THREE ESSAYS ON THE VIX INDEX

by

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ABSTRACT

This dissertation comprises three essays based on the VIX Index, the Chicago Board Options Exchange's (CBOE) index representing S&P 500 Index implied volatility.

The first essay looks at the performance of VIX-related Exchange Traded Notes (ETN). These instruments benchmarked to futures prices are designed to give exposure to stock market volatility. Performance has been poor, owing to the upward sloping nature of the VIX futures curve. The introduction of the notes appears to have coincided with a fundamental change in the pricing of VIX futures, resulting in a steeper futures curve and worse performance for ETNs than would have occurred previously. A strategy of carrying out the reverse of the trades represented by the underlying benchmark is shown to be profitable. The extent to which ETN managers cover their positions appears correlated with the slope of the futures curve.

The second essay asks if volatility indices predict realized volatility. I find that the VIX Index tends to overstate subsequent realized S&P 500 Index volatility and shows mild predictive value. The VVIX Index, a measure of VIX Index implied volatility, tends to understate realized VIX Index volatility and has little predictive value. I demonstrate that the volatility indices do not, however, by construction, directly reflect interperiod volatility as measured by standard deviation.

The third essay looks at the problem of analyzing futures prices in the specific case of the VIX Index. The maturity of futures contracts change continuously, which poses problems for assessing notional futures prices of specified maturity and the shape of the futures curve.

Benchmarks for VIX-related ETNs use linear interpolation of the futures prices nearest to the desired maturity to provide a notional value for a futures contract of 30 or 90 days duration. I show that this does not reflect the true shape of the futures curve. I apply a method designed by Charles Nelson and Andrew Siegel for modeling yield curves to the VIX futures term structure. This gives a better estimate for prices of notional contracts of specific maturity than linear interpolation, and reflects the non-linear, asymptotic nature of the futures curve.

LIST OF ABBREVIATIONS AND SYMBOLS

CBOE	Chicago Board Options Exchange
c.d.f	Cumulative distribution function
CSV	Cross-sectional volatility
ETF	Exchange Traded Fund
ETN	Exchange Traded Note
ETP	Exchange Traded Product
Libor	London Interbank Offered Rate
OLS	Ordinary least squares
p.d.f.	Probability density function
R^2	R-squared
S&P	Standard & Poors
TNA	Total net assets
TSV	Time series volatility
VIX	CBOE index, a measure of S&P 500 Index volatility
VVIX	CBOE index, a measure of VIX Index volatility

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CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

This dissertation comprises three essays on the VIX Index, the measure of implied thirty day volatility published by the Chicago Board Options Exchange (CBOE) derived from options prices on the S&P 500 Index.

Since its inception in 1993, the VIX Index has generated growing interest amongst the financial press and investors. It is strongly negatively correlated with movements of the S&P 500 Index, leading it to be dubbed "The Fear Index". CBOE began trading VIX Index futures in March 2004.

The VIX Index itself is calculated as the square root of a weighted sum of options prices. Consequently it cannot be dynamically replicated. Exchange Traded Notes (ETN) began to appear in 2009 to meet a demand from investors for exposure to the VIX Index, but as the Index itself cannot be traded, these instruments are based on a rolling portfolio of futures contracts, in the case of the short-term instruments the first and second nearby contracts, those nearest to maturity, in proportion so as to provide a notional maturity of 30 days. To physically cover positions requires that the second nearby be bought and first nearby be sold on a daily basis, until the first nearby matures, the second nearby becomes the first nearby, and so the process continues.

VIX Index futures are generally in contango, meaning that the greater the maturity of the contract, the higher the price. This has resulted in significant losses for holders of the Exchange

Traded Notes. The first essay looks at this phenomenon, at ways in which profits might be made by trading counter to the Exchange Traded Notes benchmark, and how the contango problem has worsened since the introduction of the Notes.

The second essay analyses how well the VIX Index and VVIX Index represent and predict the volatility of the S&P 500 Index and VIX Index respectively, the VVIX Index being a measure of implied volatility of the VIX Index. The VIX Index is shown to have mild predictive value and tends to overstate S&P 500 Index volatility, whereas the VVIX Index appears to understate VIX Index volatility. The reasons for this are examined and discussed.

The third essay turns to the technical problem of analyzing VIX futures prices when the maturity of a futures contract changes constantly. The benchmarks used by VIX-related ETNs are, de facto, linear interpolations of prices of contracts of maturity either side of the desired maturity, either 30 or 90 days. The VIX futures curve is not, however, generally found to be linear, and appears asymptotic in nature. The curve fitting method designed by Charles Nelson and Andrew Siegel of the University of Washington to model yield curves is applied to the VIX futures curve and found to provide a better estimate of prices for notional contracts of intermediate maturity than linear interpolation. It also has other properties that may provide useful tools for analyzing VIX futures prices. As the VIX Index itself is non-arbitrageable, futures prices may contain useful information not found in the futures of those assets where arbitrage is possible.

CHAPTER 2

AN EXAMINATION OF VIX-RELATED TRACKER FUNDS AND THEIR RELATIONSHIP TO THE FUTURES MARKET IN WHICH THEY OPERATE

2.1 Introduction

Passive investment funds designed to track an underlying asset or index have grown in popularity and range since mutual funds following the major stock indices first appeared in the 1970s. Other than proportional adjustments according to the inflow and outflow of funds as units are sold and bought, changes to the underlying portfolio are made only when index constituents are replaced.

The 1990s saw the introduction of Exchange Traded Funds (ETF), which, unlike mutual funds, are traded continuously on securities markets. Shares in ETFs can be created or broken up into the underlying assets at any time, thus the market value always remains close to the net asset value of the fund as arbitrage opportunities would otherwise be created and the differential erased.

Along with Exchange Traded Funds, Exchange Traded Notes (ETN), which have bond-type features, are now common.

The assets of the first ETFs comprised the components of the index being tracked, such as the State Street S&P 500 Index tracker, which, as at September 2017, has an asset value of almost $250bn. Holding the underlying shares effectively eliminates the risk of failure to match the performance of the underlying index, known as tracking error, other than at times when the constituents of the index change.

Some Exchange Traded Products (ETP), however, do not hold their underlying assets directly. For those following indices with a large number of underlying stocks, sampling is sometimes used, whereby only a representative proportion of the constituents of the underlying index are held, increasing the risk of tracking error. Synthetic tracking, where futures or options are held instead of the underlying asset, is also now common.

As interest in ETPs grew, funds tracking the prices of assets other than stock indices began to appear, including commodity funds. Whilst hard commodities such as precious metals can readily be traded to provide asset cover for an ETP, where a commodity is perishable or has a high storage cost, holding the asset is impractical. Consequently, managers of soft commodity ETPs generally use futures to give exposure to the movement in the underlying asset. This has led in many cases to the so-called contango problem, resulting in poor performance.

Unable to hold the underlying asset, funds take positions in a portfolio of futures contracts, generally the first and second nearby contracts. For physically settled contracts, the first nearby is not held to expiry, to avoid both exercise and increasing price fluctuations owing to delivery-based optionality and other issues as maturity approaches. The contango problem arises when the futures curve is upward sloping, i.e. the market is in contango, so that where the two-month nearby is bought and then sold one month thereafter, the premium of the two-month nearby over the one-month nearby is lost.

This essay examines the effect that the introduction of ETNs related to the VIX Index, a measure of S&P 500 Index implied volatility, appears to have had on the VIX futures market. The reasons for the poor performance of VIX-related ETNs is analyzed. I look at the correlation between the trading activity of the ETN managers and futures prices, and at potentially viable trading strategies that might exploit the predictable trading pattern of ETN managers. I demonstrate that such trading strategies have been profitable, and that the level of potential profits has increased following the introduction of VIX-related ETNs. I show that there appears at times to be insufficient liquidity to trade in the futures market for managers to cover their positions, and that the asset value of ETNs and its relationship to futures open interest are indicative of the level of profits to be made by carrying out reverse trades to those of the ETN managers.

The essay proceeds as follows. Section 2.2 examines the difficulties in using futures to replicate non-arbitraged asset prices and indices. Section 2.3 reviews the existing literature, and section 2.4 looks at the performance of VIX-related ETNs. Section 2.5 considers cash flows from trading strategies. Section 2.6 examines the relationship between fund size and futures prices, and section 2.7 the effect of ETN trading on the futures market.

Prices for the VIX Index and VIX-related ETNs are from Yahoo Finance, and futures data from Commodity Systems Inc. ETN net asset figures come from Morningstar. Libor rates are from the Federal Reserve Bank of St. Louis.

2.2 The VIX Index

The Chicago Board Options Exchange (CBOE) VIX Index, introduced in January 1993, reflects the implied volatility of the 30-day forward price of the S&P 500 Index with the aim of providing a measure of expected volatility. It is calculated as the square root of a weighted sum of short-term at-the-money and out-of-the-money option prices, both puts and calls. The VIX Index increases (decreases) as the implied one-month volatility of the S&P 500 Index increases (decreases). Both options and futures on the VIX Index are traded, and are promoted by CBOE as a means to hedge against volatility.

For any asset on which futures are traded, where carry arbitrage is possible, i.e. where the underlying asset can be both bought to hold and borrowed to sell short, then the intertemporal cost of carry, the difference between futures contracts of different maturities, will reflect the cost, including the rate of interest, of holding that asset, as described by Working (1949).

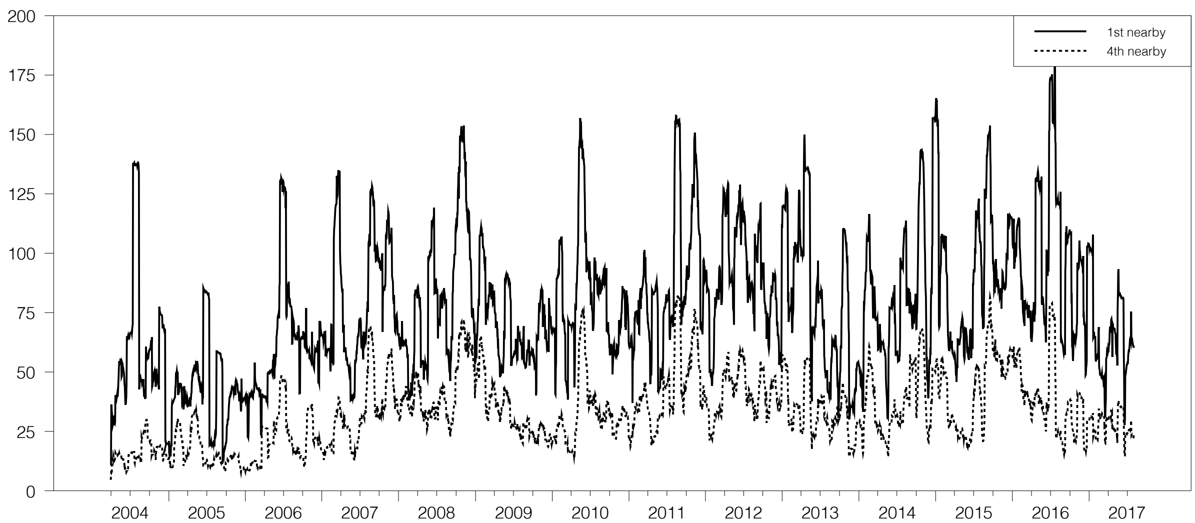
Where carry arbitrage does not hold, the futures price may reflect the expected spot price at maturity as well as an adjustment for risk unique to the particular futures contract. Reasons why carry arbitrage might not hold include an inability to borrow the underlying asset, high storage costs, or, as with the VIX Index, because the underlying benchmark is neither traded nor replicable.

The inability to replicate the underlying asset implies that carry arbitrage should not hold, and that therefore the predictions of the Samuelson (1965) hypothesis likely apply. Specifically, the longer the maturity of a futures contract, the lower will be the volatility of the contract. This is not the case for a fully arbitraged market where arbitrage ensures that futures of different maturities move in tandem with one other, given constant carry cost.

Amongst assets on which futures are traded, the VIX Index is unique in that it cannot be replicated statically, being the root of a weighted sum of its underlying traded assets, S&P 500 Index options.

Figure 2.1 shows a plot of the annualized standard deviation in closing price for the previous 21 working days of the first nearby and fourth nearby VIX futures contracts from 3/26/2004 to 8/4/2017. These are calculated as $\sigma_t = \sqrt{\frac{\sum_{i=0}^{20}(p_{FD,t-i} - \mu)^2}{21}} * \sqrt{365 * \frac{5}{7}}$ where μ is the mean daily closing price log change in the futures contract over the previous 21 days, and $p_{FD,t}$ is the closing price log change from time t-1 to t. The latter term on the right hand side of the equation converts the daily standard deviation into an annual equivalent.

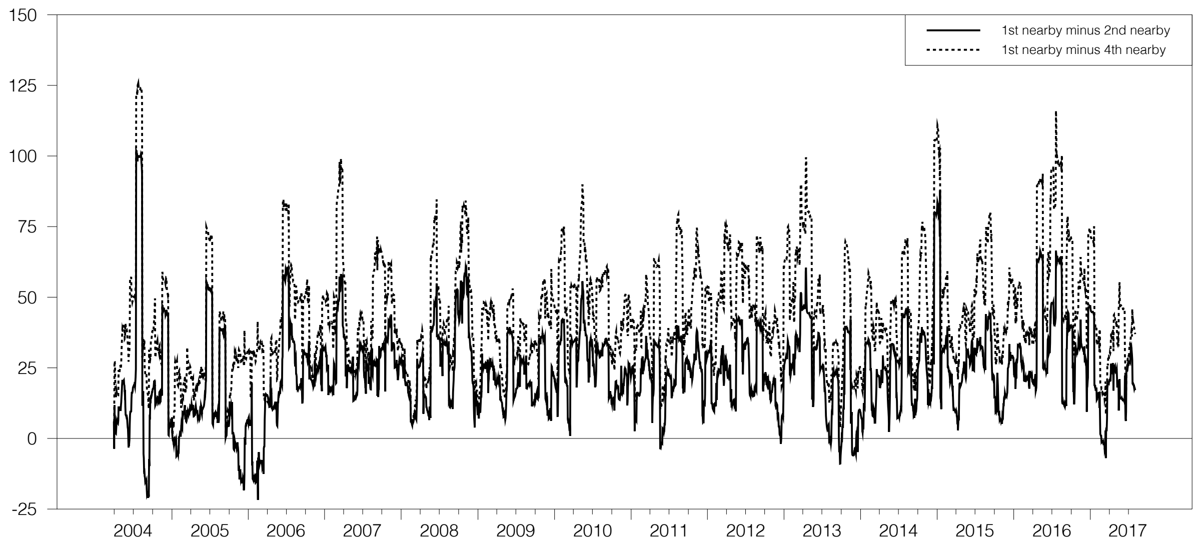
Figure 2.1: Annualized one-month standard deviations of 1st and 4th nearby



It can be seen that the volatility of the longer maturity contract is always less than that of the shorter maturity contract, confirming that the market is indeed characterized as non-arbitraged.

Differences of the rolling one-month standard deviation of the first nearby with the second and fourth nearbys are shown in Figure 2.2. From this can be seen that the standard deviation of the first nearby is greater than that of the second nearby most of the time, and always greater than that of the fourth nearby. Furthermore, the difference between standard deviations of the first and fourth nearbys is greater than that between the first and second nearbys.

Figure 2.2: Annualized one-month standard deviation, 1st nearby minus 2nd nearby and 4th nearby



As a measure of volatility, the VIX Index is likely mean reverting (Harvey & Whaley, 1992). Mean reversion implies that futures prices should be anchored to the long run mean to an extent, depending on the length of time that volatility shocks are seen to persist.

As a result, the VIX futures market should display backwardation (contango) when the VIX index stands above (below) its long run mean.

Figure 2.3: VIX index, VIX Index futures 1st nearby and 4th nearby

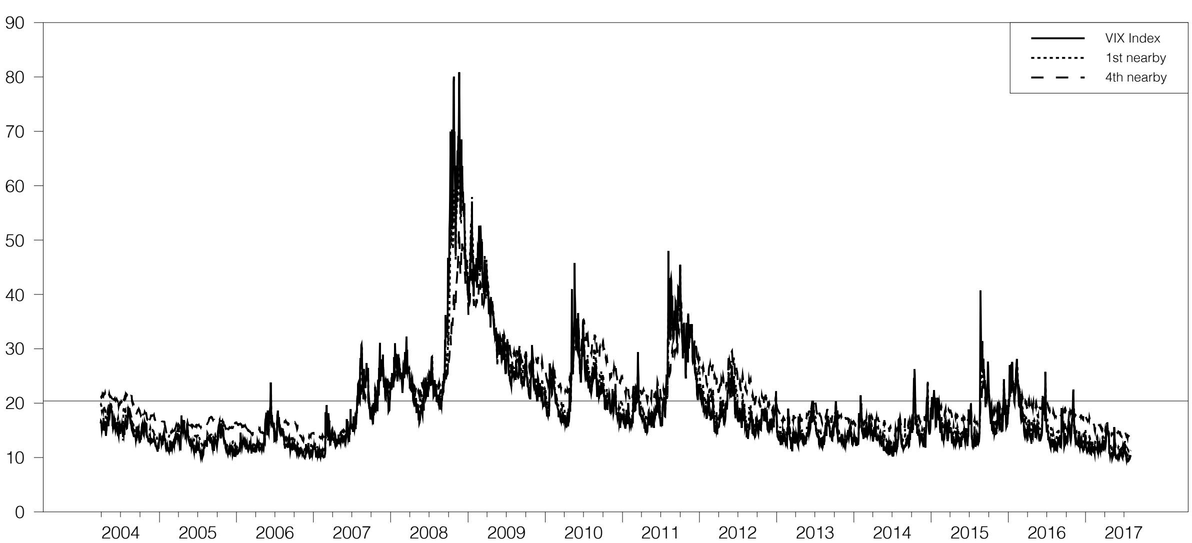


Figure 2.3 compares the level of the VIX index and the first nearby future, which are naturally close, along with the fourth nearby future. This relationship can be seen to hold until 2009, after which the market is generally in contango, i.e. price increases with maturity and hence the futures curve is upwards sloping.

There is strong negative correlation between movements in the VIX Index and the underlying S&P 500 Index, leading to the dubbing of the former amongst market commentators as the "Fear Index". This is demonstrated by a simple ordinary least squares (OLS) regression of daily VIX Index movements (*MVIX*) on S&P 500 Index returns (*RSPX*) for the period 1/2/1990 to 12/1/2017 with the constant insignificant at the 5% level, the index parameter significant at the 1% level, and adjusted R^2 of 0.1735:

$$MVIX = 0.001 - 3.010RSPX$$

Where carry arbitrage does not hold, the forward price of an asset that is negatively correlated with the S&P 500 Index should exceed the expected future spot value of that asset, subject to risk adjustment, as, according to the Capital Asset Pricing Model, the expected return of an asset negatively correlated with the stock index, in this case a VIX Index future, should be less than the risk-free rate. Consequently we should see the futures market for such an asset, where carry arbitrage does not hold and given a low risk-free rate, in contango. Of all the assets on which futures are traded, the VIX Index has perhaps the greatest negative correlation with the S&P 500 Index, so whilst the contango problem exists with soft commodities where the correlation might be expected to be close to zero, the high negative correlation of the VIX Index with the S&P 500 Index means that the futures curve is likely to tend towards being upward sloping, and hence be particularly antithetic to attempts to provide practical exposure to the underlying VIX Index using futures.

2.3 Literature Review

A number of papers have examined the relationship between the level of the VIX Index and futures prices and the term structure of VIX futures prices.

Zhang & Zhu (2006) and Zhang & Zhu (2007) develop a stochastic variance VIX futures pricing model based on VIX Index historical time series data. The model suggests that futures prices were low compared to predictions for the period up until 2005, but that this discount narrowed towards the end of the period.

Zhang & Huang (2010) examines the relationship between the level of the VIX Index and variance futures, which are based on realized variance, rather than implied volatility as represented by the VIX Index. Using a stochastic volatility model developed by Heston (1993), they find that variance futures prices are a linear function of the square of the VIX Index, which, as a measure of standard deviation, is the square root of implied variance derived from options prices.

Fassas (2012) examines the relationship between VIX Index futures and equity returns, finding correlation between the VIX Index futures term structure and subsequent S&P 500 Index returns.

Various studies have looked at the problems suffered by VIX-related ETNs. The contango problem has been examined by Liu & Dash (2012), whose results are related below. Hancock (2013) suggests that potential ETN investors can do better by investing in VIX Index futures directly. Alexander & Korovilas (2013) conclude that front running by speculators is potentially responsible for exacerbating the poor performance of VIX-related Exchange Traded Products.

Whaley (2013) notes that long VIX-related ETNs are "virtually guaranteed to lose money through time" and considers that some investors may believe that they are actually buying the VIX Index itself. He explains that the futures market is generally in contango and that this is the primary reason for the poor performance of VIX-related ETNs.

Clowers & Jones (2016) show that VIX-related ETNs are poorly correlated with the VIX Index, and also suggest that some investors may be confusing the products with the VIX Index itself.

Bahaji & Aberkane (2016) question the popularity of VIX-related ETNs, noting that despite combined losses of over $5bn since their introduction, demand for VIX-related ETNs continues to expand. They assert that volatility hedging is a sophisticated operation that cannot be accomplished by buying and holding VIX-related ETNs. They claim, however, as do Chen & Chung (2010) and Jones (2011), that use of VIX Index futures directly can improve risk-adjusted portfolio returns. Caloiero & Guidolin (2017) agree that the iPath S&P500 VIX Short-Term Futures Index ETN worsens portfolio performance, but also cast doubt on the efficacy of using direct investment in VIX Index futures as well.

Jones & Allen (2015) find that VIX Index futures prices are highly volatile owing to the absence of a no-arbitrage condition for the underlying VIX Index, particularly at times near to expiry, which has significant implications for the performance of ETNs based on those futures.

Eraker & Wu (2017) study returns from VIX Index futures, ETNs and variance swaps. They find that portfolios of 1-month maturity VIX Index futures lose around 30% per year on average, but that these returns are consistent with the dynamic equilibrium models that they develop, although not with the traditional Capital Asset Pricing Model.

This paper contributes to the literature by examining the effect that the introduction of VIX-related ETNs has had on the term structure of the VIX Index futures market. It looks at returns from trading counter to the short-term VIX–related ETN benchmark and how those returns are related to the value of outstanding ETN shares, both absolutely and relative to the value of open interest in the underlying futures contracts.

2.4 VIX-related Tracker Funds

Barclays launched the iPath S&P500 VIX Short-Term Futures Index Exchange Traded Note on January 30, 2009. The note benchmark comprises first and second nearby futures on the VIX Index, rebalanced daily so as to give a constant weighted average maturity of one calendar month.

As a proxy for the VIX Index, it has suffered a severe contango problem. Since inception to December 1, 2017, a period that saw the VIX Index decline by 74.5%, the notes stand at just 0.03% of their first day closing price, as shown in figures 2.4 and 2.5.

Figure 2.4: iPath VIX Short-Term Futures ETN v. VIX Index, rebased

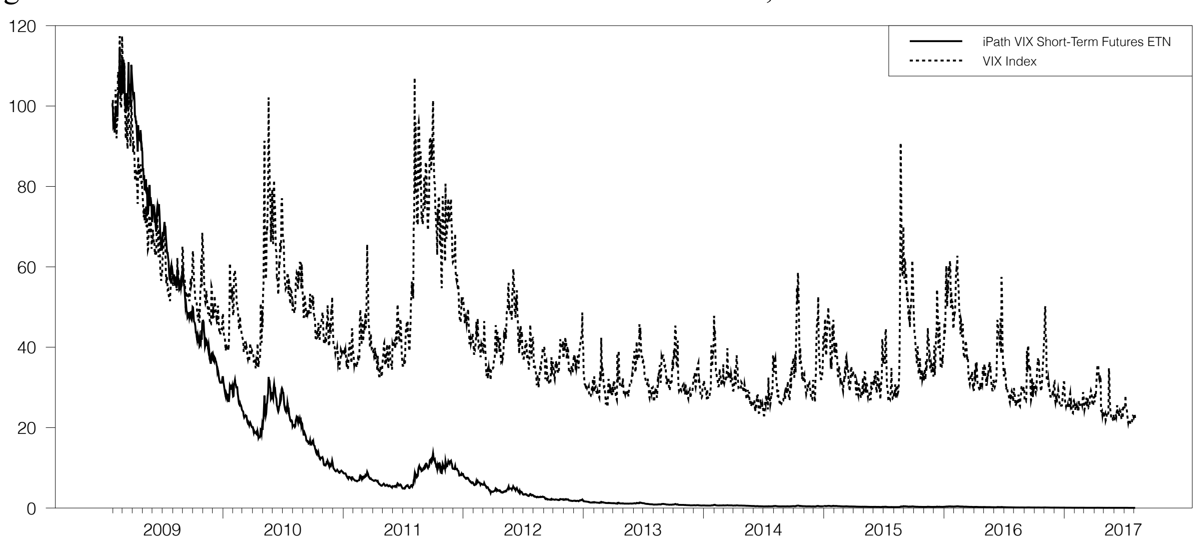
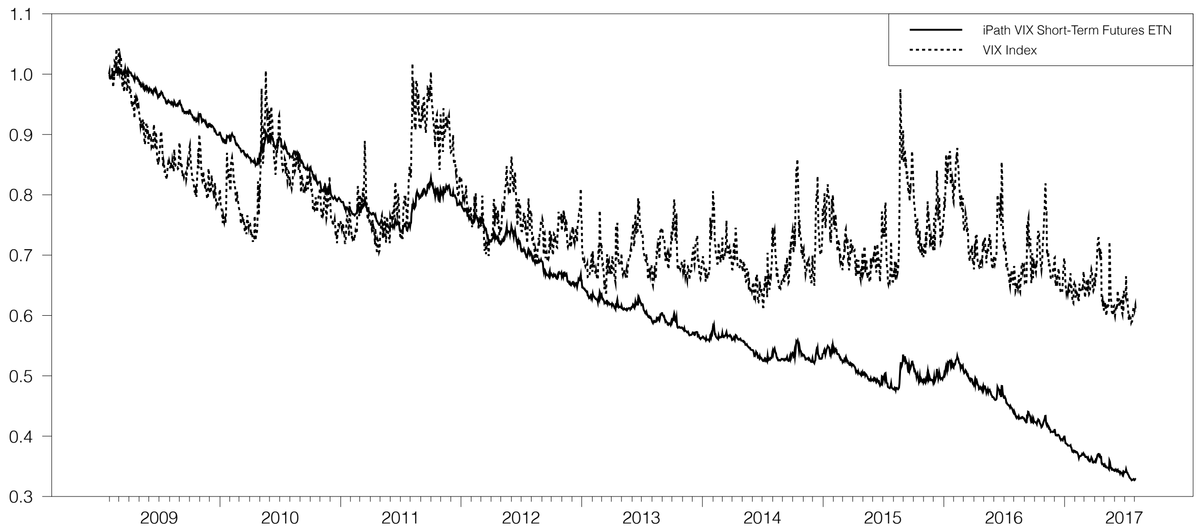


Figure 2.5: iPath VIX Short-Term Futures ETN v. VIX Index, log scale, rebased



Neither the performance of the note, nor the fee of 0.89% pa, appears to deter investors, however, with ETNs outstanding as at December 1, 2017 of 34m with a market value of $1.1bn.

A number of competitor ETNs have been issued subsequent to the iPath product, shown in Table 2.1. As can be seen from Total Net Assets (TNA) as at 12/1/2017, the iPath ETN is the largest, with a share of 53.5% of the market. Annual charges range from 0.85% for the ProShares VIX Short-Term Futures ETP to 1.65% for the VelocityShares Daily 2x VIX Short-Term ETN. All of the products use the first and second nearbys in the manner described above as their benchmark, maintaining an average maturity of one month.

Table 2.1: Price performance of VIX-related Exchange Traded Notes from date of issue

This table shows the performance of selected Short-Term VIX Exchange Traded Notes from date of issue to 12/1/2017. Total net assets (TNA) in millions of dollars as at 12/1/2017 and the change in the VIX Index over the period are also recorded.

ETN	Date Issued	TNA($m)	Performance	VIX
iPath S&P 500 VIX ST Futures ETN	1/30/09	1,145.2	-99.97%	-67.55%
VelocityShares Long VIX ST ETN	11/30/10	12.2	-99.74%	-51.40%
VelocityShares Long 2x VIX ST ETN	11/30/10	301.9	-100.00%	-51.40%
ProShares VIX Short-Term Futures ETN	1/4/11	161.8	-99.70%	-34.23%
ProShares Ultra VIX Short-Term Futures ETN	10/4/11	518.7	-100.00%	-72.00%

Liu & Dash (2012) find that the S&P 500 VIX Short-Term Futures Index, on which the iPath Short-Term VIX Index product is based, lost 0.18% per day on average over the period December 20, 2005 to August 31, 2011, and the S&P 500 VIX Mid-Term Futures Index, on which the iPath Mid-Term VIX Index product is based, lost an average of 0.07% per day over the same period.

The contango problem faced by iPath and other VIX Index ETNs can be seen graphically from Figure 2.6, a plot of the log difference between the first and second nearby futures against the one month forward one-month Libor rate, calculated from the 1 and 2 month US Dollar Libor rates, representing the cost of financing over a period of one month beginning one month hence and which is typically used as a measure of the cost of carry for a fully arbitraged non-dividend paying asset. The second nearby is at a premium to the first nearby of between 0% and 20% for most of the period, with reversals for short periods in 2006, 2007, 2008 (financial crisis), 2010 and 2011. The cost of carry for a fully arbitraged financial non-dividend paying asset is virtually indistinguishable from the horizontal axis at zero owing to the wide variation in observed intertemporal cost of carry between the first and second nearby VIX Index futures. As can be seen from Figure 2.7, the forward rate ranges from near zero to around 0.45% per month, but is swamped by the difference of the first and second nearby futures contracts, the true cost of carry.

Figure 2.6: Log difference, 2nd nearby over 1st nearby v. financing cost based on Libor rate

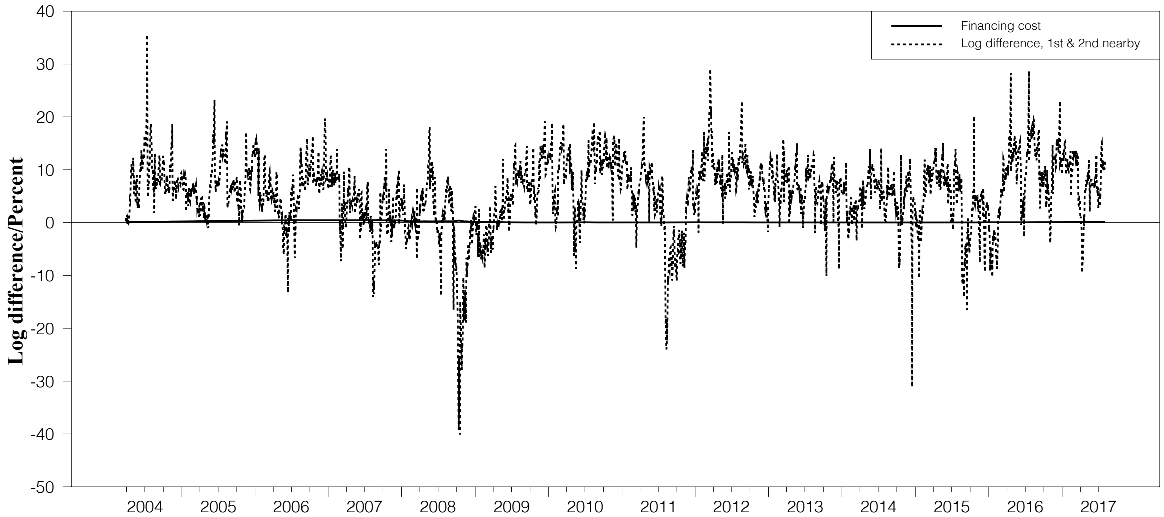


Figure 2.7: One month forward one-month Libor rate

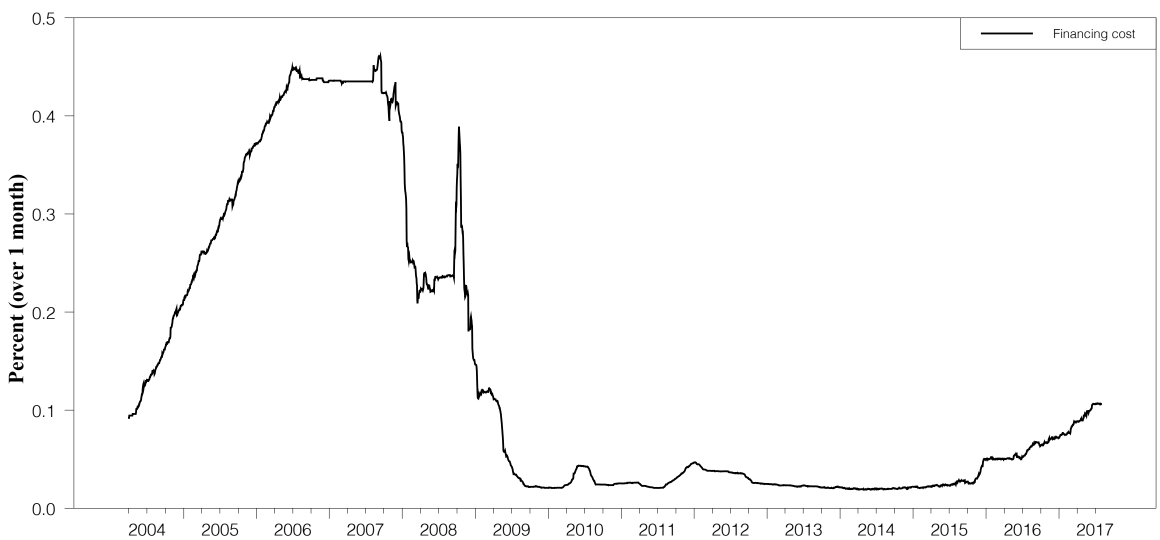
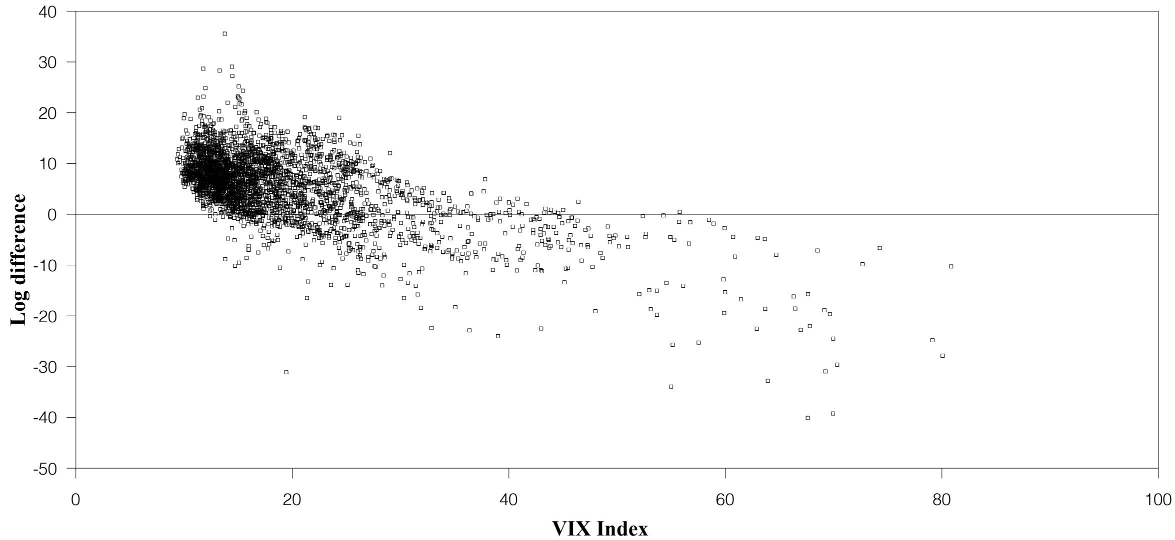


Figure 2.8 shows a scatter plot of the natural log of the second nearby future less the natural log in percent of the first nearby future against the level of the VIX Index. Confirming the pattern of Figure 2.6, most observations show a premium of between 0% and 20% of the second nearby over the first nearby value, which are seen to be concentrated around lower levels

of the VIX Index. It is for higher levels of the VIX that the reverse is seen, with a discount of the second nearby to the first of up to 40%.

Figure 2.8: Log difference, 2nd over 1st nearby v. level of the VIX Index



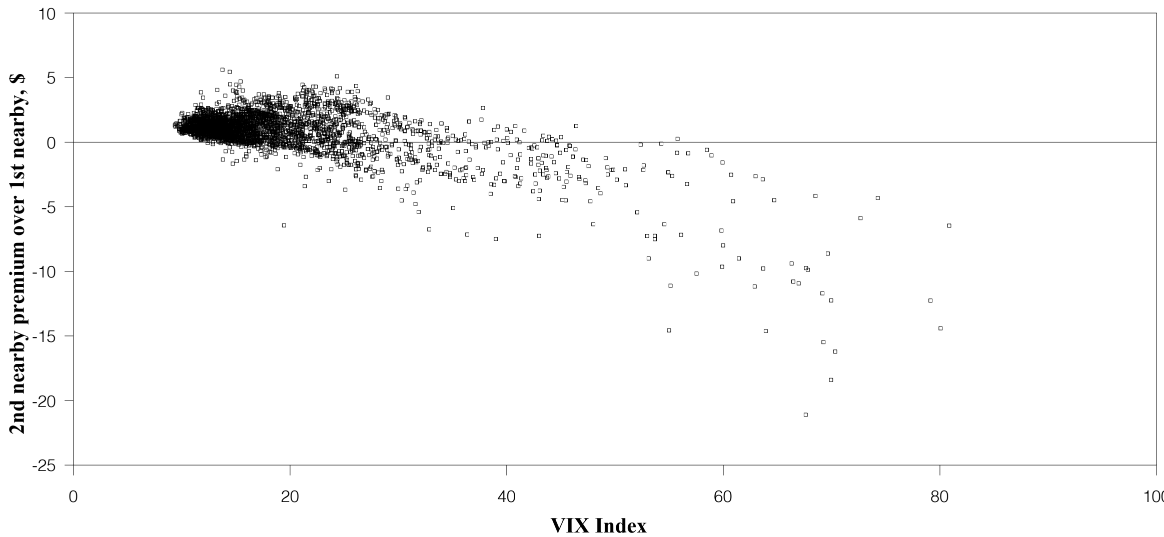
An OLS regression for the period from January 30, 2009, the date at which the iPath S&P 500 VIX Short-Term Futures Index ETN was introduced, to August 4, 2017, of the log difference [ln(2*nb*/1*nb*)] on the level of the VIX Index, expressed in percentage points:

$$\ln(2nb\,/\,1nb) = 14.325 - 0.461VIX$$

gives an adjusted R^2 of 0.3873 with both constant and VIX coefficient significant at the 1% level.

Figure 2.9 is a similar plot based on the simple difference between the second nearby and first nearby. A premium of $0 to $5 is common at lower levels of the VIX Index, with a discount of up to $22 at high levels of the VIX Index.

Figure 2.9: 2nd nearby price less 1st nearby price v. level of the VIX Index



Thus a strong negative correlation between the intertemporal cost of carry and the level of the VIX Index is seen, as expected given mean-reverting behavior.

Three weeks following the introduction of the iPath S&P500 VIX Short-Term Futures Index Exchange Traded Note, Barclays launched a mid-term version, investing in the fourth through to the seventh nearby VIX Index futures to give an average maturity of five months.

Figure 2.10: Log differences, 2nd over 1st nearby, 7th over 4th nearby

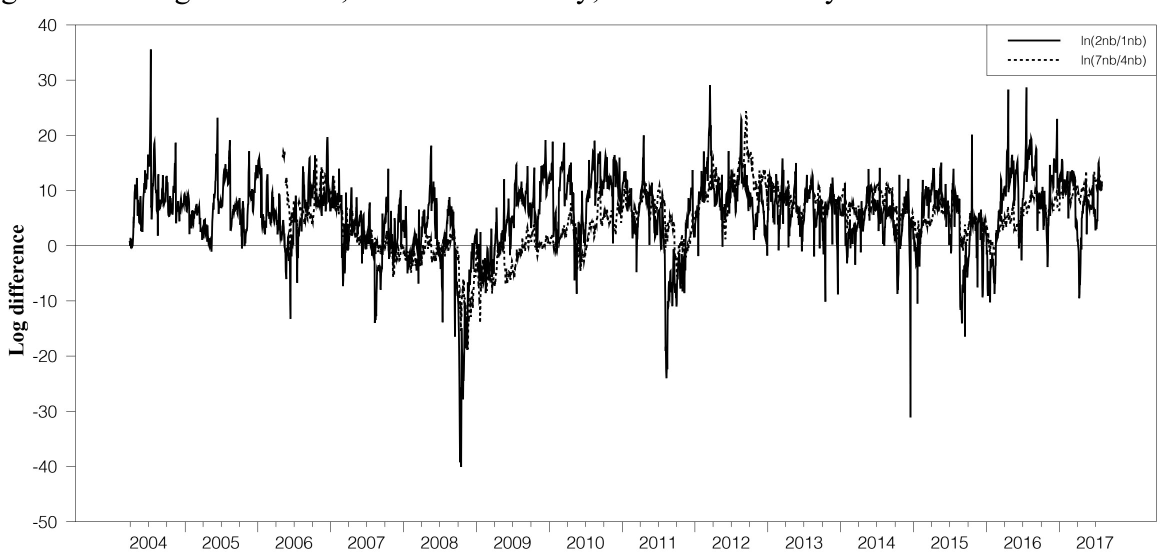


Figure 2.10 compares the log premium of the seventh nearby over the fourth nearby, ln(*7nb/4nb*), with the log premium of the second nearby over the first nearby, ln(*2nb/1nb*). The sample period is shorter as the seventh nearby did not trade prior to April 5, 2006. As the Samuelson hypothesis predicts, the intertemporal cost of carry does not fluctuate as much for the longer-term as for the shorter-term contracts. Scatter plots of the log and simple differences shown in Figures 2.11 and 2.12 display a similar pattern to those for the first and second nearbys, as expected.

Figure 2.11: Log difference of 7th over 4th nearby v. level of the VIX Index

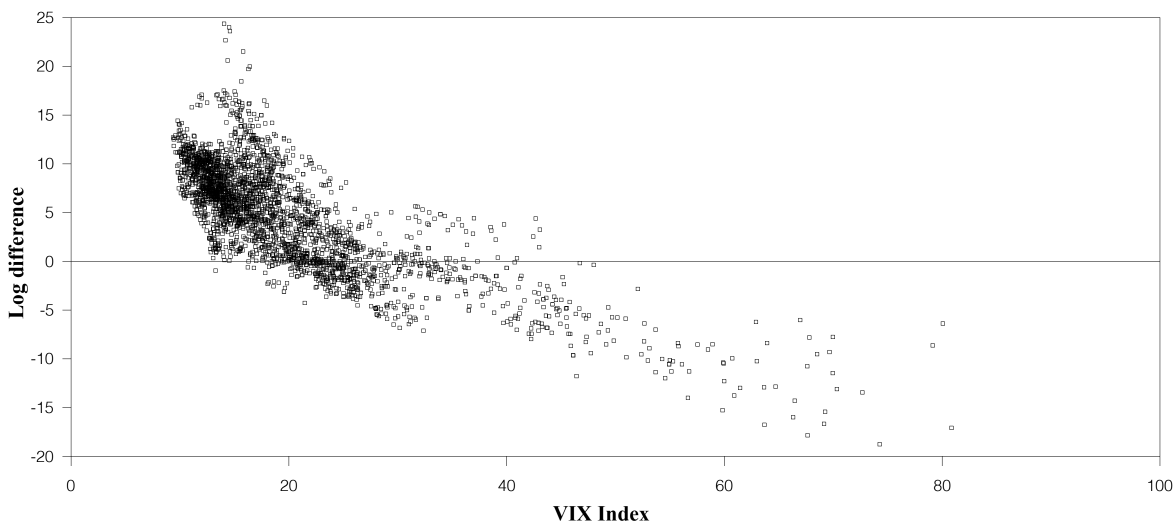
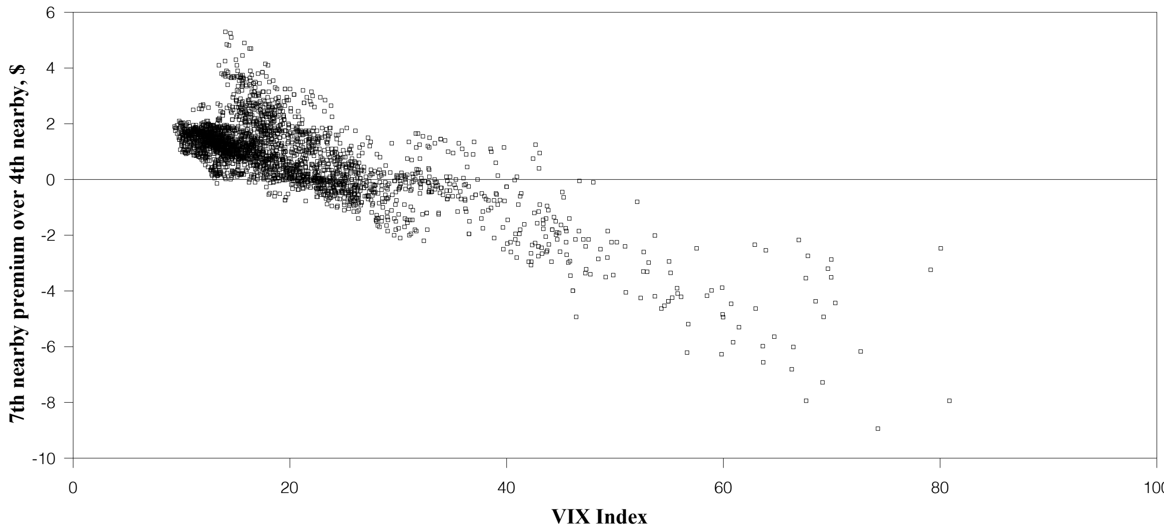


Figure 2.12: 7th nearby price less 4th nearby price v. level of the VIX Index



An OLS regression of the log difference [ln(7*nb*/4*nb*)] expressed as percentage points on the VIX Index:

$$\ln(7nb\,/\,4nb) = 13.502 - 0.443VIX$$

gives an adjusted R^2 of 0.6002 with coefficients significant at the 1% level.

The Mid-Term Notes have not fared as poorly as the Short-Term Notes, declining in value by 81.05% over a period during which the VIX Index fell by 73.39%, the contango problem partly mitigated as each contract is held for three months rather than one; the 7th nearby is bought daily, and the 4th nearby sold. Hence the portfolio comprises around a third in each of the 6th and 5th nearbys, with the balance divided between the 7th and 4th in proportion to give an average five month maturity equivalent.

2.5 Trading Strategies

VIX Index futures expire on the Wednesday that is 30 days prior to the third Friday of the following month, and hence the nth nearby becomes the (n-1)th nearby on or around the 18th day of each month.

The iPath S&P 500 VIX Short-Term Futures Index ETN, and others operating in the same manner, effectively buys the second nearby and sells the first nearby futures contract on a daily basis so as to give an average maturity of one calendar month.

The iPath S&P 500 VIX Mid-Term Futures Index Exchange Traded Note buys the seventh nearby and sells the fourth nearby futures contract on a daily basis so as to give an average maturity of five calendar months.

From January 2009, therefore, iPath, where seeking to hedge its position, would have been a buyer of the second and seventh nearbys and a seller of the first and fourth nearbys.

Table 2.2 shows the mean cash flow in dollars per underlying unit of index from strategies of selling the (n+1)th nearby and buying the nth nearby futures contract on a daily basis for n from 1 to 6, also from selling the seventh and buying the fourth nearby, along with the standard deviation of cash flows and an adjusted mean, defined as mean cash flow divided by standard deviation. The standard contract size for VIX futures is $1,000, so the mean cash flow from taking a short position in the second nearby and a long position in the first nearby for the full period is $1,000 x 0.844 = $844 per contract. The range of cash flows is given, and the percentage of cash flows that were positive.

Hence, for example, a daily strategy of selling the second nearby and buying the first nearby futures contract ("–2+1"), the opposite of the trades carried out for the iPath short-term index and thus effectively providing liquidity for the ETN trades, for the period from 3/30/2004 to 8/4/2017 has produced an average positive flow of $844 per contract (multiplier $1,000), with a standard deviation of $1,788. Flows range from –$21,100 to $5,610 with a positive flow from 83.8% of trades. There is a positive mean return in all cases, as would be expected owing to the negative correlation of the VIX Index with the S&P 500 Index.

Table 2.2: Cash flows from trading strategies

This table shows the mean cash flows from selling the n+1th nearby and buying the nth nearby daily, along with the cash flows from selling the 7th nearby and buying the 4th nearby. Mean and standard deviation figures are in $ by quoted price. Contract size is 1,000, so the mean cash flow from selling the second nearby and buying the first nearby daily for the full period is $1,000 x 0.844 = $844

Full data set, 3/29/2004–8/4/2017

Strategy	Mean	Std. deviation	Adj. Mean	Range	% Positive	Obs.
"-2+1"	0.844	1.788	0.472	-21.10/5.61	83.8%	3375
"-3+2"	0.653	1.137	0.575	-10.25/4.05	81.8%	3375
"-4+3"	0.466	0.776	0.600	-4.88/2.40	81.3%	3330
"-5+4"	0.355	0.638	0.556	-4.92/2.40	78.9%	2885
"-6+5"	0.288	0.525	0.548	-2.72/1.95	78.5%	2875
"-7+6"	0.286	0.433	0.660	-2.8/1.85	78.7%	2840
"-7+4"	0.915	1.389	0.658	-8.94/5.30	79.2%	2840

Period preceding appearance of first VIX-related ETN, 3/29/2004–1/29/2009

Strategy	Mean	Std. deviation	Adj. Mean	Range	% Positive	Obs.
"-2+1"	0.382	2.178	0.175	-21.10/5.61	80.3%	1211
"-3+2"	0.313	1.254	0.249	-10.25/2.59	76.9%	1211
"-4+3"	0.247	0.962	0.257	-4.88/1.68	72.3%	1166
"-5+4"	-0.079	0.776	-0.102	-4.92/1.19	57.0%	721
"-6+5"	-0.042	0.593	-0.070	-2.72/1.43	58.5%	711
"-7+6"	0.112	0.488	0.229	-2.80/1.44	60.5%	676
"-7+4"	-0.110	1.606	-0.069	-8.94/2.72	56.7%	676

Period from appearance of first VIX-related ETN, 3/30/2009–8/4/2017

Strategy	Mean	Std. deviation	Adj. Mean	Range	% Positive	Obs.
"-2+1"	1.102	1.464	0.753	-15.61/5.45	85.7%	2164
"-3+2"	0.844	1.017	0.830	-6.44/4.05	84.6%	2164
"-4+3"	0.584	0.623	0.937	-2.30/2.40	86.1%	2164
"-5+4"	0.499	0.509	0.981	-1.70/2.40	86.2%	2164
"-6+5"	0.396	0.451	0.878	-1.70/1.95	85.0%	2164
"-7+6"	0.340	0.399	0.852	-1.15/1.85	84.4%	2164
"-7+4"	1.235	1.139	1.085	-3.45/5.30	86.3%	2164

Carrying out such strategies would require the maintenance of a net short position over time, which would incur margin costs. Combined initial and maintenance margin has ranged from around $4,000 to as much as $20,000 per contract since VIX Index futures have been traded, and as at 1/10/2018 stands at around $12,000 per contract. There is theoretically little interest cost, as interest is generally paid on margin accounts, or indeed none at all, where, for example, Treasury bills are posted as margin. Were the strategy to be carried out, going long and shorting one contract each day, however, positions totaling around 60 contracts would be open at any one time, in which case the opportunity cost of posting the margin requirement would need to be considered. The main concern of this paper is on the positivity and fluctuation in cash flows, however, and therefore the cost of the margin requirement, which would apply equally to the reverse strategy, as represented by the ETN benchmark, has not been incorporated into the calculations.

In line with the predictions of the Samuelson Hypothesis, standard deviation of cash flows decreases with maturity. The longer maturity returns are not truly comparable as there are a number of missing observations, however adjusted mean cash flows all lie within a relatively narrow range of 0.548 to 0.660.

The data are then separated into the periods before and after the launch of the iPath S&P 500 VIX Short-Term Futures Index ETN in January 2009.

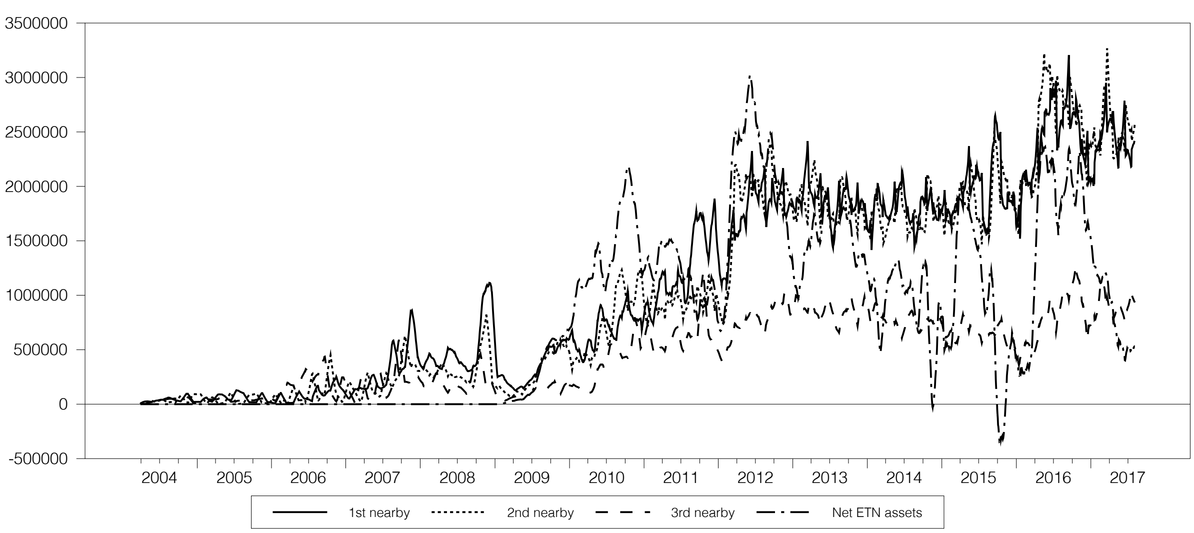
The mean return from the "–2+1" strategy is much higher for the later period than the earlier, although the adjusted mean, which by construction is the t-statistic for the test of the mean being different from zero, shows no significance. Means for the remaining strategies are also higher, which likely reflects a change in the futures curve as a whole between the two periods. However, the differences seen between different strategies across the two periods suggest there may be localized effects across maturities.

The greatest proportional increases in adjusted mean are to be seen with the "–2+1" and "–7+4" strategies, both of which engage in the reverse of the iPath trades. There is therefore evidence that the market is working against the iPath funds, exacerbating the contango issue. Furthermore, there is evidence that the potential returns from a "–2+1" strategy have increased substantially in the presence of ETN trading. Given the negative correlation between the VIX Index and the S&P 500 Index, this strategy is, however, highly correlated with S&P 500 Index performance.

2.6 Size Effects

Figure 2.13 shows the open interest in the first, second and third nearbys by value, along with the combined net asset value of ETNs investing in the first and second nearbys.

Figure 2.13: Open interest of 1st, 2nd and 3rd nearbys, ETN net asset value, 21-day rolling window

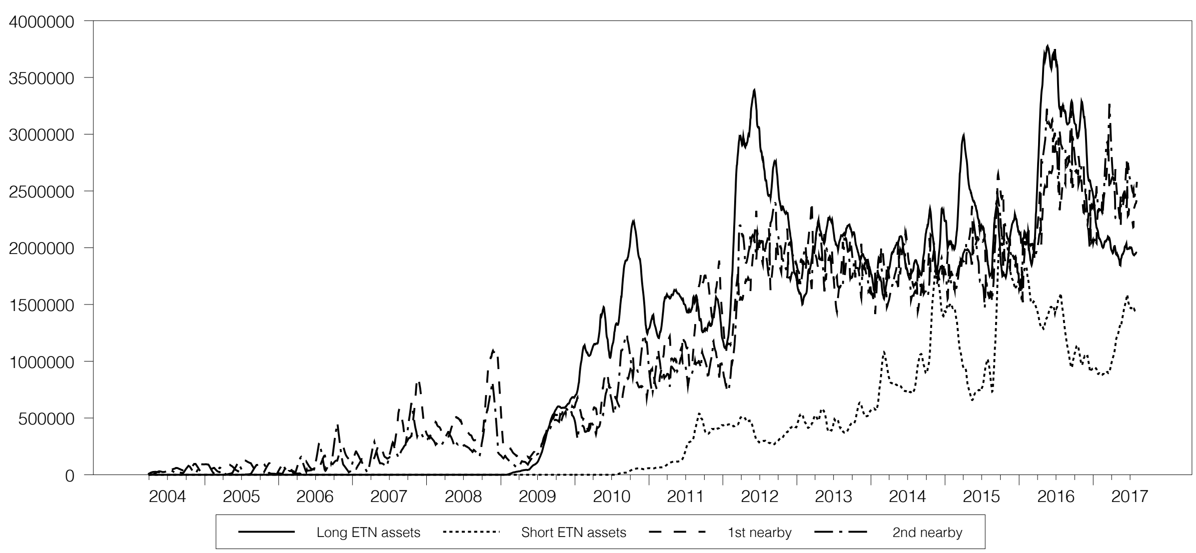


The introduction of the iPath S&P500 VIX Short-Term Futures Index ETN coincided, as would be expected, with an increase in the open interest in both the first and second nearby futures contracts. The third nearby open interest also increases in value, but by a lesser amount.

The ETN net assets figure is calculated as the sum of the total net assets of the funds using the first and second nearbys, with twice the asset figure used for the leveraged funds: the VelocityShares Daily 2x VIX Short Term ETN and ProShares Ultra VIX Short Term Futures ETN, less the asset values of inverse short-term VIX related ETPs and ETNs: the iPath Inverse S&P500 VIX Short Term Futures ETN, ProShares Short VIX Short Term Futures ETN and VelocityShares Daily Inverse VIX Short Term ETN. The net asset figure becomes negative for a period in 2015 owing to the increasing popularity of inverse short-term VIX related ETNs, which have had varying success in replicating the inverse return of those ETNs offering long exposure.

Figure 2.14 shows ETN net asset value divided into short and long issues, from which it can be seen that the long issues have a closer relationship to futures open interest than do total net assets, which includes the (negative) value of short issues.

Figure 2.14: Long and short ETN net asset value, 21-day rolling window



This is confirmed by ordinary least squares regressions of the sum of the value of open interest in the first and second nearbys on long only ETN total assets (LongTA), compared with short ETN assets alone (ShortTA) and total net assets (NetTA).

For long issues, with open interest being the sum of the value of open interest in the first and second nearbys, calculated by multiplying the number of contracts outstanding of each by the closing price of the relevant futures contract, in $m, and LongTA being net assets in long short-term VIX ETNs in $m:

$$\text{Open interest} = 345 + 1.51\ \text{LongTA}$$

with adjusted R^2 of 0.7062

For short issues alone, with net assets as negative figures:

$$\text{Open interest} = 1{,}969 - 1.86\ \text{ShortTA}$$

with adjusted R^2 of 0.5167

and for total net assets,:

$$\text{Open interest} = 2{,}189 + 0.80\ \text{NetTA}$$

with adjusted R^2 of 0.1511.

It may be noted that if long assets are equal to short assets, then no futures contract holdings are required at all in aggregate to fully hedge overall positions. All the larger managers issue both long and short ETNs, which will offset for hedging purposes, but the relatively close relationship between open interest and long ETN assets suggests that it is these that have the greatest effect on the futures market. This may be due to different managers having the bulk of the long (iPath) and inverse (Credit Suisse) ETNs in issue.

A summary of open interest and net asset data in $m for the period beginning 3/30/2004 to 8/4/2017 is shown in Table 2.3. The most broadly traded futures contracts are the first two nearbys, and there is also substantial volume in the third, fourth and fifth nearbys when traded. The analysis that follows is restricted to the first three nearbys, these being the most relevant for the effects of short-term tracking funds. There are multicollinearity issues with the longer maturity contracts, and their exclusion does not affect the overall result.

Table 2.3: Futures open interest by value and Exchange Traded Note net asset data 3/30/2004–8/4/2017

A summary of the futures open interest by value and net asset value data. Open interest is number of contracts outstanding multiplied by the closing price. Total ETN net assets (NA) is the sum of market capitalizations of all short-term VIX-related Exchange Traded Notes. Long ETN assets (A) is the sum of the market capitalizations of all long short-term VIX-related Exchange Traded Notes, i.e., those designed to give positive exposure to the VIX futures market. Short ETN assets is the sum of the market capitalizations of all inverse short-term VIX-related Exchange Traded Notes, i.e., those designed to give negative exposure to the VIX futures market. All figures are in $m.
Number of observations (Obs), the mean, standard deviation (SD), minimum and maximum are recorded.

Variable	N	Mean	SD	Min	Max
1st nearby	3,332	1,099.99	1,024.69	1.15	4,690.95
2nd nearby	3,330	1,060.25	1027.83	0.11	5,496.55
3rd nearby	3,337	462.11	351.72	0.06	1,815.68
4th nearby	3,295	333.97	268.94	0.5	1,183.90
5th nearby	2,848	321.00	245.45	0.14	1202.52
6th nearby	2,802	247.57	207.73	0.07	994.81
7th nearby	2,799	139.31	126.83	0.02	786.87
8th nearby	2,539	46.41	47.36	0.02	431.25
9th nearby	1,712	11.31	15.49	0.02	126.34
10th nearby	90	1.12	1.18	0.02	4.91
Total ETN NA	2,132	1,223.63	716.21	–851.66	3948.98
Long ETN A	2,132	1,865.95	816.16	5.00	4,741.39
Short ETN A	1,768	774.56	533.35	4.00	2,356.68

Summary statistics for the variables of interest over the full data set from 3/30/2004 to 8/4/2017 are shown in Table 2.4, and the same figures for the period from the introduction of the iPath short-term ETN in Table 2.5. In the latter period can be seen the substantially higher mean volume by value in the first and second nearbys, and higher cash flow coupled with higher mean and lower standard deviation of cash flow for the "–2+1" strategy, as seen in Table 2.2.

Table 2.4: Strategy and open interest summary data, 3/30/2004–8/4/2017

A summary of the data used in Tables 2.6 to 2.13.
Strategy "–2+1" is the daily cash flow from selling the first nearby and buying the second nearby. Mean and standard deviation are in $ per contract. The VIX Index and log daily changes in the VIX Index, VIX first difference, are included.
Futures open interest for the first three nearbys are as shown in Table 2.3, the number of contracts outstanding multiplied by the closing price recorded in $m. Total ETN net assets (NA) is the sum of market capitalizations of all short-term VIX-related Exchange Traded Notes. Long ETN assets (A) is the sum of the market capitalizations of all long short-term VIX-related Exchange Traded Notes, i.e., those designed to give positive exposure to the VIX futures market. Short ETN assets is the sum of the market capitalizations of all inverse short-term VIX-related Exchange Traded Notes, i.e., those designed to give negative exposure to the VIX futures market. All ETN figures are in $m.
Number of observations (Obs), mean, standard deviation (SD), minimum and maximum are recorded.

Variable	N	Mean	SD	Min	Max
Strategy "–2+1"	3,341	854.55	1752.37	–21,100.00	5,610.00
VIX Index	3,341	18.75	9.00	9.36	80.06
VIX first difference	3,341	0.00	0.07	–0.30	0.64
1st nearby	3,332	1,099.99	1,024.69	1.15	4,690.95
2nd nearby	3,330	1,060.25	1027.83	0.11	5,496.55
3rd nearby	3,337	462.11	351.72	0.06	1,815.68
Total ETN NA	2,132	1,223.63	716.21	–851.66	3948.98
Long ETN A	2,132	1,865.95	816.16	5.00	4,741.39
Short ETN A	1,768	774.56	533.35	4.00	2,356.68

Table 2.5: Strategy and open interest summary data, 1/30/2009–8/4/2017

Identical to Table 2.4, but restricted to the period following the introduction of VIX-related Exchange Traded Notes on 1/30/2009.

Variable	N	Mean	SD	Min	Max
Strategy "–2+1"	2,132	1,120.09	1389.24	–7,500.00	5,450.00
VIX Index	2,132	18.78	7.39	9.36	52.65
VIX first difference	2,132	0.00	0.08	–0.30	0.50
1st nearby	2,123	1,605.58	953.08	103.64	4,690.95
2nd nearby	2,130	1,569.55	956.65	37.06	5,496.55
3rd nearby	2,129	646.62	300.29	29.11	1,815.68
Total ETN NA	2,132	1,223.63	716.21	–851.66	3948.98
Long ETN A	2,132	1,865.95	816.16	5.00	4,741.39
Short ETN A	1,768	774.56	533.35	4.00	2,356.68

Results of regressions of the "–2+1" strategy are shown in Table 2.6. Column 1 gives the result of an Ordinary Least Squares regression across the whole sample of the net assets of short-term ETNs on the daily cash flow of selling the second nearby and buying the first nearby future, and shows a positive relationship between cash flow from the strategy and the value of the funds. Net assets prior to the introduction of the ETNs and missing values for open interest are taken as zero.

Column 2 includes the VIX Index level and first difference of the VIX Index, indicating a negative relationship in each case between the "–2+1" strategy and the two VIX Index parameters. This is as expected as the futures curve tends to move into backwardation when the index is substantially higher than its long run mean.

Table 2.6: Determinants of strategy profit, ETN net assets, 3/30/2004–8/4/2017

Results of Ordinary Least Squares regressions of the daily cash flow of selling the second nearby and buying the first nearby in $ on Exchange Traded Notes total net assets in $m. Net assets prior to introduction of VIX-related Exchange Traded Notes on 30/1/2009 are taken as zero. ETN is an indicator variable that takes the value 1 from this date. Adjusted R^2 is shown as r2_a.

Dependent Variable −2+1 strategy in $, period 3/30/04 − 8/4/17

	(1)	(2)	(3)	(4)	(5)	(6)
	A	B	C	D	E	F
Total ETN net assets	**0.6264*** **			**0.4847*** **	**1.1872*** **	**1.2256*** **
	(0.0353)			(0.0282)	(0.0501)	(0.0372)
VIX Index		−119.0368***	−119.2053***	−113.0600***		−120.0973***
		(2.6637)	(2.5726)	(2.5773)		(2.3516)
VIX first difference		−1125.3757***	−1090.9664***	−1684.6935***		−1955.8686***
		(329.3810)	(318.1283)	(317.4612)		(286.1885)
ETN			744.6974***			
			(47.9213)			
1st nearby value					−0.3530***	−0.3670***
					(0.0372)	(0.0275)
2nd nearby value					0.2120***	0.0132
					(0.0541)	(0.0402)
3rd nearby value					−1.5426***	−1.6173***
					(0.1772)	(0.1310)
Constant	365.4043***	3088.7143***	2616.8000***	2599.5047***	802.7311***	3290.9892***
	(40.0210)	(55.2705)	(61.4144)	(60.1674)	(47.2694)	(59.9654)
Observations	3341	3341	3341	3341	3319	3319
r2_a	0.0857	0.3813	0.4229	0.4313	0.1581	0.5405

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

An indicator variable ETN that takes a value of 1 for dates following the introduction of the iPath S&P500 VIX Short-Term Futures ETN on 1/30/2009 is added in column 3. The coefficient is positive and strongly significant, confirming that the strategy has a significantly greater return following the introduction of the ETN compared to the prior period.

Columns 4 to 6 show that the significance of the effect of fund net assets is maintained when the VIX Index measures and values of open interest in the first three nearbys are included as controls. When all controls are included, an increase of $1m in net asset value corresponds to an increase of $1.2256 in cash flow from the strategy with adjusted R^2 of 0.5405.

Restricting the regression to the period following the introduction of the iPath S&P500 VIX Short-Term Futures Index ETN on 1/30/2009, shown in Table 2.7, the coefficient on Net TNA when controls are included is 0.8871, indicating an increase in cash flow of $0.8871 for an increase of $1m in ETN net assets, with adjusted R^2 of 0.5536.

For the period from 1/1/2010 onwards, when the iPath S&P500 VIX Short-Term Futures Index ETN had become more established, the coefficient on ETN net assets when all controls are included is 0.9785, indicating an increase of $0.9785 in cash flow, again significant at the 1% level with adjusted R^2 of 0.5569 (Table 2.8), and from 1/1/2011, eliminating those observations where net assets exceeds the value of open interest in the first two nearbys, the coefficient on ETN net assets is 0.8434, corresponding to an increase in cash flow of $0.8434 per $1m of assets, significant at the 1% level with adjusted R^2 of 0.6040 (Table 2.9).

Table 2.7: Determinants of strategy profit, ETN net assets, 1/30/2009–8/4/2017

Results of Ordinary Least Squares regressions of the daily cash flow of selling the second nearby and buying the first nearby in $ on Exchange Traded Notes total net assets in $m for the period following the introduction of the first VIX-related Exchange Traded Notes. Variables are as for Table 2.6.

Dependent Variable −2+1 strategy in $, period 1/30/09 – 8/4/17

	(1) A	(2) B	(3) C	(4) D	(5) E
Total ETN net assets	**0.6515*** **		**0.5087*** **	**0.9080*** **	**0.8871*** **
	(0.0396)		(0.0387)	(0.0417)	(0.0317)
VIX Index		**−78.9404*** **	**−64.0536*** **		**−136.3575*** **
		(3.6952)	(3.7304)		(3.5271)
VIX first difference		**−1493.4608*** **	**−2438.6111*** **		**−1393.8181*** **
		(361.6260)	(355.1972)		(275.4025)
1st nearby value				**−0.2522*** **	**−0.5779*** **
				(0.0307)	(0.0245)
2nd nearby value				**0.2059*** **	**−0.1452*** **
				(0.0452)	(0.0351)
3rd nearby value				**−1.4621*** **	**−1.9294*** **
				(0.1544)	(0.1162)
Constant	**324.1838*** **	**2606.5227*** **	**1706.2769*** **	**1036.9988*** **	**4999.6786*** **
	(56.0925)	(74.3820)	(99.0267)	(72.1916)	(117.3565)
Observations	2131	2131	2131	2119	2119
r2_a	0.1126	0.1891	0.2498	0.2037	0.5536

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 2.8: Determinants of strategy profit, ETN net assets, 1/1/2010–8/4/2017

Results of Ordinary Least Squares regressions of the daily cash flow of selling the second nearby and buying the first nearby in $ on Exchange Traded Notes total net assets in $m for the period from 1/1/2010 to 8/4/2017. Variables are as for Table 2.6.

Dependent Variable -2+1 strategy in $, period 1/1/10 - 8/4/17

	(1) A	(2) B	(3) C	(4) D	(5) E
Total ETN net assets	0.7617***		0.8076***	0.8600***	0.9785***
	(0.0421)		(0.0371)	(0.0410)	(0.0315)
VIX Index		-99.3510***	-96.5201***		-141.4841***
		(4.7695)	(4.2699)		(3.9781)
VIX first difference		-1078.0412**	-2358.5768***		-1504.4518***
		(358.9705)	(326.5636)		(272.6964)
1st nearby value				-0.2563***	-0.4891***
				(0.0307)	(0.0243)
2nd nearby value				0.1908***	-0.1486***
				(0.0432)	(0.0340)
3rd nearby value				-1.4391***	-1.4589***
				(0.1559)	(0.1178)
Constant	97.2174	2841.5544***	1718.5699***	1108.5724***	4416.4946***
	(62.8734)	(87.1698)	(93.5203)	(100.4506)	(122.6750)
Observations	1903	1903	1903	1891	1891
r2_a	0.1465	0.1984	0.3581	0.2240	0.5569

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 2.9: Determinants of strategy profit, ETN net assets, 1/1/2011–8/4/2017

Results of Ordinary Least Squares regressions of the daily cash flow of selling the second nearby and buying the first nearby in $ on Exchange Traded Notes total net assets in $m for the period from 1/1/2011 to 8/4/2017. Variables are as for Table 2.6.

Dependent Variable −2+1 strategy in $, period 1/1/11 – 8/4/17

	(1) A	(2) B	(3) C	(4) D	(5) E
Total ETN net assets	**0.7247***** **(0.0404)**		**0.7293***** **(0.0302)**	**0.7537***** **(0.0417)**	**0.8434***** **(0.0295)**
VIX Index		**−142.5225***** **(4.4466)**	**−137.1807***** **(3.8281)**		**−152.5774***** **(3.8438)**
VIX first difference		**−363.7193** **(316.9437)**	**−1655.0301***** **(277.5866)**		**−1259.5052***** **(262.2385)**
1st nearby value				**−0.0701*** **(0.0352)**	**−0.2928***** **(0.0253)**
2nd nearby value				**0.2621***** **(0.0437)**	**−0.0751*** **(0.0317)**
3rd nearby value				**−0.9915***** **(0.1724)**	**−0.9863***** **(0.1203)**
Constant	**−8.4208** **(60.5023)**	**3316.0283***** **(77.5717)**	**2266.2914***** **(79.5522)**	**348.2370*** **(136.2304)**	**3816.2786***** **(131.6703)**
Observations	**1654**	**1654**	**1654**	**1647**	**1647**
r2_a	**0.1627**	**0.3916**	**0.5505**	**0.1866**	**0.6040**

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

The results of carrying out the same regression with long only ETN assets in place of total net total assets are shown in Table 2.10. The regression for the full period from 3/30/04 to 8/4/17 with all controls shows an increase in cash flow from the strategy of $1.5293 with adjusted R^2 of 0.5151. Tables 2.11 to 2.13 show the results for the periods from 2009, 2010 and 2011, the coefficients of Long TNA higher than that for Net TNA in each case, ranging from 0.9468 to 1.2239 with adjusted R^2 from 0.4757 to 0.5461. Given that long only ETN assets must always be greater than or equal to the total ETN net assets figure, as inverse ETN assets are subtracted from total net assets to give long ETN assets, the economic significance of using long only ETN assets is therefore greater than that for total ETN net assets.

Table 2.10: Determinants of strategy profit, long ETN assets, 3/30/2004–8/4/2017

Results of Ordinary Least Squares regressions of the daily cash flow of selling the second nearby and buying the first nearby in $ on long only Exchange Traded Notes assets in $m. Assets prior to introduction of VIX-related Exchange Traded Notes on 30/1/2009 are taken as zero. ETN is an indicator variable that takes the value 1 from this date. Adjusted R^2 is shown as r2_a.

Dependent Variable -2+1 strategy in $, period 3/30/04 - 8/4/17

	(1) A	(2) B	(3) C	(4) D	(5) E	(6) F
Long ETN assets	0.3267***			0.1397***	1.7530***	1.5293***
	(0.0268)			(0.0219)	(0.0675)	(0.0523)
VIX Index		-119.0368***	-119.2053***	-115.4126***		-113.1017***
		(2.6637)	(2.5726)	(2.7082)		(2.4286)
VIX first difference		-1125.3757***	-1090.9664***	-1269.9417***		-1527.4718***
		(329.3810)	(318.1283)	(328.2212)		(293.0888)
ETN			744.6974***			
			(47.9213)			
1st nearby value					-0.9641***	-0.8574***
					(0.0487)	(0.0376)
2nd nearby value					-0.5720***	-0.6335***
					(0.0643)	(0.0495)
3rd nearby value					-1.0590***	-1.0307***
					(0.1707)	(0.1315)
Constant	465.5693***	3088.7143***	2616.8000***	2854.7437***	920.5913***	3247.2665***
	(43.5382)	(55.2705)	(61.4144)	(66.0541)	(46.8572)	(61.6180)
Observations	3341	3341	3341	3341	3319	3319
r2_a	0.0424	0.3813	0.4229	0.3886	0.1820	0.5151

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

These results suggest that the slope of the futures curve between the first and second nearby, is steeper following the introduction of VIX-related ETNs in January 2009. Since that time, the size of the ETNs operating in the futures market has a positive correlation with the slope of the futures curve between the first and second nearby. The greater the ETN assets, both net and for the long VIX-related ETNs alone, the greater the intertemporal carry cost and the more profitable the strategy of dealing against the benchmark, and the poorer the performance of the ETN's benchmark index.

Table 2.11: Determinants of strategy profit, long ETN assets, 1/30/2009–8/4/2017

Results of Ordinary Least Squares regressions of the daily cash flow of selling the second nearby and buying the first nearby in $ on long only Exchange Traded Notes assets in $m for the period following the introduction of the first VIX-related Exchange Traded Notes. Variables are as for Table 2.10.

Dependent Variable -2+1 strategy in $, period 1/30/09 - 8/4/17

	(1) A	(2) B	(3) C	(4) D	(5) E
Long ETN assets	0.1964***		-0.2529***	1.0231***	0.9468***
	(0.0366)		(0.0398)	(0.0643)	(0.0503)
VIX Index		-78.9404***	-94.5058***		-138.0544***
		(3.6952)	(4.4061)		(3.8232)
VIX first difference		-1493.4608***	-1032.2096**		-770.3674**
		(361.6260)	(365.6155)		(296.4089)
1st nearby value				-0.5819***	-0.8845***
				(0.0419)	(0.0335)
2nd nearby value				-0.2604***	-0.5808***
				(0.0564)	(0.0446)
3rd nearby value				-0.8637***	-1.3458***
				(0.1579)	(0.1233)
Constant	755.1640***	2606.5227***	3369.8139***	1112.2758***	5146.6904***
	(74.6175)	(74.3820)	(140.9851)	(76.0065)	(127.4617)
Observations	2131	2131	2131	2119	2119
r2_a	0.0129	0.1891	0.2039	0.1292	0.4757

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 2.12: Determinants of strategy profit, long ETN net assets, 1/1/2010–8/4/2017

Results of Ordinary Least Squares regressions of the daily cash flow of selling the second nearby and buying the first nearby in $ on long only Exchange Traded Notes total net assets in $m for the period from 1/1/2010 to 8/4/2017. Variables are as for Table 2.6.

Dependent Variable -2+1 strategy in $, period 1/1/10 - 8/4/17

	(1) A	(2) B	(3) C	(4) D	(5) E
Long ETN assets	0.3023***		0.0943*	1.0039***	1.2239***
	(0.0473)		(0.0450)	(0.0667)	(0.0527)
VIX Index		-99.3510***	-96.3821***		-144.6768***
		(4.7695)	(4.9715)		(4.3256)
VIX first difference		-1078.0412**	-1214.3396***		-901.4381**
		(358.9705)	(364.5029)		(293.6574)
1st nearby value				-0.5699***	-0.8835***
				(0.0400)	(0.0326)
2nd nearby value				-0.2698***	-0.7194***
				(0.0551)	(0.0449)
3rd nearby value				-0.7828***	-0.6962***
				(0.1631)	(0.1274)
Constant	492.2997***	2841.5544***	2596.6216***	1070.8317***	4394.8977***
	(101.8332)	(87.1698)	(145.7793)	(111.6222)	(134.5503)
Observations	1903	1903	1903	1891	1891
r2_a	0.0205	0.1984	0.1998	0.1460	0.4792

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 2.13: Determinants of strategy profit, long ETN net assets, 1/1/2011–8/4/2017

Results of Ordinary Least Squares regressions of the daily cash flow of selling the second nearby and buying the first nearby in $ on long only Exchange Traded Notes total net assets in $m for the period from 1/1/2011 to 8/4/2017. Variables are as for Table 2.6.

Dependent Variable -2+1 strategy in $, period 1/1/11 - 8/4/17

	(1) A	(2) B	(3) C	(4) D	(5) E
Long ETN assets	**0.6212*** **		**0.4374*** **	**0.8790*** **	**1.1001*** **
	(0.0481)		**(0.0391)**	**(0.0676)**	**(0.0490)**
VIX Index		**-142.5225*** **	**-132.9237*** **		**-157.9570*** **
		(4.4466)	**(4.3738)**		**(4.1292)**
VIX first difference		**-363.7193**	**-976.7580** **		**-721.8813** **
		(316.9437)	**(310.5687)**		**(278.6820)**
1st nearby value				**-0.3046*** **	**-0.6128*** **
				(0.0448)	**(0.0331)**
2nd nearby value				**-0.1355* **	**-0.5915*** **
				(0.0567)	**(0.0422)**
3rd nearby value				**-0.2255**	**-0.1108**
				(0.1775)	**(0.1274)**
Constant	**-384.2331*** **	**3316.0283*** **	**2218.9689*** **	**66.0708**	**3567.8489*** **
	(107.5106)	**(77.5717)**	**(123.4360)**	**(147.6418)**	**(142.5922)**
Observations	**1654**	**1654**	**1654**	**1647**	**1647**
r2_a	**0.0911**	**0.3916**	**0.4340**	**0.1156**	**0.5461**

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

2.7 Effect Of ETN Trading On The Futures Market

Evidence for the influence of funds on the relative open interest in the first and second nearbys can be seen in Figures 2.15 to 2.17. These show the open interest in contracts of the first nearby per open contract in the second nearby with multiples greater than 10 excluded. Prior to introduction of the iPath fund in January 2009, the relationship was highly erratic, but since around 2010 a smooth transfer from the first nearby to the second nearby on a monthly basis, with a jump as the second nearby becomes the first nearby, can be seen, consistent with fund trading. Figure 2.16, for the period from January 2010 to present, and Figure 2.17, for calendar year 2016, demonstrate the regularity of the pattern.

Figure 2.15: Open interest, 1st nearby as a proportion of 2nd nearby, 2004 to 2017

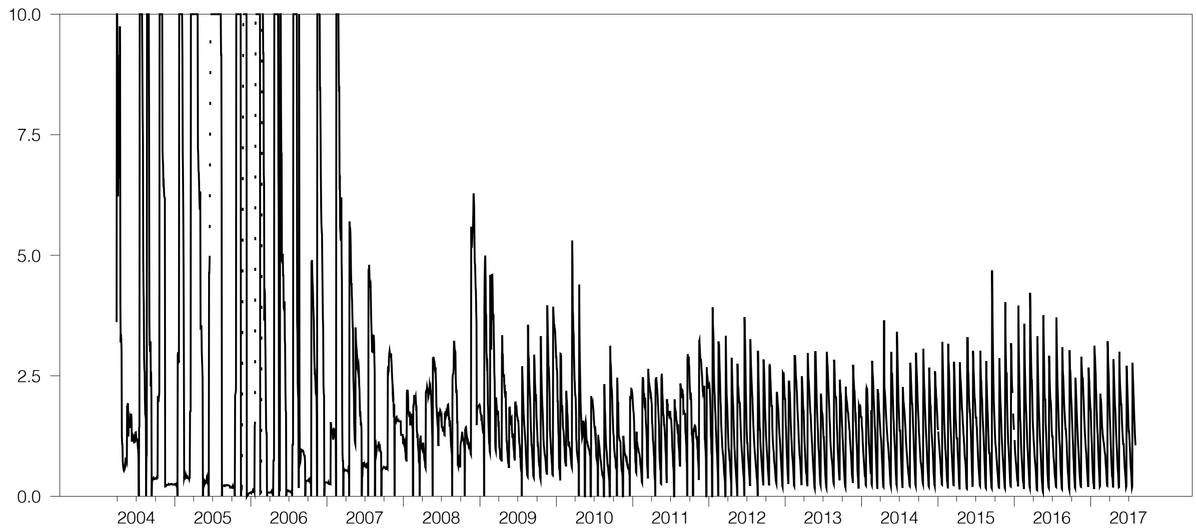


Figure 2.16: Open interest, 1st nearby as a proportion of 2nd nearby, 2010 to 2017

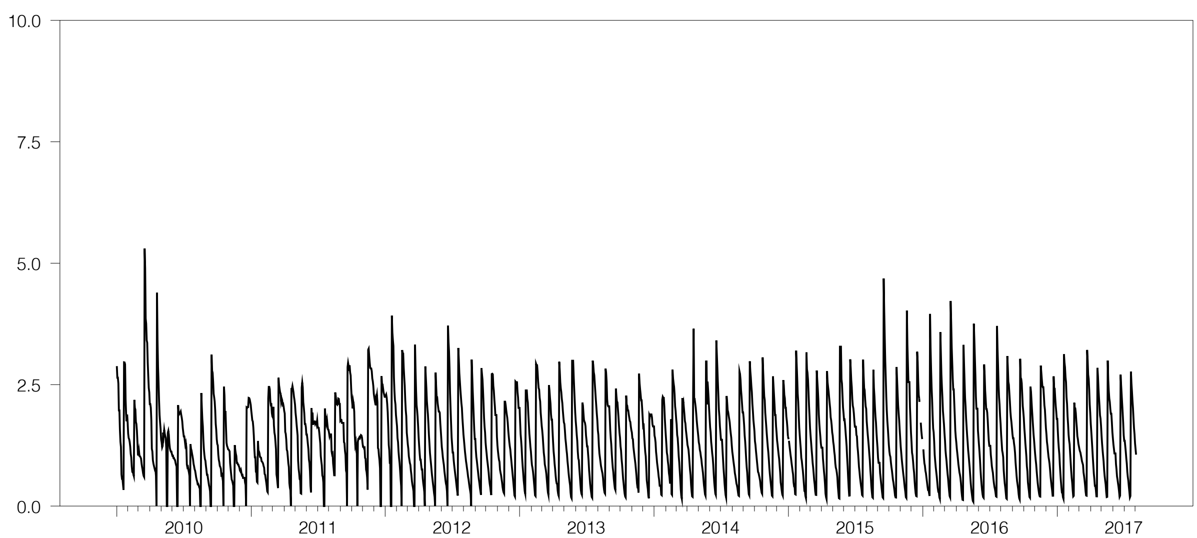
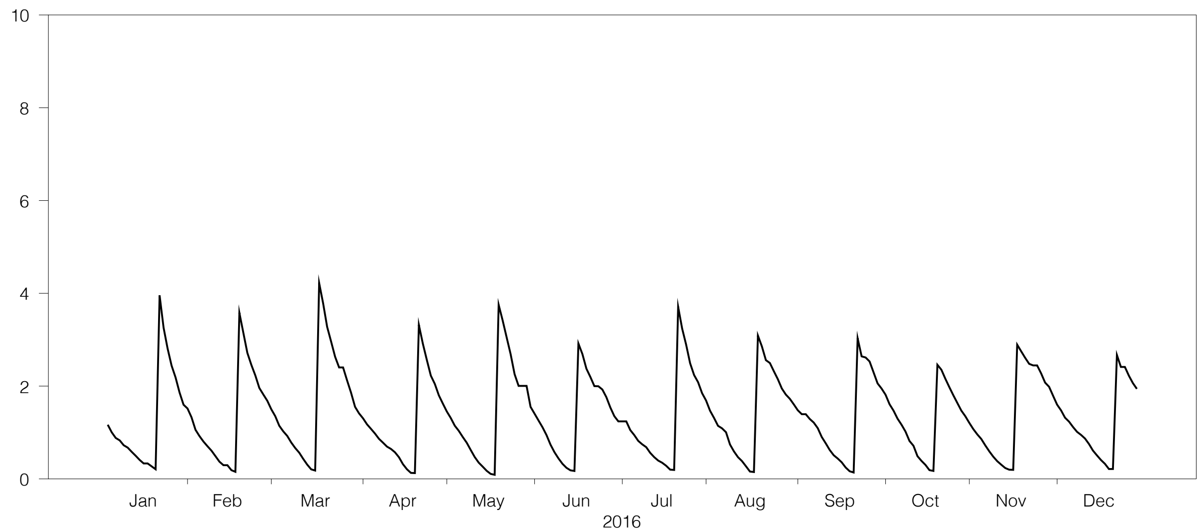


Figure 2.17: Open interest, 1st nearby as a proportion of 2nd nearby, 2016



The extent to which the ETN managers cover their positions is unknown. In order to give full coverage, for every 1,000 (unleveraged) ETN net shares in issue there must be a total of one futures contract divided between the first and second nearby. In the period before a roll date, when the second nearby becomes the first nearby, the second nearby will be the larger position, and the first nearby the larger position in the period after.

The sum of the open interest in the first and second nearbys should exceed the equivalent number of shares (in '000s) if ETN positions are being fully covered. It is not known what proportion of open interest in futures is accounted for by ETN managers, although Figures 2.13 and 2.14 above suggest that ETN-related trading dominates the market. The extent to which open interest is represented by ETN managers, and also the extent to which those managers cover their positions, may affect futures prices.

Not knowing the precise composition of a fund on any given day, a variable TNAdiff is defined as net total assets as above, divided by the sum of the open interest in the first and second nearbys by value.

Table 2.14: Strategy cash flows by ETN assets relative to futures open interest 1/30/2009–8/4/17

Cash flows from the "–2+1" strategy of selling the second nearby and buying the first nearby on a daily basis divided into bands according to the size of total ETN net assets as a proportion of the total value of open interest in the first and second nearbys combined (TNAdiff). Mean and standard deviation are given per contract, along with the mean and standard deviation of the VIX Index for those bands.

TNAdiff	Observations	Mean($)	SD($)	VIX mean	VIX SD
>1.7	3	523	2,407	23.14	8.62
1.6-1.7	3	1,817	295	19.90	1.10
1.5-1.6	4	1,800	434	20.34	2.45
1.4-1.5	6	1,858	858	22.03	2.25
1.3-1.4	14	1,549	1,165	21.71	2.10
1.2-1.3	36	1,961	1,059	20.20	3.96
1.1-1.2	30	2,511	887	20.51	3.13
1.0-1.1	35	2,437	889	23.20	4.96
0.9-1.0	41	2,890	1,508	24.13	4.88
0.8-0.9	53	1,987	1,187	20.76	4.83
0.7-0.8	95	1,731	987	19.89	4.22
0.6-0.7	198	1,823	1,268	19.79	4.89
0.5-0.6	216	1,642	1,150	18.74	5.14
0.4-0.5	310	1,068	1,106	17.36	6.08
0.3-0.4	436	979	1,343	16.64	6.75
0.2-0.3	254	712	1,353	17.30	7.38
0.1-0.2	220	299	1,178	21.12	11.92
0.0-0.1	127	-12	1,284	20.21	11.26
<0.0	51	48	819	17.63	3.43

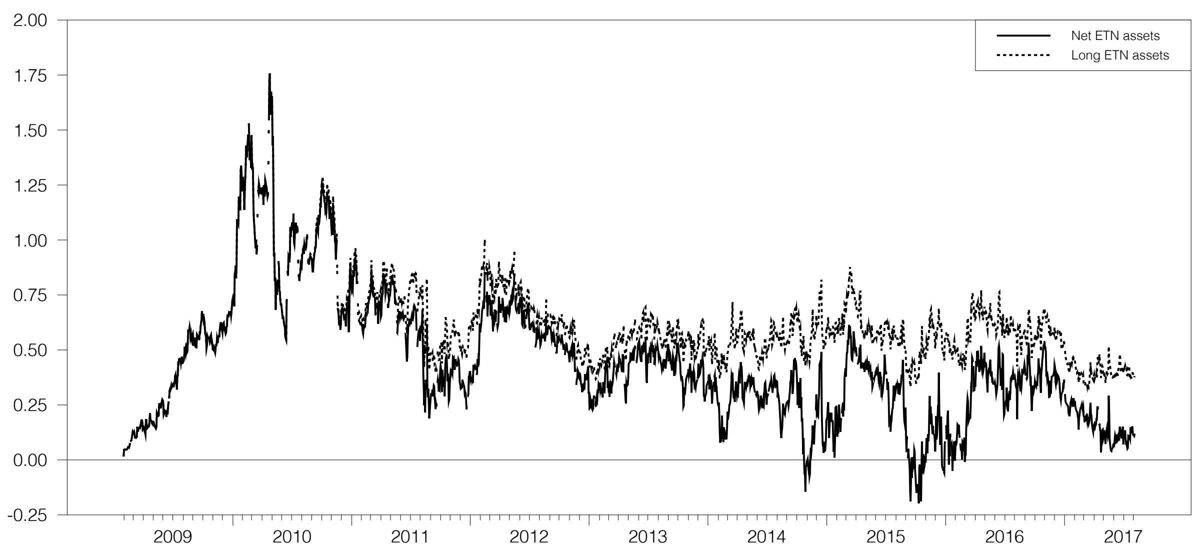
Table 2.14 gives the mean cash flows and standard deviations of the "–2+1" strategy divided into bands of the TNAdiff variable. It can be seen that the strategy has been most profitable when the total net assets of the funds are of the same value of the open interest in the first and second nearbys combined, at which times either the funds hold close to the entire market (unlikely) or are underexposed. As the funds get relatively larger or smaller, cash flows decline. Where TNAdiff is greater than 1, the total net asset value of the funds exceeds the sum of the total values of the open interest in the first and second nearbys combined, at which times

fund managers must be adopting some of the risk of their positions, in effect holding a "–2+1" counter position to their fund.

Observations are concentrated around the 0.2 to 0.5 range of TNAdiff, at which level the funds, if fully covered, represent between 20% and 50% of the market.

As can be seen from Figure 2.18, TNAdiff is greater than 1 for much of 2010. Figure 2.13 shows that this was a time of rapid increase in the size of the funds, and it may be that sufficient liquidity in the futures market was simply unavailable at that time to enable managers to cover their positions.

Figure 2.18: Net and long ETN assets as a proportion of total first and second nearby open interest by value



Restricting the analysis to the period from 1/1/2010 onwards in Table 2.15 eliminates many of the smaller observations of TNAdiff. The average TNAdiff for 2009 was 37%, i.e. total net fund assets represented 37% of the combined open interest in the first and second nearbys, a time when the funds were becoming established, but had reached 73% by the start of 2010. It could be argued that 2010 could also be disregarded, a period during which the funds appeared to outgrow the futures market, the results for which are shown in Table 2.16. It can be seen that, although the range of TNAdiff is markedly reduced, the overall pattern of returns is unchanged.

Table 2.15: Strategy cash flows by ETN assets relative to futures open interest 1/1/2010–8/4/17

Cash flows from the "–2+1" strategy of selling the second nearby and buying the first nearby on a daily basis divided into bands according to the size of total ETN net assets as a proportion of the total value of open interest in the first and second nearbys combined. Mean and standard deviation are given per contract, along with the mean and standard deviation of the VIX Index for those bands.

TNAdiff	Observations	Mean($)	SD($)	VIX mean	VIX SD
>1.7	3	523	2,407	23.14	8.62
1.6-1.7	3	1,817	295	19.90	1.10
1.5-1.6	4	1,800	434	20.34	2.45
1.4-1.5	6	1,858	858	22.03	2.25
1.3-1.4	14	1,549	1,165	21.71	2.10
1.2-1.3	36	1,961	1,059	20.20	3.96
1.1-1.2	30	2,511	887	20.51	3.13
1.0-1.1	35	2,437	889	23.20	4.96
0.9-1.0	41	2,890	1,508	24.13	4.88
0.8-0.9	53	1,987	1,187	20.76	4.83
0.7-0.8	93	1,709	986	19.87	4.27
0.6-0.7	171	1,722	1,290	19.31	4.95
0.5-0.6	150	1,367	1,145	17.35	4.03
0.4-0.5	291	975	1,059	16.87	5.94
0.3-0.4	417	951	1,352	16.13	6.44
0.2-0.3	229	669	1,392	15.82	6.11
0.1-0.2	166	473	1,092	14.97	4.91
0.0-0.1	109	364	898	16.08	5.12
<0.0	51	48	819	17.63	3.43

Table 2.16: Strategy cash flows by ETN assets relative to futures open interest 1/1/2011–8/4/17

Cash flows from the "–2+1" strategy of selling the second nearby and buying the first nearby on a daily basis divided into bands according to the size of total ETN net assets as a proportion of the total value of open interest in the first and second nearbys combined. Mean and standard deviation are given per contract, along with the mean and standard deviation of the VIX Index for those bands.

TNAdiff	Observations	Mean($)	SD($)	VIX mean	VIX SD
>1.0	0				
0.9-1.0	1	2,450		16.89	
0.8-0.9	12	1,815	577	17.40	1.51
0.7-0.8	83	1,771	915	18.94	2.65
0.6-0.7	139	1,661	1,272	18.59	4.18
0.5-0.6	144	1,337	1,152	17.11	3.80
0.4-0.5	291	976	1,059	16.87	5.94
0.3-0.4	417	951	1,352	16.13	6.44
0.2-0.3	229	669	1,392	15.82	6.11
0.1-0.2	166	473	1,092	14.97	4.91
0.0-0.1	109	364	898	16.08	5.12
<0.0	51	48	819	17.63	3.43

Results based on long ETN assets only, shown in Table 2.17, show a similar pattern. Observations are concentrated around 0.4 to 0.7, implying asset coverage of 40% to 70%. The mean returns from the strategy for lower levels of asset coverage appear somewhat random, but all of the results below asset coverage of 0.3 come from 2009, when ETNs were first introduced.

Results for the periods from 1/1/2010 and 1/1/2011 are shown in Tables 2.18 and 2.19. Again, the range of differences is reduced but the overall pattern of results is largely unchanged.

Table 2.17: Strategy cash flows by long ETN assets relative to futures open interest, 1/30/2009–8/4/17

Cash flows from the "–2+1" strategy of selling the second nearby and buying the first nearby on a daily basis divided into bands according to the size of long only ETN assets as a proportion of the total value of open interest in the first and second nearbys combined (LAdiff). Mean and standard deviation are given per contract, along with the mean and standard deviation of the VIX Index for those bands

LAdiff	Observations	Mean($)	SD($)	VIX mean	VIX SD
>1.7	3	523	2,407	23.14	8.62
1.6-1.7	3	1,817	295	19.90	1.10
1.5-1.6	4	1,800	434	20.34	2.45
1.4-1.5	6	1,858	858	22.03	2.25
1.3-1.4	14	1,549	1,165	21.71	2.10
1.2-1.3	40	2,092	1,090	20.12	3.78
1.1-1.2	31	2,433	814	20.75	3.09
1.0-1.1	33	2,315	982	23.18	5.15
0.9-1.0	52	2,760	1,433	22.68	5.25
0.8-0.9	136	1,735	1,268	19.48	4.71
0.7-0.8	171	1,479	1391	19.12	4.94
0.6-0.7	432	1,312	1,150	17.40	4.36
0.5-0.6	651	1,017	1,173	16.89	5.98
0.4-0.5	329	473	1,532	17.76	7.48
0.3-0.4	130	884	973	15.22	6.78
0.2-0.3	25	1107	779	30.90	2.19
0.1-0.2	25	299	1,178	21.12	11.92
0.0-0.1	18	-2289	84	45.19	2.83

Table 2.18: Strategy cash flows by long ETN assets relative to futures open interest, 1/1/2010–8/4/17

Cash flows from the "–2+1" strategy of selling the second nearby and buying the first nearby on a daily basis divided into bands according to the size of long only ETN assets as a proportion of the total value of open interest in the first and second nearbys combined. Mean and standard deviation are given per contract, along with the mean and standard deviation of the VIX Index for those bands.

LAdiff	Observations	Mean($)	SD($)	VIX mean	VIX SD
>1.7	3	523	2,407	23.14	8.62
1.6-1.7	3	1,817	295	19.90	1.10
1.5-1.6	4	1,800	434	20.34	2.45
1.4-1.5	6	1,858	858	22.03	2.25
1.3-1.4	14	1,549	1,165	21.71	2.10
1.2-1.3	40	2,092	1,090	20.12	3.78
1.1-1.2	31	2,433	814	20.75	3.09
1.0-1.1	33	2,315	982	23.18	5.15
0.9-1.0	52	2,760	1,433	22.68	5.25
0.8-0.9	136	1,735	1,268	19.48	4.71
0.7-0.8	169	1,464	1393	19.10	4.97
0.6-0.7	405	1,235	1,124	17.04	4.19
0.5-0.6	585	876	1,115	16.07	5.72
0.4-0.5	310	350	1,478	17.32	7.48
0.33-0.4	111	764	926	13.05	4.57

Table 2.19: Strategy cash flows by long ETN assets relative to futures open interest, 1/1/2011–8/4/17

Cash flows from the "–2+1" strategy of selling the second nearby and buying the first nearby on a daily basis divided into bands according to the size of long only ETN assets as a proportion of the total value of open interest in the first and second nearbys combined. Mean and standard deviation are given per contract, along with the mean and standard deviation of the VIX Index for those bands.

LAdiff	Observations	Mean($)	SD($)	VIX mean	VIX SD
>1.0	1	250		19.09	
0.9-1.0	13	2,301	880	17.62	2.19
0.8-0.9	107	1,626	1,173	18.38	4.01
0.7-0.8	151	1,411	1392	18.63	4.41
0.6-0.7	378	1,197	1,096	16.58	3.52
0.5-0.6	582	870	1,115	16.01	5.67
0.4-0.5	310	350	1,478	17.32	7.48
0.33-0.4	111	764	926	13.05	4.57

In column 1 of Table 2.20, an Ordinary Least Squares regression across the whole sample of TNAdiff on the daily cash flow of selling the second nearby and buying the first nearby future shows a positive relationship between cash flow from the strategy and TNA diff. As previously, net assets prior to the introduction of the ETNs are taken as zero.

Table 2.20: Determinants of strategy profit, ETN net assets relative to open interest, 3/30/2004–8/4/2017

Results of Ordinary Least Squares regressions of the daily cash flow of selling the second nearby and buying the first nearby in $ on total ETN assets relative to the total value of open interest in the first and second nearby futures combined (TNAdiff). Adjusted R^2 is shown as r2_a.

Dependent Variable −2+1 strategy in $, period 3/30/04 – 8/4/17

	A	B	C	D	E
TNAdiff	1888.0553***		2054.6258***	2085.4575***	2517.1185***
	(90.4400)		(66.7565)	(95.2473)	(67.8466)
VIX Index		−119.0368***	−122.1516***		−129.4001***
		(2.6637)	(2.3542)		(2.2905)
VIX first difference		−1125.3757***	−1444.3710***		−1266.0531***
		(329.3810)	(290.8251)		(275.9150)
1st nearby value				−0.1109**	−0.1249***
				(0.0358)	(0.0254)
2nd nearby value				0.4428***	0.2487***
				(0.0544)	(0.0386)
3rd nearby value				−1.2937***	−1.4970***
				(0.1773)	(0.1256)
Constant	316.5949***	3088.7143***	2563.7990***	511.6556***	3134.7488***
	(38.4612)	(55.2705)	(51.7717)	(49.4561)	(58.2178)
Observations	3321	3341	3321	3319	3319
r2_a	0.1158	0.3813	0.5198	0.1399	0.5690

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

The coefficient for TNAdiff is positive and significant in all cases, confirming that the size of the funds relative to the open interest in their underlying futures contracts is significantly correlated with the cash flow from the strategy. An increase of 0.1 in TNAdiff implies an increase in the cash flow from the strategy of $251.71 when controls are included, with adjusted R^2 of 0.5690.

Restricting the regression to the period following the introduction of the iPath S&P 500 VIX Short-Term Futures Index ETN on 1/30/2009 in Table 2.21, the coefficient on TNAdiff with controls included is 1,865.3, implying an increase in cash flow of $186.53 for an increase in TNAdiff of 0.1 with adjusted R^2 of 0.4993.

For the period from 1/1/2010 onwards in Table 2.22, the coefficient on TNAdiff when all controls are included is 2,789.3, indicating an increase of $279.83 in cash flow for a 0.1 increase in TNAdiff, again significant at the 1% level with adjusted R^2 of 0.5445, and from 1/1/2011, Table 2.23, the coefficient is 3,128.9, indicating an increase of $312.89 in cash flow, significant at the 1% level with R^2 of 0.6031.

Table 2.21: Determinants of strategy profit, ETN net assets relative to open interest, 1/30/2009–8/4/2017

Results of Ordinary Least Squares regressions of the daily cash flow of selling the second nearby and buying the first nearby in $ on total ETN assets relative to the total value of open interest in the first and second nearby futures combined (TNAdiff) for the period from 1/30/09 to 8/4/2017.

Dependent Variable −2+1 strategy in $, period 1/30/09 − 8/4/17

	A	B	C	D	E
TNAdiff	2065.5496***		2310.7220***	2458.6001***	1865.3410***
	(92.5432)		(78.6576)	(100.8961)	(85.4405)
VIX Index		−78.5358***	−86.4010***		−120.4653***
		(3.7049)	(3.1313)		(3.8639)
VIX first difference		−1490.3462***	−2061.1776***		−1189.3784***
		(361.5399)	(304.7559)		(291.3696)
1st nearby value				0.1126***	−0.2394***
				(0.0314)	(0.0283)
2nd nearby value				0.5327***	0.1486***
				(0.0458)	(0.0394)
3rd nearby value				−1.1789***	−1.4976***
				(0.1483)	(0.1207)
Constant	203.7087***	2599.7725***	1718.0400***	−226.5234*	3673.3139***
	(49.2692)	(74.5095)	(69.4577)	(97.6538)	(151.1674)
Observations	2119	2130	2119	2118	2118
r2_a	0.1901	0.1869	0.4226	0.2394	0.4993

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 2.22: Determinants of strategy profit, ETN net assets relative to open interest, 1/1/2010–8/4/2017

Results of Ordinary Least Squares regressions of the daily cash flow of selling the second nearby and buying the first nearby in $ on total ETN assets relative to the total value of open interest in the first and second nearby futures combined (TNAdiff) for the period from 1/1/2010 to 8/4/2017.

Dependent Variable −2+1 strategy in $, period 1/1/10 − 8/4/17

	A	B	C	D	E
TNAdiff	1849.4181***		2691.7817***	2628.6405***	2798.3474***
	(93.1413)		(73.4178)	(119.9697)	(93.9980)
VIX Index		−99.3510***	−140.2542***		−135.0861***
		(4.7695)	(3.8127)		(4.0294)
VIX first difference		−1078.0412**	−1386.8179***		−1490.3753***
		(358.9705)	(273.7168)		(276.6348)
1st nearby value				0.2164***	0.0279
				(0.0361)	(0.0291)
2nd nearby value				0.5556***	0.2559***
				(0.0458)	(0.0368)
3rd nearby value				−0.7539***	−0.7007***
				(0.1539)	(0.1190)
Constant	272.7008***	2841.5544***	2326.5218***	−883.2481***	2196.5969***
	(50.6791)	(87.1698)	(68.0149)	(156.1460)	(157.4574)
Observations	1892	1903	1892	1891	1891
r2_a	0.1722	0.1984	0.5322	0.2375	0.5445

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 2.23: Determinants of strategy profit, ETN net assets relative to open interest, 1/1/2011–8/4/2017

Results of Ordinary Least Squares regressions of the daily cash flow of selling the second nearby and buying the first nearby in $ on total ETN assets relative to the total value of open interest in the first and second nearby futures combined (TNAdiff) for the period from 1/1/2011 to 8/4/2017.

Dependent Variable −2+1 strategy in $, period 1/1/11 − 8/4/17

	A	B	C	D	E
TNAdiff	2145.5320***		2811.2154***	2952.8820***	3128.8757***
	(147.3950)		(102.5741)	(153.2762)	(109.7751)
VIX Index		−142.5225***	−155.4719***		−148.5934***
		(4.4466)	(3.7066)		(3.8472)
VIX first difference		−363.7193	−1068.6589***		−1286.4959***
		(316.9437)	(263.5101)		(262.6877)
1st nearby value				0.2726***	0.0828**
				(0.0364)	(0.0266)
2nd nearby value				0.5969***	0.2909***
				(0.0458)	(0.0336)
3rd nearby value				−0.9463***	−0.9055***
				(0.1699)	(0.1200)
Constant	153.5069*	3316.0283***	2488.6761***	−1063.7016***	2243.6523***
	(62.0087)	(77.5717)	(70.8942)	(160.1574)	(146.8250)
Observations	1648	1654	1648	1647	1647
r2_a	0.1135	0.3916	0.5837	0.2043	0.6031

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Similar results arise from regressions on long ETN assets alone, with variable LAdiff defined as above, and are shown in Table 2.24. For the period from 3/30/04 to 8/4/17, a coefficient of 2,419.9 corresponds to an increase in cash flow from the strategy of $241.99 for an increase of 0.1 in LAdiff, significant at the 1% level with R^2 of 0.5375. For the periods from 2009, 2010 and 2011, shown in Tables 2.25 to 2.27, the results are economically greater than those for TNAdiff for the later two periods, with increases in cash flow of $328.96 and $431.57 respectively given an increase of 0.1 in LAdiff.

Table 2.24: Determinants of strategy profit, long ETN assets relative to open interest 3/30/2004–8/4/2017

Results of Ordinary Least Squares regressions of the daily cash flow of selling the second nearby and buying the first nearby in $ on long only ETN assets relative to the total value of open interest in the first and second nearby futures combined (LAdiff). Adjusted R^2 is shown as r2_a.

Dependent Variable -2+1 strategy in $, period 3/30/04 - 8/4/17

	A	B	C	D	E
LAdiff	1526.1892***		1406.6759***	2087.8820***	2419.8949***
	(84.4815)		(65.1922)	(102.0331)	(74.4854)
VIX Index		-119.0452***	-116.9833***		-126.8216***
		(2.6639)	(2.5000)		(2.3681)
VIX first difference		-1126.1548***	-1247.2677***		-926.9601**
		(329.4064)	(308.6580)		(285.8207)
1st nearby value				-0.2301***	-0.2626***
				(0.0368)	(0.0268)
2nd nearby value				0.2858***	0.0676
				(0.0549)	(0.0401)
3rd nearby value				-1.1718***	-1.3186***
				(0.1780)	(0.1295)
Constant	258.1640***	3089.1726***	2501.9716***	531.7582***	3118.6206***
	(43.9211)	(55.2781)	(58.6467)	(49.8394)	(60.4118)
Observations	3320	3340	3320	3318	3318
r2_a	0.0893	0.3814	0.4588	0.1260	0.5375

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 2.25: Determinants of strategy profit, long ETN assets relative to open interest 1/30/2009–8/4/2017

Results of Ordinary Least Squares regressions of the daily cash flow of selling the second nearby and buying the first nearby in $ on long only ETN assets relative to the total value of open interest in the first and second nearby futures combined (LAdiff) for the period from 1/30/2009 to 8/4/2017.

Dependent Variable -2+1 strategy in $, period 1/30/09 - 8/4/17

	A	B	C	D	E
LAdiff	2341.2743***		2109.7979***	2379.8931***	1354.8600***
	(123.7935)		(112.7071)	(125.7200)	(110.7183)
VIX Index		-78.5358***	-70.6634***		-125.6113***
		(3.7049)	(3.4535)		(4.2246)
VIX first difference		-1490.3462***	-1872.5158***		-630.4687*
		(361.5399)	(334.8988)		(310.0993)
1st nearby value				-0.0316	-0.3860***
				(0.0315)	(0.0290)
2nd nearby value				0.3363***	-0.0349
				(0.0465)	(0.0406)
3rd nearby value				-0.9614***	-1.3122***
				(0.1545)	(0.1287)
Constant	-306.8011***	2599.7725***	1162.7683***	-186.1819	4175.2694***
	(80.5588)	(74.5095)	(103.0126)	(113.3267)	(176.2734)
Observations	2119	2130	2119	2118	2118
r2_a	0.1441	0.1869	0.3026	0.1670	0.4269

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 2.26: Determinants of strategy profit, long ETN assets relative to open interest 1/1/2010–8/4/2017

Results of Ordinary Least Squares regressions of the daily cash flow of selling the second nearby and buying the first nearby in $ on long only ETN assets relative to the total value of open interest in the first and second nearby futures combined (LAdiff) for the period from 1/1/2010 to 8/4/2017.

Dependent Variable -2+1 strategy in $, period 1/1/10 - 8/4/17

	A	B	C	D	E
LAdiff	**2350.9660*** **		**3427.4281*** **	**3166.4264*** **	**3289.6075*** **
	(136.3403)		(111.6892)	(184.0145)	(148.9768)
VIX Index		**-99.4708*** **	**-134.3087*** **		**-134.1874*** **
		(4.7667)	(4.0559)		(4.3552)
VIX first difference		**-1093.2357** **	**-1123.0235*** **		**-1055.4402*** **
		(358.8205)	(292.2329)		(297.8611)
1st nearby value				**0.1459*** **	**-0.0571**
				(0.0378)	(0.0316)
2nd nearby value				**0.3795*** **	**0.0655**
				(0.0458)	(0.0383)
3rd nearby value				**-0.2847**	**-0.2238**
				(0.1654)	(0.1329)
Constant	**-391.9485*** **	**2842.4876*** **	**1251.7861*** **	**-1622.1205*** **	**1484.0453*** **
	(91.7493)	(87.1127)	(87.9572)	(224.4137)	(212.1926)
Observations	1891	1902	1891	1890	1890
r2_a	0.1355	0.1992	0.4662	0.1729	0.4678

Standard errors in parentheses

* p<0.05, ** p<0.01, *** p<0.001

Table 2.27: Determinants of strategy profit, long ETN assets relative to open interest 1/1/2011–8/4/2017

Results of Ordinary Least Squares regressions of the daily cash flow of selling the second nearby and buying the first nearby in $ on long only ETN assets relative to the total value of open interest in the first and second nearby futures combined (LAdiff) for the period from 1/1/2011 to 8/4/2017.

Dependent Variable -2+1 strategy in $, period 1/1/11 - 8/4/17

	A	B	C	D	E
LAdiff	2905.5284***		4046.8584***	3963.4974***	4315.7479***
	(241.3086)		(171.8186)	(257.6791)	(188.9088)
VIX Index		-142.5460***	-156.3498***		-151.1878***
		(4.4463)	(3.8806)		(4.0978)
VIX first difference		-364.6536	-675.6588*		-860.4795**
		(316.9175)	(273.9376)		(278.1943)
1st nearby value				0.2775***	0.0873**
				(0.0387)	(0.0290)
2nd nearby value				0.4272***	0.1070**
				(0.0454)	(0.0343)
3rd nearby value				-0.1535	-0.0636
				(0.1746)	(0.1271)
Constant	-742.7258***	3315.7446***	1188.4015***	-2571.7715***	627.7838**
	(143.5783)	(77.5655)	(112.4436)	(249.8377)	(206.8274)
Observations	1647	1653	1647	1646	1646
r2_a	0.0804	0.3918	0.5468	0.1477	0.5499

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Thus the size of fund assets relative to open interest in the futures used to back those assets is a strong indicator of intertemporal carry cost between the first and second nearbys, and also therefore of profits to be made from trading counter to the trades of the ETN managers. The steepness of the futures curve between the first two nearbys may reflect the extent to which fund managers are failing to cover their positions. It is unclear, however, whether managers are reacting to the slope of the futures curve by reducing their coverage, or if the market is reacting to anticipated manager demand.

2.8 Conclusion

As the VIX index cannot be replicated, exposure to the index can be achieved only indirectly; in the case of the various VIX-related ETNs, via the futures market.

There is evidence that a fundamental change in the futures curve for the first and second nearbys occurred since the time that the iPath VIX-related ETN was introduced. This has resulted in a significantly worse performance for this and other ETNs than would have been the case had the change not taken place.

Liquidity providers appear to be benefitting at the expense of ETN investors. The size of the funds both absolutely and relative to the value of open interest in the first and second nearby futures contracts is significantly positively correlated with returns from a strategy of selling the second nearby and buying the first nearby futures contract, the reverse of the trades represented by the short-term ETN benchmark.

It is, of course, the case that liquidity providers and speculators expect compensation, and, in the traditional futures market, hedgers are prepared to pay it. The inverse correlation of the VIX Index with the S&P 500 Index implies that exposure to the VIX Index provides a transfer of risk, and hence ETN investors are, indirectly, hedging via the futures market and some cost to them would therefore be expected.

If the VIX Index is assumed to be fully mean reverting in the long run, it can be argued that any investment vehicle highly correlated with the VIX Index, and therefore inversely correlated with the S&P 500 Index, should be priced to give a negative return and hence is bound to underperform the VIX Index itself so long as the stock market premium is greater than the risk-free rate. The arrival of the ETN providers in the futures market appears to have pushed the cost of liquidity up significantly. As a result, VIX-related ETNs have performed poorly against any reasonable measure, yet they continue to attract a considerable amount of investment.

The value of outstanding inverse short-term VIX-related ETNs is currently around 65% of the value of short-term VIX-related funds. These may provide liquidity to the direct ETNs as they comprise directly offsetting positions, but perhaps less so than in aggregate as different issuers are involved.

There is evidence that managers are not fully covering their positions, and it may be that ETN providers are at times effectively taking counter positions to their funds.

CHAPTER 3

DO VOLATILITY INDICES PREDICT REALIZED VOLATILITY? AN EXAMINATION OF THE VIX AND VVIX INDICES AND THEIR RELATIONSHIP TO S&P 500 INDEX VOLATILITY

3.1 Introduction

This essay examines the predictive power of the VIX and VVIX indices for future volatility of the S&P 500 Index and VIX Index respectively. It contributes to the existing literature by showing that the VIX Index may overstate S&P 500 Index volatility and that the VVIX Index understates VIX Index volatility. It then shows that the VIX Index overstates S&P 500 Index 30-day return volatility and the VVIX Index understates VIX Index 30-day return volatility at higher levels of volatility, and the predictive power of the volatility indices is considered. A method of Whaley is adapted to examine the extent to which the VIX Index overstates S&P 500 Index volatility and the VVIX Index understates VIX Index volatility. Data are taken from the Chicago Board Options Exchange and Yahoo Finance.

While implied volatility based on options prices had been calculated and used as a measure for many years, the idea of an index of volatility for use as a benchmark appears first to have been mooted by Brenner and Galai (1989). They proposed a “Sigma Index” for the equity market, bond market and foreign exchange market on which options and futures could be based, and that these would provide investors with a pure hedge against volatility without incurring the price risk of using option strategies sensitive to volatility such as straddles. Indeed, Brenner and Galai demonstrated that options based on a volatility index cannot be replicated with conventional options other than by impractically expensive dynamic hedging.

The American Stock Exchange conducted a feasibility study on a Sigma Index based on Brenner and Galai's ideas in 1992, but the Chicago Board Options Exchange Volatility Index, the VIX Index, designed by Whaley and introduced in January 1993, was to become the accepted benchmark. This was designed to measure 30-day return volatility as reflected in the implied volatility of at-the-money S&P 100 Index option prices.

The calculation method was changed in 2003, based on the work of Demeterfi (1999), with the VIX Index now based on a weighted average of put and call options on the S&P 500 Index over a wider range of strike levels. Previously only near-the-money options were used which did not reflect the significant volatility skew seen in far out-of-the-money put contracts. Furthermore, S&P 500 Index option trading volumes had by this time surpassed those on the S&P 100 Index by a considerable margin. VIX Index levels were calculated back to 1990 using the new method. In March 2004, CBOE introduced the first exchange-traded futures contract, with options appearing in February 2006.

In March 2012, CBOE introduced the VVIX Index, a volatility index of the VIX Index, or implied volatility of implied volatility of the S&P 500 Index, based on options traded on the VIX Index using the same methodology as that for the VIX Index itself.

Until January 30, 2014, S&P 500 Index options had monthly expiration dates, with the two series closest to expiration being used for the calculation.

On January 30, 2014, CBOE introduced options with weekly expiration dates, settling at the close on the last trading day of each week, although those expiring on the third Friday of each month, the settlement date for options with monthly expiry dates, continue to settle at 8:30a.m. The options used in the VIX Index calculations are now those with more than 23 days and fewer than 37 days to expiration, the shorter maturity called the near-term and the longer maturity called the next-term.

The formula used by CBOE to calculate both VIX Index and VVIX Index can be expressed as:

$$VIX = 100 \text{ x } \sqrt{\left\{ T_1\sigma_1^2\left[\frac{N_{T_2}-N_{30}}{N_{T_2}-N_{T_1}}\right] + T_2\sigma_2^2\left[\frac{N_{30}-N_{T_1}}{N_{T_2}-N_{T_1}}\right]\right\} \text{ x } \frac{N_{365}}{N_{30}}}$$

with the subscripts 1 and 2 referring to the near-term and next-term options respectively. This represents the square root of an annualized, weighted interpolation of implied variance from the full range of near-term and next-term out-of-the-money options.

T is the time to expiration in years, calculated to the nearest minute.

N_{T1} is the time to settlement of the near-term options in minutes.

N_{T2} is the time to settlement of the next-term options in minutes.

N_{30} = 43,200, the number of minutes in 30 days.

N_{365} = 525,600, the number of minutes in 365 days.

Each volatility term, $\sigma_j, j = 1,2,$ is computed as:

$$\sigma_j = \sqrt{\frac{2}{T}\sum_i \frac{\Delta K_{ji}}{K_{ji}^2} e^{RT} Q(K_{ji}) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2}$$

where F equals the forward index level based on the strike prices for calls and puts where the absolute difference between the call and put premium is the smallest. This may be the same or different for the near-term and next-term options. Then:

$$F = \text{Strike Price} + e^{RT} \text{ x (Call Price – Put Price)}$$

Hence F adjusts for the difference between the implied level of the index at the expiry date and the strike price of the options nearest to that date.

K_0 = first strike below F

K_i = the strike price of the ith out-of-the-money option; calls for $K_i \geq K_0$ and puts for $K_i \leq K_0$

ΔK_i is the interval between strike prices, defined as half the interval between the next highest and next lowest strike price.

R is the risk-free interest rate to expiration, based on U.S. Treasury bill yields. Hence there are usually different risk-free rates for the near-term and next-term option calculations.

$Q(K_i)$ is the midprice of the option with strike K_i

Neither the VIX Index nor the VVIX Index is a tradable instrument. Because the indices are the square roots of weighted sums, unlike traditional indices, they can be replicated only dynamically and not statically. As shown in Chapter 2, attempts to provide exposure to the VIX Index, as with soft commodity ETFs, have met with limited success owing to the usually upward sloping term structure of VIX Index futures prices, thus futures contract values generally decrease over time. Futures contracts will naturally approach the spot value of their underlying instrument as maturity approaches. Consequently, if a futures contract is at a significant premium to the spot value, and the spot value does not increase significantly before maturity, the premium on the future erodes as time passes.

3.2 Interpretation Of The VIX Index And Literature Review

By construction, the VIX Index is a weighted average of the implied volatility of a range of out-of-the-money options, both puts and calls, on the S&P 500 Index. It is frequently referred to as a measure of expected 30-day volatility. CBOE (2009) cites “the market’s expectation of future volatility implied by S&P 500 stock index options prices" as being "what VIX truly measures," and that "VIX is a measure of expected future volatility." The Chicago Board Options Exchange website states that the VIX Index "is a key measure of market expectations of near-term volatility conveyed by S&P 500 stock index option prices." Implied volatility is not, however, expected volatility, as expectations in financial markets are never objectively measured. It may provide a crude proxy for some sort of average of option market participants' views, but it is conditional on several tenuous model assumptions.

Whaley (2000) notes "VIX is an *implied* volatility in the parlance of the securities industry" (his italics). He adds "This implied volatility is the market's 'best' assessment of the expected volatility of the underlying stock index over the remaining life of the option."

The VIX Index at that time was calculated using American style S&P 100 Index options that can be exercised at any time up to expiration. S&P 500 Index options traded on the Chicago Board Options Exchange, however, are European style, meaning that they are exercisable only at expiration. This in turn means that, since the change in VIX Index calculation method in 2003, the only volatility measure relevant to the option prices on which the VIX Index is now based is the range of possible outcomes for the S&P 500 Index at expiry. Volatility of the S&P 500 Index over the "remaining life of the option" is therefore theoretically no longer directly relevant to the implied volatility of the option, only the range of possible outcomes at expiry.

Whaley (2009), writing after the change in calculation method, says "the level of the VIX Index is implied by the current prices of options on the S&P 500 Index and represents expected future stock market volatility over the next 30 calendar days." He does not offer a precise interpretation of what a particular level of the VIX Index might be, preferring that "The real benefit from an index…comes from comparing its current level to some historical benchmark(s)…Thus, to gauge the normal behavior of the VIX, we should look to its history."

In his subsequent analysis of the VIX Index as a predictor of S&P 500 Index volatility he compares the level of the VIX Index with the level of the S&P 500 Index 30 calendar days hence. This is the true implied volatility that the VIX Index is measuring, rather than the volatility of the market in period leading up to option expiry.

Huang & Shaliastovich (2014) state that the VIX Index "provides a risk-neutral forecast of the aggregate index volatility over the next 30 days," furthermore that the VVIX Index "directly measures the risk-neutral expectations of the volatility of volatility in the financial markets." Park (2015) similarly says that the VVIX Index "represents a risk-neutral expectation of volatility of the 30-day forward VIX Index."

Liu & Dash (2012) say that the VIX Index "is a widely used measure of the implied volatility of S&P 500 Index options. It represents the market expectation of market volatility over the next 30-day period." Chow, Jiang & Li (2014) say that the VIX Index is "designed to capture the market's aggregate expectation of future volatility over the next 30 days."

Zang, Ni, Huang & Wu (2006) say of the VIX Index that as "a measure for market's expectation of 30-day implied volatility of S&P 500 index, VIX provides rich information for the prediction of the market's future trend."

Chow, Jiang & Li (2014) object to the VIX Index as a predictor of volatility alone as it also reflects skewness, the third moment of returns, and not merely the second. Hence, they argue, the VIX Index does not measure expected volatility, but rather a linear-moment combination. They suggest that when markets decline, demand for put options from portfolio insurers makes put options relatively more expensive than call options, whilst the price of market return skewness, replicable by a combination of long out-of-the-money calls and short out-of-the-money puts, becomes negative, and that consequently the VIX Index understates the true value of volatility due to the negative impact of skewness.

While this appears to violate the put-call parity relationship whereby put and call prices for the same option series must be jointly determined, the put-call parity relationship requires a fully liquid market. Grover & Thomas (2012) find, however, that liquidity across option series can vary greatly, with spreads on S&P 500 Index options in September 2007, a time when market volatility was relatively low, ranging from near 0% up to as much as 200%, with even greater variation in more volatile times. Under such conditions, particularly where there is significantly more demand for, say, puts over calls, it is possible for the put-call relationship, which is based on arbitrage, to break down. Park (2015) notes that illiquidity is particularly severe for deep out-of-the-money S&P 500 Index put options, those favored by portfolio insurers who, as Whaley (2009) notes, are the prime drivers behind movements in the VIX Index.

Given that the implied volatility of relevance to European style options is the range of potential outcomes at expiry, then it is reasonable that skewness of returns is a factor. This does not conflict with VIX being a measure of a range of possible outcomes, only with VIX as a measure of interperiod volatility, which by construction it is not.

There are two distinct volatility measures to consider: (1) Time series volatility over the 30-day period denoted TSV here. (2) Cross-sectional volatility at the end of the 30-day period denoted CSV here. The VIX Index and VVIX Index theoretically address CSV and not TSV. The extent to which the VIX Index and VVIX Index are a measure of TSV rather than CSV depends upon the correlation of outcomes between TSV and CSV. This in turn depends upon the distribution of returns of the underlying index, the S&P 500 Index in the case of the VIX Index and the VIX Index in the case of the VVIX Index. Here there is a clear distinction, in that the

VIX Index has been shown to be mean-reverting by Whaley (2009), whereas the S&P 500 Index is not.

Evidence that the VIX Index shows mean reversion, in contrast to the S&P 500 Index, can be seen by carrying out a Dickey-Fuller unit root test on the two indices. The test statistic for the S&P 500 Index over the period 1/2/90 – 12/1/17 is 0.765 with no basis for rejecting the null hypothesis of the existence of a unit root. The VIX Index gives a test statistic of –8.096, however, considerably greater in absolute terms than the 1% critical value of –3.430, suggesting strongly that there is no unit root in the evolution of the VIX Index. The absence of a unit root implies mean-reverting behavior. Thus the S&P 500 Index does not appear to be mean-reverting, whereas the VIX Index does exhibit such behavior.

Short-term volatility of a mean-reverting process should have less effect on longer-term return than a random walk process such as the evolution of share index levels is generally taken to be. If an up movement is more likely to be followed by a down movement than another up movement, then volatility as measured by standard deviation of returns will be lower over longer periods for a given unit period standard deviation than it would be for a standard normal distribution, where a unit period standard deviation would be multiplied by the square root of time. The magnitude of the mean-reverting effect will depend upon the persistence of a shock on the mean-reverting process.

In contrast, for a process characterized by a momentum effect, where an up movement is more likely to be followed by another up movement than by a down movement, longer term volatility is greater for given unit of time standard deviation than would be the case for a standard normal distribution. My results confirm that the VIX Index is strongly mean-reverting, whereas the results for the S&P 500 Index suggest that a mild momentum effect may be present.

A complication is that, as the weighted sum of options expiring either side of the desired 30-day point, the VIX Index is an interpolation of a shorter and longer implied volatility period. This is less problematic now that weekly expiration dates are available, but the volatility that the VIX Index is intended to measure, that of 30 days regardless of the expiration dates of the underlying options, is not directly measureable, a true calculation for which would require continuous option expiration.

For the VVIX Index, as the VIX Index is non-replicable, put-call parity, as a non-arbitrage condition, need not hold for the VIX Index options on which the VVIX Index is based. Indeed, Dzekounoff (2010) states that it does not, with steep call volatility skew in evidence.

There is little in the existing literature on the VVIX Index itself, although a number of papers model VIX Index option pricing without specific reference to the VVIX Index. Lian & Zhu (2011) develop a stochastic volatility model including jump processes for both volatility and the underlying VIX Index level from which VIX Index option prices are derived. Cont & Kokholm (2013) develop a model of variance swap rates, which they then apply to VIX Index option pricing. Goard & Mazur (2013), Mencia & Sentana (2013) and Lin (2013) provide further VIX Index option pricing models.

Zang, Ni, Huang & Wu (2015) note the simultaneity of jumps in the VVIX Index and VIX Index, and use the VVIX Index in developing a dynamic model of the VIX Index. Huang & Shaliastovich (2014) find that the VIX Index and VVIX Index have separate dynamics and are only weakly related, but that the VVIX Index has applications for delta hedging of option portfolios. They further suggest that the VVIX Index predicts future VIX Index option gains.

Park (2015) uses the VVIX Index as a predictor for tail-risk hedging returns.

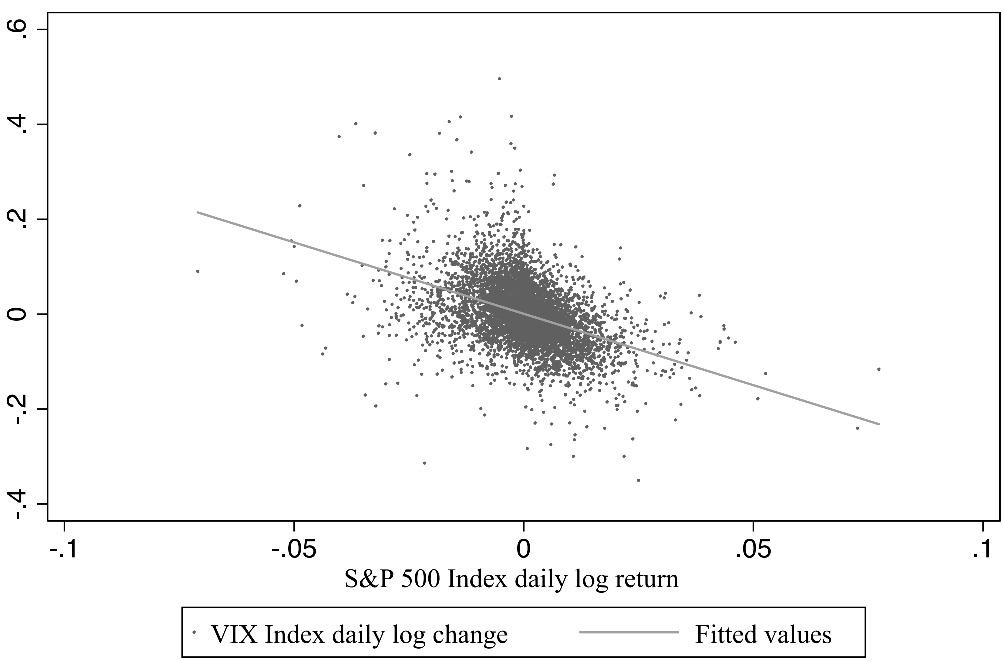
3.3 Relationships Between The Indices

The negative correlation between movements in the VIX Index and the underlying S&P 500 Index is well documented, leading to the dubbing of the former as the "Fear Index". This is demonstrated, as in Chapter 2 above, by a simple OLS regression of daily VIX Index log returns (*MVIX*) on S&P 500 Index log returns (*RSPX*) for the period 1/2/1990 – 12/1/2017, with the index parameter significant to the 1% level, the constant coefficient statistically insignificant and adjusted R^2 of 0.1735:

$$MVIX = 0.001 - 3.010RSPX$$

Figure 3.1, a scatter plot of VIX Index daily log changes against S&P 500 Index daily changes shows a relatively wide spread of VIX returns, particularly around zero.

Figure 3.1: VIX Index daily log change v. S&P 500 Index daily log return, 1/2/1990–12/1/2017

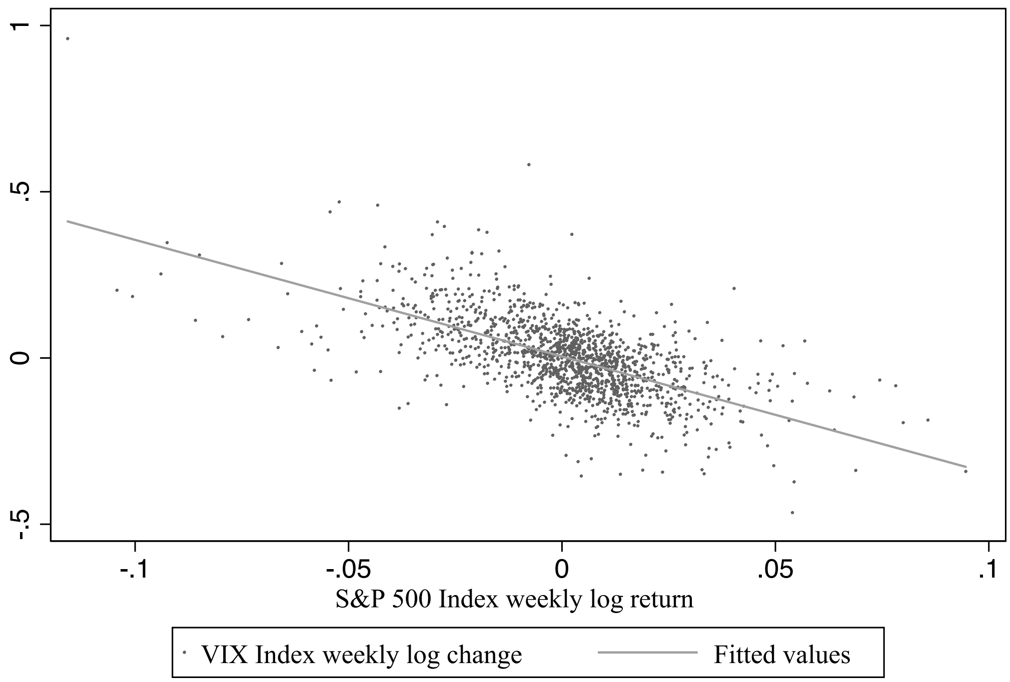


This is unsurprising as there can be quiet days in times of high volatility. The correlation coefficient between daily log returns on the VIX Index and S&P 500 Index for this period is -0.417. For weekly returns shown in Figure 3.2 the relationship is:

$$MVIX = 0.004 - 3.508 RSPX$$

with adjusted R^2 of 0.3830, the S&P 500 Index return coefficient significant at the 1% level, and the constant not significantly different from zero.

Figure 3.2: VIX Index weekly log change v. S&P 500 Index weekly log return, 1/2/1990–12/1/2017

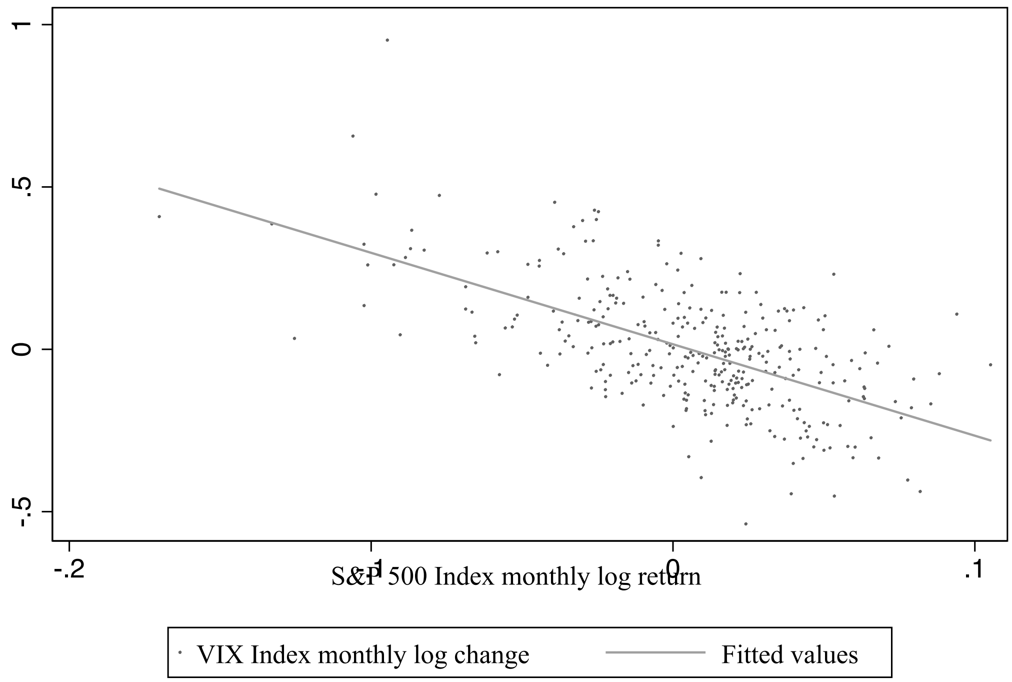


The correlation coeffient is –0.6192. For monthly returns, shown in Figure 3.3:

$$MVIX = 0.016 – 2.816 RSPX$$

with adjusted R^2 of 0.3684, and, as before, the S&P 500 Index return coefficient significant at the 1% level and the constant not significantly different from zero. Here the correlation coefficient is –0.6085.

Figure 3.3: VIX Index monthly log change v. S&P 500 Index monthly log return 1/1/1990–12/1/2017



Bollen & Whaley (2004) show that a major factor in the movement of the VIX Index is demand for put options by portfolio insurers. Whaley (2009) finds that, whereas in 1992 trading in S&P 100 Index call and put options were roughly balanced, over the first 10 months of 2008, in the midst of the financial crisis, S&P 500 Index put option daily volumes were 72% greater than that for call options (the S&P 500 Index having surpassed the S&P100 Index in terms of volume of option trading).

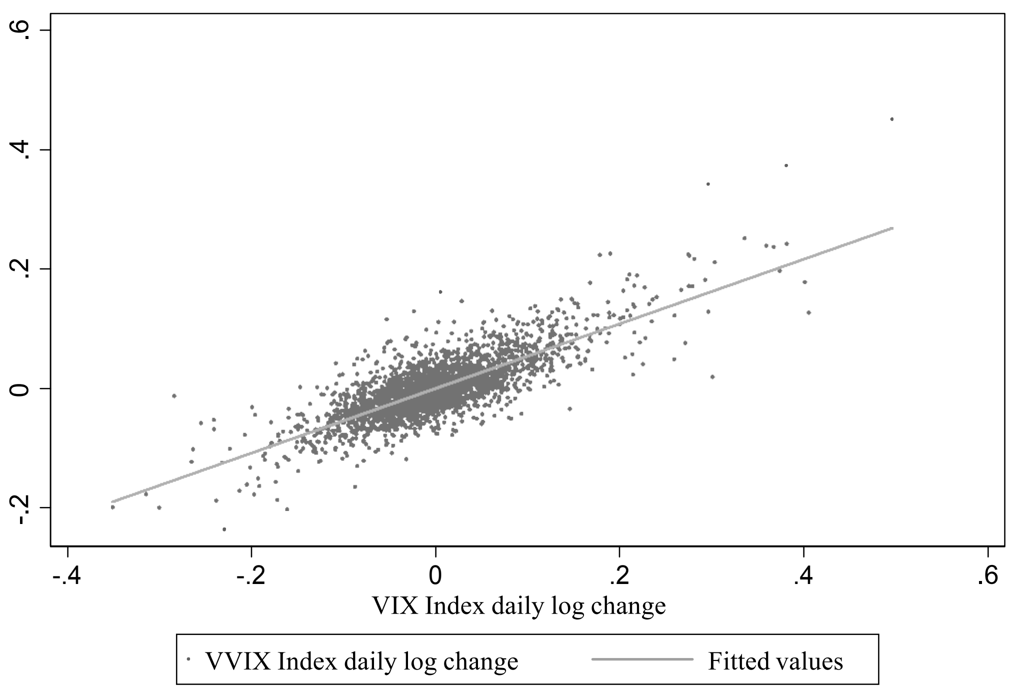
An explanation mooted by CBOE (2014) is that demand for puts from portfolio insurers will increase significantly on a falling market, whereas there is a lesser effect on calls in a rising market. Increased demand lifts premiums and hence implied volatility. As Whaley (2009) points out, the VIX Index is an indicator that reflects the price of portfolio insurance. It may be noted, however, that bear markets are generally more volatile than bull markets. Furthermore, the transition from a bull market to a bear market, evoking fear and possibly panic in investors' minds, tends to be a more violent affair than the relatively muted transition as a bear market comes to an end, where caution might be considered the primary emotion. Instinct is to flee disaster but to return tentatively.

There is a strong, positive correlation between movements in the VIX Index and VVIX Index. For the period 1/3/2007 – 12/1/2017, a simple OLS regression of daily log changes of the VVIX Index (*MVVIX*) on the VIX Index (*MVIX*) gives:

$$MVVIX = 0.000 + 0.541 MVIX$$

with the VIX coefficient significant at the 1% level. Adjusted R^2 is 0.6244, suggesting that the correlation between the VIX Index and VVIX Index is stronger than the negative correlation between the VIX Index and the S&P 500 Index. This can be seen from a scatter plot of the daily log change in the VVIX Index plotted against the log change in the VIX Index shown in Figure 3.4.

Figure 3.4: VVIX Index daily log change v. VIX Index daily log change, 1/3/2007–12/1/2017



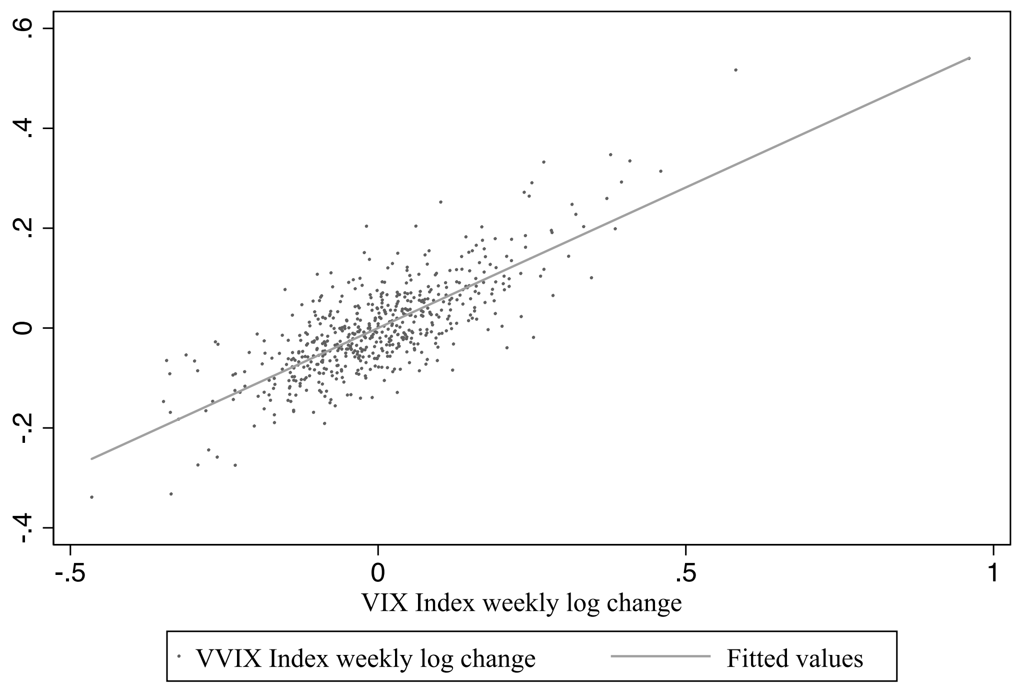
The correlation coefficient between daily log returns on the VIX Index and VVIX Index is 0.790, also indicating a stronger relationship than that between the VIX Index and S&P 500 Index, with its correlation coefficient of –0.417.

For weekly changes, shown in Figure 3.5, the relationship is:

$$MVVIX = 0.000–0.563MVIX$$

with adjusted R^2 of 0.6100, the VVIX Index return coefficient significant at the 1% level, the constant insignificant at the 5% level and correlation coeffient 0.7815.

Figure 3.5: VVIX Index weekly log change v. VIX Index weekly log change, 1/3/2007–12/1/2017

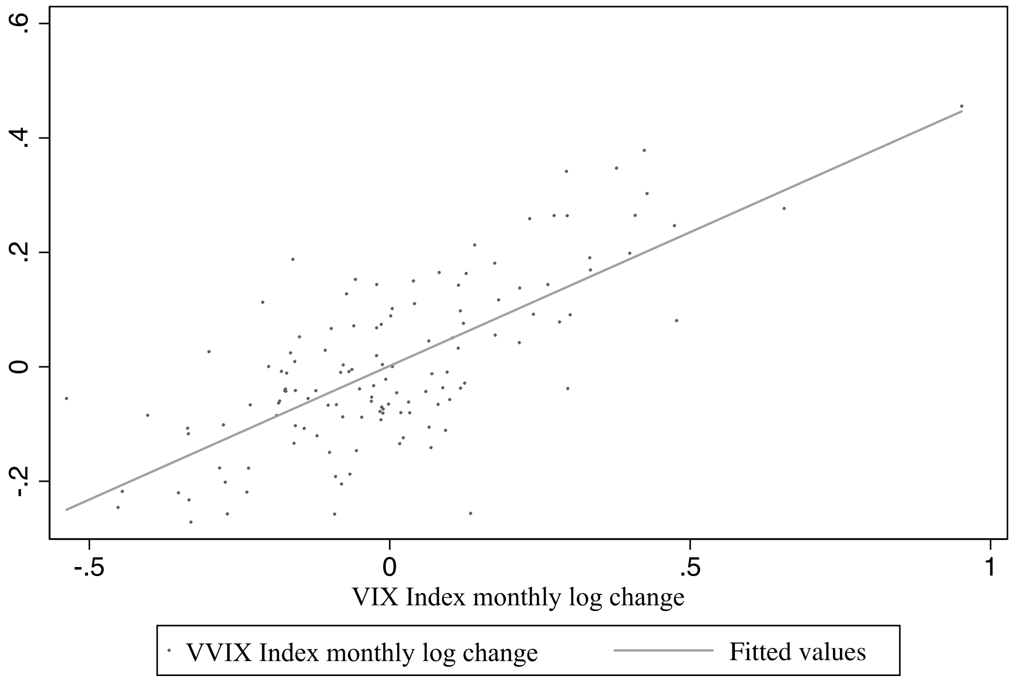


For monthly changes in Figure 3.6:

$$MVVIX = 0.016 - 0.467MVIX$$

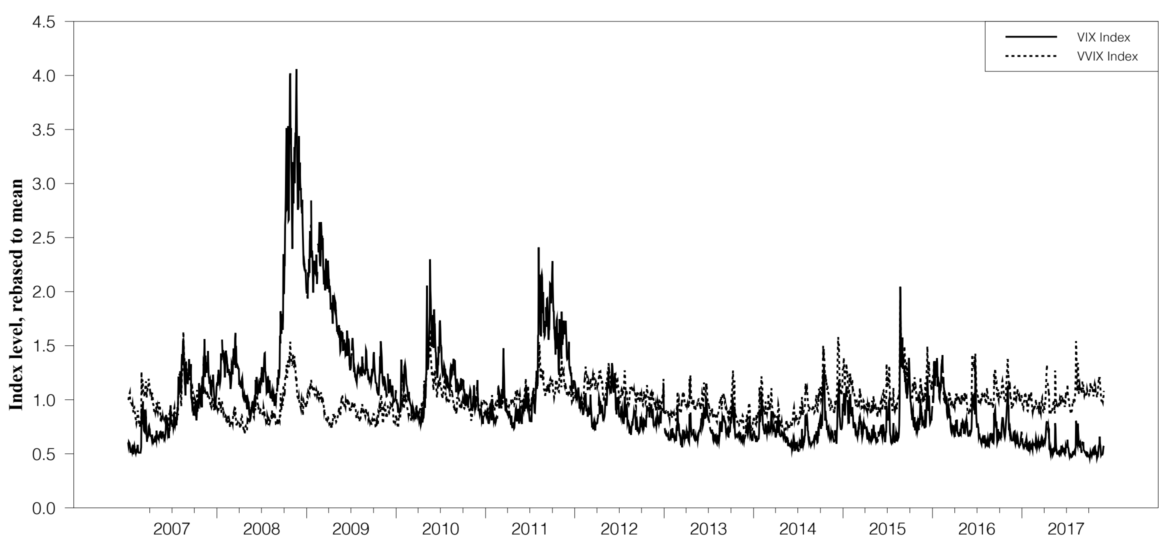
The VVIX Index movement coefficient is again significant at the 1% level and constant not significantly different from zero, with adjusted R^2 of 0.5245. The correlation coefficient is 0.7267.

Figure 3.6: VVIX Index monthly log change v. VIX Index monthly log change 2/1/2007–12/1/2017



A plot of closing values of the two volatility indices normalized by their means shown in Figure 3.7 suggests that the VVIX Index, the "volatility of S&P 500 Index volatility" is generally less volatile than the S&P 500 Index, as measured by the VIX Index.

Figure 3.7: VIX Index, VVIX Index, rebased, 1/3/2007–12/1/2017



For the period up until around 2012, the VIX Index appears to show long cycles of generally increasing or decreasing levels, with significant peaks in 2008, 2010 and 2011. Volatility of the VVIX Index appears to increase at times of increased VIX Index volatility, as expected, but to a lesser extent.

The positive correlation between daily changes in the VVIX Index and VIX Index shows that the implied volatility of the VIX Index increases as the VIX Index increases. As shown below, the VVIX Index, as a representation of VIX Index implied volatility, appears to understate realized VIX Index volatility. Hence, as VIX Index volatility increases, the disparity between implied volatility as represented by the VVIX Index and realized volatility becomes greater, and so a more pronounced movement in the VVIX Index might be expected when the VIX Index is rising than on a fall in VIX Index level/volatility, when the disparity is narrowed.

In contrast to S&P 500 Index options, call option volumes on the VIX Index are much greater than put volumes as mentioned by Dzekounoff (2010). Park (2015) states that demand for VIX Index calls outstrips that for puts due to their use in tail risk hedging strategies. As movements in the VIX Index are inversely correlated with those of the S&P 500 Index, an investor wishing to hedge against a fall in the S&P 500 Index might look to take a long position in VIX Index call options.

3.4 Volatility Indices As Indicators

The VIX Index represents implied volatility in the 30-day return of the S&P 500 Index, expressed as an annualized percentage standard deviation. Hence, for example, a VIX Index level of 60% gives an implied standard deviation of the S&P 500 Index 30-day log return of

$$60*\left(\frac{30}{365}\right)^{\frac{1}{2}}=17.2\%$$

The VVIX Index is calculated using the same methodology, using options on the VIX Index to produce an implied annualized percentage standard deviation of 30-day return of the VIX Index.

Of interest is whether the VIX Index and VVIX Index act as leading indicators of volatility in their underlying indices or are primarily a reflection of past volatility. CBOE (2009) claims that, when S&P 500 Index 30-day volatility suggested a VIX Index level of 100 in October 2008, an actual VIX Index reading of 80 predicted that volatility in the market would decline, a prediction that proved correct, and that "as a measure of future volatility, VIX worked very well." Cumby, Figlewski and Hasbrouck (1993) and Canina & Figlewski (1993) found, however, that past volatility is a better predictor of future volatility than many forward-looking models including implied volatility from option prices.

A simple OLS regression of annualized standard deviation is carried out of the level of the VIX Index on S&P 500 Index returns over the previous 30 days (SP_{hist}) and that of 30 days after (SP_{fwd}) for the period 2/2/2007 to 11/2/2017. The standard deviations are taken as the standard deviation of daily log changes in the indices over the previous and subsequent 21 trading days, including the day itself, multiplied by $\sqrt{365*\frac{5}{7}}$

This multiplier converts the daily standard deviation based on five trading days per week into an annual figure, thus rendering it comparable with the VIX Index which represents annual standard deviation by construction. Summary statistics are set out in Table 3.1.

Table 3.1: Summary statistics, S&P 500, VIX and VVIX Indices, forward and historic 21-day standard deviation, 2/2/2007–11/2/2017

	Obs.	Mean	Std Dev.	Min	Max
S&P 500	2688	1593.8	447.1	695.3	2581.1
VIX	2688	20.12	9.69	9.19	80.86
VVIX	2688	87.87	13.02	59.74	168.75
S&P σ_{hist}	2688	13.53	7.74	3.44	60.38
S&P σ_{fwd}	2688	14.34	7.77	3.44	60.38
VIX σ_{hist}	2688	111.26	42.34	41.10	269.68
VIX σ_{fwd}	2688	110.34	41.29	41.10	269.68

Regressing the VIX Index on historic 21 trading day S&P 500 Index volatility gives an adjusted R^2 of 0.7515, with both coefficients significant at the 1% level:

$$VIX = 5.448 + 1.084 SP_{hist}$$

Forward volatility alone gives an adjusted R^2 of 0.6118, again with both coefficients significant at the 1% level:

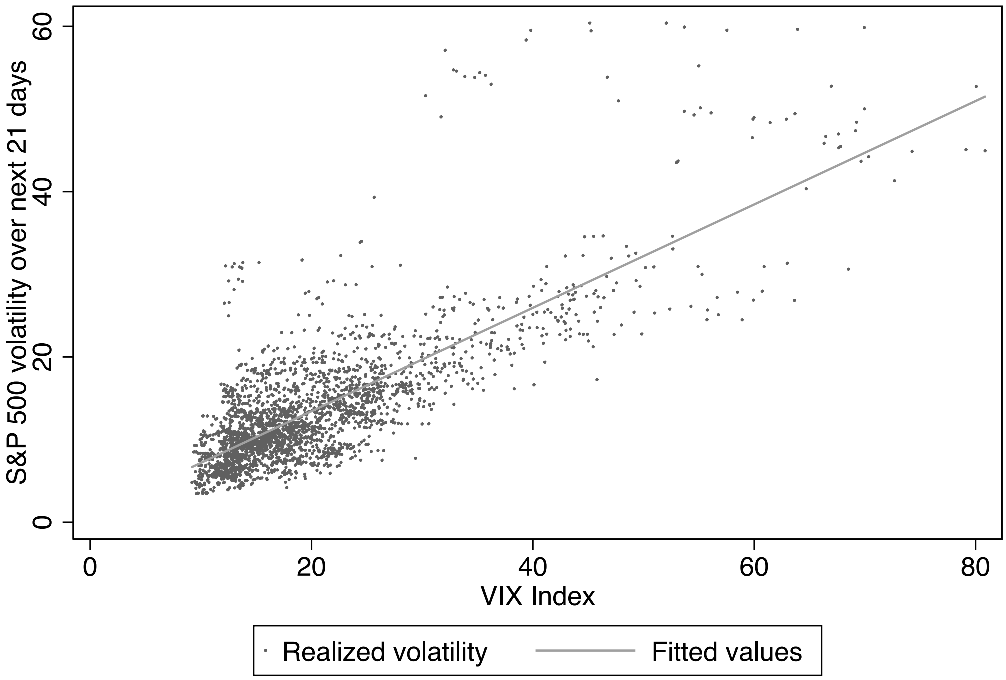
$$VIX = 6.976 + 0.975 SP_{fwd}$$

Hence both forward and historic volatility appear to be correlated with the level of the VIX Index. It is possible that the VIX Index has a mild predictive value, although this may merely be an artifact of persistence of volatility. Including historic volatility as a control:

$$VIX = 3.923 + 0.789 SP_{hist} + 0.410 SP_{fwd}$$

suggests that future S&P 500 Index volatility has a bearing on the level of the VIX Index, but that historic volatility is the greater determinant. All coefficients are significant at the 1% level with R^2 of 0.8037. A scatter plot of the level of the VIX Index against realized S&P 500 Index volatility over the subsequent 21 trading days is shown in Figure 3.8.

Figure 3.8: Realized S&P 500 Index 21-day forward volatility v. VIX Index level



The constant of 3.923 in the combined regression, which has a 95% confidence interval of 3.58 to 4.27, along with the sum of the remaining coefficients of 1.199, suggests that the VIX Index overstates realized volatility as measured by standard deviation. This may be explained if purchasers of options are primarily hedgers, with speculators providing liquidity and requiring a risk premium, which agrees with the assertion of Whaley (2009) that portfolio insurers drive the options market. A graph of the VIX Index against annualized volatility of the S&P 500 Index over the previous 21 trading days, shown in Figure 3.9, confirms this, with the VIX Index generally lying above the volatility line. There is little difference between using the historic or forward volatility figures here, as the former is merely a 21-day lag of the latter. This difference changes over time, which can be seen from Figure 3.10, a graph of a 21-day moving average of the difference between the level of the VIX Index and S&P 500 Index volatility. The difference between the two declined over the period from around 2011 onwards, going into reverse around

2015/16, but in recent times the VIX Index and realized S&P 500 Index volatility appear once again to be diverging.

Figure 3.9: VIX Index v. annualized S&P 500 Index 21-day historic volatility 2/2/07–2/1/17

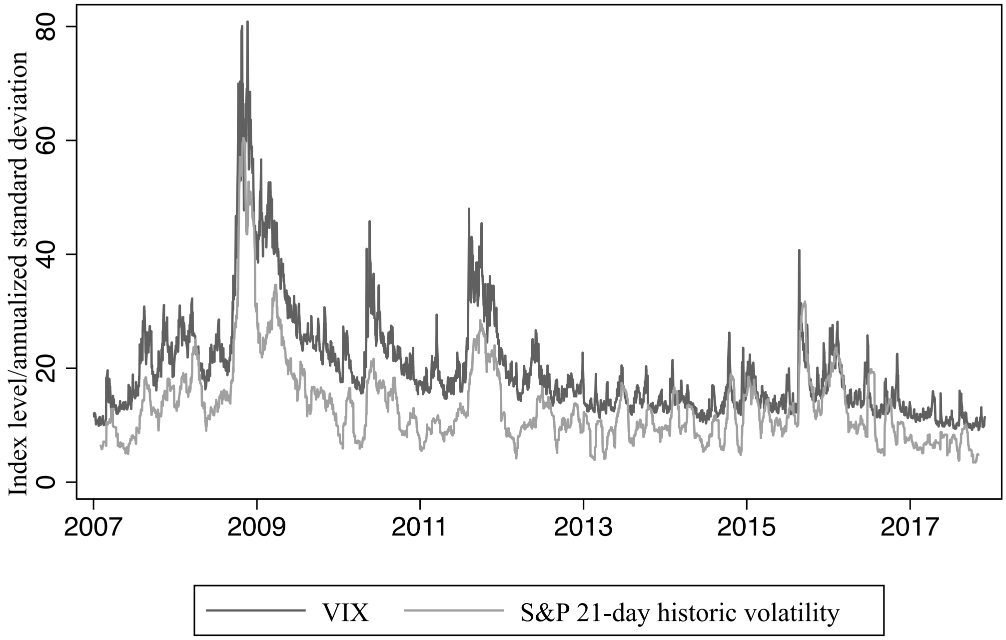


Figure 3.10: 21-day MA of VIX index less S&P historic 21-day volatility 2/2/07–11/2/17

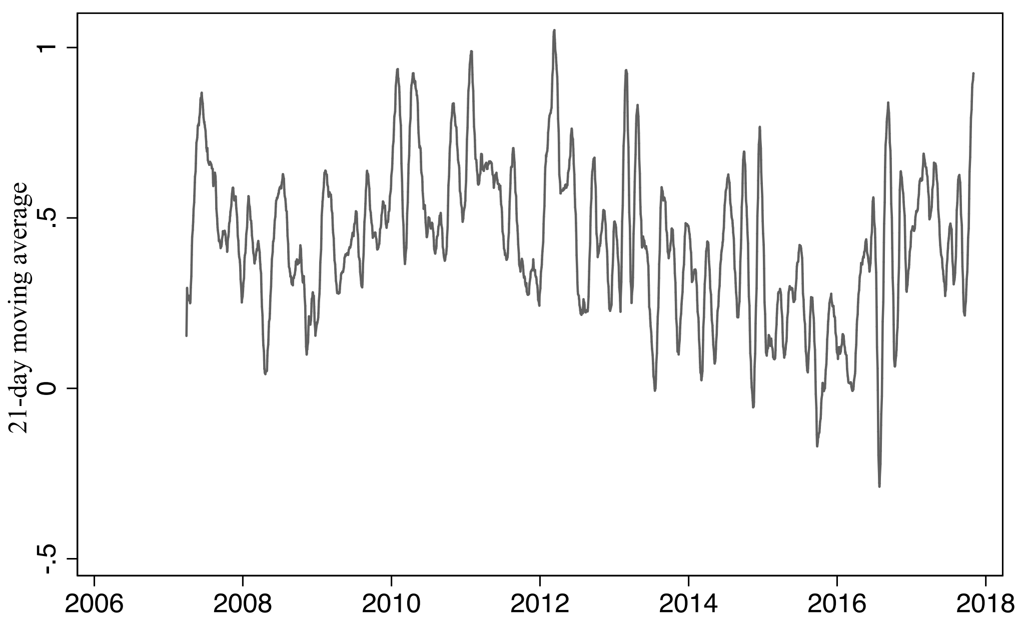


Table 3.2

Results of Ordinary Least Squares regressions of the level of the VIX Index on S&P 500 Index volatility

				VIX = constant + SP_{hist}			VIX = constant + SP_{hist} + SP_{fwd}				
	Obs	Mean premium	Mean premium %	Regression constant	Historic SD coefficient	adj. R^2	Regression constant	Historic SD coefficient	Forward SD coefficient	Sum	adj. R^2
2007*	210	7.32	72.60%	7.001	1.028	0.6224	4.594	0.724	0.501	1.225	0.7253
2008	253	8.62	45.20%	7.558	1.044	0.8614	5.505	0.892	0.230	1.122	0.8840
2009	252	11.70	62.30%	6.548	1.259	0.7281	5.841	0.410	0.947	1.357	0.8857
2010	252	10.00	91.40%	10.506	0.960	0.5956	7.107	0.849	0.386	1.235	0.6899
2011	252	9.23	74.20%	8.161	1.072	0.7679	6.292	0.772	0.416	1.188	0.8128
2012	250	7.24	79.00%	16.841	0.091	0.0078	14.136	-0.009**	0.354	0.345	0.1082
2013	252	4.39	58.60%	12.042	0.222	0.1395	10.122	0.245	0.174	0.419	0.2219
2014	252	3.70	47.00%	10.037	0.395	0.2829	5.923	0.472	0.301	0.773	0.4690
2015	252	2.13	20.36%	7.814	0.610	0.5749	6.376	0.581	0.125	0.706	0.5980
2016	252	3.48	44.08%	9.888	0.481	0.3914	6.279	0.347	0.465	0.812	0.6941
2017*	216	4.47	75.04%	3.605	1.134	0.8222	3.047	0.627	0.599	1.226	0.8905
Full period	2693	6.59	60.48%	5.936	1.096	0.7712	3.902	0.789	0.411	1.200	0.8066

* Part years

** No significance

VIX = level of the VIX index

SP_{hist} = annualized standard deviation of the S&P500 index over the previous 21 trading days

SP_{fwd} = annualized standard deviation of the S&P500 index over the following 21 trading days

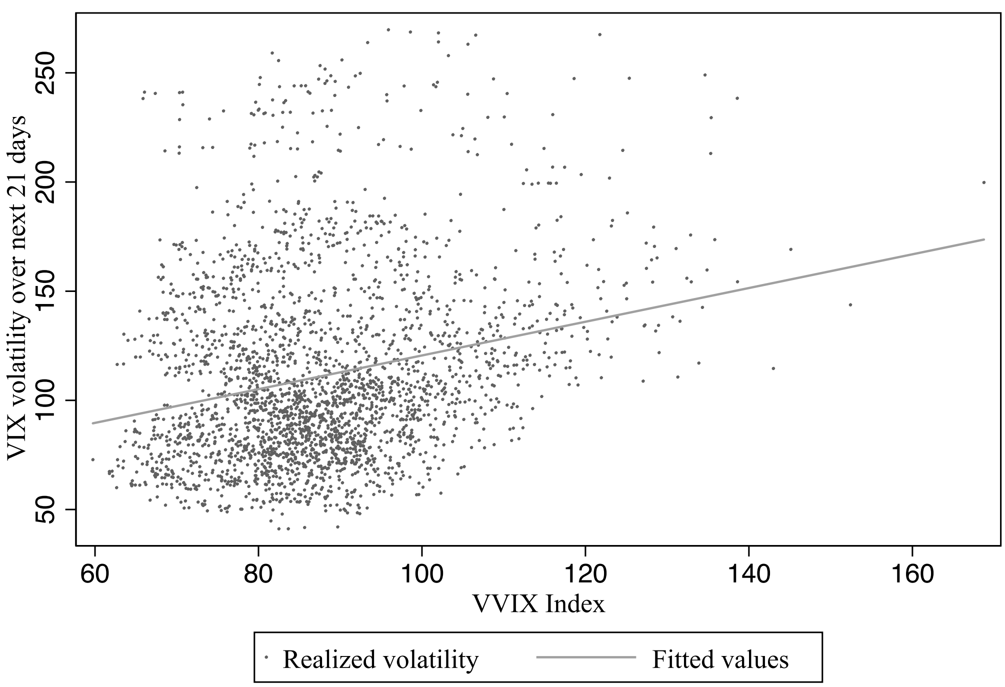
All coefficients significant at the 1% level unless stated

Regression results for each individual calendar year set out in Table 3.2 show how the mean premium of the level of the VIX Index over S&P 500 Index historic 21 trading day volatility in terms of points, and to a lesser extent percentage, has declined and then increased again. There is, however, a significant premium for each year over the period.

If the premium of the VIX Index over realized S&P 500 Index volatility is the result of option overpricing, then the decline in premium until around 2015/16 might have represented the gradual elimination of that overpricing. Alternatively, however, it may merely reflect a reduction in put option premiums over a long period of a relatively calm equity market that has now gone into reverse as nervousness over the length of the current bull market run increases.

The scatter plot of the level of the VVIX Index against subsequent 21 trading day volatility in the VIX Index shown in Figure 3.11 gives a similar but less clear relationship than for the VIX Index and S&P 500 Index volatility.

Figure 3.11: Realized VIX 21-day forward volatility v. VVIX Index level



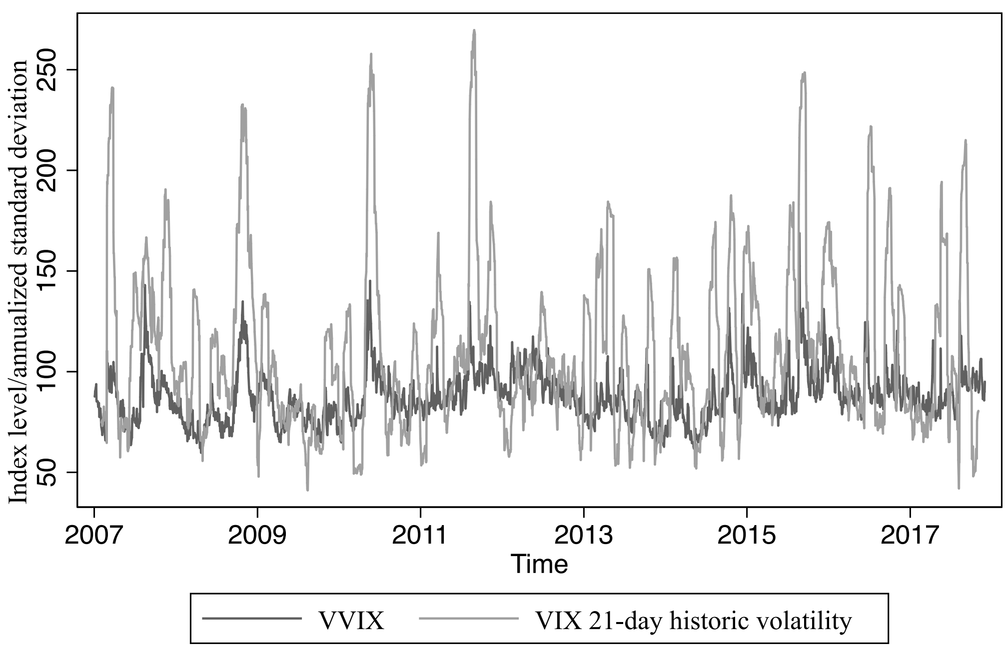
As above, a simple OLS regression with robust standard errors is carried out of annualized standard deviation of the VVIX Index on VIX Index changes over the previous 21 trading days (VIX_{hist}) and that of the subsequent 21 trading days (VIX_{fwd}).

A regression of the VVIX Index on VIX Index historic 21 trading day volatility alone gives an adjusted R^2 of 0.2855 with both coefficients significant at the 1% level

$$VVIX = 69.378 + 0.166 VIX_{hist}$$

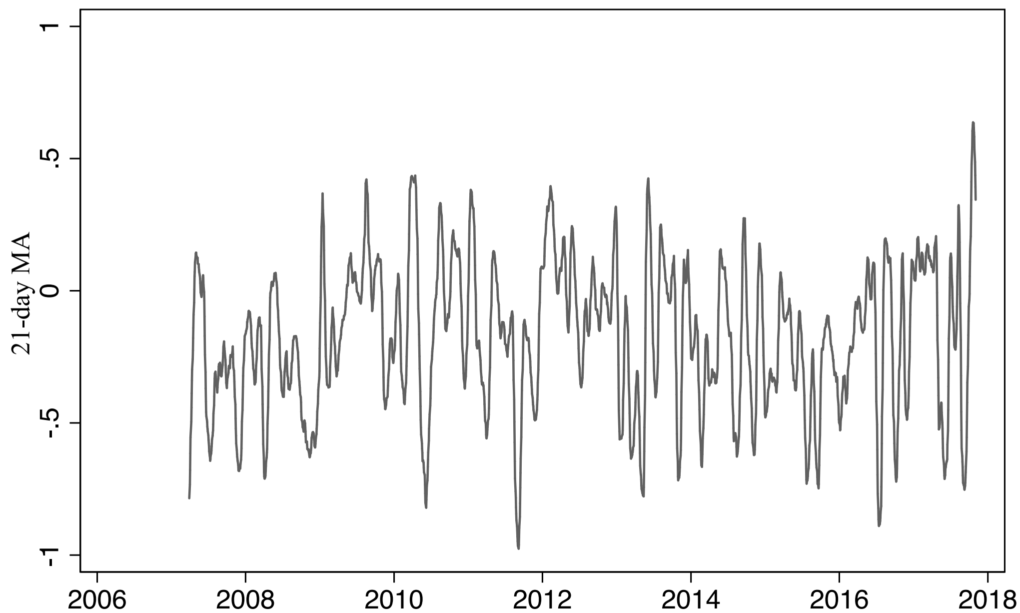
The large constant and small coefficient on the VIX Index volatility parameter suggests that there is a looser relationship between the level of the VVIX Index and realized VIX Index volatility than for the VIX Index and S&P 500 Index volatility. A plot comparing the VVIX Index with the annualized realized 21-day historic volatility of the VIX Index, shown in Figure 3.12, confirms that the VVIX Index understates VIX Index volatility significantly at times of high volatility.

Figure 3.12: VVIX Index v. annualized VIX Index 21-day historic volatility 2/2/07–12/1/17



A graph of the VVIX Index less a 21-day moving average of realized VIX Index volatility in Figure 3.13 shows that, unlike the VIX Index, the VVIX Index generally understates the volatility of the VIX Index. In contrast to the relationship between the VIX Index and S&P 500 Index volatility, there does not appear to be any changing trend in the relationship over recent times.

Figure 3.13: 21-day MA of VVIX index less VIX historic 21-day volatility 2/2/07–11/2/17



Regressing the VVIX Index on forward 21 day trading volatility alone gives a much smaller adjusted R^2 of 0.0746, although again with both coefficients significant at the 1% level:

$$VVIX = 78.240 + 0.087 VIX_{fwd}$$

Including historic volatility as a control:

$$VVIX = 63.239 + 0.157 VIX_{hist} + 0.065 VIX_{fwd}$$

gives an adjusted R^2 of 0.3255. Hence there appears to be a mild correlation between the level of the VVIX Index and both historic and future VIX Index volatility, but this is insignificant in terms of scale in the case of the latter. Adjusted R^2 values for univariate regressions of the VVIX Index on historic volatility (0.2855) and future volatility (0.0746) also suggest that historic volatility is a greater determinant of expected volatility, as is the case for S&P 500 Index volatility and the VIX Index.

While there is an argument to be made for the VIX Index having some predictive power regarding subsequent S&P 500 Index volatility, there is insufficient evidence to make a similar claim for the VVIX Index. For all regressions on the VVIX Index, the constant is much greater and volatility coefficients much smaller than is the case between the VIX Index and S&P 500 Index volatility, as can be seen from Table 3.3. This may suggest that shocks to the VIX Index are less persistent than those to the S&P 500 Index and that mean reversion of the VVIX Index is greater than that of the VIX Index, which the line chart of VIX Index 21-trading day volatility in Figure 3.12 appears to confirm.

Volatility over a 30-day period is, however, only a proxy for what the VIX Index and VVIX Index represent: the expected standard deviation of 30–day returns, not the expected standard deviation of the underlying index over the coming 30 days.

In order to demonstrate the difference between realized index volatility and the volatility of index returns, for the period 1/31/2007 to 12/01/2017, the standard deviation, the generally accepted measure of volatility, was calculated for the previous 21 trading days for daily movements on the S&P 500 Index and VIX Index. This was then compared with the 21-day overall change for the index over that period. The outcomes were then measured in terms of standard deviation and the number of occasions on which various standard normal distribution confidence intervals were exceeded was calculated.

Table 3.3

Results of Ordinary Least Squares regressions of the level of the VVIX Index on VIX Index volatility

				VVIX = constant + VIX_{hist}			VVIX = constant + VIX_{hist} + VIX_{fwd}				
	Obs	Mean discount	Mean discount %	Regression constant	Historic SD coefficient	adj. R^2	Regression constant	Historic SD coefficient	Forward SD coefficient	Sum	adj. R^2
2007*	210	42.50	26.40%	68.290	0.158	0.2543	67.274	0.158	0.008**	0.166	0.2511
2008	253	35.31	25.20%	45.507	0.310	0.7324	39.247	0.277	0.086	0.363	0.7821
2009	252	6.85	2.80%	63.389	0.189	0.2408	56.747	0.177	0.090	0.267	0.2858
2010	252	15.98	2.50%	67.368	0.201	0.6193	64.030	0.191	0.043	0.234	0.6453
2011	252	30.34	16.10%	77.587	0.125	0.3481	71.343	0.121	0.054	0.175	0.4087
2012	250	1.50	5.20%	85.977	0.095	0.0365	75.040	0.105	0.102	0.207	0.0860
2013	252	24.94	13.90%	74.683	0.055	0.0511	68.865	0.062	0.050	0.112	0.0090
2014	252	26.68	17.90%	57.432	0.233	0.3515	36.233	0.266	0.154	0.420	0.5082
2015	252	37.17	23.97%	61.777	0.250	0.5119	61.157	0.25	0.005**	0.255	0.5101
2016	252	25.81	14.08%	84.311	0.072	0.0819	72.646	0.092	0.082	0.174	0.1913
2017*	216	14.82	0.20%	80.717	0.080	0.0909	76.823	0.077	0.040	0.117	0.1096
Full period	2693	23.39	12.51%	69.378	0.166	0.2855	63.239	0.1573	0.065	0.222	0.3255

* Part years

** No significance

VVIX = level of the VVIX index
VIX_{hist} = annualized standard deviation of the VIX index over the previous 21 trading days
VIX_{fwd} = annualized standard deviation of the VIX index over the following 21 trading days
All coefficients significant at the 1% level unless stated

Given normally distributed daily movements, it would be expected that, for example, a 95% confidence interval would be exceeded approximately 5% of the time, as for a Value-At-Risk exercise.

The results are shown in Table 3.4. The number of occasions on which the index exceeded the two-sided confidence interval indicated in the first column to the upside (positive) and downside (negative) is shown, together with the total and percentage of times overall on which that confidence level was breached.

Table 3.4: Analysis of 21-day returns and period standard deviation, 1/31/2007–12/1/2017

Standard deviation of daily returns/changes on the S&P 500 Index and VIX Index were calculated for periods of 21 days, and compared with the return/change over that period. Two-sided confidence intervals were applied, and the number of occasions on which the return/change exceeded the confidence interval calculated.

S&P 500 Index

Confidence interval	Positive	Negative	Total	Percent
50%	1050	475	1525	55.9%
80%	504	181	685	25.1%
90%	302	81	383	14.0%
95%	203	49	252	9.2%
99%	47	6	53	1.9%

VIX Index

Confidence interval	Positive	Negative	Total	Percent
50%	354	504	858	31.4%
80%	71	119	190	7.0%
90%	15	44	59	2.2%
95%	4	18	22	0.8%
99%	0	6	6	0.2%

S&P 500 Index returns exceeded all of the two-sided confidence intervals more frequently than a standard normal distribution of returns would suggest.

This may be indicative of a momentum effect, with returns in one direction tending to cluster, and it can be seen that outsize returns are generally to the upside. The average standard deviations for those periods where the downside level was breached are, however, more than twice as large on average as on those occasions when the upside level was breached, confirming that falling markets tend to be more volatile than rising markets.

The daily return over the period was positive for 52.1% of observations, and the 21-day return positive for 63.2% of observations. Given a generally rising market, which was the case for most of the period under consideration, the relatively large number of upside breaches is unsurprising. Daily returns are roughly symmetrical about a mean of 0.022% with negative skewness of –0.121, and leptokurtic relative to a normal distribution with excess kurtosis of 6.678. A histogram of the distribution of S&P 500 Index daily log returns is shown in Figure 3.14. A probability density function for a normal distribution based on the same mean (0.0300%) and standard deviation (0.9890%) as the S&P 500 Index daily returns over the period is also shown. The empirical cumulative distribution of the returns is compared to the cumulative distribution function of a normal distribution in Figure 3.15, and the difference between the two in Figure 3.16. A Doornik-Hansen normality test gives a chi-squared of 1,643 with rejection of normality at the 1% level.

Figure 3.14: Distribution of S&P 500 Index daily log returns

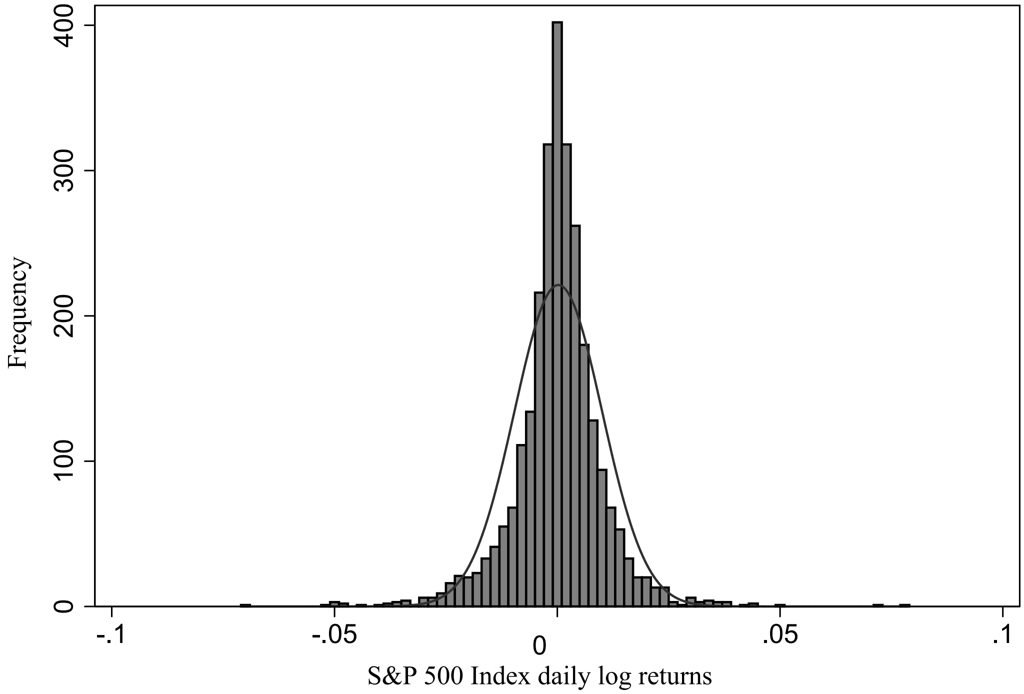


Figure 3.15: S&P 500 Index daily log return empirical cumulative distribution 1/3/1990 - 12/1/2017

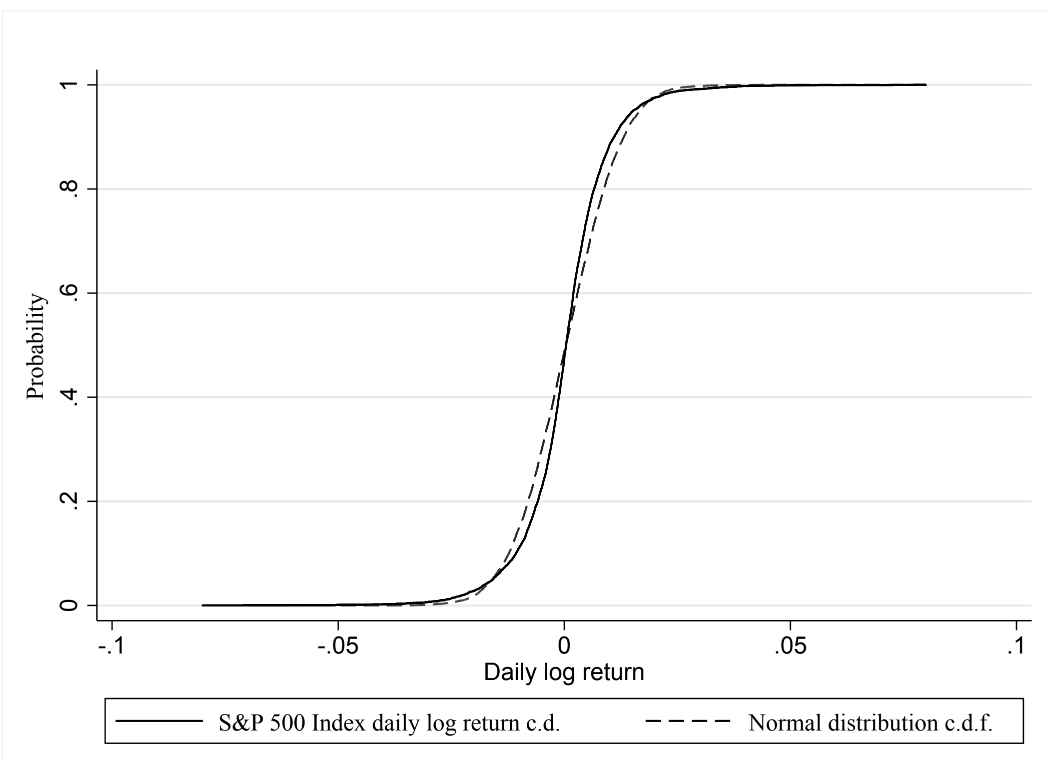
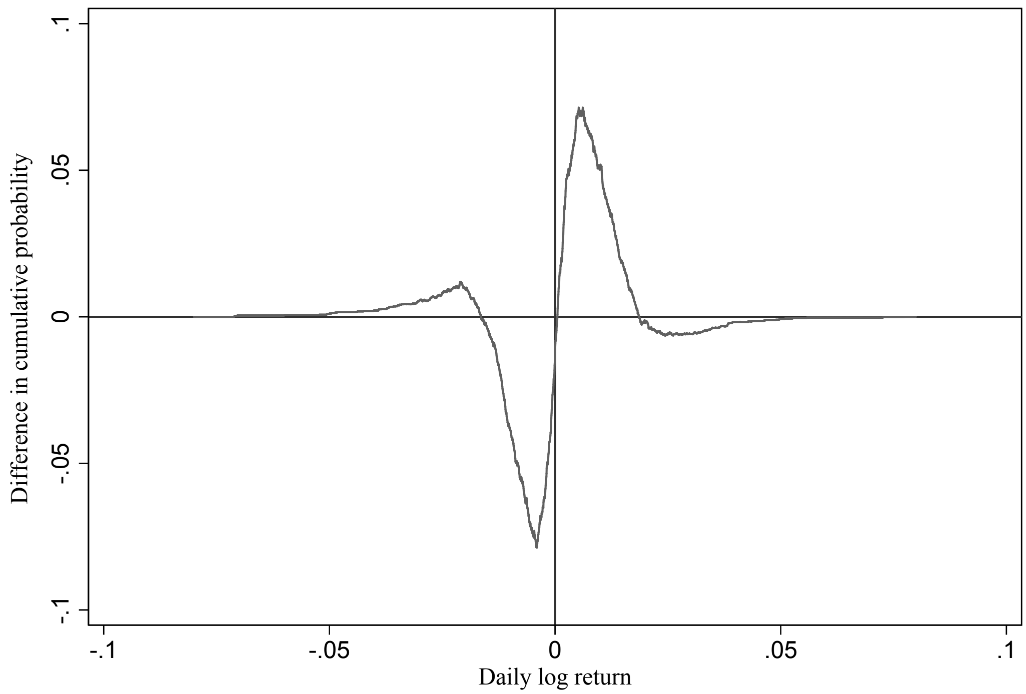


Figure 3.16: Difference between S&P 500 Index daily return empirical cumulative distribution and normal distribution c.d.f.



The probability density and cumulative distribution graphs for 21-day returns are shown in Figure 3.17 to 3.19. There is negative skewness of –1.525 about a mean of 0.438% with excess kurtosis of 6.668. A Doornik-Hansen test gives a chi-squared of 587, again with rejection of normality at the 1% level.

Figure 3.17: Distribution of S&P 500 Index 21-day log changes, 1/3/1990–12/1/2017

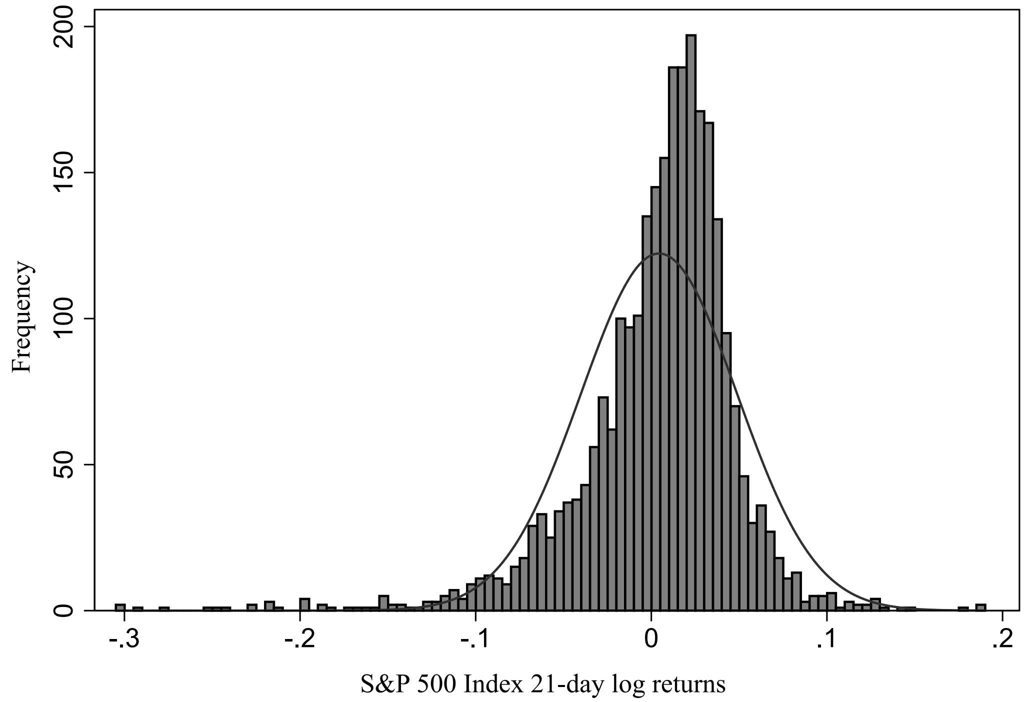


Figure 3.18: S&P 500 Index 21-day log return empirical cumulative distribution
1/3/1990 - 12/1/2017

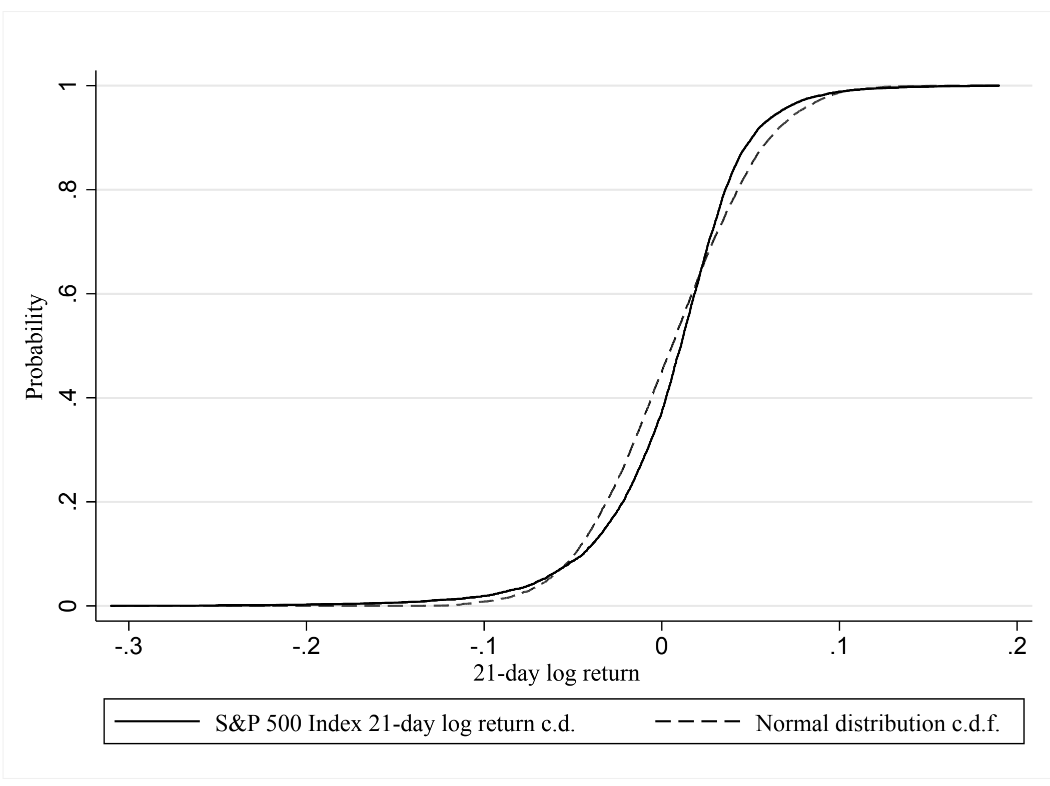
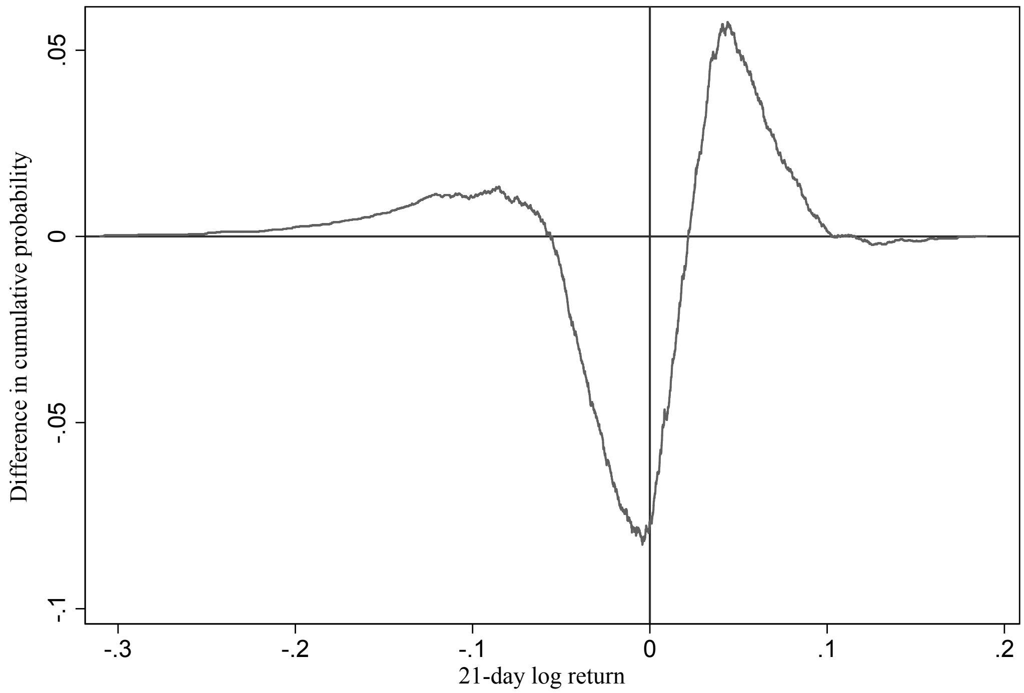


Figure 3.19: Difference between S&P 500 Index 21-day return empirical cumulative distribution and normal distribution c.d.f.



In contrast to the S&P 500 Index, VIX Index 21-day changes exceeded all confidence intervals less frequently than a standard normal distribution would suggest. This is characteristic of mean-reverting behavior.

Outsize movements are generally to the downside, and average standard deviations for observations where the upside level was breached are larger on average than when the downside level was breached. The daily change over the period was negative for 53.8% of observations, and the 21-day movement was negative for 55.3% of observations. Daily changes have a positive skewness of 0.744 about a mean of −0.007%, and are also leptokurtic relative to a normal distribution with excess kurtosis of 3.976.

The probability density of VIX Index daily log changes is shown in Figure 3.20, the cumulative distribution in Figure 3.21, and a comparison with the normal distribution cumulative distribution function in Figure 3.22. A Doornik-Hansen normality test gives a chi-squared of 479 with rejection of normality at the 1% level.

Figure 3.20: Distribution of VIX Index daily log changes, 1/3/1990–12/1/2017

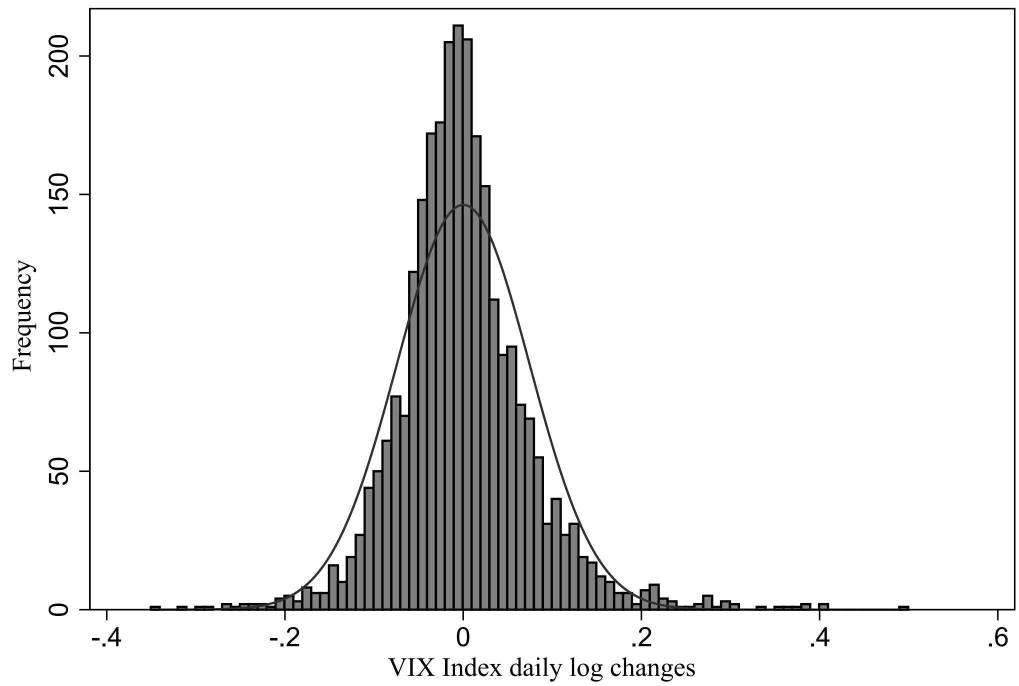


Figure 3.21: VIX Index daily log change empirical cumulative distribution 1/3/1990–12/1/2017

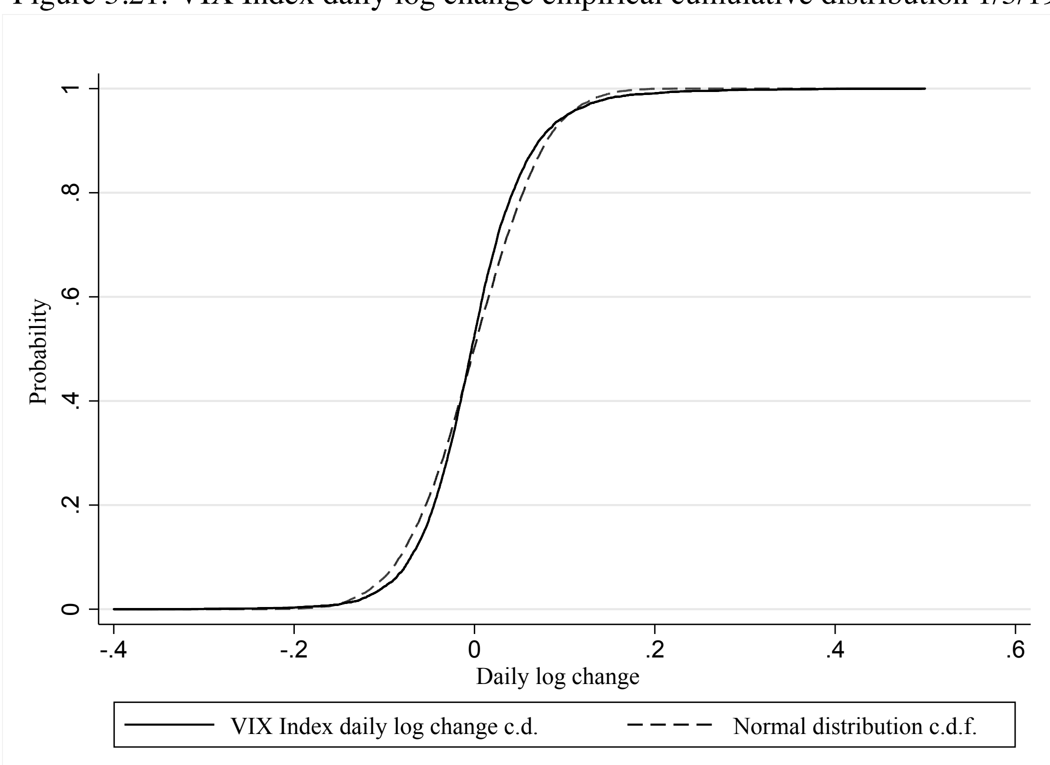
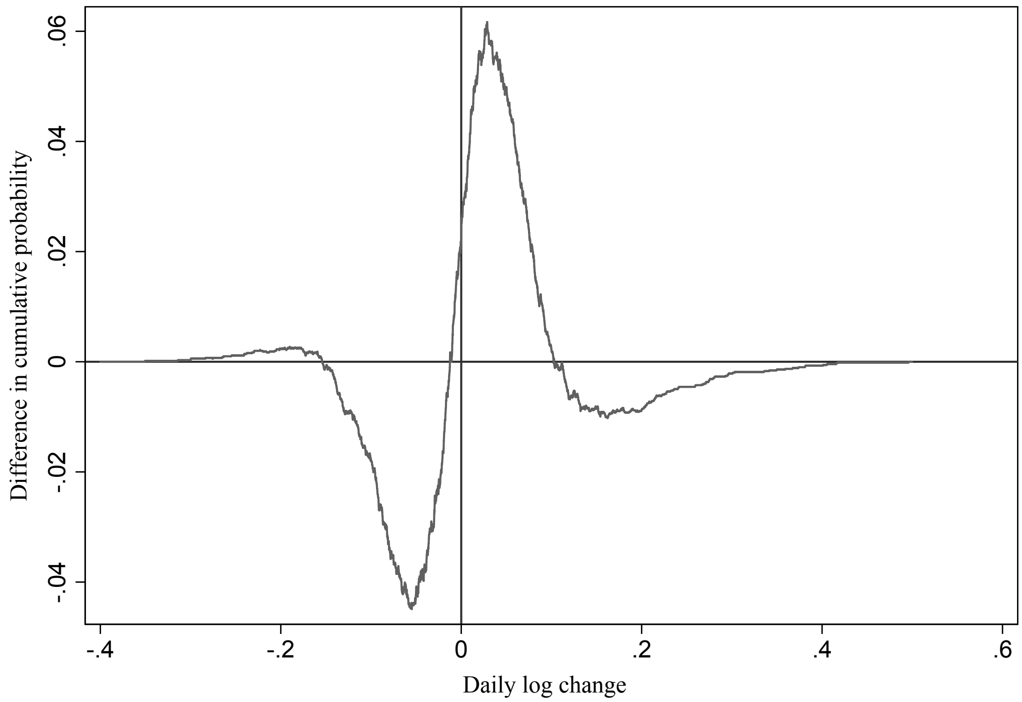


Figure 3.22: Difference between VIX Index daily log change empirical cumulative distribution and normal distribution c.d.f.



The 21-day returns show positive skewness of 0.820 and excess kurtosis of 1.946 about a mean of –0.02%, and a Doornik-Hansen normality test gives a chi-squared of 239 with rejection of normality at the 1% level. Probability density and cumulative distribution are shown in Figures 3.23 to 3.25.

Figure 3.23: Distribution of VIX Index 21-day log changes, 1/3/1990–12/1/2017

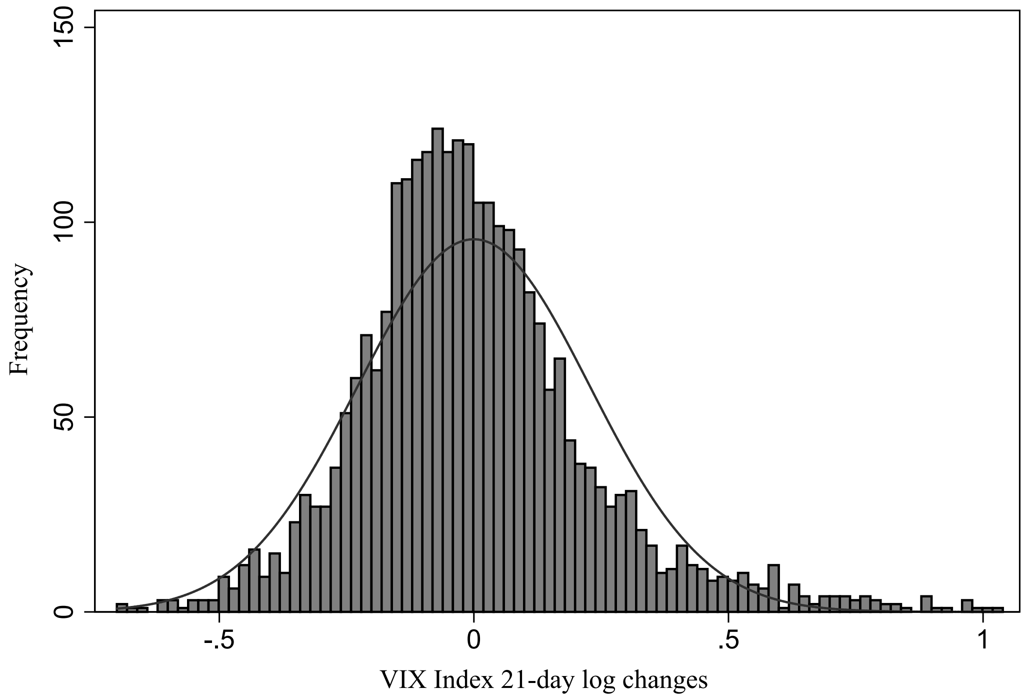


Figure 3.24: VIX Index 21-day log change empirical cumulative distribution 1/3/1990 - 12/1/2017

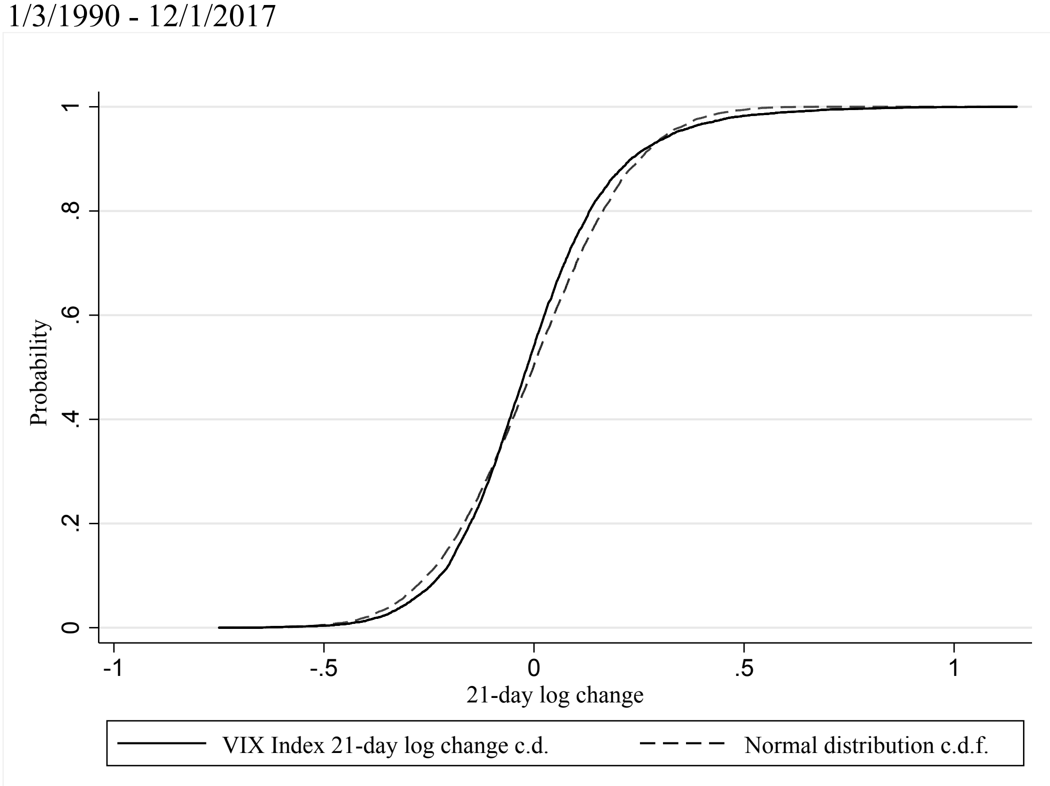
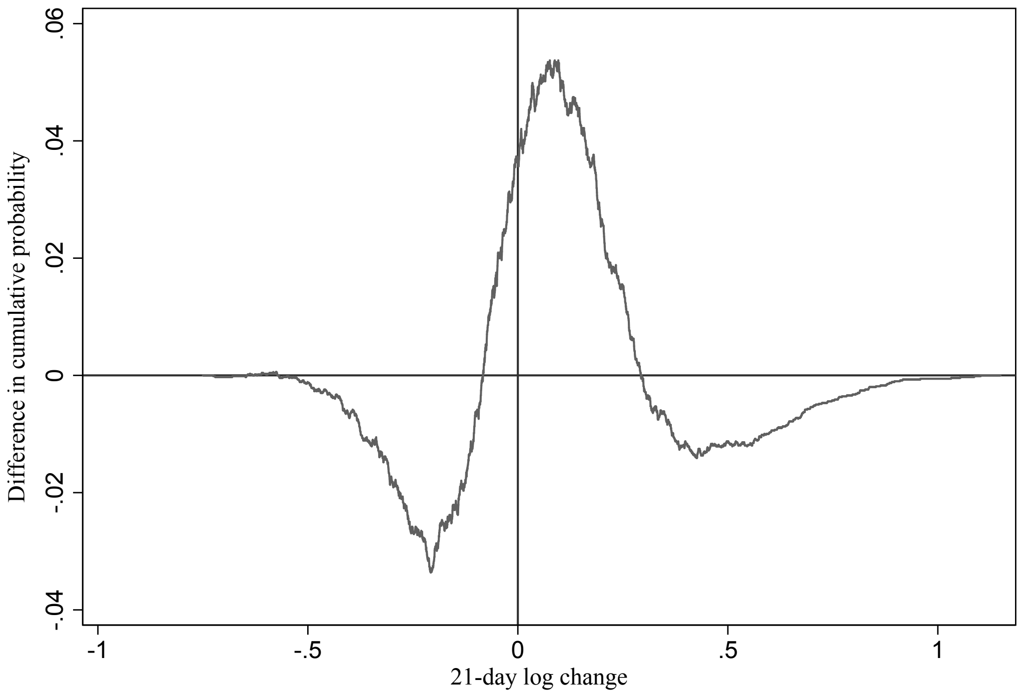


Figure 3.25: Difference between VIX Index 21-day log change empirical cumulative distribution and normal distribution c.d.f.



A comparison of daily and 21-day returns for the S&P 500 Index in Table 3.5 shows a marked difference in skewness, indicating a momentum effect.

The difference between the realized 21-day volatility of both the S&P 500 Index and the VIX Index with the expected range of 21-day change outcomes based on those volatilities help explain why the VIX Index and VVIX Index are poor indicators of daily volatility in their underlying instruments. The primary factor in the determination of European option prices must be CSV, the potential range of outcomes for the underlying instrument at expiry. Where daily movement distributions are not normal, as is shown to be the case for both the S&P 500 index and the VIX Index, a volatility index calculated from option premiums cannot directly reflect the volatility of the underlying asset or TSV. Thus the VIX Index will tend to overstate S&P 500 Index volatility, and the VVIX Index will understate VIX Index volatility, as shown empirically.

Table 3.5: S&P 500 Index and VIX Index daily and 21-day returns, 1/31/2007–12/1/2017

The mean, median, standard deviation, skewness and kurtosis of daily and 21-day returns/changes on the S&P 500 and VIX Index. The marked difference in skewness for the S&P 500 Index indicates a possible momentum effect.

	S&P Daily	S&P 21-day	VIX Daily	VIX 21-day
Mean	0.022%	0.438%	-0.004%	-0.023%
Median	0.032%	1.157%	-0.567%	-2.406%
Std. Dev.	0.894%	4.451%	7.444%	22.769%
Min	-7.099%	-30.486%	-35.059%	-70.118%
1st percentile	-2.780%	-15.153%	-17.868%	-48.290%
99th percentile	2.646%	9.591%	22.802%	73.502%
Max	7.731%	18.888%	49.601%	102.252%
Skewness	-0.122	-1.525	0.737	0.820
Kurtosis	9.678	9.668	6.953	4.946

Whaley (2009) tests the performance of the VIX Index as a predictor of 30-day volatility by calculating the proportion of times the return on the S&P 500 Index falls outside the confidence interval suggested by the implied volatility of S&P 500 Index options at the start of each month as reflected in the VIX Index.

For the period 1/31/1990 – 12/1/2017, the proportion of times that the S&P 500 Index 30 day return, assuming a 21 working day period, lies outside various confidence intervals as implied by the level of the VIX is shown in Table 3.6. The number of occasions on which the index level was exceeded to the upside or the downside is also recorded.

Table 3.6: S&P 500 Index 21-day return compared to VIX Index level, 1/31/1990–21/1/2017

The return over 21 days on the S&P 500 Index is compared with the expected standard deviation based on the level of the VIX Index at the start of the period, and the number of occasions on which the return exceeds various confidence levels is calculated.

S&P 500 Index

Expected level	Positive	Negative	Total	Percent
1%	0	38	38	0.5%
5%	29	113	142	2.0%
10%	94	202	296	4.2%
20%	308	380	688	9.8%
50%	1816	1068	2884	41.1%

As can be seen, extreme movements in the S&P 500 index are more likely to be seen on the downside, as would be expected, whereas for the 50% level, upside movements are in the majority. All of the realized figures are significantly below their expected levels, suggesting that implied volatility is greater than actual volatility, and that therefore option premiums might be higher than they should be. This concurs with the evidence presented above and the findings of Huang & Shaliastovich (2014). Hence option writing should be profitable, suggesting that hedgers are tending to the long side. This does indeed appear to be the case, agreeing with Bollen and Whaley (2004), with demand for S&P 500 Index options being largely driven by portfolio insurers and demand for puts being the main driver of S&P 500 Index option premiums.

Looking at the later period 1/31/2007 to 12/1/2017 in Table 3.7, for which VVIX index data are available, it is seen that option premiums are still lower overall than expected, although a greater proportion of 1% expected level extreme events occurred. This is perhaps unsurprising, as although the bear market of 2000-2003 is not included, the data begin around the start of the financial crisis and the relatively stable 1990s are now excluded.

Table 3.7: S&P 500 Index 21-day return compared to VIX Index level, 1/31/2007–12/1/2017

The return over 21 days on the S&P 500 Index is compared with the expected standard deviation based on the level of the VIX Index at the start of the period, and the number of occasions on which the return exceeds various confidence levels is calculated.

S&P 500 Index

Expected level	Positive	Negative	Total	Percent
1%	0	24	24	0.9%
5%	0	63	63	2.3%
10%	11	100	111	4.1%
20%	68	169	237	8.7%
50%	715	424	1139	41.7%

One can therefore conclude that the VIX Index, in terms of its performance as an indicator, tends to overstate realized volatility of the S&P 500 Index, or that expectations of volatility as based on implied volatility are biased upwards.

As noted above, the VVIX Index, representing implied volatility of the VIX Index, is calculated in an identical fashion to the way the VIX is calculated in relation to implied volatility of the S&P 500 Index. Like the S&P 500 Index options traded on CBOE, VIX Index options are also European style, exercisable only on expiry. Hence it is also appropriate to think of the VVIX Index as reflecting the range of possible changes between the present and 30 days hence on the VIX Index rather than a measure of volatility in the VIX Index during a 30-day period.

For the period 1/31/07 - 12/1/17, the proportion of occasions on which 30-day changes on the VIX Index exceed expected intervals suggested by the VVIX Index are shown in Table 3.8.

The distribution of returns is clearly different from that seen for the S&P 500 Index. Breaches of smaller confidence intervals again occur less frequently than the implied volatility of the VIX Index predicts, but extreme returns are far more numerous than expected. Both relative to historic and future 30-day volatility, the VIX Index exceeds the realized level of volatility far more frequently than the VVIX Index predicts. The implied volatility of VIX Index options is therefore lower than the true volatility of the index, suggesting both that VIX Index option premiums may be low and that VVIX Index, at least in terms of an absolute interpretation of the index, understates the subsequently realized volatility of the VIX Index.

Table 3.8: VIX Index 21-day change compared to VVIX Index level, 1/31/2007–12/1/2017

The return over 21 days on the S&P 500 Index is compared with the expected standard deviation based on the level of the VIX Index at the start of the period, and the number of occasions on which the return exceeds various confidence levels is calculated.

VIX Index

Expected level	Positive	Negative	Total	Percent
1%	91	0	91	3.3%
5%	147	26	173	6.3%
10%	205	68	273	10.0%
20%	295	180	475	17.4%
50%	579	660	1239	45.4%

If the models are valid, then the relationship between the indices and standard deviation models should be the same for the S&P 500 Index and VIX Index models given that the calculation methodologies are identical in relation to the option premiums on the underlying indices. There are a number of possible explanations for the apparent differences.

Demand for S&P 500 Index and VIX Index options have different motivations. As Whaley (2009) explains, far greater volume is seen in puts than calls in the S&P 500 Index options market, primarily due to the growth in portfolio insurance strategies over the last 20 years. Indeed, this was one reason for the change in calculation methodology of 2003. At the beginning of the 1990s, puts and calls were traded in roughly equal volumes, yet during the first 10 months of 2008, average daily volume of puts was 72% greater than that for calls, with high demand for puts both at-the-money and out-of-the-money. Bollen and Whaley (2004) show that demand for puts is a major determinant of movement in the VIX Index.

For the VIX Index, however, the daily volume of calls exceeds that of puts. Demand for VIX Index options is also centered around at-the-money series and hence premium levels are not distorted by excess demand for out-of-the-money puts. Conversely, put volume has been very low compared to call volume, particularly up to around 2010, which may provide its own distortion.

CBOE promotes VIX Index options as a hedge against volatility, which can explain call trade. Puts may be of interest to option traders, but they would likely prefer to hedge by taking counter positions in options held rather than adopt basis risk by using VIX Index put option positions. The primary practical use for VIX Index options is to enable S&P options traders to hedge vega, the effect on option prices of a change in the volatility of the underlying index.

It may be that the calculation methodology is incorrect. The implied volatility derived from the option prices does not rely on any particular options model. That is not to say, however, that the calculation accurately reflects true beliefs about volatility. There is still an assumption of lognormal returns and hence fat tails may go some way to explaining a discrepancy in implied and realized volatility.

The problem of option liquidity highlighted by Grover & Thomas (2012) affects the value of the VIX Index. Where there is persistent excess demand for puts, this may contribute to a persistent mispricing. Far out-of-the-money puts have higher implied volatilities than those with strike prices nearer to the current price of the underlying index, the source of the so-called volatility skew which appeared following the October 1987 stock market crash, as described by Rubinstein (1994). A possible reason for the relatively high demand for far out-of-the-money puts is that if the market falls sharply, this can prompt further selling in a self-perpetuating vicious cycle. This is frequently cited as a factor in the severity of the October 1987 crash, for example by Shiller (1988), Jacklin, Kleidon & Pfleiderer (1992), Jacobs (1999), MacKenzie (2004) and others. Thus a large fall has an increased likelihood of becoming an even larger fall.

If S&P 500 Index and VIX Index options are correctly priced, and the calculation methodology correctly reflects implied volatility, then prices may reflect some multi-sigma event yet to occur, something that will affect the share index far more strongly than it would the volatility of the index. Here may lie a plausible explanation for the discrepancy in VIX Index and VVIX Index valuations. If the share market were to suffer a major collapse, ongoing volatility would likely not be affected to as great an extent as would the price level. A one-off economic shock could see a major shift in the equity market, yet if the perceived probability of such a shock occurring in the future does not increase, then volatility need not be affected. Hence equity option premia would be higher than VIX Index option premia. The puzzle of why VIX Index option premia appear underpriced remains, however.

Dzekounoff (2010) states that VIX Index options have a steep call skew, citing the main reasons for this skew as being institutional hedging demand creating upward pressure on call prices, and that, as shown above, spikes in S&P 500 Index volatility are higher on the upside than the downside, thus boosting call premiums. It may be that the resulting disconnect between put and call prices causes the pattern seen in realized VIX Index volatility as related to levels of the VVIX Index, whereby extreme predicted outcomes on the upside are more common, and moderate predicted outcomes less common, than a normal distribution of outcomes would suggest. The movement of the VVIX Index suggests a greater mean reversion effect than with the VIX Index.

3.5 Conclusion

It appears that the VIX Index may have a moderate predictive value as regards future S&P 500 Index volatility, but that any information on future VIX Index volatility contained in the VVIX Index is of too little magnitude to be of practical use. Being based on European style option prices, however, the VIX Index and VVIX Index should reflect the market's belief in the potential range of market outcomes 30 days hence and not directly reflect volatility in the intervening period.

A comparison of actual index outcomes against volatility index levels shows that the VIX Index tends to overstate S&P 500 Index volatility at times of both high and low volatility, and that the VVIX Index tends to understate VIX Index volatility at times of high volatility. It also appears that realized VIX Index volatility has a much flatter and more skewed distribution that S&P 500 Index volatility, with a greater incidence of extreme positive outcomes.

These results suggest that profits are to be made by buying VIX Index options and selling S&P 500 Index options. This may partly be explained by excess demand for S&P 500 Index out-of-the-money put options by portfolio insurers to the extent that put-call parity is stretched as a result of widening option spreads for far out-of-the-money put contracts. Significant hedging demand for shorting VIX Index options might also produce this effect, but there is no evidence for this. The relative persistence of possible future shocks to the S&P 500 Index compared to that for the VIX Index might be a contributing factor.

Put-call parity does not necessarily hold for VIX Index options as the underlying index is non-replicable, as Dzekounoff (2010) has shown, resulting in a steep call skew. Given this skew, the distilling of VIX Index implied volatility across puts and calls over a range of maturities to a single figure may render the VVIX Index to be of limited value as a volatility measure.

CHAPTER 4

MODELING VIX INDEX FUTURES PRICES: APPLYING THE NELSON SIEGEL CURVE-FITTING METHOD

4.1 Introduction And Literature Review

A long-standing problem with analysis of financial instruments with a fixed maturity date is that the maturity of a particular instrument is constantly changing. Where it is desired to find a price or a yield for a particular term, but no instrument is traded for that specific date, then a way must be found to establish a price or yield based on similar instruments of different maturities.

A common example is the term structure of interest rates as they relate to bond yields. Where the pricing of an asset depends upon the prevailing bond yield of a particular maturity, yet no bond exists for that maturity, an estimate for the yield must be found based on traded bonds for which yields are available. While various theories for the shape of the bond yield curve had long been proposed, an early example of a practical attempt to provide intermediate yields is to be found in Durand (1942), who discusses the difficulties in arriving at a best estimate for such yields. Various different models were designed over the years, a frequent feature of which was the relatively large number of parameters involved. Many also involved polynomials that result in unbounded estimates for longer maturities, thus limiting the range of maturities to which they can be usefully applied, or spline polynomial functions that increase the complexity of the models, such as to be found in Cohen, Kramer & Waugh (1966), Echols & Elliott (1976), Dobson (1978) and Chambers, Carleton & Waldman (1984).

This led Milton Friedman (1977), as quoted by Nelson Siegel (1987), to suggest that a model for the yield curve of relatively few parameters would be desirable. Charles Nelson and Andrew Siegel of the University of Washington introduced their method of modeling yield curves in Nelson & Siegel (1987), which provided a model of relatively few parameters that could be applied to yield curves, with their range of possible shapes.

It has since been adapted and applied in other areas. Shaw, Murphy & O'Brien (2014) and Castellanos, Constantinou & Ng (2015) apply the Nelson-Siegel model to the term structure of credit default swap spreads. Barunik & Malinska (2016) adapt the Nelson-Siegel model to apply to the term structure of crude oil futures, and it comprises part of a model to value endowment policies in Recchioni & Screpante (2014).

This paper contributes to the existing literature by applying the Nelson Siegel method to modeling the VIX Index futures curve.

The VIX Index is a measure of 30-day implied volatility of the S&P 500 Index derived from first near term and second near term out-of-the-money option prices. As a measure representing standard deviation, it is calculated as the square root of a weighted sum of options prices, and consequently cannot be dynamically replicated. This means that arbitrage is not possible and that therefore the cost of carry relationship between futures prices of different maturities, as described by Working (1949), need not apply. The price of a futures contract will then likely reflect an expectation of the future price of the underlying instrument at maturity, with perhaps an adjustment for risk. Futures prices thus contain information about investor expectations not to be found for those assets and instruments where no-arbitrage conditions apply.

Price data has been taken from Commodity Systems Inc. Only data from 2010 onwards have been used, although price data are available from when VIX Index futures first began trading on the Chicago Board Options Exchange in March 2004. In the first few years, VIX Index futures traded sporadically, particularly the longer maturity contracts, with low volumes. Average daily volume for 2004 was only 464 contracts, rising to 1,707 in 2006 and 4,504 in 2009. In 2010, by which time VIX Index-related Exchange Traded Notes had become established, average daily volume leapt to 17,279, although this is still a small fraction of the daily volume for 2016 of around 236,000. Quotes based on small volumes can be erratic, and may be estimated by committee in the absence of any recent trade. The results obtained from the shortened data set are stronger and more relevant to the current environment.

As discussed in Chapter 2, Figures 2.1 and 2.2, the volatility of VIX futures prices declines with maturity in accordance with the Samuelson (1965) hypothesis, which is characteristic of futures prices with non-arbitrageable underlying instruments.

Further evidence of the Samuelson effect for VIX Index futures prices is found by taking first differences of daily closing prices for each nearby contract, and running ordinary least squares regressions on each nearby with the next nearby, the results from which are shown in Table 4.1. The daily movement of each maturity contract is highly correlated with the movement of the contract next to it in maturity, as would be expected. In each case, the daily movement in the shorter maturity contract is greater than that for the longer maturity contract, within relatively narrow 95% confidence ranges and adjusted R^2 values ranging between 0.8084 and 0.9531. This accords with the prediction of the Samuelson hypothesis that shorter maturity contracts should be more volatile than longer maturity contracts. It is also the case, with the possible exception of the first two results, where the first nearby price close to expiry can be particularly volatile, that the difference in daily movement between each nearby reduces with maturity, which further accords with the Samuelson hypothesis, and suggests that the futures curve may be asymptotic with increasing maturity. It is notable that the adjusted R^2 value for the first regression is significantly lower than those for the remainder.

Table 4.1: Regressions of VIX Index futures contracts first differences

Ordinary least squares regressions are run of log first differences of true closing prices on each of the first six nearby futures contracts on the log first difference of the true closing price of the next longest nearby. In each case the constant coefficient is indistinguishable from zero and has not been noted. $C_{NB}1_{TFD}$ is the log first difference for the first nearby, $C_{NB}2_{TFD}$ the log first difference for the second nearby, and so on.

				95% confidence interval	Adj. R^2
$C_{NB}1_{TFD}$	=	1.281	$C_{NB}2_{TFD}$	1.254 – 1.309	0.8084
$C_{NB}2_{TFD}$	=	1.280	$C_{NB}3_{TFD}$	1.267 – 1.293	0.9467
$C_{NB}3_{TFD}$	=	1.162	$C_{NB}4_{TFD}$	1.151 – 1.174	0.9531
$C_{NB}4_{TFD}$	=	1.108	$C_{NB}5_{TFD}$	1.097 – 1.119	0.9510
$C_{NB}5_{TFD}$	=	1.086	$C_{NB}6_{TFD}$	1.075 – 1.097	0.9507
$C_{NB}6_{TFD}$	=	1.043	$C_{NB}7_{TFD}$	1.032 – 1.055	0.9421

The VIX Index displays mean-reverting behavior (Harvey & Whaley, 1992). Consequently as the VIX Index moves away from its mean, one would expect the longer contracts to reflect the expectation that the index will move back to its mean in the long-run, which is consistent with the fact that the longer maturity contracts are less volatile than the shorter. This also suggests that the futures curve should be close to asymptotic, tending towards a long-run mean as the maturity of contracts increases.

The analysis of futures prices poses a similar problem to that of bond prices, in that the maturity of a contract changes continuously. VIX Index futures are traded in monthly maturities of up to ten months, although quotes are frequently not available for the longer maturities as trading can be sporadic for these contracts. VIX Index futures for a particular month expire on the Wednesday that comes 30 days before the third Friday of the following month, so the contract with the shortest time to maturity, the first nearby, has a maximum maturity of 35 calendar days, reducing by one each day until expiry, at which point the formerly second nearby becomes the first nearby, with a remaining life of either 35 or 28 calendar days, other than on occasions when an expiration Wednesday is a holiday. At the same time, the former third nearby becomes the second nearby, with a remaining life of 63 or 56 days, the fourth nearby becomes the third nearby, and so on. Days on which the first nearby contract expires are known as roll dates.

Prices of contracts determined by which nearby they are, as used in the regressions of Table 4.1, have no adjustment for the change in the maturity of a contract at roll dates. The average daily movement in the first nearby over the period from 1/1/10 to 8/4/17, which would be expected to be close to zero for a mean-reverting underlying asset, is positive 0.036%, whereas the average change on roll dates, when the second nearby becomes the first nearby, is positive 7.181%. As noted by Whaley (2013) and others, the VIX Index futures curve is generally upward sloping, and hence the n+1th nearby will generally be higher than the nth nearby, and thus a roll date will tend to coincide with an upward jump in the price of the nth nearby price, which is now the contract that was the n+1th nearby the previous day. Around 4% of the first difference data points represent a change in nearby, and will therefore likely show a jump in price unrelated to the daily movement in the overall curve.

Any futures contract has a limited life, and consequently time series analysis of futures prices will tend to focus on nearby status and not specific contracts. This paper considers two methods for estimating notional contracts of specified maturity; simple linear interpolation, and the Nelson Siegel curve-fitting method, designed for yield curves, and assess their relative suitability for the analysis of VIX Index futures prices.

4.2 Linear Interpolation

Futures prices may be thought of as having a term structure in the same way as bond yields. One method of estimating price for a notional contract of intermediate maturity is by linear interpolation, which assumes a constant slope in the futures curve between the two contracts nearest to the desired maturity and is calculated as the sum of the prices of those contracts, weighted inversely according to the number of days between maturity of the contract and the maturity of the notional contract being estimated. This is the way in which the benchmark for short-term VIX Index-related Exchange Traded Notes, the S&P 500 VIX Short-Term Futures Index Total Return, is calculated, using prices of the first and second nearbys in proportion so as to give an average maturity of 30 calendar days.

Linear pseudo prices for a notional contract with maturity of 30 calendar days are created by linear interpolation of settlement prices for the first and second nearbys using the formula w_{C1}=(M2-30)/(M2-M1), where w_{C1} is the weight for the first nearby, (1-w_{C1}) the weight for the second nearby and M1 and M2 the number of days remaining before settlement date for the first and second nearbys respectively.

A notional contract with maturity of 60 calendar days is calculated using the second and third nearbys with w_{C2}=(M3-60)/(M3-M2), and notional contracts for maturities of 90, 120, 150, 180 and 210 days similarly.

In this manner, notional prices can be created for a contract of any maturity, treating the spot price, or level of the VIX Index itself, as a future with zero days to maturity in order to estimate prices for notional contracts shorter than the maturity of the first nearby. The resultant pseudo price will always be bounded by the prices of the nearby contracts nearest in maturity to it.

The regressions from Table 4.1 are repeated using first differences of linearly generated pseudo prices, shown in Table 4.2. These results show the same pattern as for the true prices, confirming the predictions of the Samuelson Hypothesis, however the confidence intervals are narrower and adjusted R^2 results higher. The coefficients are also falling with greater maturity, as before, including those relating to the first and second nearbys. In using linearly interpolated notional prices rather than true prices, the problem with roll dates has been eliminated or at least significantly ameliorated.

Table 4.2: Regressions of linearly derived notional VIX Index futures contracts first differences

VIX Index futures prices are derived for notional contracts of fixed maturity in multiples of 30 days maturity using linear interpolation. Ordinary least squares regressions are run of log first differences of each contract on the contract next longest in maturity to it. In each case the constant coefficient is indistinguishable from zero and has not been noted. C_M30_{LFD} is the log first difference for the linearly derived notional contract of 30 days maturity, C_M60_{LFD} the log first difference for the linearly derived notional contract of 60 days maturity, and so on.

	95% confidence interval	Adj. R^2
$C_M30_{LFD} = 1.320\ C_M60_{LFD}$	1.308 – 1.332	0.9572
$C_M60_{LFD} = 1.214\ C_M90_{LFD}$	1.206 – 1.222	0.9781
$C_M90_{LFD} = 1.139\ C_M120_{LFD}$	1.128 – 1.150	0.9519
$C_M120_{LFD} = 1.107\ C_M150_{LFD}$	1.097 – 1.117	0.9745
$C_M150_{LFD} = 1.073\ C_M180_{LFD}$	1.064 – 1.081	0.9790
$C_M180_{LFD} = 1.060\ C_M210_{LFD}$	1.050 – 1.069	0.9763

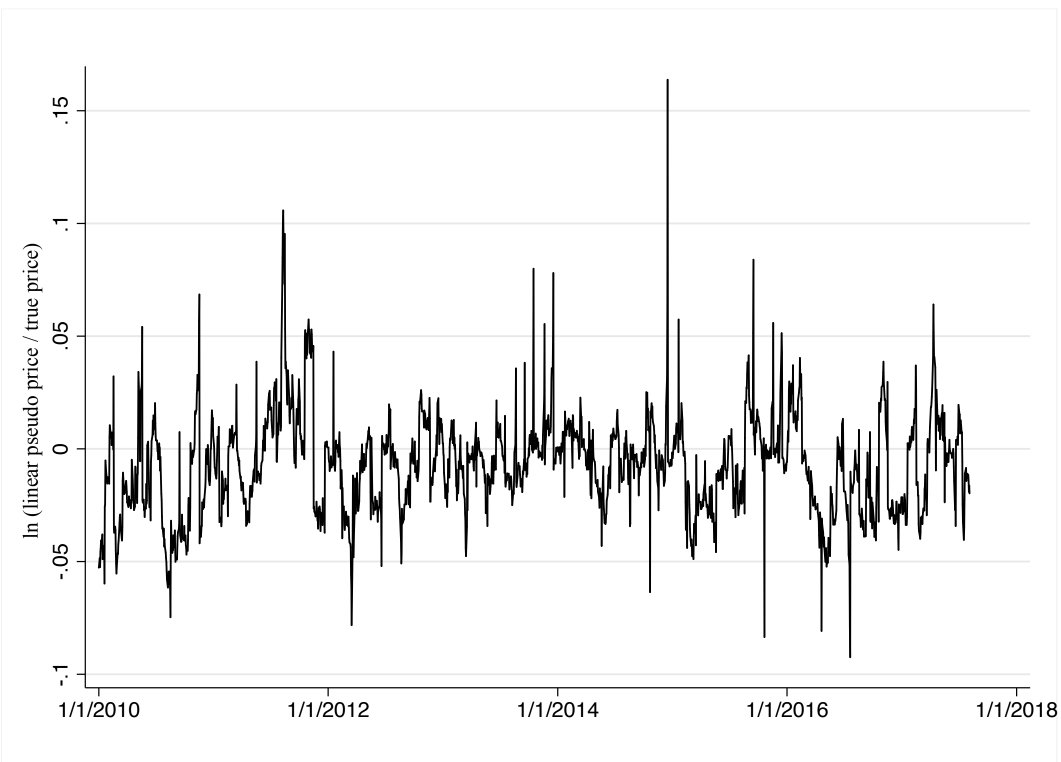
As no traded price exists that can be used test the accuracy of linear interpolation between two consecutive nearby contracts, the validity of linear interpolation as a method for determining notional futures prices of intermediate maturity is examined by comparing the daily quoted closing price of the second nearby VIX Index futures contract with a notional price for the second nearby calculated as a weighted average of the first and third nearbys, using the formula w_{C1}=(M2-M3)/(M1-M3). As the maturity of the second nearby is approximately a month removed from both the first and third nearbys, this weight is close, but not always equal, to 0.5.

A graph of the log difference between the linearly generated second nearby price and the true quote in Figure 4.1 shows that at times there is a significant difference between the two and that the true second nearby price is generally lower than the interpolated price. An ordinary least squares regression of the linear pseudo price ($C2_L$) on the true price ($C2_T$) confirms this:

$$C2_L = -0.104 + 1.000\ C2_T$$

with adjusted R^2 = 0.9906 and the constant significant at the 5% level.

Figure 4.1: Log premium of linear pseudo second nearby price over true second nearby price



As mentioned above, one consequence of these results concerns Exchange Traded Notes (ETN) relating to the VIX Index, a measure of S&P 500 Index implied volatility of returns. The benchmark for short-term VIX-related Exchange Traded Notes, such as the iPath S&P 500 VIX Short-Term Futures Index ETN, is constructed as a weighted average of the first and second nearbys in this fashion, i.e. a linear interpolation between them, to give the price of a notional 30-day contract, and the product is thus designed to reflect 30-day implied volatility of the S&P 500 Index. Given that the a linear interpolation between the first and third nearbys gives a generally inaccurate estimate of the true price of the second nearby contract, this suggests that a notional thirty day contract price calculated in this manner from the first and second nearbys may also be different from what the true price of a futures contract with thirty days to maturity might actually be.

The log difference between the third and first nearbys is an indicator of the slope of the futures curve between them. If the log difference is positive, the curve is, on average, upward sloping, or, if negative, downward. The log difference between the linear pseudo price and the true price of the second nearby can be thought of as a measure of the curvature of the slope. If positive, where the linear pseudo price is higher than the true price, the slope has, on average, positive curvature.

An ordinary least squares regression of the log difference between the linearly generated second nearby price and the true quote, $\ln(C2_L/C2_T)$, on the log difference between the prices of the third and first nearbys, $\ln(C3_T/C1_T)$:

$$\ln(C2_L/C2_T) = 0.010 - 0.155 \ln(C3_T/C1_T)$$

gives a positive constant and negative coefficient for the $\ln(C3_T/C1_T)$ parameter, with both coefficients significant at the 1% level. Adjusted R^2 is 0.3929. This shows that there is a negative relationship between the slope of the futures curve and curvature, in other words that the absolute value of the slope is declining with maturity, i.e. tending towards the horizontal, over this portion of the futures curve. This is consistent with mean-reverting behavior of the underlying instrument, the VIX Index, and suggests that the curve is close to horizontally asymptotic.

When positive and negative log differences between the third and first nearbys are regressed separately against the log difference between the linearly generated and true second nearby, it can be seen that the difference between the price of the true second nearby and its linearly generated estimate is greater when the curve is downward sloping, with both coefficients other than the constant significant at the 1% level and adjusted R^2 of 0.3990:

$$\ln(C2_L/C2_T) = 0.008 - 0.137\ \ln(C3_T/C1_T)^+ - 0.215\ \ln(C3_T/C1_T)^-$$

From this we can see that, in the particular case of the first and third nearbys, linear estimates of the second nearby price tend to be less accurate when the slope of the futures curve is downward sloping, in other words the overall curvature of the slope between the first and third nearbys is greater than when the slope of the curve is negative. This also suggests that the curve returns more quickly with increasing maturity towards its long-run mean when downward sloping than when upward sloping. More generally, we can conclude that there commonly exists a nonlinear relationship between the prices of the first three nearbys, and that a linearly derived estimate of the second nearby from the first and third nearbys is generally biased to the downside. This suggests that using linear estimates from the first and second nearbys to provide the price of a notional contract of 30-day maturity may be similarly flawed.

4.3 The Nelson Siegel (1987) Model

An alternative approach is to use the Nelson Siegel (1987) curve-fitting method designed to model yield curves. As noted above, its creation was motivated by a desire to design a yield curve model with relatively few parameters, which also makes it appropriate for modeling other curves with limited data points, such as the VIX Index futures curve.

The basic model equation is:

$$y(m)=\beta_0+\beta_1\frac{\left[1-\exp(-m/\tau)\right]}{m/\tau}+\beta_2\left(\frac{\left[1-\exp(-m/\tau)\right]}{m/\tau}-\exp(-m/\tau)\right)$$

where *m* is the maturity of the instrument being modeled in days, and β_0, β_1, β_2 and τ are parameters to be estimated, being the line, slope, curvature and decay factors respectively. In the yield curve model, *y* is the yield at maturity *m*, whereas here it will be an estimated price for a future of maturity *m*.

With the decay parameter τ estimated separately, leaving only three parameters, this is suitable as a model for the VIX futures curve as for much of the period prior to 2010 there are price quotes for as few as the first three nearbys only, particularly during the first few years of trading.

It may be noted that, because the multipliers for β_1 and β_2 tend to zero with increasing maturity, positive β_1 implies a negative slope and also curvature towards the level parameter, regardless of the curvature parameter, as the model cannot generate an unbounded curve. Positive β_2 implies greater curvature for positive values of β_1 and reduced curvature for negative values of β_1. Hence the construction is horizontally asymptotic with the model price tending to the level parameter β_0 as time tends to infinity. On the assumptions that the VIX index is mean-reverting and that futures prices reflect in some way future expectations of the VIX index, the index itself being characterized as non-arbitrageable, the level parameter may be interpreted as a form of long-run expectation of the VIX Index level and hence of long-run S&P 500 Index implied volatility.

The model is applied to the data using ordinary least squares regressions on each of the daily closing prices of the quoted VIX futures to give level, slope and curvature parameters. These may then be used to find the price of a notional contract of any maturity by inputting the relevant number of days to maturity.

Nelson and Siegel find that little precision is lost in applying their model to interest rates if the τ parameter is fixed, and I have followed their example. As they explain, τ is a time constant that determines the rate at which the slope and curvature parameters decay. I find that a relatively high mean adjusted R^2 value is achieved by setting tau equal to 42 days.

Adjusted R^2 values for the fitted curves have a mean of 0.9670 with standard deviation of 0.0891. The individual values range from 0.9998 to as low as 0.0972. Just 18 dates out of a total of 1,981 observations give an adjusted R^2 value of below 0.5, all but one of which come from before 2012. The model explains 95% of the variation in the futures curve in 88% of observations, and over 99% of the variation in 55% of observations

The lowest adjusted R^2 observations occur where futures prices across maturities are fairly close to one another, i.e. the curve is flat, and where the effects of noise or asynchronous pricing are seen. Charts for the four lowest R-squared results in Figures 4.2–4.5 show that the curves are still a relatively close fit in absolute terms but there appear to be one or two outliers in each case.

The chart for 1/26/2016, the date giving the lowest adjusted R^2 figure after 2012, in Figure 4.6, shows that the problem appears to be an outlying price for the ninth nearby, which is skewing the results. This is an extreme example, however. The curve is again relatively flat in absolute terms, and the generated pseudo price for the ninth nearby gives a value more consistent with the remaining maturities.

Figure 4.2: VIX futures prices 2/5/2010

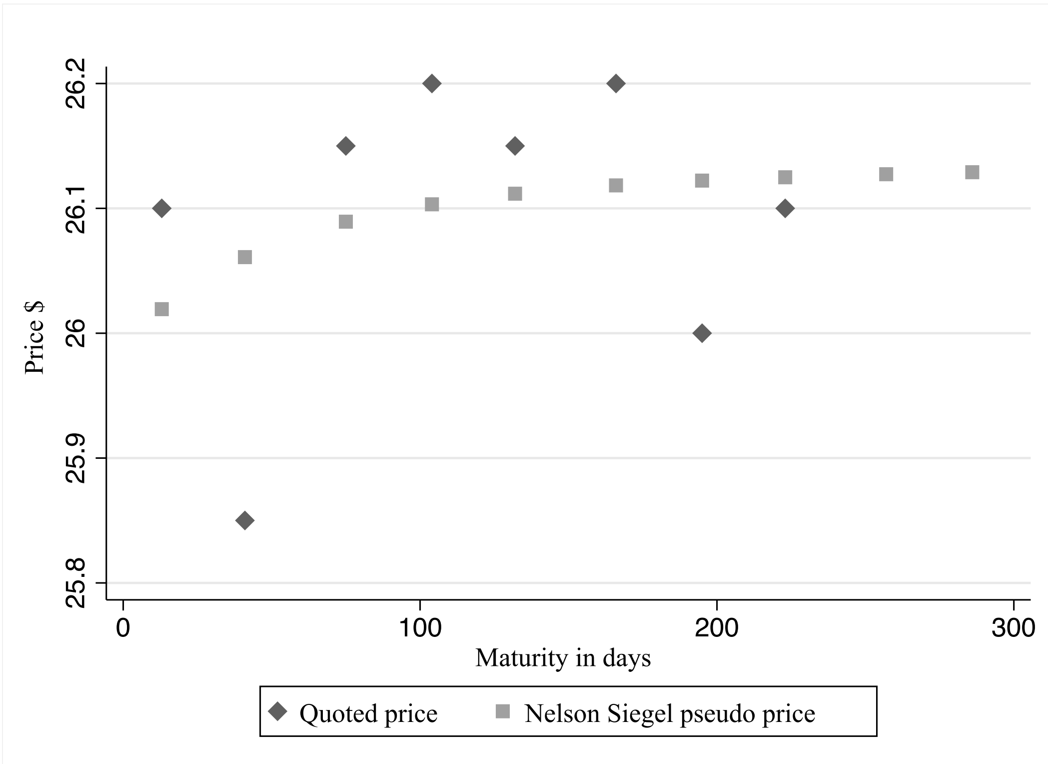


Figure 4.3: VIX futures prices 11/11/2011

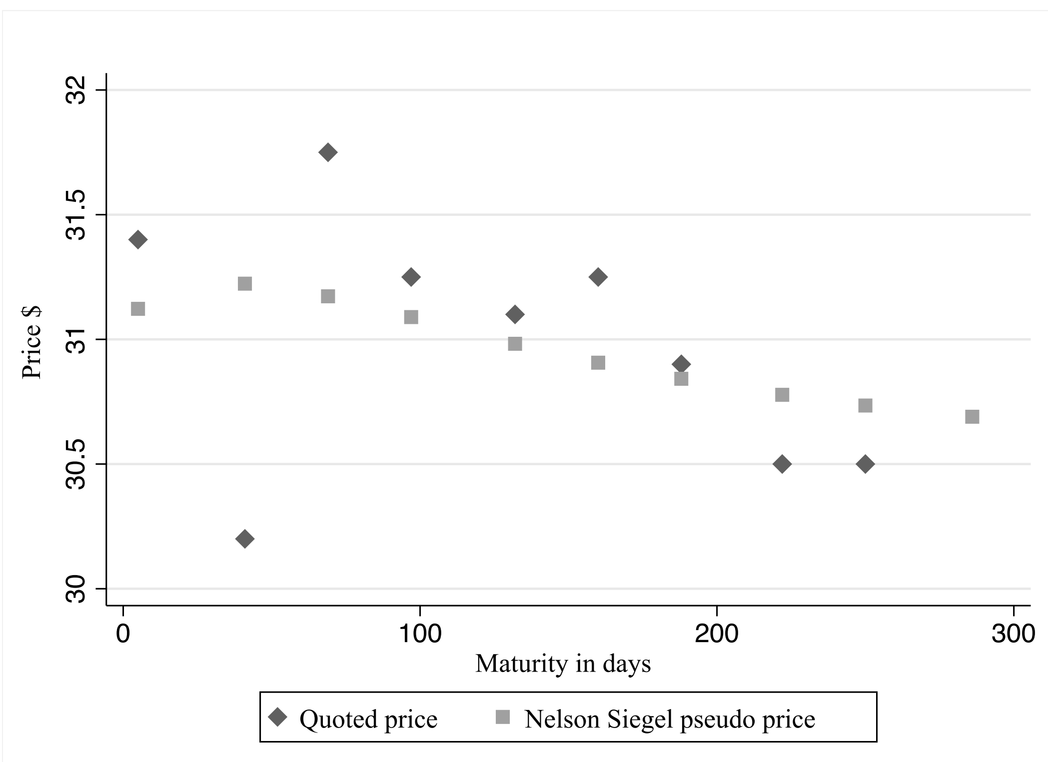


Figure 4.4: VIX futures prices 11/15/2011

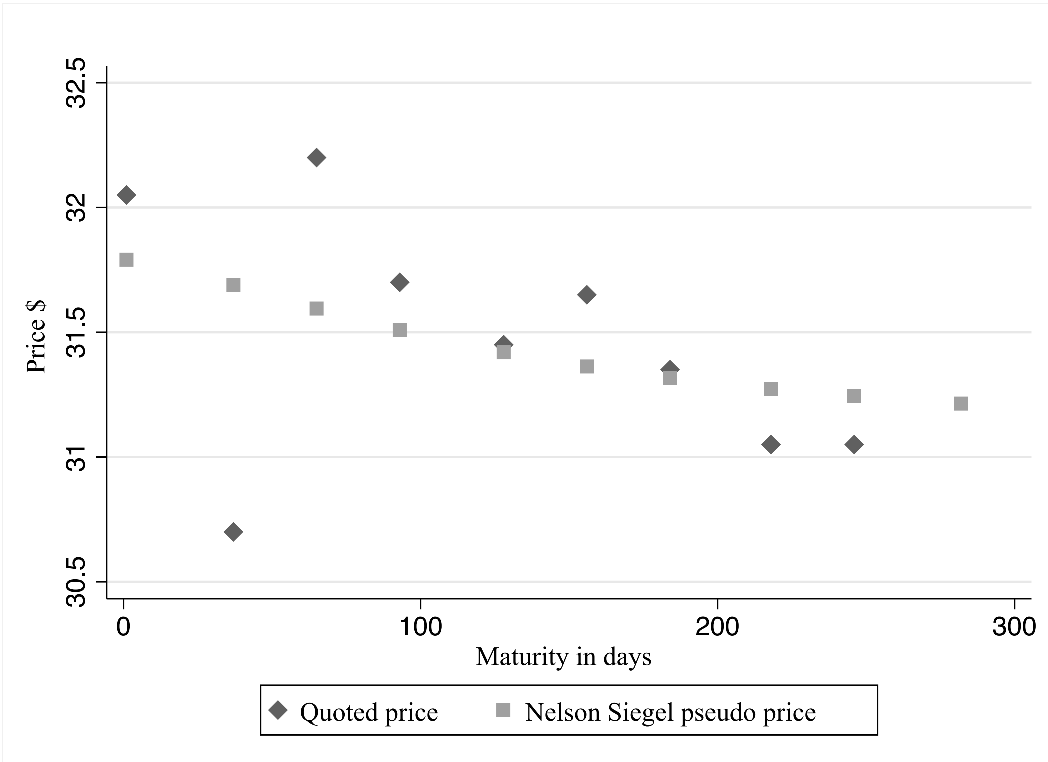


Figure 4.5: VIX futures prices 5/26/2010

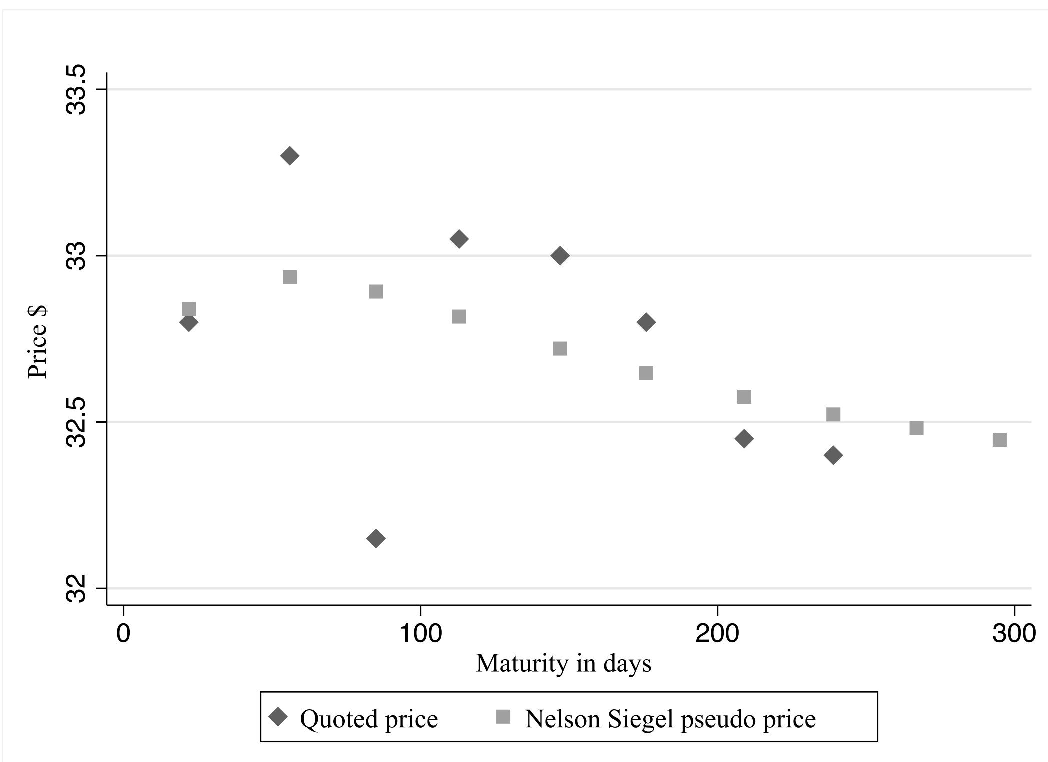
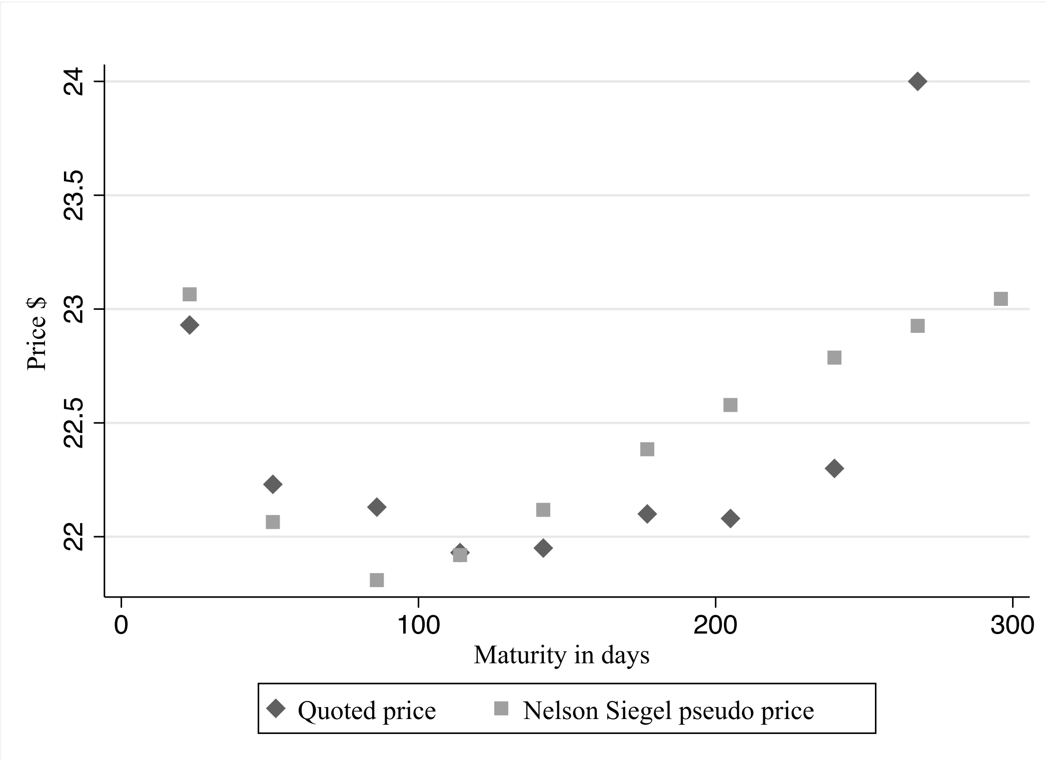


Figure 4.6: VIX futures prices 1/26/2016



Nelson Siegel (1987) introduces a second order curvature term, which allows for a greater flexibility for model curvature. In evaluating interest rate term structure, however, there are many more data points per observation than with VIX futures. For the reduced data set from 2010 onwards, at there are quotes for at least seven futures contracts each day. For the prior data, however, there are 40 days where prices for the first three nearbys alone are available, where the fourth parameter would be undefined, and in the 469 observations with only four prices quoted, the curve will fit precisely.

Furthermore, the slope and curvature of the VIX Index futures curve are generally observed to have no more than one change of sign, which the standard three factor model can replicate, and therefore there seems little advantage in adding a further parameter that will allow, for example, an S-shaped or humped curve that can be found in the bond yield curves for which the model was originally designed.

From the parameters, pseudo prices for any length maturity contract may be estimated. The Nelson Siegel method also provides prices for longer contracts where prices may not be available. To apply the linear model in this case would require extrapolation, which would be unbounded and thus, given non-zero curvature and the asymptotic nature of the futures curve, as is generally the case, is likely give increasingly inaccurate estimates as the maturity of the longest available contract becomes more distant.

In order to test the accuracy of out of sample predictions, the data were restricted to those observations where at least 10 contracts of the ninth nearby were traded, reducing the number of observations to 1137. The regressions for each day were then rerun using only the first six nearbys to generate the level, slope and curvature parameters. For this sample, the mean adjusted R^2 is 0.9713 and minimum R-squared 0.2374. All but 7 observations give adjusted R^2 greater than 0.5, and 68 lower than 0.9.

The predicted values of the ninth nearby $C9_{NS}$ were then regressed on the quoted values $C9_T$:

$$C9_{NS} = -0.299 + 1.011\ C9_T$$

Adjusted R-squared is 0.9969, which suggests that the curve fitting method produces relatively accurate estimates.

In order to compare the accuracy of the Nelson Siegel method with simple linear interpolation, an ordinary least squares regression was carried out as before on the linearly generated, from the first and third nearbys, second nearby pseudo price $C2_L$ against the true daily closing price $C2_T$, as previously, giving:

$$C2_L = -0.104 + 1.000\ C2_T$$

with constant significant at the 1% level, the second nearby coefficient not significantly different from 1, and adjusted R^2 of 0.9906.

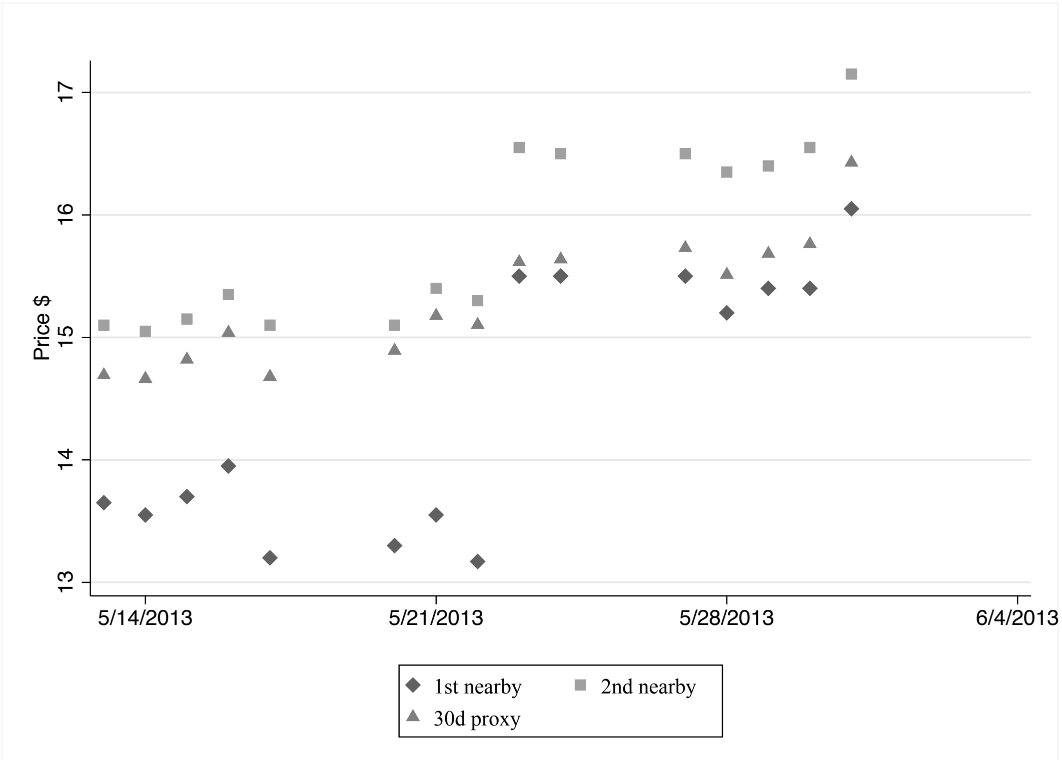
The Nelson Siegel regressions were conducted on the same reduced sample with the second nearby price removed from the data, and pseudo prices for the second nearby generated from the remaining data. These were then regressed on the true prices to give:

$$C2_{NS} = -0.164 + 1.003\ C2_T$$

with the constant significant at the 1% level, the second nearby coefficient not significantly different from 1 and adjusted R^2 of 0.9947. Although the constant is larger in absolute terms than with the linearly interpolated pseudo price, they are not statistically distinguishable, so, although the Nelson Siegel second nearby pseudo price is still generally lower than the true price, it produces a higher adjusted R^2 than simple linear interpolation. It may be noted that for the full data set, from 3/26/2004, the Nelson Siegel method gives a smaller constant, the coefficient on the second nearby parameter closer to 1 and a higher adjusted R^2 figure.

Roll dates pose a particular problem when examining futures curves. This is the date on which the first nearby expires and the second nearby becomes the first nearby, the third nearby becomes the second nearby and so on. As noted above, the average daily change for first nearby prices on roll dates is 7.181%, compared to an average daily change of 0.036%. This issue is shown graphically in Figure 4.7, which shows the prices of the first nearby, the second nearby and the Nelson Siegel generated 30-day maturity pseudo price for the three-week period beginning May 13, 2013. The first nearby future expired on May 23, so between May 22 and May 23 both the first nearby and second nearby prices appear to jump when the second nearby becomes the first nearby. Little change is seen in the price of the second nearby as it becomes the first nearby, and this is reflected in the 30-day pseudo price derived from the Nelson Siegel model.

Figure 4.7: Rolldate effects



Repeating regressions for the first differences of pseudo prices shown using the Nelson Siegel derived prices, shown in Table 4.3, give higher adjusted R^2 than simple linear interpolation in all cases. As the Nelson Siegel model assumes an asymptotic level to which futures contracts will tend with increasing maturity, the coefficients of movement between contiguous contracts tend to one as maturity increases, as can be seen. This is also seen in the results from first differences of true futures prices in Table 4.1, providing evidence that this characteristic of the Nelson Siegel generated curve makes it a suitable model for the VIX futures curve.

Table 4.3: Regressions of Nelson Siegel derived notional VIX Index futures contracts first differences

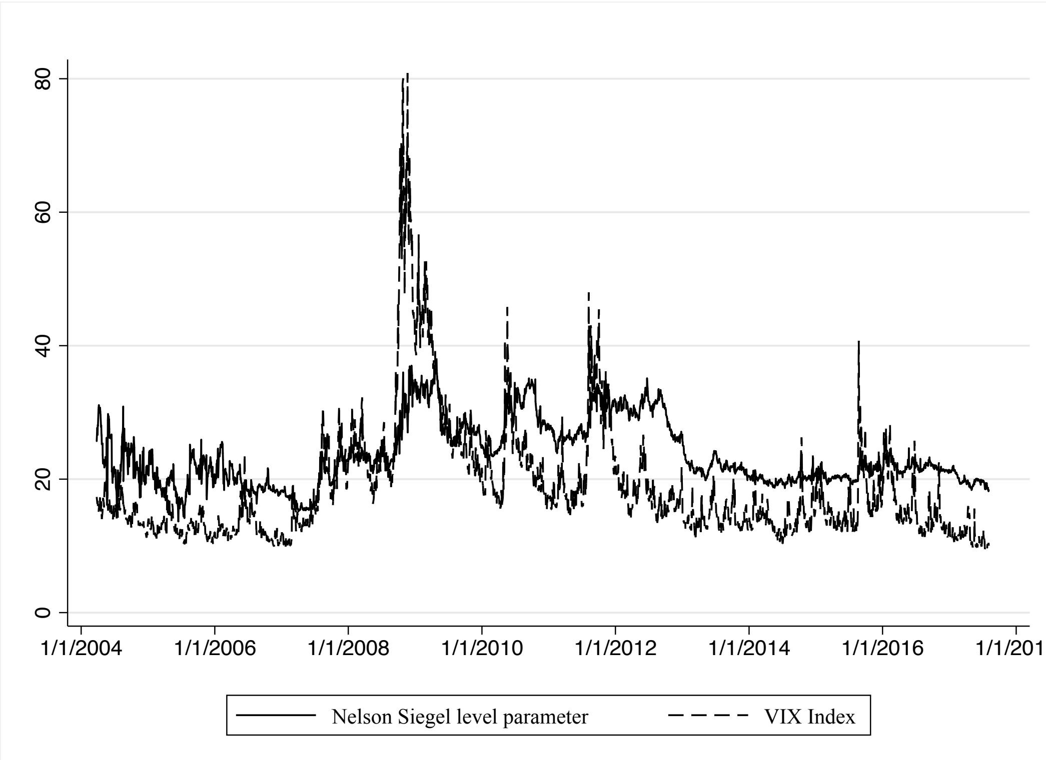
VIX Index futures prices are derived for notional contracts of fixed maturity in multiples of 30 days maturity using the Nelson Siegel (1987) curve fitting method. Ordinary least squares regressions are run of log first differences of each contract on the contract next longest in maturity to it. In each case the constant coefficient is indistinguishable from zero and has not been noted. C_M30_{NSFD} is the log first difference for the Nelson Siegel derived notional contract of 30 days maturity, C_M60_{NSFD} the log first difference for the Nelson Siegel derived notional contract of 60 days maturity, and so on.

	95% confidence interval	Adj. R^2
$C_M30_{NSFD} = 1.287\ C_M60_{NSFD}$	1.273 – 1.302	0.9388
$C_M60_{NSFD} = 1.221\ C_M90_{NSFD}$	1.214 – 1.228	0.9829
$C_M90_{NSFD} = 1.177\ C_M120_{NSFD}$	1.171 – 1.182	0.9903
$C_M120_{NSFD} = 1.131\ C_M150_{NSFD}$	1.125 – 1.137	0.9876
$C_M150_{NSFD} = 1.088\ C_M180_{NSFD}$	1.082 – 1.094	0.9857
$C_M180_{NSFD} = 1.054\ C_M210_{NSFD}$	1.049 – 1.059	0.9867

As has been shown, the Nelson Siegel curve fitting method as applied to VIX Index futures allows for a price for a notional future or any maturity to be estimated based on traded contract prices. Furthermore, as the VIX Index is non-arbitrageable, futures prices may be viewed as an expected future level of the VIX Index, with perhaps an adjustment for risk and a premium to reflect the negative correlation of the VIX Index with the S&P 500 Index.

Valuation models for stock prices will generally include some sort of measure of volatility to represent risk. This is often calculated using historic standard deviation of a stock index, such as the S&P 500 Index, or alternatively, since its creation, the VIX Index, which is designed to be forward looking, but only 30 days hence. For stock price models, long run expectations of volatility may be of greater import than either historic or short-run volatility. The level parameter of the Nelson Siegel model, shown in Figure 4.8 together with the level of the VIX Index, can provide just such a long-run estimate.

Figure 4.8: Nelson Siegel level parameter and VIX Index



The Nelson Siegel level parameter appears well behaved compared to the VIX Index, and may provide insight into the differences between short-run and long-run expectations of volatility. For example, it appears that expectations of long-run volatility remained high for some time after the VIX Index subsided following the turbulent period of late 2011, not declining until some months later. As returns on the VIX Index are negatively correlated with the S&P 500 Index, as shown by Whaley (2000) and others, it may be argued that a long run VIX Index future should stand at a premium to the future expected value of the VIX Index, in accordance with the Capital Asset Pricing Model. The extent to which this is the case and the rate at which an asymptotic level is approached is a possible area for future research.

As can be seen from Figure 4.8, the level parameter tends to stand at a premium to the VIX Index at lower levels and is also less volatile. This is confirmed by an ordinary least squares regression of the level parameter (*LevelNS*) on the VIX Index (*VIX*):

$$LevelNS = 15.290 + 0.514VIX$$

with the constant coefficient significant at the 1% level and the VIX Index coefficient significantly different from 1, also at the 1% level, and adjusted R^2 of 0.4476. The large constant term and small VIX Index coefficient shows that the Nelson Siegel level term generally lies above the VIX Index, suggesting that the long run VIX Index future generally stands at a premium to the VIX Index at levels of the VIX Index below around 30, which the VIX Index has been for over 94% of the period from 1/10/17 to 8/4/17. It is also consistent with the generally upward sloping VIX Index futures curve as described in Whaley (2013). The VIX Index coefficient of around half confirms that the Nelson Siegel level term is more stable than the VIX Index itself.

Of interest also might be the peaks and troughs of the VIX Index futures curve, which may be thought to reflect the investors' expectations of the point at which market volatility will begin to return to a more normal state. Maxima and minima for fitted Nelson Siegel curves may easily be derived.

Distribution charts for maxima and minima of Nelson Siegel derived curves from 1/1/2010 to 8/4/2017 are shown in Figures 4.9–4.12. As the VIX futures market is generally in contango, i.e. upward sloping, as shown by Whaley (2013), the majority of minima are very short maturities, and the majority of maxima long. This is particularly the case with the data set from January 2010 onwards, from when it was argued in Chapter 2 that the introduction of VIX-related Exchange Traded Funds resulted in a steeper VIX Index futures curve which would result in more monotonic curves. Figures 4.11 and 4.12 eliminate the endpoints (0s and 210s).

Figure 4.9: Distribution of minima for fitted curves, 1/1/2010–8/4/2017

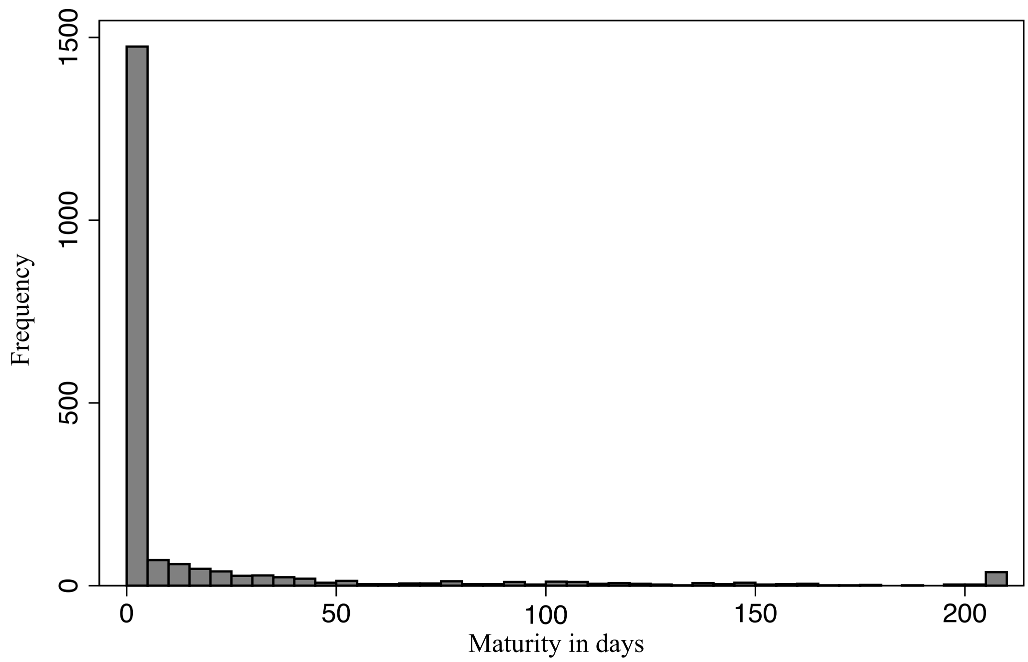


Figure 4.10: Distribution of maxima for fitted curves, 1/1/2010–8/4/2017

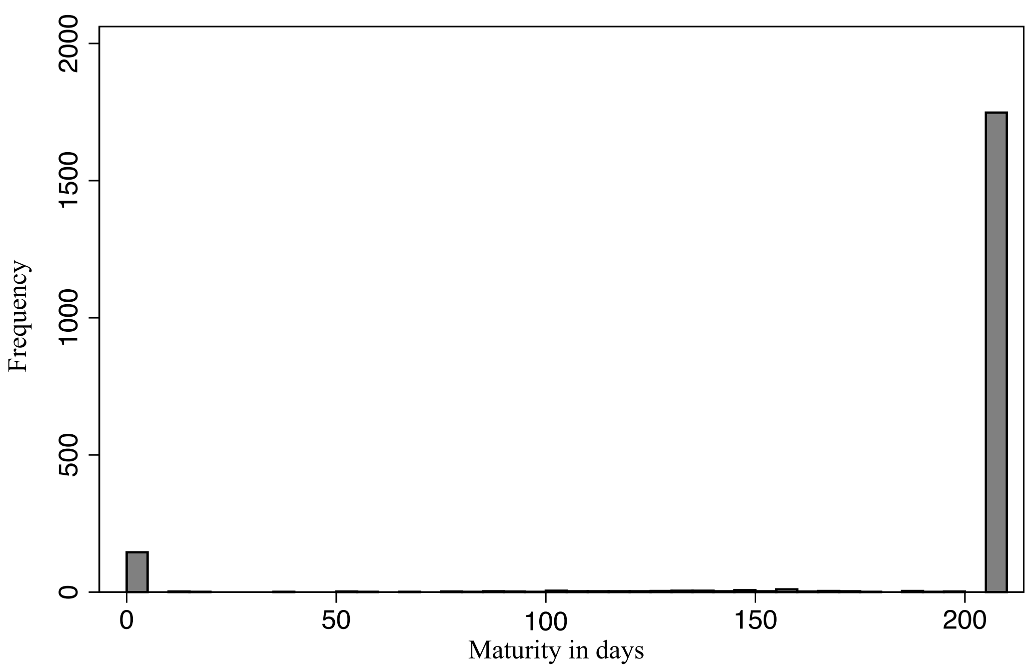


Figure 4.11: Distribution of minima for non-monotonic fitted curves, 1/1/2010–8/4/2017

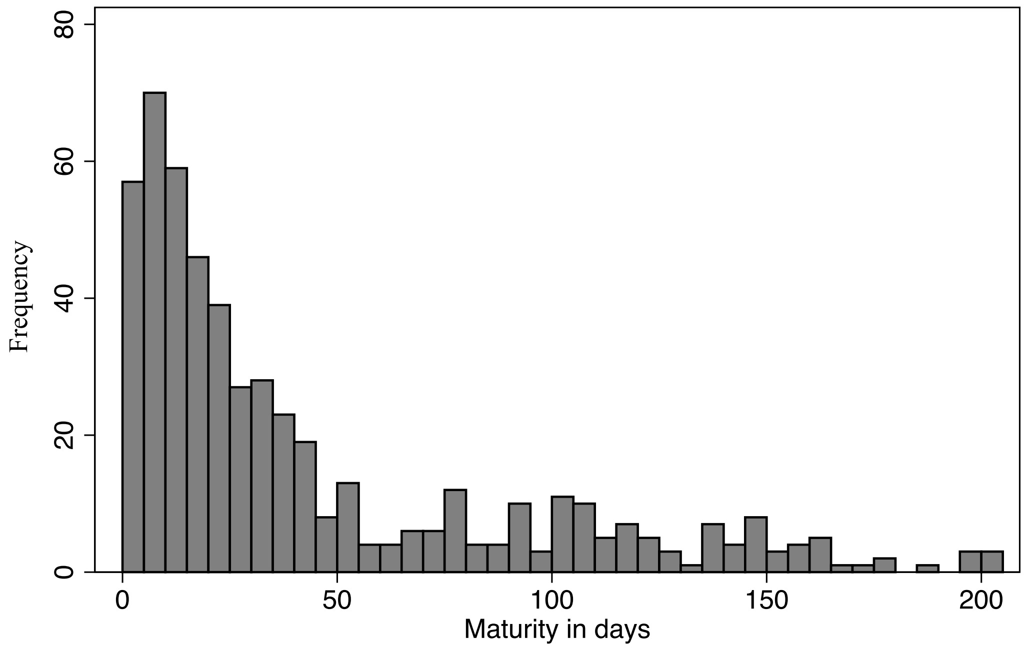
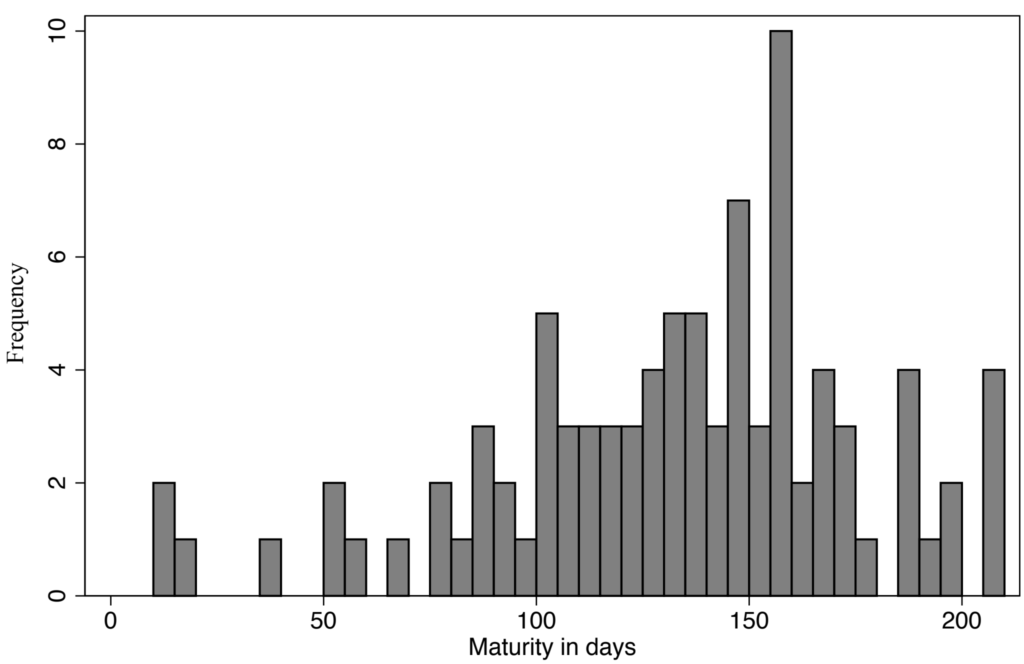


Figure 4.12: Distribution of maxima for non-monotonic fitted curves, 1/1/2010–8/4/2017



4.4 Conclusion

The VIX Index has become a popular indicator of implied S&P 500 Index volatility, earning the moniker in the financial press of the "Fear Index", owing to its inverse correlation with the S&P 500 Index.

The VIX Index is shown to be non-arbitrageable, and therefore VIX Index futures prices may hold more information of interest to investors than do those of arbitrageable assets. As futures contracts have a fixed maturity, it is problematic to conduct time series analysis on futures prices directly, so a method of estimating a price of a notional contract of fixed time to maturity is desirable.

It is questionable whether the coefficients resulting from running regressions on true prices over time have any useful meaning, at least for the shorter maturities, as the maturity of a contract changes daily. A change in an estimated price for a contract of a fixed time to maturity, however, may be interpreted as the change in the expectation of the level of the VIX Index at a specific future point in time, and is likely to be of greater use in estimating the effect on future expectations of, for example, a current shock to the spot price than a change in the quoted price of a contract of changeable maturity.

Short-term VIX-related Exchange Traded Notes are based on a benchmark that effectively uses linear interpolation to calculate a notional futures contract of 30 days maturity. Estimates obtained by linear interpolation evince expected characteristics, but are shown to be potentially inaccurate, particularly when the futures curve has greater curvature, generally at times when the curve is steep. Estimates are realistically limited to maturities close to the furthest maturity for which a price quote is available, and such longer maturity contracts can be infrequently traded and with low volume. Distant estimates are likely to be increasingly inaccurate as maturity increases owing to the constant slope and hence unboundedness of curves derived by linear extrapolation, other than on those occasions when the slope is close to zero.

The Nelson Siegel (1987) curve-fitting method, designed to model the term structure of interest rates, has a number of characteristics that makes it a good candidate for modeling the VIX Index futures curve. It has only four parameters, which are reduced to three by fixing the decay parameter. Although VIX Index futures are tradable in monthly maturities of up to 10 months in duration, the longer maturities trade infrequently, and, in the early years of their existence, sometimes only the first three nearbys were quoted.

The curve generated is horizontally asymptotic, which, subject to the caveat mentioned below, would be typical given an underlying non-arbitrageable mean-reverting instrument. The resulting asymptote, the level coefficient β_0, may be considered as a long run expectation of implied volatility. Useful information may also be contained within the slope and curvature parameters.

A smooth curve is generated, so although outlying quotes, as a result of asynchronous pricing or incorrectly reported data, will have an effect on the resulting coefficients, that effect will be ameliorated. The curve is shown generally to give better estimates for notional contract prices than linear interpolation.

There is an argument that the VIX Index futures curve should not, in fact, be strictly horizontally asymptotic, even if the VIX Index is fully mean reverting. Given the negative correlation of the VIX Index with the S&P 500 Index, exposure to the VIX Index perhaps ought carry a premium owing to its insurance aspect, in accordance with the Capital Asset Pricing Model. This premium would increase with maturity, resulting in a long-run upward sloping curve. This is an area for further research, which may suggest an adjustment to the model.

CHAPTER 5

CONCLUSION

This dissertation has looked at three aspects of the VIX Index. It looked at the problems that investors and asset managers have had trying to create exposure to the index, and how the futures market has itself may have been affected by attempts to do so. The ability of the VIX Index and the VVIX Index to reflect and predict volatility in their underlying indices is considered. Finally, the difficulties in analyzing futures prices is addressed with respect to VIX Index futures, and the Nelson Siegel (1987) curve fitting method is applied. Thus notional futures contracts of any maturity may be modeled, potentially opening up further avenues of research.

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