 4 control forces/moments used to control the extension, torsion, in-plane bending, and out of plane bending. In order to reduce this control scheme, a parametric study will be completed which examines the various combinations of control forces to reduce the required control cost while maintaining the desired drag reductions.

4.1.1 Reduction Algorithm

The methodology will follow the flow chart presented in Figure 4.1. Starting with a choice of the number of actuators to use based on the number in the fully distributed scheme. Once a number of actuators is chosen, all possible combinations of that number will be explored. For example when 5 elements are chosen the full list of combinations is found using the MatLab command $\text{COMBNK}(V,K)$. For each combination, the corresponding control matrices, $B_f^i$ and $B_m^u$ must be reduced to only include the sections from desired elements (i.e $B_{i\text{reduced}}^f$ and $B_{u\text{reduced}}^m$). With the updated reduced control influence matrix the baseline optimization can be run again to determine the new optimum shape which can be achieved within the reduced control framework. Once the current combination is analyzed, a new combination can be studied with a repeating pattern until
Choose Number of Elements (4,5,etc)

Choose Active Elements
\[ c = \text{combnk}(1:10,5) \]

Run Minimum Drag Optimization

All Combinations Completed?

Determine Best Combination:
- Minimum Drag?
- Minimum Control Effort?
- Most Closely Resembles Original Optimum Shape?

All Numbers of Elements Completed

Determine Best Reduction Scheme

Figure 4.1: Control reduction algorithm
all combinations are studied. Once all combinations have been analyzed, the best combination must be determined.

4.1.2 Element Combination Selection

Using the algorithm presented above, the results for all of the element combinations are shown below. The plots in figures 4.2 and 4.3 show the drag and control cost of every possible combination within a certain number of active elements. The plots in figures 4.4 and 4.5 show the same data but normalized using the following transformations, where $\alpha$ and $\beta$ are plotted.

\[
D = D_{\min} + \alpha(D_{\max} - D_{\min}) \\
U_c = U_{c\min} + \beta(U_{c\max} - U_{c\min})
\]  

(4.1)

From this plot the data point which is closest to the origin is selected as the best possible combination of elements to use for control reduction. Within the plots these points are highlighted in red. The data from these combinations is presented in table 4.1. The table highlights the percent increase in drag associated with the new control scheme as well as the percent reduction in control cost. It can be seen that each combination produces a fairly equal increase in drag, which is mostly negligible. However, the decrease in control cost is great as the 4 element scheme shows a control cost reduction of 85% compared to the 7 element reduction of only 66%.

4.1.3 Drag vs Control Optimization for Preferred Schemes

The selection of the best reduced actuation scheme out of the resulting schemes presents another challenge. One could simply select the scheme which reduces the drag the most, or select the scheme which reduces the control cost the most. Additionally some combination of the two could be selected. For this study we chose the scheme which best reduced both quantities weighted equally. The combination selected as the overall best was the 4 element combination of elements 3, 4, 6, and 9. This offered the lowest drag, while greatly reducing the control cost associated with the geometry. In order to further explore
Figure 4.2: Drag and control cost data for actuation reduction schemes: 1 - 6 elements
Figure 4.3: Drag and control cost data for actuation reduction schemes: 7 - 9 elements
Figure 4.4: Normalized data for actuation reduction schemes: 1 - 6 elements
Figure 4.5: Normalized data for actuation reduction schemes: 7 - 9 elements
Table 4.1: Control reduction results

<table>
<thead>
<tr>
<th>Number of Elements</th>
<th>Combination</th>
<th>Drag, N Value</th>
<th>% Change</th>
<th>Control Cost Value</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-</td>
<td>51.3368</td>
<td>-</td>
<td>5.13E+06</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>2 3 4 5 6 7 8 9 10</td>
<td>52.1192</td>
<td>1.524053</td>
<td>4.00E+06</td>
<td>-22.16</td>
</tr>
<tr>
<td>8</td>
<td>3 4 5 6 7 8 9 10</td>
<td>51.8222</td>
<td>0.945521</td>
<td>2.25E+06</td>
<td>-56.13</td>
</tr>
<tr>
<td>7</td>
<td>2 3 4 5 6 7 8 7 8</td>
<td>51.8187</td>
<td>0.938703</td>
<td>1.74E+06</td>
<td>-66.00</td>
</tr>
<tr>
<td>6</td>
<td>2 4 5 6 8 9 7 8 8</td>
<td>51.8161</td>
<td>0.93169</td>
<td>9.79E+05</td>
<td>-80.93</td>
</tr>
<tr>
<td>5</td>
<td>5 6 7 8 10</td>
<td>51.8127</td>
<td>0.927015</td>
<td>1.06E+06</td>
<td>-79.40</td>
</tr>
<tr>
<td>4</td>
<td>3 4 6 9</td>
<td>51.7606</td>
<td>0.825529</td>
<td>7.47E+05</td>
<td>-85.44</td>
</tr>
<tr>
<td>3</td>
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<td>-89.03</td>
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<tr>
<td>2</td>
<td>4 7</td>
<td>51.8413</td>
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<td>5.63E+05</td>
<td>-89.02</td>
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<td>1</td>
<td>9</td>
<td>51.7997</td>
<td>0.901692</td>
<td>4.01E+05</td>
<td>-92.18</td>
</tr>
</tbody>
</table>

Figure 4.6: Trade-off between drag and control cost
this combination, a multi-objective study similar to the one in section 3.1.1 was repeated. This time though the reduced actuation scheme was utilized. It can be seen in fig 4.6 that the reduced scheme produces results similar to the previous study. The graph shows the trade-off associated between drag reduction and control cost minimization. The important point to note with this plot is that compared to fig 3.1 the control cost values are nearly an order of magnitude smaller, while the changes in drag are much smaller. This shows how effective the actuation scheme reduction was at reducing the required control cost.

4.2 Steady Level Flight Velocity Range Optimization

Similar to the previously discussed studies, the optimization with control cost constraint is carried out with a range of flight speeds from 20 to 28 m/s. Each of these cases is treated as an individual steady level flight case, meaning only the symmetric modes are considered as design variables. The aircraft is again trimmed using the traditional control surfaces for each flight speed in order to have a point of comparison with the optimum solution as well as an initial set of design variables. The optimum shapes are compared with the trim conditions for each case in Table 4.2. It can be seen that for each speed, the optimization produces a wing geometry that reduces the drag, thus the required thrust to maintain the trim. The size of the reduction varies with the speed range with a maximum reduction of 12.7% occurring when the speed is 27 m/s and a minimum reduction of 2.1% when the speed is 20 m/s. The geometries can be seen in Figure 4.7. Aside from U = 28 m/s, it can be seen that the shapes do not have any drastic changes as the velocity changes. This provides some positivity going forward with the controller development as the changes between data points are minimal.
Table 4.2: Optimum wing geometry for a range of speeds

<table>
<thead>
<tr>
<th>Speed, m/s</th>
<th>Baseline</th>
<th>Optimum Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Thrust, N</td>
<td>Body AOA, deg</td>
</tr>
<tr>
<td>20</td>
<td>87.468</td>
<td>4.528</td>
</tr>
<tr>
<td>21</td>
<td>80.110</td>
<td>3.699</td>
</tr>
<tr>
<td>22</td>
<td>73.756</td>
<td>2.973</td>
</tr>
<tr>
<td>23</td>
<td>68.245</td>
<td>2.333</td>
</tr>
<tr>
<td>24</td>
<td>63.454</td>
<td>1.766</td>
</tr>
<tr>
<td>25</td>
<td>59.282</td>
<td>1.260</td>
</tr>
<tr>
<td>26</td>
<td>55.665</td>
<td>0.803</td>
</tr>
<tr>
<td>27</td>
<td>52.521</td>
<td>0.389</td>
</tr>
<tr>
<td>28</td>
<td>49.811</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Figure 4.7: Optimum wing geometries
5.1 Conclusions

Determination of the optimum wing geometry of a highly flexible aircraft under varying flight conditions was explored in this paper. Given the flexible nature of high aspect ratio aircraft, a modal based approach was used in determining the optimum wing bending and torsion geometry. The magnitudes of the modes were used as design variables within the optimization. Additionally, a distributed control actuation was formulated by assuming each element of the main wing could be actuated. This gave an insight into the forces and moments required to generate a specific wing geometry. The distributed force calculation was included within the optimization and it was verified using a steady level flight case.

Once the optimization was completed and tested, it was updated to include multi-objective functions. These studies included the trade-off between drag and control cost, as well as the trade-off between drag and root bending moment due to a gust load. The three parameters were then all considered within one optimization to further demonstrate the trade-offs associated with each objective.

The reduction of the actuation scheme was also explored. A reduction from 10 actuators per wing to 4 actuators was realized to have benefits in control cost, whilst limiting the associated sacrifice in flight performance. The chosen reduced scheme performed well at not just minimizing drag but also in successfully generating optimum geometries for a multi-objective study balancing control cost and drag. These results were in line with similar results using a fully distributed scheme. The optimization was then applied over a range of velocities in preparation for applying principles of Linear Parameter Varying (LPV) control to the problem to develop a robust flight controller which would
generate the desired optimum wing geometry over an entire flight path.

It should be noted that the optimization was computed using the gradient-based optimizer \texttt{fmincon} in MATLAB, which can only produce a local minimum. Despite not being a global minimum, the results produced were consistent and showed significant improvement over the baseline solution.

5.2 Future Work / Recommendations

As is the case with all research projects, the work never truly ends. This project specifically has few loose ends that need to be addressed as well as some additional topics which could be explored. The first loose end which needs completed is the topic of the LPV controller. The control matrices for a velocity range have been determined, now the actual controller needs to be developed and properly tested. This would include testing on the reduced model which the controller will be based on as well as linear and non-linear cases using the full aircraft model. This process could also be explored over a range of flight altitudes or during a turning maneuver as well. A second loose end would be to further explore the actuation reduction techniques to better determine the best actuation scheme or further test the capabilities of the scheme presented here.

Some suggestion for future work associated with this project include updates to all the various levels of the optimization process. One important update would be to the optimization algorithm itself. Currently the solver can only guarantee a local minimum is achieved. A more powerful solver could potentially lead to the true global minimum. One such thought would be the use of some sort of genetic algorithm. Additionally the aerodynamic model used should be updated as well. The calculation of drag in this formulation is limited to induced drag but the reduction of other forms of drag could be more important. One suggestion would be some form of vortex based solver such as the vortex lattice method which would provide more accurate aerodynamic solutions while still maintaining a relatively fast solution process. A more robust method of determining the locations and number of actuators required is also desired. A method similar to the ones
addressed in the literature review would provide a more powerful methodology if a finer mesh is used in the model. The methods outlined there would also provide additional insight into which actuators are best for maintain certain shapes such as bending or torsion. The control cost associated with these actuators should also be studied further. The current work only considers the required forces and not the energy associated with maintaining a geometry compared to the energy saved through drag reduction. Finally the future work should improve upon the gust load alleviation analysis. A more complete modeling would include time domain analysis and potentially include random gusts such as the Dryden gust model.
REFERENCES


