DEVELOPMENT AND TESTING OF A BRIDGE WEIGH-IN-MOTION METHOD CONSIDERING VEHICLES TRAVELING AT VARIABLE SPEED

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ABSTRACT

Government stakeholders rely upon traffic information such as the weight of trucks on the roadways to provide and maintain safe and reliable highway/bridge infrastructure. Bridge weigh-in-motion (BWIM) provides an alternative to conventional static weigh stations for obtaining vehicle axle weights. Traditional BWIM algorithms are capable of predicting the axle weights of vehicles traveling at constant speed across a bridge with known influence line, but they often lose accuracy when measuring vehicles are traveling at nonconstant speed. This thesis presents a methodology to improve BWIM accuracy when measuring a vehicle traveling at nonconstant speed by transforming variable speed response data to constant speed data.

A BWIM package capable of determining vehicle speed and axle spacing, calculating the influence lines of a bridge, and predicting the axle weights of a vehicle crossing the bridge is developed in MATLAB. A numerical study is performed using finite element analysis in MATLAB to evaluate the performance of the BWIM package when measuring loads traveling at constant speed and variable speed. The results of the numerical study show the speed correction is able to improve BWIM accuracy for a variable speed vehicle to nearly the accuracy level of a constant speed vehicle. A field study is also performed. A vehicle with known weight was used as a calibration vehicle to measure the influence line of a bridge on the University of Alabama campus. A different vehicle was then driven across at constant speed, then again at variable speed to generate data for various study cases. Results of the field study showed that correcting variable speed response data can significantly improve the accuracy of axle weight predictions, but more
research is required to reach the accuracy level BWIM is able to achieve when measuring constant speed vehicles.
DEDICATION

I would like to dedicate this thesis to my grandfather Marvin Lansdell. He saw me begin my journey through college, and I know he would have loved to see the end. I have never known a man so loved by so many people. The true mayor of Petersville.
I would like to express the most genuine gratitude to my advisor Dr. Wei Song for the constant support he showed me through my years of undergraduate and graduate research. His motivation, patience, and knowledge are the reason I have been able to complete this thesis. Under his guidance I was able to learn skills that, when I arrived at the University of Alabama, I never believed I would obtain by the time I graduated.

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CHAPTER 1

INTRODUCTION

Part of the effort to maintain bridge infrastructure is the ability to monitor the traffic which crosses it. Reliable measurement of the gross, axle, and axle group weights of the trucks using a bridge can help guide and/or improve routine maintenance, pavement design, and load limit enforcement.

The most widespread method for monitoring the weight of vehicles on the highway is static weigh stations. Although they provide accurate weight measurement, static weigh stations have the disadvantage of requiring all traffic to be weighed to leave the highway. Weigh-in-motion (WIM) and bridge weigh-in-motion (BWIM) are two alternatives to static weigh stations which can measure a moving load. WIM systems are commonly placed in or under roadway pavement and will measure a vehicle’s weight as it crosses that specific point. BWIM was first introduced in the late 1970s by Moses [1]. Instead of instrumenting areas of roadway, BWIM systems are placed under existing bridges and measure a vehicle’s weight as it crosses. A few of the advantages [2] BWIM systems hold over WIM systems include:

- Safer to implement since no work is done in the traffic lanes
- System can be easily relocated to different bridges
- Less susceptible to vehicle dynamics caused by uneven pavement
- Harder for vehicles to divert around, since bridges are often critical parts of highway infrastructure
Some disadvantages [2] of BWIM include:

- Increased error when multiple large vehicles are on the bridge at once. This can be minimized by choosing to instrument short span bridges.
- Some roadways may not have a bridge which is suitable for BWIM testing.

Another potential drawback of current popular BWIM algorithms is they often assume the vehicles being measured are traveling at a constant speed. This is a reasonable assumption for short-span highway bridges, but will yield large error for long span highway or railroad bridges. To address this challenge, this thesis proposes a methodology to improve BWIM accuracy when measuring axle weights of a vehicle traveling at variable speed. In conducting the experimental study of this thesis, vehicle speed is evaluated for the leading and trailing halves of a bridge using information obtained via strain measurements. This method can also be applied to offer more refined evaluation of varying speed, provided that a sufficient number of strain transducers are available.

This thesis is organized as follows: In CHAPTER 2, a literature review is performed to discuss the developments of BWIM in the past. The methodology proposed in this thesis is discussed in CHAPTER 3. In CHAPTER 4, numerical simulations are first conducted to verify the proposed BWIM method, and results from a numerical application of the BWIM methodology on a campus pedestrian bridge are shown as a proof of concept. CHAPTER 5 describes a field test that is performed with a pedestrian bridge on the University of Alabama campus. Strain data from the field test is analyzed to find the travelling speed, axle weights, and gross vehicle weight (GVW) of the test vehicle. The accuracy of the field test results are also analyzed and discussed. CHAPTER 6 presents the conclusion and future research needs.
CHAPTER 2

BACKGROUND

The concept of BWIM was first introduced as a feasibility study for the Federal Highway Administration (FHWA) by Moses in 1979 [1]. The FHWA was searching for alternatives to conventional WIM methods. The bridge used in testing was a three span continuous beam-slab design. The proposed BWIM system utilized strain transducers installed on the bottom flange of girders at the middle of an exterior span. The system required axle detectors on the road, so tape switches were placed before the bridge to determine axle spacing and velocity. Moses was able to conclude that truck weight predictions using an instrumented bridge span could produce promising results.

The ideas of Moses were later expanded in Australia by Peters in 1986 [3]. Peters instrumented short span culverts with BWIM technology. Culverts offer protection from dynamic forces due to a layer of soil between the road surface and the strain transducer. Peters correlated the peak strain values of unknown vehicles and a calibration vehicle. Since Peters advised only instrumenting culverts less than 3 meters in span, the problem of multiple vehicles on the test span at once was virtually eliminated. Due to high site availability and low cost of installation, the Culway system has been put into operation at over one hundred locations in Australia [4].

The COST 323 Management Committee began a six year study in 1993 to standardize BWIM operation in Europe until an official standard is set [5]. The COST 323 report covered aspects of model approval, site determination, and accuracy rating. The final report was published
in 2002. During the years the COST 323 Management Committee was working to standardize BWIM, it became clear that more advanced research was necessary to meet the requirements of decision makers [2]. As a result, the Weigh-in-Motion of Axles and Vehicles for Europe (WAVE) research project was submitted to the European Commission in 1995 to address the need for additional BWIM study. The WAVE initiative sought, amongst numerous other goals, to remove axle detectors from the roadway. Axle detectors on the roadway pose a risk to workers during installation and maintenance, and they are exposed to more wear and tear than any other component of a BWIM system [6]. The proposed replacement for roadway axle detectors, Free-of-Axle Detector (FAD), relies on additional strain measurements to determine vehicle speed and axle spacing. WAVE also expanded many BWIM principles previously used on orthotropic steel deck bridges for use on slab bridges.

SiWIM is a commercial BWIM hardware and software package introduced by the WAVE project [7]. Early revisions of the software sought to improve the hardware, ease-of-use, and reliability of its algorithm. More recent revisions have expanded the package to work on bridges once thought unsuitable for BWIM, and FAD support has been implemented. As of 2008, SiWIM is in use in over ten countries [8].

In summary, the development of BWIM systems is closely related to axle detection and influence line (IL) determination techniques. Axle detection techniques have changed over the years to remove axle detectors for the road surface, and instead rely upon additional strain transducers to determine vehicle speed and axle spacing [9]. IL determination has evolved from using peak strain values [3] and adjusted theoretical ILs [16] to using ILs calculated with field tests [18]. Some BWIM algorithms also account for the lateral distribution of wheel loads across
multiple traffic lanes. The topics of axle detection, influence lines, and 2-dimensional algorithms are introduced as follows.

2.1 Axle Detection

Roadway axle detectors are commonly used by government stakeholders to monitor the traffic on a section of highway. Early BWIM systems relied on detectors placed on the road surface to determine axle spacing and vehicle velocity. These detectors include removable options such as pneumatic rubber tubes or tape switches, or they can include permanent piezo or optic-fiber sensors [9]. Pneumatic tubes are one example of roadway axle detectors, and can be seen in Fig 2.1. Pneumatic tubes send a pulse of air down the tube when an axle strikes. That pulse of air is then converted into an electronic signal by a pneumatic converter [4]. The permanent sensors are often similar to sensors used in pavement WIM systems but are not of high enough quality for standalone use.

![Pneumatic road tubes](image)

*Fig 2.1: Pneumatic road tubes [7]*

The concept of Free-of-Axle Detector (FAD) systems was introduced by the Weigh-in-Motion of Axles and Vehicles for Europe (WAVE) project [7]. FAD BWIM systems remove axle
detectors from the road surface. The roadway sensors are often replaced by additional strain
transducers under the bridge deck. Dempsey et al. demonstrated in 1998 that FAD technology can
be applied to bridges with orthotropic steel decks [10]. The results showed encouraging accuracy.
FAD was expanded to concrete slab bridges by Znidaric et al. in 2002 [9]. Znidaric came to the
conclusion that FAD installation on thin slab spans less than 10 meters long can yield results
similar to those obtained with axle detectors.

Some early methods of FAD implementation used sensors which were able to capture
vehicle velocity, but were unable to determine the number of axles or axle spacing of a vehicle
([11] [12]). These methods led to accurate predictions of gross vehicle weight (GVW), but cannot
provide estimation in regards to individual axle weights. Optical sensors placed on the vertical
members of a Warren truss bridge were used by Ojio et al. in 2000 as a way to capture vehicle
velocity [11]. This method produced GVW results within 10% of weights determined by a static
weigh station. Ojio and Yamada instrumented the reinforcing stringers of a plate girder bridge with
a FAD BWIM system in 2002 to monitor bridge fatigue [12]. Strain transducers placed under
neighboring spans were compared to determine the vehicle speed, but this system also did not
measure axle weights.

The University of Connecticut and the Connecticut Department of Transportation
(ConnDOT) have conducted research on FAD technology. Cardini and DeWolf worked with
ConnDOT in 2009 to implement a BWIM system on a bridge already part of ConnDOT’s extensive
bridge health monitoring program [13]. The system was similar to that described previously by
Ojio and Yamada [12], but the main girders were instrumented instead of stringers. Wall et al. [14]
expanded upon the ideas of Ojio [12] and Cardini [13] in 2009 by calculating GVW as described
in the two previous reports, but axle weights were also calculated. Strain response was double differentiated to show peaks in curvature at the sensor location. The relative height of these peaks (each peak corresponding to a particular axle) were compared to determine the percentage of GVW carried by each axle.

This thesis will adopt the technique for determining vehicle speed and axle spacing described by Wall [14]. This method is seen as advantageous, because it only requires additional strain transducers to be mounted under the bridge. This allows for safer installation since no workers are required to enter the roadway, and traffic never needs to be stopped. This method also has the ability to be scaled up to track the vehicle velocity over more spans.

2.2 Influence Line Determination

One of the key factors that can affect the accuracy of BWIM measurements is the determination of the influence line of the bridge. Many BWIM algorithms are based on the idea that a load on the bridge will cause a bending moment response proportional to the product of the load’s magnitude and the IL ordinate at the position of the load [15]. However, bridges often do not behave exactly as the theoretical IL describes due to uncertainties in boundary conditions and material properties. Early BWIM systems such as the Culway system by Peters [3] correlated peak values in measured response from a calibration vehicle to peak response values from an unknown vehicle. This method has the disadvantage of disregarding most of the response data measured, since it compares only peak values. Znidaric et al. addressed this issue in 1998 by adjusting the theoretical IL to more closely describe measured truck data [16]. The IL was adjusted by smoothing peaks and adjusting boundary conditions. McNulty and O’Brien in 2003 proposed a
point-by-point method for adjusting the theoretical IL based upon measured data [17]. However, this approach is manual, and its effectiveness is limited by the skill level of the operator. O’Brien et al. proposed a method in 2006 to calculate the IL from direct strain measurements [18]. This method compares internal moment in the girders, determined via measured strain transducers, to external moment caused by the loads. The difference between the two curves is then minimized via least squares method. This algorithm was shown to be effective.

The determination of the bridge’s IL in this thesis will adopt the method set forth by O’Brien [18]. This method uses the entire response curve of the calibration vehicle to determine the IL. Determining the IL via direct measurements is also expected to yield a more accurate IL for bridges with complex geometry, such as the bridge studied in this thesis.

2.3 2-Dimensional Algorithms

Quilligan et al. in 2002 addressed the issue of two trucks crossing a BWIM instrumented bridge in adjacent lanes [15]. An algorithm was developed to correctly recognize two vehicles crossing pneumatic tube axle detectors at the same time. An influence surface of the bridge was calculated to replace the IL of 1-dimensional methods, and a digital camcorder analyzed reflective strips in the roadway to determine the position of a passing vehicle. This method showed results within 10% of static weighed values for times when multiple trucks were passing. The 2-dimensional algorithm also yielded improved results for the single truck case, when compared a 1-dimensional algorithm on the same bridge.

In 2014, Zhao et al. made modifications to the IL calculation described by O’Brien [18] to account for the distribution of wheel loads (of a single vehicle) onto multiple girders, even girders
not directly under the truck lane [19]. This method calculates individual ILs for each girder, and it allows for all girders to have independent material properties. The results showed increased accuracy of truck weight predictions when considering the load applied to all four girders, as opposed to only considering the load carried by the two girders directly under the lane of travel.

Even though the bridge in this thesis has only two bottom chords, the transverse load distribution described by Zhao [19] will be adopted herein. This is to avoid assuming both chords carry an equal proportion of the vehicle load.

2.4 Summary

This thesis proposes a BWIM method to consider nonconstant vehicle speed. To accomplish this goal, the approach described going forward is based upon the work presented by Wall, O’Brien, and Zhao ([14] [18] [19]). Wall’s [14] implementation of additional strain transducers to gather vehicle speed and axle spacing was chosen due to its computational efficiency. Calculation of the IL via direct strain measurements, as described by O’Brien [18], is seen as the optimal method for a bridge with complex geometry. Finally, Zhao’s [19] method of accounting for lateral distribution of load is also applied, because it allows for each girder to have a unique IL. The methods described going forward can also be easily implemented on other bridges.
CHAPTER 3
THEORETICAL BACKGROUND

The methodology proposed in this chapter uses strain measurements from transducers installed on the bottom chord of a truss bridge to estimate the axle weights and GVW of a vehicle. This methodology can also account for varying speed of the vehicle. Vehicle speed is evaluated over several segments of the bridge span, and corrections are applied to transform variable speed data to constant speed data. The correction algorithm can be scaled to account for any number of sections. However, more strain transducers would be required to obtain a more refined estimation of vehicle speed.

Fig 3.1 shows a diagram of the steps necessary to predict the axle weights of an unknown vehicle. Section 3.1 will discuss how strain measurements are used to determine speed and axle spacing (Wall [14]). In Section 3.2, the lateral distribution factors for each girder are calculated in accordance with Zhao [19]. Section 3.3 describes the method for calculating the influence line for each girder, as explained by O’Brien [18]. In Section 3.4, axle weight calculations as described by Zhao [19] are presented. Finally, Section 3.5 presents corrections to account for nonconstant vehicle speed across the span.
3.1 Speed and Axle Detection

This section presents the method outlined by Wall [14] for determining vehicle speed and axle spacing using only strain measurements.

Consider a simple span bridge with a single axle load crossing as represented by Fig 3.2.
As load $P$ moves from point A to point B, the largest internal moment in the span will occur as the load is at point C. The influence line for moment at any particular point, such as point C, represents the internal moment at the point in question as load $P$ moves across the beam. The influence line for the moment at midspan of the beam in Fig 3.2 can be seen in Fig 3.3, and the corresponding equation is shown in eq. (3.1).

Fig 3.2: Simply supported beam representing a simple span bridge

Fig 3.3: Influence line for moment at midspan of simple beam
\[ M_c = \begin{cases} \frac{Px}{2} & 0 < x \leq \frac{L}{2} \\ \frac{PL}{2} \left(1 - \frac{x}{L}\right) & \frac{L}{2} < x < L \end{cases} \] (3.1)

Here, \( M_c \) is the internal moment at point \( C \), \( x \) is the position of load \( P \) measured from point \( A \), and \( L \) is the length of the beam.

The internal stress distribution caused by any internal moment \( M \) can be stated as follows:

\[ \sigma = \frac{M_c}{I} \] (3.2)

Here, \( \sigma \) is normal stress at the sensor location, \( M \) is the internal moment, \( c \) is the distance from the centroid of the shape to the sensor location, and \( I \) is the moment of inertia of the cross-section.

Since BWIM systems most often measure strain under the bridge, stress can be related to strain by Hooke’s Law.

\[ \sigma = E\varepsilon \] (3.3)

Here, \( E \) is the Young’s Modulus of the cross-section, and \( \varepsilon \) is the strain at the location of \( \sigma \).

By substituting eq. (3.1) and eq. (3.3) into eq. (3.2), a relationship can be seen for the strain at sensor location as load \( P \) moves across.

\[ \varepsilon_c(x) = \begin{cases} \frac{Pcx}{2EI} & 0 < x \leq \frac{L}{2} \\ \frac{PcL}{2EI} \left(1 - \frac{x}{L}\right) & \frac{L}{2} < x < L \end{cases} \] (3.4)
If the point load is assumed to move at a constant speed \( v \), position \( x \) can be related to time \( t \) by equation eq. (3.5).

\[
x = vt
\]  

(3.5)

Substituting eq. (3.5) into eq. (3.4) allows the strain at point C to be represented in relation to time, instead of position.

\[
\varepsilon_c(t) = \begin{cases} 
\frac{P_cvt}{2EI} & 0 < t < \frac{L}{2v} \\
\frac{P_cL}{2EI} \left( 1 - \frac{vt}{L} \right) & \frac{L}{2v} < t < \frac{L}{v} 
\end{cases}
\]

(3.6)

The first derivative of eq. (3.6) is shown in eq. (3.7).

\[
\frac{d\varepsilon_c}{dt}(t) = \begin{cases} 
\frac{P_cv}{2EI} & 0 < t < \frac{L}{2v} \\
-\frac{P_cv}{2EI} & \frac{L}{2v} < t < \frac{L}{v} 
\end{cases}
\]

(3.7)

If the time between any two \( \varepsilon_c(t) \) samples is taken as \( \Delta t \), the second derivative of eq. (3.6) can be described as shown in eq. (3.8).

\[
\frac{d^2\varepsilon_c}{dt^2}(t) = \begin{cases} 
\frac{P_cv}{2EI\Delta t} & t = 0, \frac{L}{v} \\
-\frac{P_cv}{EI\Delta t} & t = \frac{L}{2v} \\
0 & \text{elsewhere}
\end{cases}
\]

(3.8)

The plots for eq. (3.6) eq. (3.7) and eq. (3.8) respectively can be seen in Fig 3.4.
It is important to notice that the peaks in eq. (3.8) occur as the load reaches the beam, crosses the sensor location, and leaves the beam. If a different sensor were to capture the strain at another location as load $P$ moves across the beam, the shape of the strain plot would be similar. A series of plots can be seen in Fig 3.5 showing how the same data presented in Fig 3.4 would look if captured by sensor located at the quarter point instead of midpoint.

Fig 3.4: Strain influence line at midspan for a simple beam
If the distance between the two sensors is $d$ then the average velocity of the moving point load between the $(y-1)^{th}$ and $y^{th}$ sensors can be described by rearranging eq. (3.5).
\[ v = \frac{d}{t_y - t_{y-1}} \quad (3.9) \]

where \((t_y - t_{y-1})\) is the difference in time between when load reaches the \((y - 1)^{th}\) sensor and when the load reaches the subsequent \(y^{th}\) sensor.

Superposition can be used to account for the presence of multiple point loads at once, such as a vehicle load.

\[
\varepsilon_y = \sum_{i=1}^{N} \varepsilon_{i,y}(t - t_{i,y}) \quad (3.10)
\]

\[
\frac{d\varepsilon_y}{dt} = \sum_{i=1}^{N} \frac{d\varepsilon_{i,y}}{dt}(t - t_{i,y}) \quad (3.11)
\]

\[
\frac{d^2\varepsilon_y}{dt^2} = \sum_{i=1}^{N} \frac{d^2\varepsilon_{i,y}}{dt^2}(t - t_{i,y}) \quad (3.12)
\]

where \(N\) is the number of axles, \(\varepsilon_{i,y}\) is the strain at the \(y^{th}\) sensor location caused by the \(i^{th}\) axle, and \(t_{i,y}\) is the time difference between when the first axle enters the bridge and when the \(i^{th}\) axle reaches \(y^{th}\) sensor location. Fig 3.6 depicts the case of a two axle vehicle crossing a simply supported bridge. The vehicle in this case has a back axle which is slightly heavier than its front axle.
Axle spacing can be defined as the distance from the first axle to the $i^{th}$ axle. Axle spacing can be calculated knowing vehicle velocity from eq. (3.9).

$$D_i = v \ast (t_{i,y} - t_{1,y})$$  \hspace{1cm} (3.13)
3.2 Load Distribution Factor

The following section will discuss the calculation of lateral load distribution factors as described by Zhao [19]. Applying the lateral distribution factor allows the bridge to be modeled as a 2-dimensional structure. Modeling as a 2-dimensional structure can be important if the bridge in question is not symmetric along its longitudinal axis, such as the bridge in this thesis. Although there are other methods for considering theoretical load distribution, the method presented by Zhao [19] uses actual measured strain data to formulate the distribution factors. While it is important to consider that all parts of a bridge can have different properties, such as transverse connectivity, which can lead to varying lateral distribution factors along the length of the bridge, it was chosen to only consider midspan strain readings, and thus the lateral distribution at the midspan. Limiting each girder to only have only a single distribution factor can reduce the complexity of the model and calibration process.

Fig 3.7 shows the point loads caused by a truck crossing a simple bridge span. $Q_1, Q_2, \ldots Q_j, \ldots Q_g$ represent lateral distribution factors for each girder $G_1, G_2, \ldots G_j, \ldots G_g$, where $g$ is the total number of girders. From the instant the front axle of the vehicle reaches the bridge to when the last axle leaves, there are $S$ total scans. The maximum strain of each girder can then be determined, and the girder with the highest maximum strain is referred to as the reference girder. The 50 greatest strain values of the reference girder are then found alongside the corresponding strains from the other girders. For example, the largest recorded strain on the reference girder which occurs at time step $k$ can be represented as $\varepsilon_{j,k}$. The corresponding strains $\varepsilon_{1,k}, \varepsilon_{2,k}, \ldots \varepsilon_{j,k}, \ldots \varepsilon_{g,k}$ can be evaluated as shown in eq. (3.14).
\[ Q_{j,k} = \frac{\varepsilon_{j,k}}{\sum_{j=1}^{g} \varepsilon_{j,k}} \]  \hspace{1cm} (3.14)

Here, \( Q_{j,k} \) is the percentage of strain experienced by girder \( j \) at time step \( k \) in reference to the strain across all girders at time step \( k \), such that \( Q_{1,k} + \cdots + Q_{j,k} + \cdots + Q_{g,k} = 1 \).

The time instants \( k \) corresponding to the 50 largest strains recorded on the reference girder are evaluated according eq. (3.14). Each \( Q_{j,k} \) is then averaged across all 50 time steps to produce a lateral distribution factor for girder \( j \).

\[ Q_j = \frac{\sum_{k=1}^{50} Q_{j,k}}{50} \]  \hspace{1cm} (3.15)

Finally, a vector \( Q \) can be used to describe the lateral distribution factors for all girders.

\[ Q = \begin{bmatrix} Q_1 \\ \vdots \\ Q_j \\ \vdots \\ Q_g \end{bmatrix} \]  \hspace{1cm} (3.16)

Fig 3.7: Lateral distribution of load across two bridge girders
3.3 Influence Line Determination

The following influence line derivation described herein follows the methods described by O’Brien [18] and Zhao [19], which minimizes the error between the theoretical strain caused by the application of multiple moving point loads (see eq. (3.24)) and the actual strain measured.

The vehicle shown in Fig 3.8 is a schematic drawing for a calibration vehicle, which is necessary for determining the IL of each girder. The axle spacing and static axle weights of the calibration vehicle must be known. To facilitate the discussion, axles are numbered in ascending order starting with the leading axle. Axle spacing is described as the distance from the leading axle to each subsequent axle by $D_i$. This means $D_1$ will be zero for any vehicle.

Influence lines are most commonly defined as function of distance along a particular structure. In the case of BWIM where constant vehicle velocity is often assumed, distance along the bridge is a function of time, and time in turn is a function of the number of scans performed by the data acquisition system (DAQ). Using this relationship, the influence line can be described as a function of the number of scans executed by the DAQ system (see (3.17)).
\[ I(x) = I(x(t)) = I(x(t(k))) = I(k) \]  

(3.17)

The relationship in eq. (3.17) means each point \( k \) can be thought of as a time step. Axle spacing can be redefined as the number of scans which occurs from the time the first axle crosses a point to when the subsequent \( i^{th} \) axle crosses that same point.

![Diagram of a three axle vehicle on a timeline of scans](Fig 3.9: Three axle vehicle shown on a timeline of scans)

\[ C_i = \frac{D_i f}{v} \]  

(3.18)

The number of scans between axles can be calculated by eq. (3.18), where \( C_i \) is axle spacing redefined in reference to DAQ scans, \( v \) is vehicle velocity, and \( f \) is the scanning frequency of the DAQ.

There is one IL ordinate for each time step \( k \) from \( k = 1 \) to \( k = S - C_N \) where \( N \) is the total number of axles of the vehicle. The moment the first axle reaches the bridge corresponds to \( k = 1 \) while \( K = S \) corresponds the moment the final axle is leaving the bridge. The length of the IL will always have \( C_N \) fewer points than the total number of time scan \( S \). This is because the test begins
at $k = 1$ when the first axle reaches the bridge, but the test must continue until the final axle leaves the bridge which will be $C_N$ time steps after the first axle left.

![Generic influence line on a timeline of scans](image)

Fig 3.10: *Generic influence line on a timeline of scans*

The moment in a girder at any time $k$ caused by internal forces can be described using basic mechanics.

$$M_k = \frac{\sigma_k I}{y}$$  \hspace{1cm} (3.19)

Here, $\sigma$ is the internal stress in the girder at time $k$, $I$ is the moment of inertia, and $y$ is the distance from the neutral axis to the sensor location. Assuming constant material and section properties along the length of the member, this relationship can be rewritten in terms of strain $\varepsilon_k^{th}$ in the bottommost fiber and modulus of elasticity $E$.

$$M_k = \frac{EI}{y} \varepsilon_k^{th}$$  \hspace{1cm} (3.20)

Section modulus is defined as follows:
The section modulus $Z$ can be substituted into eq. (3.21).

$$Z = \frac{I}{y} \quad (3.21)$$

Where $M_k$ is bending moment at the sensor location of the girder at time-step $k$ and $\varepsilon_{k}^{th}$ is the theoretical strain in the girder at time step $k$.

The moment at the sensor location (e.g. midspan) of the girder can also be described by the IL ordinates and the external forces acting on the structure.

$$M_k = EZ\varepsilon_{k}^{th} \quad (3.22)$$

$$M_k = \sum_{i=1}^{N} P_i \cdot Q_j \cdot I(k - C_i) \quad (3.23)$$

Where $N$ is the total number of axles of the calibration vehicle, $P_i$ is the known weight of the calibration vehicle’s $i^{th}$ axle, $Q_j$ is the lateral distribution factor of the corresponding girder, and $I(k - C_i)$ is the $(k - C_i)$-th IL ordinate corresponding to the location of the $i^{th}$ axle. Here, $M_k$ is the bending moment in the $j^{th}$ girder at the $k^{th}$ time step. Since the IL calculation can only be performed for a single girder at a time, the subscript $j$ has been left off some variables for the sake of succinctness.

By setting equations (3.22) and (3.23) equal and solving for $\varepsilon_{k}^{th}$ it is possible to receive a relationship for the theoretical strain in the girder in terms of the IL.
\[ \varepsilon_{k\text{th}} = \frac{1}{E \cdot Z} \sum_{i=1}^{N} P_i \cdot Q_j \cdot I(k - C_i) \]  

(3.24)

Finally, considering the example truck has three axles, \( \varepsilon_{k\text{th}} \) can be expanded.

\[ \varepsilon_{k\text{th}} = \frac{1}{E \cdot Z} [P_1 \cdot Q_j \cdot I(k - C_1) + P_2 \cdot Q_j \cdot I(k - C_2) + P_3 \cdot Q_j \cdot I(k - C_3)] \]  

(3.25)

The least squares method can be used to minimize the difference between the measured bridge response \( e^m \) and the theoretical bridge response \( \varepsilon_{k\text{th}} \). The error \( \Delta_k \) is a sum of \( N \) terms, because the position corresponding to each IL ordinate has a wheel located directly on it exactly \( N \) times. Once again, \( N \) being the total number of vehicle axles.

\[ \Delta_k = \sum_{i=1}^{N} (\varepsilon_{k+c_i}^m - \varepsilon_{k+c_i}^{\text{th}})^2 \]  

(3.26)

Considering that there should be \( N \) equations for each point on the IL, the equation for the error function at any arbitrary point along the IL is shown in eq. (3.27).

\[ \Delta_k = \left( \varepsilon_{k+c_2}^m - \frac{1}{E \cdot Z} [P_1 \cdot Q_j \cdot I(k + C_2) + P_2 \cdot Q_j \cdot I(k) + P_3 \cdot Q_j \cdot I(k - C_3 + C_2)] \right)^2 \\
+ \left( \varepsilon_{k+c_3}^m - \frac{1}{E \cdot Z} [P_1 \cdot Q_j \cdot I(k + C_3) + P_2 \cdot Q_j \cdot I(k - C_2 + C_3) + P_3 \cdot Q_j \cdot I(k)] \right)^2 \]  

(3.27)

If \((k + C_i)\) is nonpositive or greater than \((S - C_N)\) then \( I(k + C_i) = 0 \). This reduction will only occur when not all axles are on the bridge at the same time, and the zero terms will correspond to axles off the bridge.
Differentiate eq. (3.27) with respect to $I$ to find the value for $I$ which minimizes error between the measured strain and the theoretical strain to give following equation:

$$\frac{\partial \Delta_k}{\partial I_k} = 2 \left( \frac{P_1 \cdot Q_j}{E \cdot Z} \right) \left[ \varepsilon_{k}^{m} - \frac{1}{E \cdot Z} \left[ P_1 \cdot Q_j \cdot I(k) + P_2 \cdot Q_j \cdot I(k - C_2) + P_3 \cdot Q_j \cdot I(k - C_3) \right] \right]$$

$$+ 2 \left( \frac{P_2 \cdot Q_j}{E \cdot Z} \right) \left[ \varepsilon_{(k+C_2)}^{m} - \frac{1}{E \cdot Z} \left[ P_1 \cdot Q_j \cdot I(k + C_2) + P_2 \cdot Q_j \cdot I(k) + P_3 \cdot Q_j \cdot I(k - C_2 + C_3) \right] \right]$$

$$+ 2 \left( \frac{P_3 \cdot Q_j}{E \cdot Z} \right) \left[ \varepsilon_{(k+C_3)}^{m} - \frac{1}{E \cdot Z} \left[ P_1 \cdot Q_j \cdot I(k + C_3) + P_2 \cdot Q_j \cdot I(k - C_2 + C_3) + P_3 \cdot Q_j \cdot I(k) \right] \right] = 0 \quad (3.28)$$

Eq. (3.28) can be put into matrix form such that the $k^{th}$ row of the matrix corresponds to the appropriate $\Delta_k$.

$$[W_j]_{(S-C_3) \times (S-C_3)} \times [I_j]_{(S-C_3) \times 1} = [\psi_j]_{(S-C_3) \times 1} \quad (3.29)$$

$$[W_j] = Q^2 \begin{bmatrix} \sum P_i^2 & \cdots & P_2 P_1 & \cdots & P_3 P_2 & \cdots & P_3 P_1 & \cdots & 0 & \cdots & 0 & \cdots & 0 & \cdots & \cdots \\ \cdots & P_1 P_3 & \cdots & P_2 P_3 & \cdots & P_1 P_2 & \cdots & \sum P_i^2 & \cdots & P_2 P_1 & \cdots & P_3 P_2 & \cdots & P_3 P_1 & \cdots \\ \cdots & \cdots & 0 & \cdots & 0 & \cdots & 0 & \cdots & P_1 P_3 & \cdots & P_2 P_3 & \cdots & P_1 P_2 & \cdots & \sum P_i^2 \end{bmatrix} \quad (3.30)$$

The main diagonal of the $[W]$ matrix is formed by the sum of the squares of the individual axle weights. Each row, corresponding to a particular IL ordinate, is comprised of every permutation of axle pairings. The axle pairings are offset from the diagonal by the distance between the two axles. For example, the $P_2 P_3$ pairing would be offset from $\sum P_i^2$ by $(C_2 - C_3)$. Indices which exceed the bounds of the matrix correspond to axles which are not on the bridge at the time of the scan. All other entries are zero.
\[ [I_j] = \begin{bmatrix} I_{1,j} \\ \vdots \\ I_{k,j} \\ \vdots \\ I_{(S-C_3),j} \end{bmatrix} \] (3.31)

The \([I]\) vector represents the influence line for the \(j^{th}\) girder.

\[ [\psi_j] = \begin{bmatrix} E \cdot Z [P_1 \cdot Q_j \cdot \varepsilon_1^m + P_2 \cdot Q_j \cdot \varepsilon_{C_2}^m + P_3 \cdot Q_j \cdot \varepsilon_{C_3}^m] \\ \vdots \\ E \cdot Z [P_1 \cdot Q_j \cdot \varepsilon_k^m + P_2 \cdot Q_j \cdot \varepsilon_{k+C_2}^m + P_3 \cdot Q_j \cdot \varepsilon_{k+C_3}^m] \\ \vdots \\ E \cdot Z [P_1 \cdot Q_j \cdot \varepsilon_{S-C_3}^m + P_2 \cdot Q_j \cdot \varepsilon_{S-C_3+C_2}^m + P_3 \cdot Q_j \cdot \varepsilon_{S}^m] \end{bmatrix} \] (S-C_3) \times 1 (3.32)

The \([\psi_j]\) vector represents the sum of the moments caused at the sensor location of girder \(j\) when each load \(P\) passes over each IL ordinate. It can be seen that each row of \([\psi_j]\) consists of the sum of 3 moments, each moment corresponding to one axle. Each of the strain readings \(\varepsilon_{k+C_i}\) is time shifted by \(C_i\) to correspond with the \(i^{th}\) axle.

The \([\psi_j]\) matrix can also be visualized as the superposition of moment caused by three individually moving point loads.

\[ [\psi_j] = [\psi_j]_1 + [\psi_j]_2 + [\psi_j]_3 \] (3.33)

In the arrangement shown by eq. (3.33), \([\psi_j]_i\) represents the moment at the sensor location caused only by the \(i^{th}\) axle as it moves across the bridge. This can be seen in eq. (3.34).
Finally, eq.(3.35) solves for $I$ at all points $k$.

$$[I]_{(S-C_3)\times 1} = [W]_{(S-C_3)\times (S-C_3)}^{-1} \times [\psi]_{(S-C_3)\times 1}$$  

(3.35)

3.4 Axle Weight Determination

This section describes the process to obtain axle weights of a truck using a known IL obtained in Section 3.3. This derivation is based on the process described by Zhao [19].

The moment response $M^k$ is the sum of moments $M_k$ across all bridge girders at any particular time step $k$. It can be expressed using axle weights $P_1 \ldots P_N$, known lateral distribution factors $Q_1 \ldots Q_g$, and the calculated influence ordinate $I(k)$ for scan $k$. This leads to eq. (3.36).

$$M^k = \sum_{j=1}^{g} \sum_{i=1}^{N} (P_i \cdot R_{i,j}^k \cdot Q_j)$$  

(3.36)

Where $R_{i,j}^k = I_j(k-C_i)$ for the $i^{th}$ axle on the $j^{th}$ girder. For any $(k-C_i)$ which are either nonpositive or exceed $(S-C_N)$ the corresponding $R_{i,j}^k$ should be set to zero.

Eq. (3.36) can also be expressed in matrix form by writing the collection of $P_i$ and $Q_j$ terms as column vectors.
\[ M^k = [P_1 \ldots P_i \ldots P_N] \begin{bmatrix} R_{1,1}^k & \cdots & R_{1,g}^k \\ \vdots & \ddots & \vdots \\ R_{i,1}^k & \cdots & R_{i,g}^k \\ \vdots & \ddots & \vdots \\ R_{N,1}^k & \cdots & R_{N,g}^k \end{bmatrix} \begin{bmatrix} Q_1 \\ \vdots \\ Q_g \end{bmatrix} \] (3.37)

\[ M^k = [P]^T [R^k][Q] \] (3.38)

To see how eq. (3.38) can be applied to all time steps, vertically stack consecutive time steps of eq. (3.38). The resulting equation can be seen in an expanded form in eq. (3.39).

\[
[M] = \begin{bmatrix} M^1 \\ \vdots \\ M^k \\ \vdots \\ M^N \end{bmatrix} = \begin{bmatrix} P_1[R^1]_{(1,:)}[Q] + \cdots + P_i[R^1]_{(i,:)}[Q] + \cdots + P_N[R^1]_{(N,:)}[Q] \\ \vdots \\ P_1[R^k]_{(1,:)}[Q] + \cdots + P_i[R^k]_{(i,:)}[Q] + \cdots + P_N[R^k]_{(N,:)}[Q] \\ \vdots \\ P_1[R^S]_{(1,:)}[Q] + \cdots + P_i[R^S]_{(i,:)}[Q] + \cdots + P_N[R^S]_{(N,:)}[Q] \end{bmatrix} 
= P_1[L_1][Q] + \cdots + P_i[L_i][Q] + \cdots + P_N[L_N][Q] = \sum_{i=1}^{N} P_i[L_i][Q]
\] (3.39)

Where \([R^k]_{(i,:)}\) corresponds to all elements in the \(i^{th}\) row of the \([R^k]\) matrix shown in eq. (3.37) and eq. (3.38). The \([L_i]\) matrix contains the influence line ordinate for each girder at all points on the bridge.

\[
[L_i] = \begin{bmatrix} [R^1]_{(i,:)} \\ \vdots \\ [R^k]_{(i,:)} \\ \vdots \\ [R^S]_{(i,:)} \end{bmatrix} = \begin{bmatrix} R_{i,1}^1 & R_{i,2}^1 & \cdots & R_{i,g}^1 \\ \vdots & \vdots & \ddots & \vdots \\ R_{i,1}^k & R_{i,2}^k & \cdots & R_{i,g}^k \\ \vdots & \vdots & \ddots & \vdots \\ R_{i,1}^S & R_{i,2}^S & \cdots & R_{i,g}^S \end{bmatrix}
\] (3.40)

A graphical representation of \([L_i]\) for a single girder can be seen in Fig 3.11. It should be noted that each \([L_i]\) is simply a shift of \([L_1]\) by \(C_i\) scans.
The moment in the bridge as described in eq. (3.39) can be rearranged once again to consolidate the knowns and the unknowns.

\[
M = P_1 \beta_1 + P_2 \beta_2 + \cdots + P_N \beta_N
\]

(3.41)

Here, \([\beta_i]\) is a vector of length \(S\) which contain the product of the influence line of each bridge girder and the corresponding lateral distribution factor, as shown in eq. (3.42).

\[
[\beta_i] = [L_i][Q]
\]

(3.42)

All \([\beta_i]\) can be combined into a series of column vectors.

\[
[\beta] = [\beta_1 \beta_2 \cdots \beta_N]
\]

(3.43)

Combining eq. (3.41) and eq. (3.43) gives the following:

\[
\sum_{i=1}^{N} P_i \beta_i
\]
\[ M = \beta[P] \quad (3.44) \]

Finally, the axle weights \([P]\) (defined in eq. (3.37) and eq. (3.38)) can be solved by left multiplying the pseudoinverse of \([M]\) with \([\beta]\) from eq. (3.44).

\[ [P] = [M]^*[\beta] \quad (3.45) \]

The gross vehicle weight is then the sum of all axle weights.

\[ GVW = \sum_{i=1}^{N} P_i \quad (3.46) \]

3.5 Speed Correction

This section presents the methodology to correct BWIM data gathered from a vehicle which traveled at nonconstant speed across the bridge. The method presented measures the average speed of the vehicle between two or more strain transducers installed along the longitudinal axis of the bridge to determine the vehicle velocity on different portions of the span. The data corresponding to the faster portions is then scaled to match the speed of the slowest portion.

Consider a simple bridge span with strain transducers 1, 2, \(...\), \(y\), \(...\), \(Y\) installed on along the same girder. The distance between sensors \((y - 1)\) and \(y\) and the distance between sensors \(y\) and \((y + 1)\) are denoted as \(d_y\) and \(d_{y+1}\), respectively. Fig 3.12 shows the strain response at the midspan and two quarter points as a point load moves across a simple span girder. The point load increases velocity when it reaches midspan. Average velocity \(v_y\) for each segment can be calculated using the segment distance \(d_y\).
Instrumenting a girder with $Y$ strain transducers will yield $(Y - 1)$ bridge segments which have a strain transducer at their beginning and end. This will lead to $(Y - 1)$ average vehicle velocities measured over the entire bridge. The time vector produced by the DAQ with constant sampling frequency is denoted as $[T]$. Time vector $[T]$ and average vehicle velocity $v_y$ across each segment can be used in conjunction with eq. (3.5) to define the measured strain data as a function of position along the bridge. This new position vector is denoted as $[X]$. 

$\begin{align*}
  v_y &= \frac{d_y}{t_y - t_{y-1}} 
\end{align*}$ (3.47)
Defining the strain measurements as a function of position does not increase the accuracy of the BWIM model if each point is not separated by a constant displacement. Due to the varying values of $v_y$, the distance traveled by the load between each recorded strain value will vary for each bridge segment, as shown in Fig 3.12. The step distance for each segment can be known as $\delta_y$. Interpolate the position vector $[X]$ such that the step distance for all points matches the smallest $\delta_y$. Fig 3.13 illustrates how the distance traveled between DAQ scans can affect the spacing of strain readings. The point load in the figure traveled from left to right. Its speed is constant until it passes the midspan, then the speed is doubled. The top figure shows data points captured with respect to time, and the bottom figure shows data with respect to position.
Fig 3.14 shows how the spacing of data points is made uniform after interpolation of the plot with respect to position.

Fig 3.13: *Variable speed load response versus time (top) and versus position (bottom)*

Fig 3.14 shows how the spacing of data points is made uniform after interpolation of the plot with respect to position.
The strain response curve has been interpolated such that the distance between each data point is constant, as is the case with constant speed data. The corrected strain can now be correlated with the IL calculated in Section 3.3 to determine axle weights. The IL must also be interpolated such that each IL ordinate corresponds to a location in the displacement vector \([X]\). It is important to note, the IL displacement vector should be \(D_N\) (the distance from the first axle to the final axle of the unknown vehicle) shorter than the displacement vector \([X]\) for the measured strain response. This is explained in Section 3.3 with the help of Fig 3.10. The strain response can then be analyzed with the corresponding IL as described in Section 3.4.

*Fig 3.14: Interpolated data points of variable speed point load*

The strain response curve has been interpolated such that the distance between each data point is constant, as is the case with constant speed data. The corrected strain can now be correlated with the IL calculated in Section 3.3 to determine axle weights. The IL must also be interpolated such that each IL ordinate corresponds to a location in the displacement vector \([X]\). It is important to note, the IL displacement vector should be \(D_N\) (the distance from the first axle to the final axle of the unknown vehicle) shorter than the displacement vector \([X]\) for the measured strain response. This is explained in Section 3.3 with the help of Fig 3.10. The strain response can then be analyzed with the corresponding IL as described in Section 3.4.
CHAPTER 4

NUMERICAL ANALYSIS

To evaluate the performance of the algorithm, finite element analysis was performed on a model of a pedestrian bridge on the University of Alabama campus to obtain predicted axle weights of simulated vehicles. Section 4.1 will discuss the finite element program used to perform numerical analysis, while Section 4.2 showcases some of the important MATLAB functions written to perform BWIM calculations. Verification of the finite element program is shown in Section 4.3 with the use of a simply supported beam case. Finally in Section 4.4, the verified program is used to predict the axle weights of a simulated vehicle crossing a bridge model.

4.1 Finite Element Analysis Program

A finite element analysis (FEA) program using the software MATLAB was developed by the author with the guidance of the author’s advisor. The FEA package was developed before the start of the work described in this thesis to fit a variety of needs, not all of which were utilized for this analysis.

The main processes of the author’s FEA program can be seen in Fig 4.1. Select subroutines of the FEA program are described in greater detail below. Sections 4.1.1 through 4.1.4 describe subroutines which are part of the main FEA package shown in Fig 4.1.
Fig 4.1: Flowchart of FEA package
4.1.1 Local Element Stiffness/Mass Matrices

Function: std_stiffness_mass.m

The purpose of this subroutine is to construct the 12x12 stiffness and mass matrices for an element. The subroutine requires inputs such as material properties of the element in question, and it outputs the Euler-Bernoulli stiffness and consistent mass matrices with respect to the element’s local coordinate system. The subroutine is called repeatedly based upon the number of elements in the structure.

4.1.2 Coordinate Transformation

Function: coor_trans.m

The coordinate transformation subroutine builds the transformation matrix necessary for converting a matrix described with respect to an element’s local coordinates to one which references the global coordinate system. The subroutine requires inputs such as the list of nodes and list of elements, which describe the geometry of the structure. Outputs include the transformation matrix. The subroutine is called repeatedly for each element in the structure.

4.1.3 Element Strain

Function: strain_analysis.m

This subroutine is responsible for calculating the strain at particular locations along each element. Shape functions are double differentiated to calculate the curvature at a specified number of locations along each element, and strain is calculated at a specified distance from the neutral
axis. Some inputs include the list of nodes, list of elements, and displacement of each node. Bending and axial strain at a specified number of locations is output for every element.

4.1.4 Deflection Plot

Function: Plot_deform_shape.m

This plot subroutine draws the undeformed and deformed shapes of the structure. The two shapes are overlaid on each other to show differences. A scale factor is able to be specified to easily view small deflections. Shape functions are used to calculate the location of subpoints between each node. This gives a smoother looking image. Some inputs for this subroutine include the list of nodes, list of elements, and displacement of each node. The deflection curve of a simply supported beam loaded at midspan can be seen in Fig 4.2. A 500 times magnification was applied to the plot of the deformed shape to better show the curve.
4.2 Bridge Weigh-in-Motion Package

After strain data is simulated by the FEA package, the data is able to be processed by a BWIM package. Sections 4.2.1 and 4.2.2 discuss other MATLAB scripts used to determine the IL of a girder and predict vehicle axle weights.

4.2.1 Influence Line Calculation

Script: IL_W_calc.m

The purpose of this script is to calculate the influence line of a girder based upon either strain measurements or strain outputs by the FEA package. A flowchart of the operations can be seen in Fig 4.3. Strain measurements taken from the girder in question are used as input. Next, the calibration vehicle is defined by the static weights of each axle, axle spacing, sampling frequency
of the strain data, and lateral distribution factors of the bridge girders. The $[W]$ matrix shown in eq. (3.30), which helps describe axle spacing, is built in the following steps:

1) The term $\sum_{i=1}^{N} P_i^2$ is placed on the diagonal
2) Permutations of axle pairings are calculated
3) The product of the axles in each permutation is placed off diagonal based upon the relative distance between the axles.

Then, the $[\psi]$ vector (seen in eq. (3.32)) is built to describe the sum of the moments as axles pass each point on the bridge. Finally, eq. (3.35) is applied to determine the vector of IL ordinates.
4.2.2 Axle Weight Calculation

Function: axle_weight_calc.m

This function uses inputs such as the sum of strain data across all girders, all ILs, and lateral distribution factors to predict axle weights. A flowchart of the function can be seen in Fig 4.4. The \([L_i]\) matrix, which holds the IL for each girder, and the \([\beta_i]\) vector are calculated for each vehicle axle. Once all \([\beta_i]\) are found and \([\beta]\) is constructed, eq. (3.45) is able to be applied to find individual axle weights.

Fig 4.4: Flowchart of axle weight calculation
4.3 Program Verification

Multiple cases were examined to verify the accuracy of the author’s FEA and BWIM packages. A model of the simply supported beam shown in Fig 4.5 was used for all the tests in this section. First, Section 4.3.1 shows how the IL calculated via methodology described in Section 3.3 agrees with the theoretical IL. Next in Section 4.3.2, predicted axle weights are compared to axle loads defined in the model. The speed correction algorithm is demonstrated in Section 4.3.3. Finally, the influence of noise on the speed correction algorithm is examined in Section 4.3.4.

4.3.1 Verification of Influence Line Calculation

Strain response from a known calibration vehicle is required to determine the IL. The IL calculation was verified by comparing a vector of calculated IL ordinates to the theoretical IL determined via the Mueller-Breslau method. For the calculated IL case, two point loads simulating the vehicle shown in Fig 4.6, denoted as Vehicle 1, crossed the beam simultaneously. The strain at midspan was recorded for each time step, and the response curve can be seen in Fig 4.7. The IL was calculated using the MATLAB script described in Section 4.2.1. The calculated IL is expected to be equal to the theoretical IL. The two curves can be seen in Fig 4.8.
The IL plots in Fig 4.8 can be seen to line up very well, and both plots also show a peak moment of 136 kip-in. This is also in agreement with the theoretical max moment in the beam, which can be calculated as $M_{max} = \frac{PL}{4}$. This demonstrates that the FEA package can capture the response correctly and the IL determination algorithm is successful in calculating the IL ordinates.
Fig 4.7: Strain response of Vehicle 1

Fig 4.8: Mueller-Breslau theoretical IL vs IL calculated with FEA package
4.3.2 Verification of Axle Weight Calculation

A known IL and the strain response of an unknown vehicle are required to predict axle weights. Two constant speed vehicles were simulated to show the accuracy of the axle weight prediction. The strain response at midspan was recorded for both simulations and the MATLAB function described in Section 4.2.2 was run. The first vehicle simulated was Vehicle 1 shown in Fig 4.6. The second vehicle simulated, denoted as Vehicle 2, can be seen in Fig 4.9.

![Diagram of Vehicle 2](image)

Fig 4.9: Vehicle 2

The strain response of both vehicles is shown in Fig 4.10. It should be noted that Vehicle 1’s back axle is heavier than its front, while Vehicle 2’s front axle is heavier than its back. The IL determined in Section 4.3.1 and shown in Fig 4.8 was used to predict axle weights. The axle weight predictions for both vehicles can been seen in Table 4.1. Both vehicles showed no error in the calculation.
4.3.3 Speed Correction Algorithm Verification

This section demonstrates the speed correction algorithm proposed in this thesis. Another vehicle was simulated to cross the beam for this section. That vehicle, denoted Vehicle 3, can be seen in Fig 4.11. Vehicle 3 starts crossing the beam at velocity $v_1 = 528 \dfrac{in}{s}$ then it changes to velocity $v_2 = 1056 \dfrac{in}{s}$ as it passes midspan. Strain was analyzed at the midpoint and quarter points of the beam. Fig 4.12 shows the locations on the beam at which strain was measured.

<table>
<thead>
<tr>
<th>Case</th>
<th>P1 (kips)</th>
<th>P2 (kips)</th>
<th>GVW (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle 1</td>
<td>6.00 [0.00]</td>
<td>8.00 [0.00]</td>
<td>14.00 [0.00]</td>
</tr>
<tr>
<td>Vehicle 2</td>
<td>7.25 [0.00]</td>
<td>5.34 [0.00]</td>
<td>12.59 [0.00]</td>
</tr>
</tbody>
</table>

Table 4.1 Predicted axle weights of Vehicle 1 and Vehicle 2
Strain response is double differentiated as described in Section 3.1 to determine the velocity and axle spacing of Vehicle 3. The variable speed response curves at the quarter points and midspan of the beam can be seen in Fig 4.13. The peaks in the double differentiated strain curve are marked. The quarter points are separated by 132 inches (known a priori), and the sampling frequency is 500 Hz. The vehicle was simulated to double its speed as the second axle crossed midspan. The time at which each axle crosses each sensor location is marked on the figure. The velocities \( v_1 \) and \( v_2 \) can be determined, using eq. (3.47), by dividing the distance between sensor locations (132 in) by the time it takes to travel from one sensor to the next (0.25 s for the first segment). Velocities \( v_1 \) and \( v_2 \) can be calculated from Fig 4.13 as 528 in/s and 1056 in/s respectively. Since vehicle velocity is scaled to the slowest speed captured on the bridge, axle spacing should be calculated from the slowest velocity. The axle spacing of the vehicle can be calculated from the first quarter strain data. 0.17 seconds elapse from the time the first axle crosses...
the sensor to when the second axle crosses. Knowing the sensor is reading data every 0.002 seconds, it can be found that 85 sensor scans occur between when the first axle reaches the sensor and when the second axle reaches it. Therefore, \( C_2 = 85 \) scans.

Fig 4.13: Strain response of Vehicle 3 traveling at variable speed

With the axle spacing and vehicle speeds determined, the speed correction algorithm can now be applied. Three different cases were examined for Vehicle 3. A constant speed case, a
variable speed case without applying correction, and finally a variable speed case where correction is applied by interpolating the data to create a constant interval between samples, as discussed in Section 3.5.

Fig 4.14 shows the strain response curve of Vehicle 3 traveling at a constant speed across the beam. This case will be used as a baseline.

![Graph showing strain response at constant speed](image)

**Fig 4.14: Vehicle 3 strain response at constant speed**

Fig 4.15 (left) shows the strain response curve of Vehicle 3 traveling at variable speed with respect to time. Fig 4.15 (right) shows the same curve but now the response is with respect to displacement. It should be noticed that Fig 4.15 (right) has the same shape as the constant speed case.
The results of the axle weight predictions in Table 4.2 show very accurate predictions for the constant speed and corrected variable speed cases, but significant error is shown in the uncorrected case.

![2 Speed Strain Data vs Time](image1)

**Fig 4.15: Strain Response of Vehicle 3 vs time (left) and vs displacement (right)**

<table>
<thead>
<tr>
<th>Case</th>
<th>P1 (kips)</th>
<th>P2 (kips)</th>
<th>GVW (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Speed</td>
<td>6.58 [0.00]</td>
<td>9.03 [0.00]</td>
<td>15.61 [0.00]</td>
</tr>
<tr>
<td>Variable Speed: No Correction</td>
<td>8.63 [31.2]</td>
<td>7.10 [21.4]</td>
<td>15.73 [0.01]</td>
</tr>
<tr>
<td>Variable Speed: Corrected</td>
<td>6.58 [0.00]</td>
<td>9.03 [0.00]</td>
<td>15.61 [0.00]</td>
</tr>
</tbody>
</table>

Table 4.2 Predicted axle weights of Vehicle 3
4.3.4 Noise Influence

Signal noise can present a problem when data is measured in reality. To analyze the impact of noise on the model, various levels of noise were added to the Vehicle 3 strain data from Fig 4.13 and the IL from Fig 4.8. The axle spacing determination and axle weight calculations were then performed again.

To add noise to the data, first a vector of random numbers with mean of zero and variance of one ranging from -1 to 1 was generated. The vector of random numbers was then multiplied by the standard deviation of the original data and the desired noise level before it was added to the original data.

An infinite impulse response (IIR) low-pass filter with a cutoff of 10 Hz was used to filter the noise. The filtered data was then double differentiated, and the axle spacing and velocity of Vehicle 3 were determined. Fig 4.16 shows the plots of the strain response and double differentiated strain response measured at the quarter points and midpoint of the beam. The marked peaks in each plot correspond to the time when an axle crossed the sensor location. The velocity and axle spacing can be recalculated from the plot as described in Section 4.3.3.
The results of the axle weight predictions at noise levels ranging from 0% to 10% can be seen in Table 4.3. Individual axle weight predictions were mostly within 5% of the true value, while GVW stayed within 0.5% of the true value.

Fig 4.16: Filtered strain response of Vehicle 3 with 2% noise
4.4 Campus Bridge Simulation

A model of the bridge presented in Section 5.1 was constructed in MATLAB. Vehicles were simulated to cross the bridge to find the ILs of the bridge model, and predict axle weights. Section 4.4.1 will give an overview of how the bridge was modeled for FEA. The determination of the ILs will be discussed in Section 4.4.2. In Section 4.4.3 axle weights of simulated vehicles are presented. Finally in Section 4.4.4, the impact of signal noise on axle weight prediction will be discussed.

4.4.1 Bridge Model

The bridge studied in this thesis is a pedestrian bridge on the University of Alabama campus. The bridge is comprised mostly of hollow structural sections (HSS). The bridge has a twelve foot wide lightweight corrugated concrete deck. The span is 145.5 feet, and the maximum load is ten kips. Fig 4.17 shows a picture of the bridge.

<table>
<thead>
<tr>
<th>Noise</th>
<th>P1 (kips) [% Error]</th>
<th>P2 (kips) [% Error]</th>
<th>GVW (kips) [% Error]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 %</td>
<td>6.58 [0.00]</td>
<td>9.03 [0.00]</td>
<td>16.61 [0.00]</td>
</tr>
<tr>
<td>1%</td>
<td>6.77 [2.88]</td>
<td>8.84 [2.11]</td>
<td>16.61 [0.00]</td>
</tr>
<tr>
<td>2%</td>
<td>6.88 [3.13]</td>
<td>8.87 [1.81]</td>
<td>15.65 [0.26]</td>
</tr>
<tr>
<td>5%</td>
<td>6.44 [2.10]</td>
<td>9.16 [1.45]</td>
<td>15.60 [0.01]</td>
</tr>
<tr>
<td>10%</td>
<td>6.24 [5.18]</td>
<td>9.32 [3.20]</td>
<td>15.56 [0.32]</td>
</tr>
</tbody>
</table>

Table 4.3 Predicted axle weights of Vehicle 3 at different noise levels
A portion of the submittal drawing can be seen in Fig 4.18. Information such as the panel spacing, member type, and camber were used to first construct a 3-dimensional model in SAP2000. Parameters such as the list of nodes, list of elements, and list of sections were exported from SAP2000 into text files to be loaded into the MATLAB workspace.
Since the author’s FEA package does not have the ability to model the concrete deck, the decision was made to remove the deck from the model. Losing such a large mass was not seen as a problem, since the FEA package does not perform dynamic analysis on the structure. The entire model consists of 7888 nodes and 8024 elements. Most of the nodes and elements were on the two C-channels which support the deck, because the simulated loads were applied to the C-channel. A distance of 0.5 inches was required between nodes along the C-channel to maintain even node
spacing across all panels. The boundary conditions were assumed to be four fixed points, one at each corner. Plots of the bridge generated by the FEA package can be seen in Fig 4.19.

![Bridge Model](image)

C-channel

Fig 4.19 *Elevation view (top) and isometric view (bottom) of bridge plots*
4.4.2 Influence Line Determination

An accurate IL is vital to successfully predicting axle weights. The lateral distribution factors and IL of both bottom chords were determined using the strain recorded by the author’s FEA program. Fig 4.20 shows the six locations on the bottom chords at which strain was recorded for each vehicle simulation.

A known calibration vehicle, whose GVW is less than the ten kip max of the bridge, was simulated for the IL determination. That vehicle is denoted as Vehicle 4, and it can be seen in Fig 4.21.
The lateral distribution factor for each chord, denoted as Girder 1 and Girder 2, was calculated using the 50 largest strains recorded at the midspan, as described in Section 3.2. The lateral distribution factors $Q_1 = 0.497$ and $Q_2 = 0.503$ were determined for Girders 1 and 2 respectively.

The IL calculation described in Section 3.3 was then performed for each girder. The IL of Girder 1 and Girder 2 can be seen in Fig 4.22.

Fig 4.22: Strain ILs of Girders 1 and 2
4.4.3 Axle Weight Predictions

Correctly determining vehicle velocity is important whether the vehicle is traveling at constant speed or variable speed. This calculation is first shown in Section 3.1 and demonstrated in Section 4.3.3. In both of those sections, the location of maximum response for the strain transducers occurred at the sensor location. Repeated simulations of the bridge model showed that the location of maximum response for each sensor was not directly above the sensor location. This is thought to be caused by the load being applied to the C-channel while the sensor is measuring strain in the bottom chord. The location of maximum response for each sensor was determined to accurately predict vehicle speed. The IL with respect to position at each sensor location was plotted in Fig 4.23, and the locations of the peaks were marked. While the actual sensor locations are 485 inches apart, the distance between the first and second sensor peaks was found to be 517.5 inches while the corresponding value for the second and third sensors was 586.5 inches.
The axle weight calculation was tested on the bridge model by simulating the crossing of Vehicle 5, seen in Fig 4.24. Sampling frequency was set to 500 Hz. The vehicle was simulated to travel at $v_1 = 250 \frac{in}{s}$ until its rear axle reached midspan, at which point it began traveling at $v_2 = 500 \frac{in}{s}$.

![Fig 4.23: Location of maximum response at sensor locations](image)

$x = 380$ in

$x = 897.5$ in

$x = 1484$ in
The graphs displayed in Fig 4.25 show the strain response and double differentiated response at each sensor location on girder 1. The peaks corresponding to axles crossing the sensors’ locations of maximum response have been marked on each graph. The velocity for both bridge sections \( v_1 \) and \( v_2 \) can be determined with eq. (3.47), as demonstrated in Section 4.3.3. It is important that distance \( d_y \) in eq. (3.47) be taken as the distance between locations of maximum response and not the distance between sensor locations. Calculating the velocity for both sections, it can be found that \( v_1 = 250 \text{ in/s} \) and \( v_2 = 490 \text{ in/s} \). The axle spacing can also be determined as demonstrated in Section 4.3.3. The plot showing strain recorded at the first quarter point shows an axle spacing of \( C_2 = 138 \) scans, while the plot of the third quarter point strain shows \( C_2 = 69 \) scans. Since the speed correction requires all velocities calculated to be rescaled to the slowest velocity, the axle spacing should be determined from the bridge section corresponding to the slowest velocity. Therefore, \( C_2 = 138 \) scans.

Fig 4.24: Vehicle 5
Axle weight predictions for Vehicle 5 were calculated for three different cases. For the first case, the vehicle was simulated to travel at constant speed $v = 250 \text{ in/s}$. For the second case, the variable speed strain data displayed in Fig 4.25 was analyzed, but the variable speed correction described in Section 3.5 was not applied. For the final case, the same variable speed response data was subjected to speed correction before performing the axle weight prediction. The results in Fig 4.25:

- **First Quarter Strain v Time**
  - $t_{1,1} = 1.52s$, $t_{2,1} = 1.796s$

- **Midpoint Strain v Time**
  - $t_{1,2} = 3.590s$, $t_{2,2} = 3.820s$

- **Third Quarter Strain v Time**
  - $t_{1,3} = 4.878s$, $t_{2,3} = 5.016s$
Table 4.4 indicate that the speed correction algorithm can produce predictions nearly as accurate as a constant speed case, while the uncorrected case contains significantly more error.

<table>
<thead>
<tr>
<th>Case</th>
<th>P1 (kips)</th>
<th>P2 (kips)</th>
<th>GVW (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Speed</td>
<td>3.70 [0.61]</td>
<td>2.84 [0.60]</td>
<td>6.54 [0.60]</td>
</tr>
<tr>
<td>Variable Speed: No Correction</td>
<td>4.54 [22.0]</td>
<td>1.84 [35.5]</td>
<td>6.38 [2.99]</td>
</tr>
<tr>
<td>Variable Speed: Corrected</td>
<td>3.66 [1.60]</td>
<td>2.89 [1.04]</td>
<td>6.55 [0.45]</td>
</tr>
</tbody>
</table>

Table 4.4 Predicted axle weights of Vehicle 5

4.4.4 Noise Influence

Artificial noise was applied to the response data of Vehicle 5 and both ILs to see how the algorithm performs under five different noise levels. Section 4.3.4 describes the process of generating the noise added to each vector of strain response. Axle spacing, vehicle velocity, and axle weight calculations were performed again for each case. An IIR low-pass filter with a cutoff of 7 Hz was used to smooth data before the double derivative was calculated.

Five different noise levels ranging from 0% to 10% were analyzed. Fig 4.26 shows the filtered strain response of Vehicle 5 with 2% noise added. Peaks in the double differentiated data were analyzed based upon the size of the peak and the location of the peak relative to the location of the maximum strain response.
The axle weight predictions for each noise level can be seen in Table 4.5. The predictions show under 5% error for each axle until the 10% noise case. Large errors in that case are most likely attributed to an inaccurate axle spacing prediction. It is possible a less complicated bridge model may not have seen this problem until higher noise levels. It can also be seen that even when individual axle predictions have high error, GVW predictions can still be very accurate.

Fig 4.26: Vehicle 5 filtered strain response with 2% noise
<table>
<thead>
<tr>
<th>Noise</th>
<th>P1 (kips)</th>
<th>P2 (kips)</th>
<th>GVW (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>3.70 [0.61]</td>
<td>2.89 [0.60]</td>
<td>6.54 [0.60]</td>
</tr>
<tr>
<td>1%</td>
<td>3.61 [2.99]</td>
<td>2.94 [2.66]</td>
<td>6.55 [0.46]</td>
</tr>
<tr>
<td>2%</td>
<td>3.84 [3.24]</td>
<td>2.73 [4.46]</td>
<td>6.57 [0.15]</td>
</tr>
<tr>
<td>5%</td>
<td>3.55 [4.63]</td>
<td>2.99 [4.53]</td>
<td>6.54 [0.61]</td>
</tr>
<tr>
<td>10%</td>
<td>3.10 [16.6]</td>
<td>3.36 [17.6]</td>
<td>6.46 [1.82]</td>
</tr>
</tbody>
</table>

Table 4.5 Predicted axle weights of Vehicle 5 at various noise levels
CHAPTER 5
FIELD STUDY

The BWIM algorithm was applied to a field study on the University of Alabama campus. The test consisted of three distinct cases in which a two-axle vehicle was driven across the bridge while strain transducers recorded the response in the bottom chord. Section 5.1 introduces the bridge, sensors, and DAQ used in the experiment. Section 5.2 considers the first test case in which a pickup truck was driven across the bridge at constant speed. Section 5.3 highlights the second test case in which an unknown vehicle was driven across at constant speed. Section 5.4 examines the third test case in which the same unknown vehicle traveled at variable speed across the span. Finally, Section 5.5 summarizes results of the field study. Table 5.1 shows an overview of each test case.

<table>
<thead>
<tr>
<th>Case</th>
<th>Section</th>
<th>Trials</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.2</td>
<td>4</td>
<td>Determine ILs and lateral distribution factors</td>
</tr>
<tr>
<td>2</td>
<td>5.3</td>
<td>4</td>
<td>Predict axle weights of a constant speed vehicle</td>
</tr>
<tr>
<td>3</td>
<td>5.4</td>
<td>4</td>
<td>Predict axle weights of a variable speed vehicle</td>
</tr>
</tbody>
</table>

Table 5.1: Field Test Overview

67
5.1 Test Setup

The field study was executed using the same pedestrian bridge on the University of Alabama campus modeled in Section 4.4. A picture of the bridge can be seen again in Fig 5.1. The bridge has a span of 145.5 feet with a twelve foot wide lightweight concrete deck. The trusses on either side are comprised of rectangular HSS tube, and the maximum permissible load is ten kips.

![Bridge on the University of Alabama campus](image)

Fig 5.1: *Bridge on the University of Alabama campus*

The bridge was instrumented with six strain transducers placed at the quarter points. Fig 5.2 shows a diagram of strain transducers and data acquisition installed at the bridge site.
Fig 5.2: Bridge sensor and DAQ diagram
Each strain transducer is the ST350 model manufactured by Bridge Diagnostics Inc. The ST350 model is a 350 ohm full bridge strain transducer, which can be seen again in Fig 5.3

![BDI ST350 strain transducer](image)

Signal from the strain transducers was captured at a 500 Hz sampling rate using the EtherCAT fieldbus terminal manufactured by Beckhoff. A diagram of the EtherCAT can be seen in Fig 5.4. The EtherCAT terminal (EK1814) was equipped with a power booster (EL9410), voltage converter (EL9505), and six strain boards (EL3356) to receive all six strain signals and output data for processing. Data from the EtherCAT was transmitted to an xPC Target manufactured by Speedgoat. The xPC Target executed a MATLAB Simulink model which converted the data from mV/V of excitation to microstrain.
5.2 Test Case One

The objective of the first test case was to drive a calibration vehicle with known weight and axle spacing across the bridge to determine the IL. The calibration vehicle, denoted Vehicle 6, used for this case can be seen in Fig 5.5.
The vehicle was driven across at a constant speed of 5 miles per hour (mph). Due to a lack of roadway on both sides of the bridge, the vehicle had to start from rest a few feet from the bridge. The slow travel speed made it possible to perform most of the acceleration before entering the bridge. Case one was executed four times. Fig 5.6 shows a picture of Vehicle 6 on the bridge during the field test.
Due to complex bridge geometry, it was not assumed that maximum strain recorded for each sensor would occur at the location directly above the corresponding sensor. To determine the distance between positions of maximum response, an IL was generated for each sensor location. Distance between maximum response locations was then calculated as the distance between the peaks of corresponding ILs.

The results from trials three and four were chosen to determine lateral distribution factors and construct ILs due to how well the two curves aligned. The strain recorded by the Ch3 strain transducer during trials three and four can be seen in Fig 5.7. Raw data can be seen on the left, while filtered data can be seen on the right. The filtered data has been subjected to a low-pass IIR filter with a cutoff of 0.1 Hz.
Lateral distribution factors were calculated as described in Section 3.2. It was determined that $Q_1 = 0.504$ and $Q_2 = 0.496$ for girders one and two respectively. Knowing the lateral distribution factors, an IL was then constructed for each of the sensor locations, as discussed in Section 3.3. The ILs calculated using the response from trial three were averaged with the corresponding ILs calculated from trial four. The resulting IL for the Ch3 sensor location can be seen in Fig 5.8.
The distance between the locations of maximum strain response for each sensor were then calculated from the six ILs. Table 5.2 shows the resulting distances which will be used to determine vehicle speed in later cases.

![Influence Line at Ch3 Location](image)

**Fig 5.8: Influence line at midspan**

The distance between the locations of maximum strain response for each sensor were then calculated from the six ILs. Table 5.2 shows the resulting distances which will be used to determine vehicle speed in later cases.

<table>
<thead>
<tr>
<th>Sensors</th>
<th>Ch1 - Ch3</th>
<th>Ch2 - Ch4</th>
<th>Ch3 - Ch5</th>
<th>Ch4 - Ch6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance [in]</td>
<td>654</td>
<td>634.8</td>
<td>363.1</td>
<td>387.4</td>
</tr>
</tbody>
</table>

Table 5.2 *Distance between maximum strain response locations*
5.3 Test Case Two

The objective of the second test case was to predict the axle weights of a new vehicle with unknown weights using the ILs calculated from the first test case. Vehicle 7, shown in Fig 5.9, was driven across the bridge at a constant speed of 5 mph. Once again, the vehicle started from rest a few feet from the bridge due to the lack of roadway on both sides.

Due to the large dynamic response of the bridge, it was more challenging than anticipated to discern vehicle axle properties from some of the double differentiated strain curves. For case two, average speed and axle spacing were calculated with the double differentiated response of the first and third quarter point sensors. The velocity and axle spacing calculated with the Ch1 and Ch5 sensors were averaged with the corresponding data from the Ch2 and Ch6 sensors. The velocity was determined with eq. (3.5) where $x$ was taken as the distance between locations of maximum strain response for the corresponding sensors, instead of the distance between the strain

Fig 5.9: Vehicle 7

$P_1=2.32$ kips

$D_2=9.2$ ft

$P_2=1.40$ kips
transducers. The strain data from Ch1 and Ch5 can be seen in Fig 5.10. An IIR low-pass filter with a cutoff of 0.5 Hz has been applied to the signal to remove adverse noise effects.

Fig 5.10: Filtered strain response of Vehicle 7 at 1\textsuperscript{st} and 3\textsuperscript{rd} quarter points

The strain response at the midspan was interpolated to ensure data points in the response vector align with data points in the IL. The response vector at the midspan was also shifted such that the locations of initial excitation align. The shifted curve can be seen in Fig 5.11.
The axle weight calculation described in Section 3.4 was then applied. The resulting predictions from each of the four trials can be seen in Table 5.3. The results show the algorithm was able to predict the axle weights with moderate accuracy.

![Midspan Strain Response and IL vs Position](image)

**Fig 5.11: Midspan filtered strain response shifted to align with influence line**

<table>
<thead>
<tr>
<th>Trial</th>
<th>P1 (kips)</th>
<th>P2 (kips)</th>
<th>GVW (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.24 [3.34]</td>
<td>1.44 [2.63]</td>
<td>3.68 [1.08]</td>
</tr>
<tr>
<td>2</td>
<td>2.21 [4.61]</td>
<td>1.52 [8.93]</td>
<td>3.74 [0.54]</td>
</tr>
<tr>
<td>3</td>
<td>2.43 [4.65]</td>
<td>1.31 [6.24]</td>
<td>3.74 [0.54]</td>
</tr>
<tr>
<td>4</td>
<td>2.45 [5.60]</td>
<td>1.22 [12.6]</td>
<td>3.67 [1.34]</td>
</tr>
</tbody>
</table>

**Table 5.3 Axle weight predictions of Vehicle 7 at constant speed**
5.4 Test Case Three

The objective of the third test case was to predict the axle weights of Vehicle 7 as it crossed the bridge at a nonconstant speed. The third test case consisted of four trials in which Vehicle 7 started traveling at a constant 5 mph. The vehicle accelerated to 10 mph as its back axle reached midspan. The vehicle then traveled at a constant 10 mph for the remainder of the span.

Strain response data was once again subjected to an IIR low-pass filter. The cutoff frequency was 0.65 Hz. Fig 5.12 shows the strain response gathered by Ch2, Ch4, and Ch6 sensors. Velocity and axle spacing were calculated as set forth in Section 3.5, and demonstrated in Section 4.3.3. Once again, it is important that distance $d_y$ from eq. (3.47) be taken as the distance between locations of maximum response, displayed in Table 5.2, instead of the distance between sensor locations.
The data corresponding to the faster velocity was interpolated such that the distance traveled between DAQ scans was equal for the slower and faster sections of the bridge. The IL was then interpolated such that the position of each data point corresponded with a data point from the corrected response curve.

Axle weight predictions were performed twice for each trial. The first set of axle weight predictions did not consider the variable speed of Vehicle 7, so the vehicle was assumed to have

Fig 5.12: Filtered strain response of Vehicle 7 traveling at variable speed

The data corresponding to the faster velocity was interpolated such that the distance traveled between DAQ scans was equal for the slower and faster sections of the bridge. The IL was then interpolated such that the position of each data point corresponded with a data point from the corrected response curve.

Axle weight predictions were performed twice for each trial. The first set of axle weight predictions did not consider the variable speed of Vehicle 7, so the vehicle was assumed to have
traveled at a constant speed across the entire span. Not considering the variable speed produced very poor axle weight predictions, which can be seen in Table 5.4. The second set of predictions considered the nonconstant vehicle speed by applying the speed correction method described in Section 3.5. The results of this consideration can be seen in Table 5.5. Axle weight error was upwards of 100% and sometimes 200% when the speed correction was not applied. However, when the speed correction was applied, the axle weight error was significantly reduced. For trial 2, the speed correction was able bring axle weight error within 20% of the true value.

<table>
<thead>
<tr>
<th>Trial</th>
<th>P1 (kips)</th>
<th>P2 (kips)</th>
<th>GVW (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.09 [147]</td>
<td>4.63 [231]</td>
<td>3.54 [4.84]</td>
</tr>
<tr>
<td>2</td>
<td>-0.13 [106]</td>
<td>3.68 [163]</td>
<td>3.56 [4.57]</td>
</tr>
<tr>
<td>3</td>
<td>-1.06 [146]</td>
<td>4.59 [228]</td>
<td>3.53 [5.11]</td>
</tr>
<tr>
<td>4</td>
<td>0.22 [90.5]</td>
<td>3.38 [142]</td>
<td>3.60 [3.22]</td>
</tr>
</tbody>
</table>

Table 5.4 Axle weight predictions of Vehicle 7 without considering variable speed

<table>
<thead>
<tr>
<th>Trial</th>
<th>P1 (kips)</th>
<th>P2 (kips)</th>
<th>GVW (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.87 [19.5]</td>
<td>2.01 [43.3]</td>
<td>3.88 [4.30]</td>
</tr>
<tr>
<td>2</td>
<td>2.58 [11.3]</td>
<td>1.13 [19.0]</td>
<td>3.72 [0.00]</td>
</tr>
<tr>
<td>3</td>
<td>3.02 [30.3]</td>
<td>0.55 [60.47]</td>
<td>3.58 [3.76]</td>
</tr>
<tr>
<td>4</td>
<td>3.12 [34.7]</td>
<td>0.53 [62.2]</td>
<td>3.65 [1.88]</td>
</tr>
</tbody>
</table>

Table 5.5 Axle weight predictions of Vehicle 7 considering variable speed
5.5 Summary

The three test cases described in CHAPTER 5 were designed to mirror the cases set forth in the numerical study. Case one proved effective in calculating the ILs of the bridge. Case two showed the BWIM algorithm’s ability to predict the axle weights of a vehicle moving at constant speed. Finally, case three showcases the ability of the speed correction method to improve axle weight prediction accuracy for a vehicle traveling at variable speed. However, the corrected speed axle weight predictions are still not as accurate as the case two predictions.

Some possible sources of error contributing to inaccuracies in the third case include the following:

- Inaccuracies in vehicle velocity determination most likely caused by bridge dynamics. The bridge used for testing was highly flexible and therefore was subject to large amounts of dynamic excitation.

- The testing location did not have adequate space on both ends of the bridge. Vehicles were forced to begin from rest a few feet from the start of the span. This was compensated for by the slow travel speeds of five and ten miles per hour, but being able to approach the span at speed would have been preferred.
CHAPTER 6

CONCLUSION

Accurate traffic monitoring by measuring axle and gross vehicle weights of trucks on the roadway is vital to maintaining existing highway infrastructure and constructing new roadways/bridges to meet future demands. This thesis proposes a new BWIM method that is capable of considering vehicles with varying speed. This method allows for a more accurate axle load estimation for vehicles traveling at nonconstant velocity, and it is improved based on the BWIM methods set forth by Wall, O’Brien, and Zhao ([14] [18] [19]).

BWIM was first introduced by Moses [1] in the late 1970s as an alternative to traditional roadway WIM systems. While early BWIM implementations used axle detectors placed in or on the road surface, large studies such as Weigh-in-Motion of Road Vehicles for Europe [2] have pushed out roadway axle detectors in favor for FAD systems which require no work in traffic lanes to install. Additional advances in BWIM technology are discussed in CHAPTER 2. A theoretical background is presented in CHAPTER 3. The methods for axle detection, influence line determination, and axle weight prediction posed by Wall, O’Brien, and Zhao ([14] [18] [19]) respectively are presented in detail. A method for correcting strain response to account for variable vehicle speed via interpolation is raised in Section 3.5. CHAPTER 4 showcases a numerical study performed in MATLAB using a finite element analysis package. As part of the numerical study, the BWIM algorithm was applied to a simulated vehicle crossing a simply supported beam and a model of a University of Alabama pedestrian bridge. The effectiveness and accuracy of the
The proposed BWIM method is successfully verified with the simply supported beam. With the bridge model, the results of the numerical study confirmed the speed correction method is capable of supplying accurate axle weight predictions for a vehicle which changes speed on the bridge. Finally, a field study executed on the University of Alabama campus is discussed in CHAPTER 5. The experimental study was comprised of three cases which were used to i) determine the IL of the bridge, ii) validate the BWIM algorithm with a vehicle traveling at constant speed, and iii) test functionality of speed correction on a vehicle traveling at variable speed. Results of the field study showed the speed correction method to be capable of improving the accuracy of axle weight predictions for a vehicle traveling at a variable speed.

While the axle weight predictions of the vehicle traveling at variable speed were not as accurate as the predictions for the vehicle traveling at constant speed, the methods presented in this thesis show compelling promise. More research is required to raise prediction accuracy to levels more in line with results gathered from constant speed vehicles. Some suggestions for areas of future research include the following:

- Examine performance of the variable speed correction on a highway bridge at highway speeds.
- Apply alternate FAD methods which may offer more accurate velocity and axle spacing determination, while being less susceptible to bridge dynamics and/or sensor noise.
- Examine possible accuracy improvements brought about by increasing the number of segments over which velocity is measured.
- Explore performance benefits of applying variable speed correction alongside a dynamic BWIM algorithm.
REFERENCES


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