THREE ESSAYS ON MORE POWERFUL UNIT ROOT TESTS

WITH NON-NORMAL ERRORS

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ABSTRACT

This dissertation is concerned with finding ways to improve the power of unit root tests. This dissertation consists of three essays. In the first essay, we extends the Lagrange Multiplier (LM) unit toot tests of Schmidt and Phillips (1992) to utilize information contained in non-normal errors. The new tests adopt the Residual Augmented Least Squares (RALS) estimation procedure of Im and Schmidt (2008). This essay complements the work of Im, Lee and Tieslau (2012) who adopt the RALS procedure for DF-based tests. This essay provides the relevant asymptotic distribution and the corresponding critical values of the new tests. The RALS-LM tests show improved power over the RALS-DF tests. Moreover, the main advantage of the RALS-LM tests lies in the invariance feature that the distribution does not depend on the nuisance parameter in the presence of level-breaks.

The second essay tests the Prebisch-Singer hypothesis by examining paths of primary commodity prices which are known to exhibit multiple structural breaks. In order to examine the issue more properly, we first suggest new unit root tests that can allow for structural breaks in both the intercept and the slope. Then, we adopt the RALS procedure to gain much improved power when the error term follows a non-normal distribution. Since the suggested test is more powerful and free of nuisance parameters, rejection of the null can be considered as more accurate evidence of stationarity. We apply the new test on the recently extended Grilli and Yang index of 24 commodity series from 1900 to 2007. The empirical findings provide significant evidence to support that primary commodity prices are stationary with one or two

trend breaks. However, compared with past studies, they provide even weaker evidence to support the Prebisch-Singer hypothesis.

The third essay extends the Fourier Lagrange Multiplier (FLM) unit root tests of Enders and Lee (2012a) by using the RALS estimation procedure of Im and Schmidt (2008). While the FLM type of tests can be used to control for smooth structural breaks of an unknown functional form, the RALS procedure can utilize additional higher-moment information contained in non-normal errors. For these new tests, knowledge of the underlying type of non-normal distribution of the error term or the precise functional form of the structure breaks is not required. Our simulation results demonstrate significant power gains over the FLM tests in the presence of non-normal errors.

DEDICATION

This dissertation is dedicated to those who wholeheartedly supported and guided me, especially to my parents and my wife.

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INTRODUCTION

The main focus of this dissertation is to find ways to improve the power of unit root tests. The issue of whether an economic time series is best characterized by either a unit root or a stationary process has assumed great importance in both the theoretical and the applied time series econometrics literature. As a consequence, tests of the null hypothesis that a series is integrated of order one, I(1), against the alternative hypothesis that it is integrated of order zero, I(0), have received much attention. It is well known that traditional unit root tests have poor power, or in other words, the capability of traditional unit root tests to distinguish in finite samples the unit root null from nearby stationary alternatives is low. As such, the development of more powerful unit root tests has not been a trivial concern. In a seminal paper, Perron (1989) first documented that failure to account for a structural break can result in the standard Dickey-Fuller (DF) test lose power significantly. To provide a remedy, Perron suggested a modified DF unit root test that includes dummy variables to control for known structural breaks in the level and slopes. Perron's study has three assumptions that are hardly satisfied in empirical researches, which includes: (i) the structural breaks are assumed to be known a priori, (ii) the structural breaks are assumed to be steep or abrupt, and (iii) the error term is assumed to follow a standard normal distribution, respectively. Subsequent papers further modified unit root tests have been developed to relax one or more of these assumptions. See Zivot and Andrews (1992), Lumsdaine and Papell (1997), Perron (1997), Vogelsang and Perron (1998), and Lee and Strazicich (2003, 2004), among others.

In the first essay, we extends the Lagrange Multiplier (LM) unit toot tests of Schmidt and

Phillips (1992) to utilize information contained in non-normal errors. The new tests adopt the Residual Augmented Least Squares (RALS) estimation procedure of Im and Schmidt (2008). This essay complements the work of Im, Lee and Tieslau (2012) who adopt the RALS procedure for DF-based tests. We refer them as the RALS-LM tests. This essay provides the relevant asymptotic distribution and the corresponding critical values of the new tests. The RALS-LM tests show improved power over the RALS-DF tests, and the power of both RALS-DF and RALS-LM tests can increase dramatically when the error term is highly asymmetric or has fat-tails with unknown forms of non-normal distributions. Moreover, the main advantage of the RALS-LM tests lies in the invariance feature that the distribution does not depend on the nuisance parameter in the presence of multiple level-breaks, and it is expected that they can be more useful in extended models with other types of structural changes.

The second essay tests the Prebisch-Singer hypothesis by re-examining paths of primary commodity prices which are known to exhibit multiple structural breaks. In order to examine the issue more properly, we first suggest new unit root tests that can allow for structural breaks in both the intercept and the slope. In particular, we adopt a procedure to make the resulting test free of the nuisance parameter problem which trend-shifts induce otherwise. Then, we adopt the RALS procedure to gain much improved power when the error term follows a non-normal distribution. Since the suggested test is more powerful and free of nuisance parameters, rejection of the null can be considered as more accurate evidence of stationarity. We apply the new test on the recently extended Grilli and Yang index of 24 commodity series from 1900 to 2007. The main findings of this study reveal that 21 out of the 24 commodity prices are found to be stationary around a broken trend, implying that shocks to these commodities tend to be transitory. Only three relative commodity price series are found to be difference stationary. The empirical findings provide significant evidence to support that primary commodity prices are stationary with one or two trend breaks. In our trend analysis, we find that only 7 series in which the relative commodity prices display negative trend more than 50% of the time period examined; however, 8 relative commodity prices display no significant positive or negative trend in more than 90% of the time period examined. Compared with past studies, our findings provide even weaker evidence to support the Prebisch-Singer hypothesis.

The third essay extends the Fourier Lagrange Multiplier (FLM) unit root tests of Enders and Lee (2012a) by using the RALS estimation procedure of Im and Schmidt (2008). While the FLM type of tests can be used to control for smooth structural breaks of an unknown functional form, the RALS procedure can utilize additional higher-moment information contained in non-normal errors. For these new tests, knowledge of the underlying type of non-normal distribution of the error term or the precise functional form of the structure breaks is not required. By using parsimonious number of parameters to control for possible breaks in the deterministic term and use the additional information lied in nonlinear moments of the error term, the power of RALS-F-LM test increase dramatically when the error term follows a non-normal distribution.

CHAPTER 1

MORE POWERFUL LM UNIT ROOT TESTS WITH NON-NORMAL ERRORS

A recent paper of Im, Lee and Tieslau (2012) adopts the Residual Augmented Least Squares (RALS) estimation procedure of Im and Schmidt (2008) in order to improve the power of the traditional Dickey-Fuller (1979, DF) unit root tests. We refer to this test as the RALS-DF unit root test since it is an extension of the traditional DF test. The RALS procedure utilizes the information that exists when the errors in the testing equation exhibit any departures from normality, such as non-linearity, asymmetry, or fat-tailed distributions. The underlying idea of the RALS procedure is appealing because it is intuitive and easy to implement. If the errors are non-normal, the higher moments of the residuals contain the information on the nature of the non-normality. The RALS procedure conveniently utilizes these moments in a linear testing equation without the need for a priori information on the nature of the non-normality, such as the density function or the precise functional form of any non-linearity. The power gain over the usual DF tests is considerable when the error term is asymmetric or has a fat-tailed distribution.

This essay extends the work of Im, Lee and Tieslau (2012), and considers the Lagrange Multiplier (LM) version of the RALS unit root tests. We refer to them as the RALS-LM tests. We provide the relevant asymptotic distribution of these new tests and their corresponding critical values. The LM unit toot tests were initially suggested by Schmidt and Phillips (1992, SP). To begin with, consider an unobserved components model,

$$y_t = \psi + \xi t + x_t, \quad x_t = \beta x_{t-1} + e_t \tag{1.0.1}$$

The unit root null hypothesis implies $\beta = 1$, against the alternative that $\beta < 1$. Here, the parameters ψ and ξ will denote level and deterministic trend, respectively, regardless of whether y_t contains a unit root (β = 1) or not. The key difference between the LM and the DF procedures is found in the detrending method. For the LM version tests, the coefficients of the deterministic trend components are estimated from the regression in differences of Δy_t on Δz_t with $z_t = [1, t]'$. Denoting the maximum likelihood estimates (MLE) from the LM procedure as $\tilde{\psi}$ and $\tilde{\xi}$, SP (1992) suggest using the detrended form of y_t,

$$\tilde{y}_t = y_t - \tilde{\psi} - \tilde{\xi} t. \tag{1.0.2}$$

On the other hand, the DF test is based on the estimates of the coefficients from the regression of y_t in levels on $(1, t)$.[1] The LM tests show improved power over the DF tests. This essay shows that the same feature will carry-over to the RALS version tests; the RALS-LM tests show improved power over the RALS-DF tests.

The main advantage of the LM tests of SP (1992) is that they are less sensitive to the parameters related to structural changes. In particular, they are free of nuisance parameters in models with level shift, as we will explain in more detail in the next section. However, the DF version tests do not have this property. As such, there are operating advantages of using the LM version of the unit root test for models with structural changes, and the same feature can be

[1] We note that the GLS tests of Hwang and Schmidt (1996), and the DF-GLS tests of Elliott, Rothenberg, and Stock (1996) adopt a detrending method similar to that of the LM test. For the GLS tests, the coefficients of the deterministic trend components are estimated from the regression in quasi-differences of Δy_t^* (= $y_t - (1 - c/T)y_{t-1}$) on Δz_t^* (= $z_t - (1 - c/T)z_{t-1}$), where c is a nuisance parameter that takes on some small value. The GLS tests of Hwang and Schmidt (1996) use a fixed value c/T which is given a priori as a small value, such that c/T = 0.02 and $\Delta y_t^* = y_t - 0.98 y_{t-1}$. The DF-GLS tests search for the optimal small value of c/T that maximizes the power under the local alternative. When c/T is zero, these GLS-based tests are identical to the LM tests of SP (1992). In reality, the difference in the power of the LM tests and the GLS tests is not significant. The main source of the power gain for the GLS tests is its use of the LM type detrending procedure, although searching for the optimal value of c/T can lead to a marginal improvement in power.

utilized in the RALS-LM tests with level-shifts, although it would be difficult to consider the RALS-DF tests with breaks.

The remainder of the essay is organized as follows. In Section 1.1, we discuss the LM procedure and propose the RALS-LM tests. In Section 1.2, we examine the size and power properties and compare them with those of the RALS-DF tests. Section 1.3 provides concluding remarks.

1.1. LM and RALS-LM Tests

We are interested in testing the unit root null hypothesis $H_0 : \beta = 1$ against the stationary alternative hypothesis $H_a : \beta < 1$. We let z_t denote the deterministic terms, including structural changes, and rewrite the data generating process (DGP) in (1.0.1) as:

$$y_t = z_t'\delta + x_t, \quad x_t = \beta x_{t-1} + e_t. \tag{1.1.1}$$

For example, if $z_t = [1, t]'$, we have the usual no-break LM test of SP (1992). To consider a model with level shift where a break occurs at $t = T_B$, we may add a dummy variable, D_t, where $D_t = 1$ if $t \geq T_B + 1$ and $D_t = 0$ if $t \leq T_B$. Then, we have

$$y_t = \psi + \xi t + dD_t + x_t, \quad x_t = \beta x_{t-1} + e_t. \tag{1.1.2}$$

Again, the parameter d is estimated from the regression in differences of Δy_t on Δz_t,where $z_t = [1, t, D_t]'$. Here, the estimated value of d will denote the magnitude of the level shift in a consistent manner, regardless of whether y_t contains a unit root or not. We do not need to assume that $d = 0$ under the null, and the critical values of the test will not change for different values of d. More importantly, the LM tests will not depend on the nuisance parameter, $\lambda(= T_B/T)$, which denotes the location of the break, as shown in Amsler and Lee (1995). As such, the same critical values of the usual LM test (without breaks) can be used even in the presence of multiple level shifts. By contrast, Perron's (1989) tests with level shifts depend on λ, and different critical values need to be obtained for all different combinations of the break locations in the case of multiple level shifts.

While the dependency on λ might be matter of minor inconvenience for the exogenous tests of Perron (1989), the issue becomes complicated in the case of endogenous-break unit root tests for which the location of the break is determined from the data where the t-statistic on the unit root hypothesis is minimized, or the F-statistic on the dummy coefficients is maximized. The popular endogenous-break unit root tests based on the DF version models can exhibit spurious rejections unless the parameters in d, which denote the magnitude of the structural breaks (either in level-shifts or trend-shifts), take on zero values. Such an approach leads to a conceptual difficulty of not allowing for breaks under the null of a unit root. Therefore, rejections of the unit root null hypothesis will not necessarily imply trend-stationarity since the possibility of a unit root with break(s) still remains; see Nunes *et al*. (1997), and Lee and Strazicich (2001), among others for details. The LM-based tests, on the other hand, are free of this problem in models with level-shift. In light of this, the LM tests with two endogenous breaks are considered in Lee and Strazicich (2003) who allow for breaks both under the null and alternative hypotheses in a consistent manner.

It also is possible to allow for multiple breaks by employing additional dummy variables for multiple level shifts with $z_t =[1, t, D_{1t}, ..., D_{Rt}]$, where $D_{jt} = 1$ for $t \geq T_{Bj} + 1$, $j = 1, ..., R$, and zero otherwise; R is the number of structural breaks. The invariance feature of the LM tests with level-shifts is useful for extending the univariate LM tests to a panel setting. To that end, Im, Lee and Tieslau (2005) suggest panel LM unit root tests with level breaks. Without the invariance feature of the LM tests, the panel test statistic would not be feasible since the test would depend on the nuisance parameters indicating the location of the breaks. This is particularly true when each cross-section unit is likely to experience a different number and location of breaks.

This essay shows that the same invariance feature in the LM tests also will hold in the RALS-LM tests with level-shifts. In this case, the RALS-LM tests with multiple level-shifts will have the same distribution as the RALS-LM tests without breaks.[2] This is one main advantage of the RALS-LM tests. Without the invariance feature of the LM tests, it would be difficult to construct valid critical values for RALS-based tests, since the RALS procedure will induce an additional nuisance parameter. Thus, it would be extremely difficult to construct valid RALS-DF tests with breaks.

We now explain details of the RALS-LM procedure. In general, following the LM (score) principle, the LM unit root test statistic can be obtained from the following regression:

$$\Delta y_t = \delta' \Delta z_t + \phi \widetilde{y}_{t-1} + e_t, \tag{1.1.3}$$

where $\widetilde{y}_t = y_t - \widetilde{\psi} - z_t \widetilde{\delta}$, $t = 2, ..., T$; $\widetilde{\delta}$ is the vector of coefficients in the regression of Δy_t on ΔZ_t, and $\widetilde{\psi}$ is the restricted MLE of ψ given by $y_1 - z_1 \widetilde{\delta}$; and, y_1 and z_1 denote the first observation of y_t and Z_t, respectively. To control for autocorrelated errors, one can include the terms $\Delta \widetilde{y}_{t-j}$, $j = 1, ..., p$ in (1.1.3) , and the testing regression is given as:

$$\Delta y_t = \delta' \Delta z_t + \phi \widetilde{y}_{t-1} + \sum_{j=1}^{p} c_j \Delta \widetilde{y}_{t-j} + e_t. \tag{1.1.4}$$

[2] The invariance property of the LM tests does not hold in models with trend-shifts where $z_t = [1, t, D_t, tD_t]'$ is used. However, the LM based tests are much less sensitive to the parameters of trend-breaks than the DF version tests. For example, Nunes (2004) found that the critical values do not change much in the models with trend-shifts and considered a method using the same critical values regardless of different values of λ. However, the LM tests still depend on the nuisance parameter in these models, and using the same critical values can lead to mild size distortions.

Then, the LM test statistic is given by:

$$\tau_{LM} = t\text{-statistic testing the null hypothesis } \phi = 0.$$

Next, we explain how to utilize the information on non-normal errors in order to improve upon the power of the unit root test, making use of the RALS estimation procedure as suggested in Im and Schmidt (2008), and Im, Lee and Tieslau (2012). To begin with, we define $\xi_t = (\Delta\widetilde{y}_{t-1}, \Delta\widetilde{y}_{t-2}, ..., \Delta\widetilde{y}_{t-p})'$, $f_t = (\widetilde{y}_{t-1}, \xi_t')'$ and $F_t = (\Delta z_t', f_t')'$. Suppose we have the following moment conditions:

$$E[g(e_t) \otimes F_t] = 0, t = 1, 2, ..., T, \tag{1.1.5}$$

where $g(e_t)$ is a function defined as $g(e_t) = (e_t, [h(e_t) - K]')'$ with $K = E(e_t)$, and $h(e_t)$ is a nonlinear function of the error term e_t. Then the moment condition becomes:

$$E[e_t \otimes F_t] = 0, \tag{1.1.6}$$

$$E[(h(e_t) - K) \otimes F_t] = 0. \tag{1.1.7}$$

The first part is the usual moment condition of least squares estimation and the second part involves an additional moment conditions based on nonlinear functions of e_t. We let $\hat{e}_t$ denote the residuals from the usual LM regression (1.1.4). Following Im and Schmidt (2008), we define the following term:

$$\hat{w}_t = h(\hat{e}_t) - \hat{K} - \hat{e}_t\hat{D}_2, \tag{1.1.8}$$

where $h(\hat{e}_t) = [\hat{e}_t^2, \hat{e}_t^3]'$, $\hat{K} = \frac{1}{T}\sum_{t=1}^{T} h(\hat{e}_t)$, and $\hat{D}_2 = \frac{1}{T}\sum_{t=1}^{T} h'(\hat{e}_t)$. Using $m_j = T^{-1}\sum_{t=1}^{T} \hat{e}_t^j$, we define the augmented terms:

$$\hat{w}_t = [\hat{e}_t^2 - m_2, \hat{e}_t^3 - m_3 - 3m_2\hat{e}_t]'. \tag{1.1.9}$$

The RALS-LM procedure involves augmenting the testing regression (1.1.4) with $\hat{w}_t$. The first term in $\hat{w}_t$ is associated with the moment condition $E[(e_t^2 - \sigma_e^2)\widetilde{S}_{t-1}] = 0$, which is the condition of no heteroskedasticity. This condition improves the efficiency of the estimator of ϕ when the error terms are not symmetric. The second term in $\hat{w}_t$ improves efficiency unless $m_4 = 3\sigma^4$. It is possible to use higher moments using $h(\hat{e}_t) = [\hat{e}_t^2, \hat{e}_t^3, \hat{e}_t^4, .., \hat{e}_t^k]'$ with $k > 3$, and the properly defined $\hat{w}_t$ in (1.1.9) that corresponds to the higher moments. The additional efficiency gain is expected, unless $m_{k+1} = k\sigma^2 m_{k-1}$ which holds only for the normal distribution.[3] Thus, when the distribution of the error term is not normal, one may increase efficiency by augmenting the testing regression with $\hat{w}_t$, as follows:

$$\Delta y_t = \delta' \Delta z_t + \phi \widetilde{y}_{t-1} + \sum_{j=1}^{p} g_j \Delta \widetilde{y}_{t-j} + \hat{w}_t' \gamma + e_t. \tag{1.1.10}$$

The RALS-LM statistic is obtained through the usual least squares estimation procedure applied to (1.1.10). We denote the corresponding t-statistic for $\phi = 0$ as τ_{RLM}. We adopt Assumption 1 and Assumption 2 of Im, Lee and Tieslau (2012) for the error term e_t, and $g(e_t)$ in (1.1.5), respectively. Then it can be shown that the asymptotic distribution of $\tau_{RALS-LM}$ is given as follows.

Lemma 1. *Suppose that we consider the usual t-statistic on $\phi = 0$ in equation (1.1.10). Then, under the null, the limiting distribution of the RALS-LM t-statistic $\tau_{RALS-LM}$ can be derived as*

$$\tau_{RALS-LM} \rightarrow \rho \tau_{LM} + \sqrt{1-\rho^2} N(0,1), \tag{1.1.11}$$

where τ_{LM} denotes the limiting distribution of the t-statistic for the usual LM estimator in regression (1.1.4), and ρ is the correlation between e_t and $\psi(e_t)$

$$\rho = \frac{\sigma_{\psi e}}{\sigma_\psi \sigma_e}, \tag{1.1.12}$$

[3] However, we do not pursue this direction further and leave it as future research. This extension requires the assumption that the higher moments exist. In any case, the power gain is already significant enough when using the augmented terms in (1.1.9).

where $\psi(e_t) = D'C^{-1}g(e_t)$, $\sigma_\psi^2 = Var[\psi(e_t)] = Var[D'C^{-1}g(e_t)] = D'C^{-1}D$, $\sigma_{\psi e} = E[\psi(e_t)e_t]$ $= DC^{-1}E[g(e_t)e_t]$, $C = E[g(e_t)g(e_t)']$, and $D = E[g'(e_t)]$.

PROOF:

Consider the regression (1.1.10). We let $\hat{\zeta}_t = (\widetilde{\xi}_t', \hat{w}_t')'$, where $\widetilde{\xi}_t = \xi_t - T^{-1}\sum_{t=1}^T \xi_t$, and $\xi_t = (\Delta\widetilde{y}_{t-1}, \Delta\widetilde{y}_{t-2}, ..., \Delta\widetilde{y}_{t-p})'$. Following Theorem 2 in Hansen (1995), we have

$$T(\hat{\phi} - \phi) = \frac{T^{-1}(\sum_{t=2}^T \widetilde{y}_{t-1}^* e_t - \sum_{t=2}^T \widetilde{y}_{t-1}^* \widetilde{\zeta}_t' (\sum_{t=2}^T \widetilde{\zeta}_t \widetilde{\zeta}_t')^{-1} \sum_{t=2}^T \widetilde{\zeta}_t e_t)}{T^{-2}(\sum_{t=2}^T \widetilde{y}_{t-1}^{*2} - \sum_{t=2}^T \widetilde{y}_{t-1}^* \widetilde{\zeta}_t' (\sum_{t=2}^T \widetilde{\zeta}_t \widetilde{\zeta}_t')^{-1} \sum_{t=2}^T \zeta_t \widetilde{S}_{t-1}^*)},$$

From the moment condition $T^{-1}\sum_{t=1}^T \hat{w}_t \xi_t' = o_p(1)$, and $T^{-1}\sum_{t=1}^T \widetilde{\xi}_t e_t = o_p(1)$, we have

$$T(\hat{\phi} - \phi) = \frac{T^{-1}(\sum_{t=2}^T \widetilde{y}_{t-1}^* e_t - \sum_{t=2}^T \widetilde{y}_{t-1}^* \hat{w}_t' (\sum_{t=2}^T \hat{w}_t \hat{w}_t')^{-1} \sum_{t=2}^T \hat{w}_t' e_t)}{T^{-2}(\sum_{t=2}^T \widetilde{y}_{t-1}^{*2})} + o_p(1).$$

Since $1/T\sum_{t=2}^T \widetilde{y}_{t-1}^* \hat{w}_t = O_p(1)$, $1/T\sum_{t=2}^T \hat{w}_t \hat{w}_t' \to_p M > 0$, $1/T\sum_{t=2}^T \hat{w}_t e_t = o_p(1)$, then we have

$$T(\hat{\phi} - \phi) = \frac{T^{-1}\sum_{t=2}^T \widetilde{y}_{t-1}^* e_t}{T^{-2}(\sum_{t=2}^T \widetilde{y}_{t-1}^{*2})} + o_p(1).$$

Apply the lemma from Hansen (1995),

$$T(\hat{\phi} - \phi) = a(1)R\left(\rho\frac{\int_0^1 W_1^c dW_1}{\int_0^1 (W_1^c)^2} + (1-\rho^2)^{1/2}\frac{\int_0^1 W_1^c dW_2}{\int_0^1 (W_1^c)^2}\right),$$

where $R = \sigma_u/\sigma_e$. Additionally, we have

$$\begin{aligned} t(\hat{\phi}) &= \hat{\psi}\sigma_u^{-1}T(1/T^2\sum_{t=2}^T \widetilde{y}_{t-1}^{*2} - 1/T^2\sum_{t=2}^T \widetilde{y}_{t-1}^* \hat{\zeta}_t' (\sum_{t=2}^T \hat{\zeta}_t \hat{\zeta}_t')^{-1} \sum_{t=2}^T \hat{\zeta}_t \widetilde{y}_{t-1}^*)^{1/2} \\ &= T\hat{\phi}\sigma_u^{-1}(1/T^2\sum_{t=2}^T \widetilde{y}_{t-1}^{*2} - 1/T^2\sum_{t=2}^T \widetilde{y}_{t-1}^* \hat{w}_t' (\sum_{t=2}^T \hat{w}_t \hat{w}_t')^{-1} \sum_{t=2}^T \hat{w}_t \widetilde{y}_{t-1}^*)^{1/2} \\ &= \sigma_u^{-1}(1/T^2\sum_{t=2}^T \widetilde{y}_{t-1}^{*2})^{1/2} T\hat{\phi} + o_p(1), \end{aligned}$$

and $T\phi = -ca(1)$. The test statistics under the null of $\phi = 0$ can be obtained as

$$t(\hat{\phi}) = \sigma_u^{-1}\left(a(1)^{-2}\sigma_e^2\int_0^1 (W_1^c)^2\right)^{1/2}\cdot\left[-ca(1)+a(1)R(\rho\frac{\int_0^1 W_1^c dW_1}{\int_0^1 (W_1^c)^2}+(1-\rho^2)^{1/2}\frac{\int_0^1 W_1^c dW_2}{\int_0^1 (W_1^c)^2})\right]$$
$$= -\frac{c}{R}\left(\int_0^1 (W_1^c)^2\right)^(1/2)+\rho\frac{\int_0^1 W_1^c dW_1}{((W_1^c)^2)^{1/2}}+(1-\rho^2)\frac{\int_0^1 W_1^c dW_2}{((W_1^c)^2)^{1/2}}$$

The null holds when $c = 0$. Then we obtain

$$t(\hat{\phi}) = \rho t_{\phi} + \sqrt{(1-\rho^2)}N(0,1).$$

□

These results are essentially similar to those given in Im, Lee and Tieslau (2012). Also, the RALS-LM tests are asymptotically identical to the GMM estimators using the same moment conditions in (1.1.6) and (1.1.7). It is interesting to see that the limiting distribution of τ_{LM} is similar to that of the unit root tests with stationary covariates, as advocated by Hansen (1995).[4] The difference is how the parameter ρ^2 is estimated. We have a special case of Hansen's models and ρ^2 can be estimated by:

$$\hat{\rho}^2 = \hat{\sigma}_A^2/\hat{\sigma}^2, \tag{1.1.13}$$

where $\hat{\sigma}^2$ is the usual estimate of the error variance in the LM regression (1.1.4), and $\hat{\sigma}_A^2$ is the estimate of the error variance in the RALS-LM regression in (1.1.10).

Note that the asymptotic distribution of the RALS-LM test statistic $\tau_{RALS-LM}$ does not depend on the break location parameter λ_j in the model with level-shifts, following the results of Amsler and Lee (1995). Thus, we do not need to simulate new critical values, regardless of the number of level-shifts and all possible different combinations of break locations. From a

[4] A similar asymptotic result also is advocated in Guo and Phillips (1998, 2001).

practical perspective, it likely would be infeasible to obtain all possible different critical values corresponding to different break locations and values of ρ^2. For a finite number of level-shifts, we only need one set of critical values since they are asymptotically invariant to both the break magnitude and location.

Note that when $\rho^2 = 1$, we have $\tau_{RALS-LM} = \tau_{LM}$, so that the critical value for the usual LM test can be used. In Table 1.1, we report the asymptotic critical values of the RALS-LM tests, for different values of $\rho^2 = 0, 0.1, ..., 1.0$ and T = 50, 100, 300 and 1000, respectively. All of these critical values are obtained via Monte Carlo simulations using 100,000 replications. These critical values can be used even when multiple level breaks occur in the data. To see this we provide the empirical critical values of the RALS-LM tests when the number of level-shifts is 1 and 2. The results in Table 1.2 are virtually identical to the critical values of the RALS-LM tests without breaks, as reported in Table 1.1.

1.2. Simulations

In this section, we investigate the finite small sample properties of the RALS-LM unit root tests. Our goal is to verify the theoretical results presented above and examine the performance of the tests. Pseudo-iid $N(0,1)$ random numbers were generated using the RATS procedure $\%RAN(1)$ and all results were obtained via simulations in WinRATS 7.2. The DGP was given in (1.1.2), and the initial values x_0 and e_0 are assumed to be random with mean zero and variance 1. In order to examine the power when non-normal error exists, we consider seven types of non-normal errors which include: (i) a chi-square distribution with $df = 1, 2, 3, 4$, and (ii) a t-distribution with $df = 2, 3, 4$. For purposes of comparison, we also examined the case when the error term follows a standard normal distribution. The size and power property are examined with two different DGPs; (a) no break with $z_t = [1, t]'$ and $\delta' = (0, 1)$, and (b) one level shift with $z_t = [1, t, D_t]'$ and $\delta' = (0, 1, 5)$. We also let $\lambda = T_B/T$ denote the fraction of the series before the break occurs at $t = T_B + 1$. For all of these cases, we have used the same critical values in Table 1.1 of the usual RALS-LM tests without breaks. All simulation results are calculated using 10,000 replications for the sample size, $T = 100$ by using the 5% significance level.[5]

In Table 1.3, we report the size and power properties of the RALS-LM tests, and compare them with the usual LM tests, the DF tests and the RALS-DF tests. We begin by examining the model with no breaks. From Panel A in Table 1.3, we observe that, in all cases, none of the three tests shows any serious size distortions. The RALS-LM tests show significantly improved power over the usual LM tests when the errors are non-normal with either a chi-square or t-distribution. The results in Panel C for the size-adjusted power are more relevant. The gain in power of the RALS-LM tests is greater when the degrees of freedom of the chi-square distribution is smaller, implying more asymmetric patterns of the error distribution. For example, when $\beta = 0.9$ and the error term follows a $\chi^2(1)$ distribution, the size-adjusted power of the RALS-LM test is 0.976, while the power of the usual LM test is 0.294 (and 0.187 for the DF test). Also, the gain is larger when the degrees of freedom of the t-distribution is smaller, implying fatter-tails of the error term.[6] In Figure 1.1, we have provided a graph to show these results.

[5] Results for the larger sample sizes with $T = 300$ and 1,000 are omitted. They show a similar pattern with greatly improved power properties. They are available with upon request.

[6] The question of interest is the effect of using the estimated values of $\hat{\rho}^2 = \hat{\sigma}_A^2/\hat{\sigma}^2$ in (1.1.13) on the size and power property of the tests. The true value of ρ^2 is unknown, but it depends on the type and degree of non-normal errors. It seems clear that the size property is fair in all cases that we examined. The power gain would be larger when the value of ρ^2 is small. This occurs when the degrees of freedom of the chi-square distribution is smaller, implying more asymmetric patterns, and when the degrees of freedom of the t-distribution is smaller, implying fatter-tails. Our simulation results are consistent with our expectations.

When the error term follows a normal distribution, the LM tests are more powerful than the RALS-LM tests. However, the difference in power is rather small. These results prove that the RALS-LM tests have good size and power properties even when the sample size is relatively small. In Panel D of Table 1.3, we report the size and size-adjusted power of the RALS-DF tests of Im, Lee and Tieslau (2012). It seems clear that the RALS-LM tests are generally more powerful than the RALS-DF tests. The difference is as expected since the LM tests are usually more powerful than the DF tests.

Next, we examine the property of these tests when the DGP includes a structural break where the size and power properties are examined for $\lambda = 0.25$ and $\lambda = 0.5$. In this case, it is not useful to consider the RALS-DF test since the distribution of the test depends on λ. The results for the RALS-LM and traditional LM tests are presented in Table 1.4. Note that we use the same critical values for the traditional LM tests without breaks and the RALS-LM tests without breaks. Again, we do not observe any significant size distortions under the null, even when the critical values of the tests without breaks are used for the models with breaks. Regardless of the locations of breaks with either $\lambda = 0.25$, or $\lambda = 0.5$, the results on the size, power and size-adjusted power do not change appreciably. This outcome clearly shows the invariance results for both the LM and the RALS-LM tests. Also, we observe significant power gains for the RALS-LM tests when the error term follows a non-normal distribution.

1.3. Concluding Remarks

This essay develops new RALS based LM unit root tests. These new RALS-LM tests show improved power gains over the corresponding RALS-DF tests, and the power of both RALS-DF and RALS-LM tests can increase drastically when the error term is highly asymmetric or has fat-tails with unknown forms of non-normal distributions. Also, the RALS-LM tests have the feature that they are invariant to the nuisance parameter in the models with multiple level shifts, and it is expected that they can be more useful in extended models with other types of structural changes.

Overall, we conclude that the RALS-LM tests show improved performance over the corresponding RALS-DF tests. However, we should note that the power gain of the LM version tests (and also the DF-GLS version tests) will disappear when the initial value is large. In such cases, the RALS-DF version tests are more powerful than the RALS-LM version tests. As such, one may consider a fair balance between the RALS-LM and the RALS-DF tests in the presence of non-normal errors; it is incorrect to say that one version would dominate uniformly over the other. Clearly, the main advantage of the RALS-LM tests lies in the invariance feature that the distribution does not depend on the nuisance parameter in the presence of level-breaks, and that they are less sensitive in other extended break models. On the other hand, the RALS-DF version tests can be more useful in standard models without breaks, especially when the initial value is large.

Table 1.1 Critical Values of RALS-LM Test with No Break

T	%	ρ^2										
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
50	1	−2.333	−2.882	−3.089	−3.213	−3.314	−3.401	−3.477	−3.543	−3.599	−3.650	−3.707
	5	−1.640	−2.216	−2.426	−2.572	−2.689	−2.783	−2.862	−2.928	−2.992	−3.045	−3.089
	10	−1.286	−1.864	−2.081	−2.235	−2.358	−2.459	−2.545	−2.619	−2.684	−2.740	−2.793
100	1	−2.322	−2.875	−3.069	−3.200	−3.301	−3.378	−3.446	−3.499	−3.551	−3.595	−3.625
	5	−1.646	−2.216	−2.422	−2.562	−2.672	−2.764	−2.840	−2.903	−2.958	−3.010	−3.062
	10	−1.283	−1.861	−2.075	−2.230	−2.352	−2.450	−2.535	−2.606	−2.665	−2.719	−2.772
300	1	−2.332	−2.874	−3.057	−3.175	−3.265	−3.347	−3.403	−3.455	−3.509	−3.553	−3.596
	5	−1.642	−2.211	−2.413	−2.554	−2.664	−2.755	−2.830	−2.893	−2.944	−2.992	−3.028
	10	−1.276	−1.856	−2.071	−2.231	−2.351	−2.450	−2.532	−2.603	−2.664	−2.711	−2.758
1000	1	−2.345	−2.892	−3.080	−3.205	−3.299	−3.374	−3.428	−3.474	−3.510	−3.538	−3.570
	5	−1.652	−2.223	−2.428	−2.568	−2.677	−2.761	−2.836	−2.897	−2.947	−2.990	−3.031
	10	−1.291	−1.871	−2.083	−2.234	−2.352	−2.451	−2.535	−2.605	−2.667	−2.715	−2.755

Table 1.2 Critical Values of RALS-LM Test with Level Break

T	R	%	ρ^2 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
50	1	1	−2.333	−2.886	−3.077	−3.208	−3.309	−3.398	−3.478	−3.543	−3.605	−3.653	−3.711
		5	−1.640	−2.216	−2.422	−2.570	−2.688	−2.784	−2.862	−2.932	−2.990	−3.044	−3.094
		10	−1.286	−1.863	−2.081	−2.237	−2.358	−2.461	−2.546	−2.619	−2.683	−2.742	−2.794
	2	1	−2.333	−2.891	−3.078	−3.209	−3.309	−3.398	−3.467	−3.532	−3.587	−3.646	−3.695
		5	−1.640	−2.220	−2.427	−2.575	−2.696	−2.791	−2.867	−2.933	−2.991	−3.044	−3.100
		10	−1.286	−1.866	−2.080	−2.235	−2.358	−2.460	−2.546	−2.622	−2.687	−2.743	−2.794
100	1	1	−2.322	−2.876	−3.066	−3.203	−3.307	−3.383	−3.456	−3.515	−3.560	−3.609	−3.637
		5	−1.646	−2.218	−2.421	−2.563	−2.675	−2.761	−2.839	−2.903	−2.963	−3.011	−3.062
		10	−1.283	−1.861	−2.076	−2.228	−2.351	−2.451	−2.535	−2.607	−2.667	−2.720	−2.771
	2	1	−2.322	−2.877	−3.066	−3.204	−3.295	−3.372	−3.444	−3.497	−3.554	−3.583	−3.636
		5	−1.646	−2.220	−2.422	−2.566	−2.677	−2.767	−2.842	−2.908	−2.962	−3.013	−3.056
		10	−1.283	−1.862	−2.076	−2.231	−2.349	−2.450	−2.533	−2.604	−2.662	−2.717	−2.771
300	1	1	−2.332	−2.875	−3.055	−3.175	−3.270	−3.344	−3.402	−3.458	−3.505	−3.545	−3.588
		5	−1.642	−2.213	−2.413	−2.557	−2.666	−2.757	−2.833	−2.892	−2.944	−2.991	−3.028
		10	−1.276	−1.856	−2.070	−2.229	−2.349	−2.450	−2.531	−2.601	−2.660	−2.711	−2.757
	2	1	−2.332	−2.875	−3.057	−3.175	−3.273	−3.353	−3.410	−3.457	−3.506	−3.554	−3.593
		5	−1.642	−2.212	−2.414	−2.556	−2.664	−2.754	−2.831	−2.894	−2.945	−2.990	−3.023
		10	−1.276	−1.856	−2.072	−2.229	−2.349	−2.449	−2.532	−2.604	−2.664	−2.709	−2.756
1000	1	1	−2.345	−2.895	−3.080	−3.204	−3.296	−3.376	−3.427	−3.470	−3.504	−3.536	−3.565
		5	−1.652	−2.223	−2.428	−2.570	−2.677	−2.763	−2.836	−2.896	−2.948	−2.993	−3.029
		10	−1.291	−1.870	−2.082	−2.234	−2.352	−2.452	−2.534	−2.607	−2.667	−2.715	−2.755
	2	1	−2.345	−2.895	−3.077	−3.202	−3.300	−3.375	−3.427	−3.470	−3.510	−3.544	−3.571
		5	−1.652	−2.221	−2.427	−2.567	−2.675	−2.762	−2.836	−2.897	−2.950	−2.990	−3.031
		10	−1.291	−1.870	−2.082	−2.234	−2.353	−2.452	−2.536	−2.606	−2.665	−2.715	−2.756

Note: For R = 1, the break is assumed at the middle. For R = 2, the breaks are assumed at the one third and two thirds.

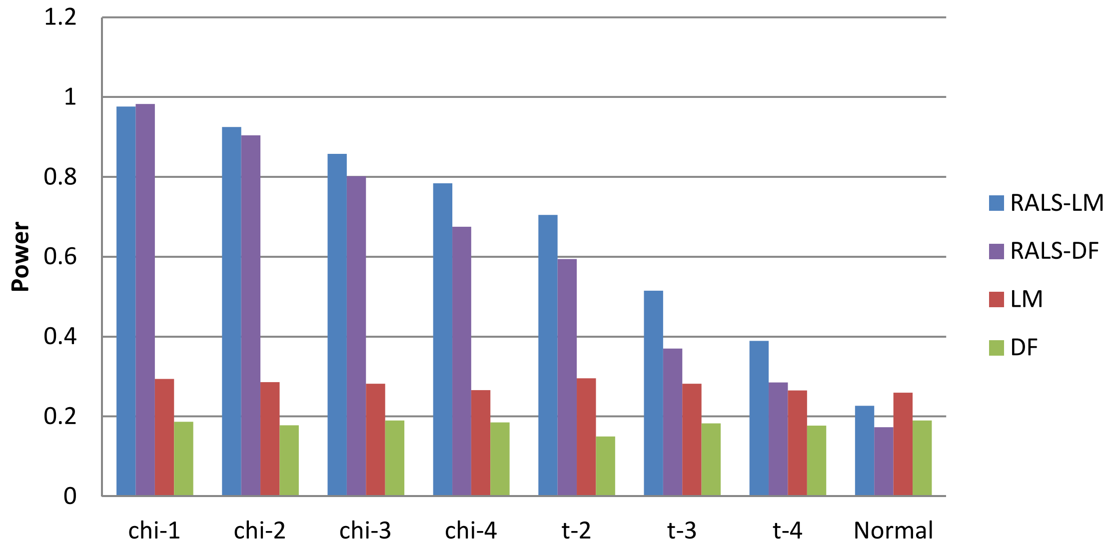
Table 1.3 Size, Power, and Size-Adjusted Power with No Break (T = 100)

β	Tests	Distribution of the Error Term							
		χ_1^2	χ_2^2	χ_3^2	χ_4^2	t_2	t_3	t_4	*N*(0,1)
		Panel A. Size							
1	RALS-LM	0.057	0.059	0.060	0.064	0.045	0.042	0.052	0.086
	LM	0.038	0.042	0.044	0.047	0.037	0.043	0.046	0.052
	DF	0.046	0.051	0.046	0.049	0.053	0.049	0.052	0.051
		Panel B. Power							
0.8	RALS-LM	0.997	0.995	0.991	0.985	0.948	0.887	0.856	0.780
	LM	0.770	0.757	0.763	0.765	0.789	0.755	0.762	0.754
	DF	0.656	0.648	0.654	0.650	0.637	0.651	0.645	0.643
0.9	RALS-LM	0.977	0.929	0.870	0.809	0.664	0.474	0.393	0.341
	LM	0.244	0.254	0.255	0.254	0.236	0.253	0.248	0.268
	DF	0.170	0.181	0.180	0.182	0.161	0.180	0.183	0.193
		Panel C. Size-Adjusted Power							
0.8	RALS-LM	0.998	0.996	0.992	0.985	0.962	0.918	0.864	0.659
	LM	0.817	0.789	0.787	0.776	0.848	0.787	0.779	0.745
	DF	0.688	0.645	0.670	0.654	0.609	0.657	0.635	0.638
0.9	RALS-LM	0.976	0.925	0.858	0.784	0.705	0.515	0.389	0.227
	LM	0.294	0.286	0.282	0.266	0.296	0.282	0.265	0.260
	DF	0.187	0.178	0.190	0.185	0.150	0.183	0.177	0.190
		Panel D. Size and Size-Adjusted Power of RALS-DF Tests							
1.0	RALS-DF	0.049	0.059	0.062	0.068	0.047	0.053	0.056	0.054
0.9	RALS-DF	0.988	0.927	0.831	0.716	0.633	0.384	0.292	0.191

Table 1.4. Size, Power, and Size-Adjusted Power with Level Break (T = 100)

β	λ	Tests	Distribution of the Error Term χ^2_1	χ^2_2	χ^2_3	χ^2_4	t_2	t_3	t_4	*N*(0,1)
			Panel A. Size							
1	0.25	RALS-LM	0.056	0.054	0.057	0.062	0.046	0.044	0.051	0.083
		LM	0.040	0.046	0.045	0.046	0.037	0.043	0.047	0.053
	0.50	RALS-LM	0.055	0.056	0.057	0.062	0.046	0.041	0.051	0.083
		LM	0.040	0.045	0.043	0.047	0.035	0.041	0.049	0.052
			Panel B. Power							
0.8	0.25	RALS-LM	0.996	0.993	0.987	0.979	0.934	0.871	0.833	0.753
		LM	0.743	0.730	0.735	0.733	0.764	0.727	0.732	0.724
	0.50	RALS-LM	0.996	0.993	0.987	0.978	0.934	0.869	0.832	0.755
		LM	0.739	0.727	0.738	0.739	0.761	0.731	0.732	0.726
0.9	0.25	RALS-LM	0.971	0.919	0.851	0.788	0.653	0.463	0.385	0.327
		LM	0.235	0.248	0.251	0.250	0.227	0.247	0.246	0.260
	0.50	RALS-LM	0.971	0.918	0.853	0.786	0.653	0.462	0.389	0.329
		LM	0.235	0.249	0.251	0.249	0.231	0.243	0.246	0.256
			Panel C. Size-Adjusted Power							
0.8	0.25	RALS-LM	0.997	0.994	0.989	0.979	0.951	0.898	0.839	0.643
		LM	0.785	0.748	0.758	0.752	0.822	0.760	0.744	0.715
	0.50	RALS-LM	0.996	0.994	0.988	0.978	0.950	0.901	0.843	0.645
		LM	0.788	0.758	0.768	0.758	0.829	0.770	0.737	0.719
0.9	0.25	RALS-LM	0.971	0.918	0.842	0.766	0.688	0.498	0.379	0.228
		LM	0.281	0.266	0.272	0.266	0.285	0.275	0.257	0.252
	0.50	RALS-LM	0.970	0.915	0.842	0.761	0.689	0.508	0.388	0.235
		LM	0.275	0.273	0.277	0.262	0.293	0.283	0.251	0.250

Figure 1.1 Size-Adjusted Power, T = 100



Note: Size adjusted power of no break tests with β = 0.9.

CHAPTER 2

THE PREBISCH-SINGER HYPOTHESIS AND RELATIVE COMMODITY PRICES: FURTUER EVIDENCE ON BREAKS AND TREND SHIFTS

The Prebisch-Singer hypothesis (PSH) postulates a secular decline in commodity prices relative to manufactured goods in the long-run (Prebisch, 1950; Singer, 1950). The basis for the declining trend includes a low income elasticity of primary commodities, productivity differentials between industrial and commodity producing countries, and asymmetric market structures (oligopolistic rents in manufacturing versus zero profit competition in primary goods). The policy implication associated with the PSH for developing countries, to the extent they export primary commodities and import manufactured goods, is deterioration in their net barter terms of trade. To counter this economic outcome, developing countries may pursue policies of export diversification away from primary commodities or import substitution to increase the domestic production of manufactured goods. Indeed, in the context of development efforts in Africa, Deaton (1999) notes that countries that rely on primary commodity exports need to understand the nature of commodity prices in order to design effective development and macroeconomic policy.

Early studies by Spraos (1980), Sapsford (1985), Thirwall and Bergevin (1985), and Grilli and Yang (1988) of the PSH focused on examining the sign and significance of the estimated trend term based on the assumption that the logarithm of relative commodity prices is stationary, typically around a linear trend, with the results supportive of the PSH. However, this seemingly simple issue is more complicated than it appears due to the possibility of nonstationarity of the data. As such, more recently, the research focus has been on determining whether relative commodity prices are trend stationary (i.e. deterministic trend) or difference stationary (i.e. stochastic trend). The distinction between trend stationary and difference stationary processes is important. If relative commodity prices contain a unit root, the standard method of least squares to test for the presence of a trend will suffer from severe size distortion. On the other hand, if relative commodity prices are generated by a trend stationary process, yet modeled as a difference stationary, the test will be inefficient and will lack power relative to the trend stationary process (Perron and Yabu, 2009). Studies by Bleaney and Greenaway (1993), Newbold and Vougas (1996), and Kim *et al*. (2003) have noticed relative commodity prices may be characterized as difference stationary and the relative commodity prices evolve according to a stochastic trend instead of a deterministic trend, rendering less support for the PSH.

One additional complication is how to deal with structural breaks in examining relative commodity prices. In light of the study by Perron (1989), which demonstrated that if a structural break is ignored the power of unit root tests is lowered, more recent studies by Cuddington and Urzúa (1989), Powell (1991), and Cuddington (1992) introduce structural breaks in testing for the PSH. However, given that break locations are not known, a number of recent studies by Leon and Soto (1997), Zanias (2005), and Kellard and Wohar (2006), utilize unit root tests with allowance for endogenously determined structural breaks.[1] However, Ghoshray (2011) notes the spurious rejection problem of the endogenous unit root tests that most of these studies employed. The spurious rejection problem occurs when endogenous break unit root tests attempt to

[1] Most popular endogenous break unit root tests include Zivot and Andrews (1992), Lumsdaine and Papell (1997), Perron (1997), Vogelsang and Perron (1998), and Lee and Strazicich (2003). Those tests feature searching for the breaks using a grid search. The location of a break is determined at the place where the t-statistic on the unit root hypothesis is minimized or the t-statistic for the break coefficient is maximized.

eliminate the dependency on the nuisance parameter by assuming that breaks are absent under the null. Then, as pointed out by Nunes *et al*. (1997) and Lee and Strazicich (2001), assuming away the nuisance parameter leads to a serious size distortion resulting in spurious rejections under the null hypothesis. The LM test suggested by Lee and Strazicich (2003) allows for structural breaks under the null hypothesis and do not suffer from the spurious rejection of the null hypothesis. As such, Ghoshray (2011) adopts the LM endogenous tests of Lee and Strazicich (2003) to provide different test results; see also Harvey *et al*. (2010), and Kejriwal *et al*. (2012). These studies provide mixed evidence regarding the PSH; however, the studies generally raise doubts as to whether relative commodity prices are best represented by a single downward trend or a shifting trend changing over time.

In this essay, we re-examine the path of relative primary commodity prices. For this, we suggest a new Residual Augmented Least Squares–Lagrange Multiplier (RALS-LM) unit root tests with trend breaks and non-normal errors. The motivation of the new approach is to utilize all possible information to maximize the power of the tests but to make the test free of nuisance parameters. In particular, we try to utilize the information on non-normal errors. Non-normal errors might have been ignored in the literature but they can provide valuable information for improving the power. Specifically, we adopt the LM unit root tests with trend breaks combined with the RALS method to utilize the information contained in non-normal errors. The RALS methodology was initially suggested by Im and Schmidt (2008). The Essay 1 develops the RALS procedure for the LM tests, but those tests do not allow for trend breaks. We showed that the RALS-LM tests gain improved power with non-normal errors and are fairly robust to some forms of non-linearity. However, these tests also lose power when existing trend breaks are not taken into account. In our analysis of the path of relative primary commodity prices, it seems very likely that trend-shifts might have occurred in the data; see Figure 2.1, for example. Thus, it is necessary to develop new tests that allow for possible trend-shifts when the RALS method is adopted. In doing so, we adopt a procedure to make the resulting test free of the nuisance parameter problem which trend-shifts induce otherwise. We will show that the resulting RALS-LM tests with breaks become much more powerful in the presence of non-normal errors.

Based on the discussion mentioned above, it is reasonable to believe the results given by the new RALS-LM tests with trend breaks will provide more accurate information on the path of relative commodity prices. We apply the new tests to the recently extended Grilli and Yang index which contains 24 time series of relative primary commodities prices over the period 1900-2007. We find more commodity price time series are stationary by utilizing possible non-normal errors which have been ignored in previous studies.

The remainder of the study is organized as follows. Section 2.1 discusses the RALS-LM tests with level and trend breaks. In section 2.2, we provide simulation results about the small sample performance of the new test. Section 2.3 presents the data and the empirical results. Concluding remarks are given in Section 2.4.

2.1. Econometric Methodology

In this study, to examine relative commodity prices, we wish to employ the most reliable and powerful tests that utilize all major factors. To begin with, we consider the following data generating process (DGP) based on the unobserved component representation:

$$y_t = \delta' Z_t + e_t, \quad e_t = \beta e_{t-1} + \epsilon_t, \tag{2.1.1}$$

where Z_t contains exogenous variables. The unit root null hypothesis is $\beta = 1$. The level-shift only, or "crash," model can be described by $Z_t = [1, t, D_t]'$, where $D_t = 1$ for $t \geq T_B + 1$ and zero otherwise, and T_B stands for the time period of the break. The trend-break model can be described by $Z_t = [1, t, DT_t^*]'$, where $DT_t^* = t - T_B$ for $t \geq T_B + 1$, and zero otherwise. A more general model which allows for both level-shift and trend-shift can be described as $Z_t = [1, t, D_t, DT_t^*]'$. This general model will be the focus of our study. To consider multiple breaks, we can include additional dummy variables such that:

$$Z_t = [1, t, D_{1t}, ..., D_{Rt}, DT_{1t}^*, ..., DT_{Rt}^*]', \tag{2.1.2}$$

where $D_{it} = 1$ for $t \geq T_{Bi} + 1$, $i = 1, ..., R$, and zero otherwise, and $DT_{it}^* = t - T_{Bi}$ for $t \geq T_{Bi} + 1$ and zero otherwise. Following the LM (score) principle, we impose the null restriction $\beta = 1$ and consider in the first step the following regression in differences:

$$\Delta y_t = \delta' \Delta Z_t + u_t, \tag{2.1.3}$$

where $\delta = [\delta_1, \delta_2, \delta_{3i}', \delta_{4i}']'$, $i = 1, ..., R$. The unit root test statistics are then obtained from the following regression:

$$\Delta y_t = \delta' \Delta Z_t + \phi \widetilde{S}_{t-1} + e_t, \tag{2.1.4}$$

where $\widetilde{S}_t$ denotes the de-trended series

$$\widetilde{S}_t = y_t - \widetilde{\Psi} - Z_t \widetilde{\delta}. \tag{2.1.5}$$

We let τ_{LM} be the t-statistic for $\phi = 0$ from (2.1.4). Here, $\widetilde{\delta}$ is the coefficient in the regression of Δy_t on ΔZ_t in (2.1.3), and $\widetilde{\Psi}$ is the restricted MLE of Ψ: that is, $\widetilde{\Psi} = y_1 - Z_1 \widetilde{\delta}$. Subtracting $\widetilde{\Psi}$ in (2.1.5) makes the initial value of the de-trended series to begin at zero with $\widetilde{S}_1 = 0$, but letting $\widetilde{\Psi} = 0$ leads to the same result. It is important to note that in the de-trending procedure (2.1.5), the coefficient $\widetilde{\delta}$ was obtained in regression (2.1.3) using first differenced data. Thus, the de-trending parameters are estimated in the first step regression in differences. Through this channel the dependency on nuisance parameters is removed in the crash model.

However, the dependency on nuisance parameters is not removed with this de-trending procedure in the model with trend breaks. In such cases, it will be difficult to combine these LM tests with the RALS procedure which will induce a new parameter. Indeed, the dependency of the tests on the nuisance parameter in the trend-shift models has been an issue in the literature, since it poses a difficulty in developing more extended tests. The same problem occurs in our case when we wish to utilize information on non-normal errors. As such, we consider a simple transformation which can make the LM unit root test statistic free of the dependency on the break location as in Lee *et al*. (2012). Specifically, we can see that usual LM tests for the model with trend-shifts will depend on λ_i^*, which denotes the fraction of sub-samples in each regime such that $\lambda_1^* = T_{B1}/T$, $\lambda_i^* = (T_{Bi} - T_{Bi-1})/T$, $i = 2, ..., R$, and $\lambda_{R+1}^* = (T - T_{BR})/T$.

However, the following transformation can remove the dependency on the nuisance parameter:

$$\widetilde{S}_t^* = \begin{cases} \dfrac{T}{T_{B1}}\widetilde{S}_t & \text{for } t \leq T_{B1} \\ \dfrac{T}{T_{B2} - T_{B1}}\widetilde{S}_t & \text{for } T_{B1} < t \leq T_{B2} \\ \vdots & \\ \dfrac{T}{T - T_{BR}}\widetilde{S}_t & \text{for } T_{BR} < t \leq T. \end{cases} \tag{2.1.6}$$

We then replace $\widetilde{S}_{t-1}$ in the testing regression (2.1.4) with $\widetilde{S}_{t-1}^*$ as follows:

$$\Delta y_t = \delta' \Delta Z_t + \phi \widetilde{S}_{t-1}^* + e_t. \tag{2.1.7}$$

We denote τ_{LM}^* as the t -statistic for $\phi = 0$ from (2.1.7). Then, the asymptotic distributions of the test statistic τ_{LM}^* will be invariant to the nuisance parameter λ_i^*.

$$\tau_{LM}^{*} \rightarrow -\frac{1}{2}\left[\sum_{i=1}^{R+1}\int_{0}^{1}\underline{V}_{i}(r)^{2}dr\right]^{-1/2}, \tag{2.1.8}$$

where $\underline{V}_i(r)$ is the projection of the process $V_i(r)$ on the orthogonal complement of the space spanned by the trend break function $dz(\lambda^*, r)$, as defined over the interval $r \in [0,1]$, where $V_i(r) = W_i(r) - rW_i(1)$, and $W_i(r)$ is a Wiener process for $i = 1, ..., R$.

Following the transformation, the asymptotic distribution of τ_{LM}^* depends only on the number of trend breaks, since the distribution is given as the sum of $R+1$ independent stochastic terms. With one trend-break ($R = 1$), the distribution of τ_{LM}^* is the same as that of the untransformed test τ_{LM} using $\lambda^* = 1/2$, regardless of the initial location of the break(s). Similarly, with two trend-breaks ($R = 2$), the distribution of τ_{LM}^* is the same as that of the untransformed test τ_{LM} using $\lambda_1^* = 1/3$ and $\lambda_2^* = 2/3$. In general, for the case of R multiple breaks, the same analogy holds: the distribution of τ_{LM}^* is the same as that of the untransformed test τ_{LM} using $\lambda_i^* = i/(R+1)$, $i = 1, ..., R$. Therefore, we do not need to simulate numerous critical values at all possible break point combinations. The critical values of τ_{LM}^* are reported in Lee *et al*. (2012), but instead of using these tests, we move on to the next step.

To improve the power of the LM statistic τ_{LM}^*, we adopt the procedure to utilize the information on non-normal errors. We adopt the RALS method as in Im *et al*. (2012). The RALS procedure is to augment the following term $\hat{w}_t$ to the testing regression (2.1.7),

$$\hat{w}_t = h(\hat{e}_t) - \hat{K} - \hat{e}_t\hat{D}_2, \tag{2.1.9}$$

where $h(\hat{e}_t) = [\hat{e}_t^2, \hat{e}_t^3]'$, $\hat{K} = \frac{1}{T}\sum_{t=1}^{T} h(\hat{e}_t)$, and $\hat{D}_2 = \frac{1}{T}\sum_{t=1}^{T} h'(\hat{e}_t)$. To capture the information of non-normal errors, we let $h(\hat{e}_t) = [\hat{e}_t^2, \hat{e}_t^3]'$, which involves the second and third moments of $\hat{e}_t$. Then, letting $\hat{m}_j = T^{-1}\sum_{t=1}^{T} \hat{e}_t^j$, the augmented term can be given as

$$\hat{w}_t = [\hat{e}_t^2 - \hat{m}_2, \hat{e}_t^3 - \hat{m}_3 - 3\hat{m}_2\hat{e}_t]' \tag{2.1.10}$$

The first term in $\hat{w}_t$ is associated with the moment condition $E[(e_t^2 - \sigma_e^2)\widetilde{S}_{t-1}] = 0$, which is the condition of no heteroskedasticity. The second term in $\hat{w}_t$ improves efficiency unless $m_4 = 3\sigma^4$. This condition improves the efficiency of the estimator of ϕ when the error terms are not symmetric. In general, knowledge of higher moments m_{j+1} are uninformative if $m_{j+1} = j\sigma^2 m_{j-1}$. This is the redundancy condition. The normal distribution is the only distribution that satisfies the redundancy condition. However, if the distribution of the error term is not normal, the condition is not satisfied. In such cases, one may increase efficiency by augmenting the testing regression (2.1.7) with $\hat{w}_t$. Then the transformed RALS-LM test statistic with trend-breaks is obtained from the regression

$$\Delta y_t = \delta'\Delta Z_t + \phi\widetilde{S}_{t-1}^* + \hat{w}_t'\gamma + u_t \tag{2.1.11}$$

We denote the corresponding t-statistic for $\phi = 0$ as $\tau_{RALS-LM}^*$. Then it can be shown that the asymptotic distribution of $\tau_{RALS-LM}^*$ is given as follows:

$$\tau_{RALS-LM}^* \to \rho\tau_{LM}^* + \sqrt{1-\rho^2}Z \tag{2.1.12}$$

where ρ reflects the relative ratio of the variances of two error terms.

It is interesting to see that the limiting distribution is similar to the distribution of the unit root tests with the stationary covariates of Hansen (1995). If the usual Dickey-Fuller (DF) tests without breaks were used, the limiting distribution would be identical. However, one can hardly combine the DF tests with breaks with the RALS procedure due to the dependency on the parameters indicating the locations of break(s), λ_i^*. But, note that the asymptotic distribution of the transformed RALS-LM test statistic $\tau^*_{RALS-LM}$ no longer depends on the break location parameter. Thus, we do not need to simulate new critical values for all the possible break location combinations, and this paves the way for us to employ the RALS procedure in the presence of trend-shifts. For a specific number of breaks, we just need one set of critical values since the critical values are asymptotically invariant to the break location. We note in passing that they are also invariant to the magnitude of breaks, contrary to some of popular endogenous unit root tests; see Lee and Strazicich (2001, 2003) and Perron (2006). The critical values of the transformed break RALS-LM unit root test with trend breaks are newly provided in Appendix Table 1, for $R = 1$ and 2, $T = 50$, 100, 300 and 1000, and $\rho^2 = 0$ to 1. All these critical values are obtained via Monte Carlo simulations using 100,000 replications. These critical values can be used when the break locations are known, or they are estimated consistently in a two-step procedure.

The new unit root tests used in this study differs from those used in the previous studies in several important aspects. First, we use linear unit root tests with structural breaks instead of non-linear unit root tests. It is true that conventional linear unit root tests will lose power in the presence of non-linearity; however, when dealing with real world data we rarely know the specific form of the non-linearity. If the forms of non-linearity are unknown and mis-specified, non-linearity tests are often less powerful than the DF type linear tests (see Choi and Moh, 2007). As our RALS-LM unit root tests are based on the linearized model specifications, the tests will not be subject to this difficulty. Moreover, non-normality can possibly mimic some unknown forms of non-linearity indirectly in a linear model framework.

Second, the new tests are free of the nuisance parameter and spurious rejection problems although the distribution of unit root tests with structural breaks depends on the parameter(s) indicating the location of break(s). Unlike the DF based tests, the usual LM tests do not suffer from spurious rejections, but they still depend on the location parameters in the model with trend-shifts. In such cases, they cannot be combined easily with the RALS procedure. As such, we adopt a simple transformation of the data which eliminates dependency on the trend breaks. As a result, the same critical values can be used at different break locations, even when the LM tests are combined with the RALS procedure to capture non-normal errors at the same time.

Third, the new LM tests select the proper number of breaks determined from the data. The endogenous break unit root tests used in several previous studies usually find and include the number of breaks as pre-specified in the model. However, as discussed in Lee *et al*. (2012), one or more unnecessary breaks will be included in the unit root test and as a result exhibits lower power. Whether or not a structural break exists is an empirical issue that must be determined from the data. As such, a two-step procedure can be more meaningful and the new LM tests are more powerful compared with the tests used in previous studies. This approach is favored in recent studies, given that the so-called endogenous unit root tests tend to exhibit spurious rejection and/or nuisance parameter problems; see Perron (2006) and Lee *et al*. (2012). The two-step procedure shows much improved performance in terms of both size and power properties. In Appendix Table 2, we have examined the properties of the new RALS-LM tests. We could observe that the performance of the tests under the null is fairly good in the presence of various trend-shifts. Also, we observe a significant increase in power when the errors are non-normal. Thus, we can achieve improved power by using the information on non-normal errors, but without using any nonlinear functional form. The transformation procedure works reasonably well.

2.2. Monte Carlo Experiment

We want to examine briefly the property of the transformed RALS-LM test using different combinations of non-normal errors and break locations. Pseudo-iid $N(0,1)$ random numbers were generated using the RATS procedure $\%RAN(1)$ and all results were obtained via simulations using WinRATS (v. 8.1). The DGP was given as the form of (1). The initial values y_0 and ϵ_0 are assumed to be random. We consider the following non-normal errors: (i - iv) χ^2 distribution with $df =$ 1 to 4, and (v - vii) t-distribution with $df =$ 2 to 4. We also include the size and power for (viii) standard normal, transformed LM test, untransformed RALS-LM and LM test as well as DF test for comparison. All simulation results are calculated using 10,000 simulations for sample size 100.[2] The size (frequency of rejections under the null when $\beta = 1$) and power (frequency of rejections under the alternative when $\beta = 0.9$ and $\beta = 0.8$) of the tests are examined using 5% critical values. The finite sample size and power property for transformed RALS-LM test and comparing tests for $T = 100$ and $R = 1$ are presented in Appendix Table 2.

[2] Results for T = 300 and 1,000 are similar and available upon request from author.

We consider DGP with two different λ^*s: $\lambda^* = 0.25$ and $\lambda^* = 0.5$, but we use only one set of critical values calculated based on $\lambda^* = 0.5$. For all the five tests examined, we see little size distortion whether $\lambda^* = 0.25$ or 0.5. For the power property, we see large power gain when RALS procedure is used than the corresponding non-RALS type test. When the non-normal error becomes normal (degree of freedom becomes larger for the χ^2 distribution and t-distribution), the power gain becomes less. One surprising result is the power for the RALS based tests are larger than the corresponding non-RALS based test even when the error term follows a normal distribution. To the contrary, the transformation procedure seems to make the tests less powerful than the un-transformed tests. However, since the LM type of unit root test statistics are dependent on the trend breaks, we need the transformation procedure to make the test valid. Fortunately, for all the non-normal distributions of error term, the power gain from the RALS procedure is much higher than the power loss from the transformation procedure.

2.3. Data and Empirical Results

In this study, we use recently extended Grilli and Yang annual data on prices for twenty-four primary commodity prices spanning 1900-2007 provided by Pfaffenzeller et al. (2007). The commodities examined are aluminum, banana, beef, cocoa, coffee, copper, cotton, hides, jute, lamb, lead, maize, palm oil, rice, rubber, silver, sugar, tea, timber, tin, tobacco, wheat, wool and zinc.

For each primary commodity i, we deflate the nominal prices with the United Nations Manufactures Unit Value Index (MUV) as follows:

$$y_{it} = \frac{\text{nominal commodity price}_{it}}{\text{MUV}_t}\,. \tag{2.3.1}$$

To examine the relative primary commodity prices in (2.3.1), we utilize the two-step LM unit root test suggested by Lee et al. (2012)[3] and three-step RALS-LM unit root test with trend shifts. Throughout, we consider a model with at most two level and trend breaks.

The two-step LM test can be summarized as follows: In the first step, we set a maximum structural break number R (in this essay R = 2) and apply the $maxF$ test to identify the break locations, test the significance of each break, and simultaneously determine the optimal lags with the corresponding number and location of breaks. If the null of no trend break is not rejected or if the null of no trend break is rejected but one of the break dummy variables is not significant based on the standard t-test, we move to the beginning of the first step with the structural break number equal to $R-1$. This procedure continues until the break number becomes zero or all the identified break dummy variables are significant. If the first step yields a break number equal to zero, we use the usual no-break LM unit root test of Schmidt and Phillips (1992); if one or more breaks are found, we use one break (or R breaks) LM unit root test of Amsler and Lee (1995) and Lee and Strazicich (2003) with the break number and location as well as the corresponding optimal lags determined in the first step. Then, we obtain the LM test statistic, denoted as τ^*_{LM}. The first two steps of the RALS-LM test are the same as the two-step LM test. We use the higher moment information obtained from the second step and augment it to the regression of the two-step LM test as the third step of the RALS-LM test denoted as $\tau^*_{RALS-LM}$. While searching for the optimal number of breaks, we use the grid search within 0.10-0.90 intervals of the whole

[3] See also Yabu and Perron (2009), and Kejriwal and Perron (2010). These papers show consistently that the two-step based unit root tests have better size and power properties than endogenous unit root tests.

sample period so that each subsample before and after the breaks will have enough observations to perform a valid test.[4] The corresponding number of lags is chosen using a general to specific approach with maximum lags equal to eight.[5]

As a prelude to the test of a unit root allowing for structural breaks, we examine the 24 primary commodity prices using Augmented Dickey-Fuller (ADF) test, LM test of Schmidt and Phillips (1992) and RALS-LM test without breaks. We report the results in Table 2.1, labeled τ_{ADF}, τ_{LM} and $\tau_{RALS-LM}$, respectively. The preliminary results show that 12 relative commodity prices reject a unit root using the ADF test at the 10% significance level; the numbers for LM test and RALS-LM test are 9 and 14, respectively. The relative commodity price series (cocoa, rubber, silver, wheat and zinc) reject the unit root using the RALS-LM test but cannot reject a unit root using the LM test with all having low ρ^2 values,[6] which indicate the extra rejections come from using the non-normal information embedded in the error term. Out of the 12 rejections using the ADF test, 11 of them are also rejected using the RALS-LM test. The non-rejection of the maize price series may be a result of not accounting for breaks in the intercept or trend.[7]

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[4] When using more than one break, we set the minimum length between two breaks to be at least 1/10 of observations for the same reason.

[5] The general to specific method can be explained as follows: For each combination of breaks, we start with first eight lags, and examine the significance of the eighth lag (or $\Delta\widetilde{S}_{t-8}$). If it is significant at 10% level (the absolute value of the t-statistics is greater than 1.645), we select eight lags as the optimal lags; if not, we try the first seven lags, and repeat the previous check. The searching procedure ends when the coefficient of the last lag $\Delta\widetilde{S}_{t-j}$, $j \geq 1$, is significant in which case we select the optimal lags to be j; otherwise, we select the optimal lags to be zero.

[6] The ρ^2 of these five commodity prices are 0.619 (cocoa), 0.512 (rubber), 0.438 (silver), 0.630 (wheat), and 0.429 (zinc), respectively. The range for ρ^2 is [0, 1]; lower ρ^2 means the RALS procedure uses more information in the non-normal errors.

[7] As we can see, after including one or two breaks in the transformed LM and RALS-LM tests, the unit root in the maize price series was rejected at the 1% significance level.

Tables 2.2 and 2.3 report unit root tests allowing for one and two trend breaks, respectively with τ^*_{LM} the test statistic using the transformed LM test and $\tau^*_{RALS-LM}$ the test statistic using transformed RALS-LM test. Using the one-break transformed LM and RALS-LM tests, the number of rejections is 21 and 19, respectively, while the rejections in the corresponding two-break tests in Table 2.3 are both 21. While it appears we obtain at least as many trend stationary series when we add more trend breaks to the model, it is not necessarily so. For example, we reject the null of a unit root in the lamb price series at the 5% significance level in the no-break LM test, however, we cannot reject the null hypothesis of a unit root with the one/two- break LM test; the same situation occurs with the cocoa, lamb, palm oil and silver price series with the no-break LM test. Moreover, we can reject the null hypothesis of a unit root for the palm oil price series at the 5% significance level using the ADF test, no-break LM test, no-break RALS-LM test, and the one-break LM test; however, we cannot reject the null hypothesis of a unit root in the one-break RALS-LM test, two-break LM test, and two-break RALS-LM test.

Next, we apply our new two-step LM unit root test and the three-step RALS-LM unit root test, the results of which are shown in Table 2.4. As discussed above, when using the two-step and three-step tests, we select the optimal number of breaks using a $maxF$ procedure suggested in Lee et al. (2012). Here, the optimal break number is determined using a general-to-specific approach. The optimal break number is set to two if both trend break coefficients are significant at the 10% level; if one of them is insignificant, we examine the trend break coefficient from the one break test; if it is insignificant, the no break test is selected. For example, the second trend break in the banana price series and the first trend break in the lead price series reveal that the pre-specified two trend breaks are not significant at the 10% level, but the trend breaks for these two series using the one-break test are significant. Thus, we select the optimal number of breaks for the banana and lead price series to be one. The three relative commodity price series found to be difference stationary are copper, lamb and palm oil.

Following Ghoshray (2011) and Kellard and Wohar (2006), we examine the prevalence of trends in the primary commodity prices using the optimal breaks identified in Table 2.4. Panel A of Table 2.5 displays the results of the estimated trend stationary models with one and two structural breaks. A trend stationary model is estimated with the error term following an ARMA(p, q) process. Out of the 21 trend stationary price series, 12 relative commodity prices are found to exhibit a significant negative trend, though not necessarily for the entire sample. The remaining 9 relative commodity price series have no evidence of a significant positive or negative trend or show a mixture of positive trend and no trend. Panel B of Table 2.5 shows the results of the estimated difference stationary models for those relative commodity price series found to exhibit unit root behavior. A difference stationary model is estimated with the error term following an ARMA (p, q) process. All three relative commodity price series considered (copper, lamb, and palm oil) display a significant negative trend or a mixture of a significant negative trend and positive trend. Then, we synthesize the results in Table 2.5 by constructing Ψ (-), Ψ(+), and Ψ(.) that measure the prevalence of a negative trend, positive trend, and trendless behavior, respectively. We define $\Psi(-) = \lambda(-)/T$, where $\lambda(-)$ equals the number of years that a statistically significant negative trend exists. Similarly, we define $\Psi(+) = \lambda(+)/T$ for a significant positive trend, and $\Psi(.) = 1 - \Psi(-) - \Psi(+)$ for no trend. The results of the trend prevalence among the relative commodity prices are shown in Table 2.6.

The prevalence of a negative trend is found in 15 commodities. That is, each of these commodities displays at least one significant negative trend segment. Out of the 15 commodities, 7 commodities display a negative trend for more than 50% of the sample period. The prevalence of a positive trend is found in 10 commodities, of which 4 commodities display a positive trend for more than 50% of the sample period. However, the prevalence of trendless behavior is found in 19 commodities, of which 8 commodities (aluminum, banana, copper, cotton, lead, palm oil, silver and tin) display trendless behavior for more than 90% of the sample period.

Overall, the results suggest that the trend is variable, and there is no evidence of a single negative trend. To visualize our empirical findings, we superimpose the level and trend breaks identified by the two-break test in Table 2.5 and plot the relative commodity prices. Linear trends are then estimated using OLS to connect the break points as shown in Figure 2.1. The optimal break points identified in Table 2.4 are concentrated in two groups. Among the 46 breaks, 20 breaks show up between 1910-1930 and 11 breaks show up between 1970-1990, which account for more than 67% of the all the identified breaks. Compared with past studies, our findings provide even weaker evidence to support the PSH. [8]

2.4. Concluding Remarks

This study employs newly developed LM unit root tests to determine whether relative primary commodity prices contain stochastic trends. Unlike the endogenous break unit root test, our two-step LM and three-step RALS-LM tests always include the appropriate number of

[8] Persson and Teräsvirta (2003) and Balagtas and Holt (2009) have adopted a flexible model and estimated a number of smooth transition autoregressions (STARs). They also find limited support for the PSH.

breaks in the model. Simulation results show a power gain from the RALS procedure when the error term follows a non-normal distribution. Since the unit root tests are free of spurious rejections, as well as more powerful than those employed in previous studies, the rejection of the null can be considered as genuine evidence of stationarity. The main findings of this study reveal that 21 out of the 24 commodity prices are found to be stationary around a broken trend, implying that shocks to these commodities tend to be transitory. Only three relative commodity price series are found to be difference stationary. There are only 7 series in which the relative commodity prices display negative trend more than 50% of the time period examined; however, 8 relative commodity prices display no significant positive or negative trend in more than 90% of the time period examined. Compared with past studies, our findings provide even weaker evidence to support the PSH. Also, we believe that in light of the significantly improved power, the newly suggested RALS-LM tests with trend-shifts can be useful in other time series applications in agricultural economics and related areas.

Table 2.1 Results using ADF Test and No-Break LM Unit Root Tests

	ADF		LM	RALS-LM		
	τ_{ADF}	k	τ_{LM}	$\tau_{RALS-LM}$	ρ^2	k
Aluminum	-3.180*	7	-3.520**	-3.593***	0.554	7
Banana	-2.390	2	-1.547	-1.568	1.015	2
Beef	-2.049	5	-1.909	-1.503	0.537	5
Cocoa	-2.557	2	-2.250	-5.317***	0.619	2
Coffee	-3.315*	0	-3.353**	-4.806***	0.648	0
Copper	-1.598	8	-1.983	-1.805	0.799	8
Cotton	-2.975	2	-2.057	-2.422	0.893	2
Hides	-3.925**	3	-3.322**	-3.527**	0.830	3
Jute	-3.285*	3	-3.044*	-3.125**	0.949	3
Lamb	-3.411*	4	-3.442**	-3.398**	0.821	4
Lead	-2.376	1	-1.858	-1.674	0.777	0
Maize	-5.667***	0	-1.817	-1.589	0.695	4
Palm oil	-4.829***	0	-4.082***	-3.035**	0.577	0
Rice	-4.024**	7	-3.546**	-4.249***	0.840	7
Rubber	-2.114	0	-2.212	-2.883**	0.521	0
Silver	-2.242	2	-2.336	-4.434***	0.438	2
Sugar	-3.923**	2	-3.920***	-6.759***	0.449	2
Tea	-2.091	7	-2.207	-2.338	0.817	7
Timber	-4.199***	3	-3.532**	-3.470**	0.938	3
Tin	-2.355	0	-2.321	-2.023	0.759	0
Tobacco	-1.414	4	-1.580	-1.721	0.911	4
Wheat	-4.042***	4	-1.503	-2.912**	0.630	6
Wool	-3.093	4	-1.554	-1.763	0.819	4
Zinc	-4.875***	1	-2.353	-4.073***	0.429	2

Notes: Since our LM test and RALS-LM test share the same procedure when searching for the optimal lags, we only report one time to save the space. k is the optimal number of lagged first-differenced terms. τ_{ADF}, τ_{LM} and $\tau_{RALS-LM}$ denote the test statistics for the ADF test, LM test and RALS-LM test respectively. *, ** and *** denote the test statistic is significant at 10%, 5% and 1% levels, respectively.

Table 2.2 Results using Transformed One-Break LM and RALS-LM Tests

	LM	RALS-LM		T_B	k
	τ^*_{LM}	$\tau^*_{RALS-LM}$	ρ^2		
Aluminum	-4.847***	-6.105***	0.404	1916	7
Banana	-3.680*	-4.016**	0.915	1966	6
Beef	-4.511***	-3.110*	0.619	1972	1
Cocoa	-1.755	-1.750	0.875	1976	2
Coffee	-4.487***	-6.899***	0.680	1975	0
Copper	-3.203	-3.156	0.780	1969	6
Cotton	-4.266**	-4.691***	0.929	1927	2
Hides	-4.115**	-3.925**	0.850	1932	3
Jute	-4.715***	-4.584***	1.012	1970	3
Lamb	-2.940	-3.196	0.820	1974	4
Lead	-3.450*	-4.525***	0.719	1978	1
Maize	-6.009***	-5.829***	0.591	1948	1
Palm oil	-4.123**	-2.871	0.540	1983	0
Rice	-4.494***	-4.039**	0.899	1972	1
Rubber	-5.512***	-8.208***	0.553	1912	1
Silver	-3.533*	-1.010	0.508	1979	6
Sugar	-4.135**	-4.800***	0.501	1920	6
Tea	-3.905**	-3.702**	0.871	1966	7
Timber	-4.307***	-4.339***	0.869	1979	3
Tin	-4.088**	-5.605***	0.798	1984	0
Tobacco	-3.596*	-3.808**	0.904	1988	4
Wheat	-4.867***	-5.649***	0.666	1919	6
Wool	-5.257***	-4.616***	0.793	1956	3
Zinc	-5.587***	-7.502***	0.561	1914	1

Notes: Since our LM test and RALS-LM test share the same procedure when searching for the break points and the corresponding optimal lags, we only report one time to save the space. k is the optimal number of lagged first-differenced terms. T_B denotes the estimated break point. τ^*_{LM} and $\tau^*_{RALS-LM}$ denote the test statistics for the transformed LM test and RALS-LM test respectively. *, ** and *** denote the test statistic is significant at 10%, 5% and 1% levels, respectively.

Table 2.3 Results using Transformed Two-Break LM and RALS-LM Tests

	LM τ^*_{LM}	RALS-LM $\tau^*_{RALS-LM}$	ρ^2	T_B		k
Aluminum	-6.995***	-6.732***	0.527	1915	1927	7
Banana	-5.274***	-5.789***	0.903	1949	1984n	8
Beef	-6.604***	-5.109***	0.888	1956	1972	8
Cocoa	-6.793***	-10.475***	0.514	1945	1955	1
Coffee	-6.056***	-8.018***	0.651	1974	1987	3
Copper	-3.449	-3.109	0.843	1915	1925	1
Cotton	-8.361***	-8.641***	0.886	1929	1951	1
Hides	-7.001***	-7.496***	0.842	1919	1930	0
Jute	-5.494***	-5.487***	0.991	1930	1965	3
Lamb	-3.740	-3.269	0.899	1936	1956	4
Lead	-4.186*	-5.539***	0.688	1920n	1978	1
Maize	-7.776***	-6.226***	0.820	1918	1947	3
Palm oil	-3.427	-1.108	0.779	1917	1927	8
Rice	-6.500***	-6.294***	0.773	1915	1980	1
Rubber	-8.242***	-9.396***	0.738	1911	1924	2
Silver	-10.254***	-13.900***	0.475	1972	1983	1
Sugar	-7.083***	-14.038***	0.313	1923	1936	1
Tea	-5.105***	-4.414**	0.904	1919	1966	7
Timber	-7.518***	-7.799***	0.930	1966	1979	1
Tin	-5.750***	-6.162***	0.866	1972	1984	1
Tobacco	-4.766**	-4.268**	0.955	1915	1988	4
Wheat	-7.989***	-8.748***	0.750	1919	1945	3
Wool	-5.640***	-4.334**	0.792	1913	1956	3
Zinc	-5.904***	-6.332***	0.449	1917	1988	1

Notes: k is the optimal number of lagged first-differenced terms. T_B denotes the estimated break point. n denotes that the identified break point was not significant at the 10% level. τ^*_{LM} and $\tau^*_{RALS-LM}$ denote the test statistics for the transformed LM test and RALS-LM test respectively. *, ** and *** denote the test statistic is significant at 10%, 5% and 1% levels, respectively.

Table 2.4 Results using Two-Step LM and Three-Step RALS-LM Tests

	LM	RALS-LM				
	τ^*_{LM}	$\tau^*_{RALS-LM}$	ρ^2	T_B		k
Aluminum	-6.995***	-6.732***	0.527	1915	1927	7
Banana	-3.680*	-4.016**	0.915	1966		6
Beef	-6.604***	-5.109***	0.888	1956	1972	8
Cocoa	-6.793***	-10.475***	0.514	1945	1955	1
Coffee	-6.056***	-8.018***	0.651	1974	1987	3
Copper	-3.449	-3.109	0.843	1915	1925	1
Cotton	-8.361***	-8.641***	0.886	1929	1951	1
Hides	-7.001***	-7.496***	0.842	1919	1930	0
Jute	-5.494***	-5.487***	0.991	1930	1965	3
Lamb	-3.740	-3.269	0.899	1936	1956	4
Lead	-3.450*	-4.525***	0.719	1978		1
Maize	-7.776***	-6.226***	0.820	1918	1947	3
Palm oil	-3.427	-1.108	0.779	1917	1927	8
Rice	-6.500***	-6.294***	0.773	1915	1980	1
Rubber	-8.242***	-9.396***	0.738	1911	1924	2
Silver	-10.254***	-13.900***	0.475	1972	1983	1
Sugar	-7.083***	-14.038***	0.313	1923	1936	1
Tea	-5.105***	-4.414**	0.904	1919	1966	7
Timber	-7.518***	-7.799***	0.930	1966	1979	1
Tin	-5.750***	-6.162***	0.866	1972	1984	1
Tobacco	-4.766**	-4.268**	0.955	1915	1988	4
Wheat	-7.989***	-8.748***	0.750	1919	1945	3
Wool	-5.640***	-4.334**	0.792	1913	1956	3
Zinc	-5.904***	-6.332***	0.449	1917	1988	1

Notes: k is the optimal number of lagged first-differenced terms. T_B denotes the estimated break point. τ^*_{LM} and $\tau^*_{RALS-LM}$ denote the test statistics for the two-step LM test and three-step RALS-LM test respectively. *, ** and *** denote the test statistic is significant at 10%, 5% and 1% levels, respectively.

Table 2.5 Estimated Trend Stationary and Difference Stationary Models

	T_{B1}	T_{B2}	Regime 1	Regime 2	Regime 3	ARMA
Panel A. Estimated trend stationary models with breaks						
Aluminum	1915	1927	0.041(0.102)	-0.164(-0.874)	-0.011(-0.680)	2,2
Beef	1956	1972	0.004***(2.861)	0.042***(3.503)	-0.020***(-3.337)	2,1
Cocoa	1945	1955	-0.009**(-2.032)	0.053***(2.534)	-0.005*(-1.681)	1,0
Coffee	1974	1987	0.005**(2.168)	-0.018(-0.954)	-0.011(-0.888)	1,0
Cotton	1929	1951	-0.031(-0.278)	0.002(0.025)	-0.032(-0.990)	2,2
Hides	1919	1930	0.068***(4.448)	0.076**(2.41)	-0.004**(-2.093)	0,1
Jute	1930	1965	-0.044(-0.421)	0.009(0.169)	-0.040*(-1.856)	1,2
Maize	1918	1947	0.031(1.324)	0.012(10.29)	-0.017***(-3.956)	0,3
Rice	1915	1980	-0.066(-0.498)	-0.008(-1.456)	-0.030**(-2.140)	2,2
Rubber	1911	1924	0.037(0.238)	-0.971***(-11.234)	-0.027***(-4.017)	2,0
Silver	1972	1983	-0.002(-1.074)	0.098***(4.725)	-0.002(-0.260)	0,3
Sugar	1923	1936	0.035(1.303)	-0.060(-1.058)	-0.010**(-2.068)	0,1
Tea	1919	1966	0.018(0.275)	0.017(1.069)	-0.024***(-2.646)	2,2
Timber	1966	1979	0.006***(3.839)	0.012(1.045)	0.002(0.361)	1,1
Tin	1972	1984	0.003(1.055)	0.010(0.594)	-0.012(-1.031)	1,0
Tobacco	1915	1988	0.035***(2.713)	0.006***(4.388)	-0.026***(-3.221)	2,2
Wheat	1919	1945	0.032**(2.246)	-0.040***(-4.876)	-0.168***(-7.402)	2,2
Wool	1913	1956	0.021(0.553)	-0.014**(-2.302)	-0.030***(-6.249)	0,1
Zinc	1917	1988	0.038**(1.995)	0.003(1.394)	0.027*(1.703)	0,1
Banana	1966	N.A.	-0.002(-0.207)	-0.008(-1.006)	N.A.	1,2
Lead	1978	N.A.	0.012(0.218)	-0.0299(-0.252)	N.A.	2,2
Panel B. Estimated difference stationary models with breaks						
Copper	1915	1925	0.041(0.727)	-0.136**(-2.008)	0.018(0.821)	0,0
Lamb	1936	1956	0.003(1.475)	-0.018*(-1.819)	0.016***(3.133)	0,3
Palm oil	1917	1927	-0.007(-0.418)	-0.260***(-3.629)	-0.006(-0.478)	2,2

Notes:T_{B1} and T_{B2} denote the first and second break dates respectively. The slope coefficients are reported for regime 1, 2 and 3. ***, ** and * denote significant at the 1%, 5% and 10% levels respectively. The final column represents the ARMA(p,q) specification. The numbers in parentheses denote the t-rations.

Table 2.6 Relative Measures of a Prevalence of a Trend

	Ψ(-)	Ψ(+)	Ψ(.)
Aluminum	0.000	0.000	1.000
Banana	0.000	0.000	1.000
Beef	0.324	0.676	0.000
Cocoa	0.907	0.093	0.000
Coffee	0.000	0.694	0.306
Copper	0.093	0.000	0.907
Cotton	0.000	0.000	1.000
Hides	0.713	0.287	0.000
Jute	0.389	0.000	0.611
Lamb	0.185	0.472	0.343
Lead	0.000	0.000	1.000
Maize	0.556	0.000	0.444
Palm oil	0.093	0.000	0.907
Rice	0.250	0.000	0.750
Rubber	0.889	0.000	0.111
Silver	0.000	0.102	0.898
Sugar	0.657	0.000	0.343
Tea	0.380	0.000	0.620
Timber	0.000	0.620	0.380
Tin	0.000	0.000	1.000
Tobacco	0.176	0.824	0.000
Wheat	0.815	0.185	0.000
Wool	0.870	0.000	0.130
Zinc	0.000	0.343	0.657

Figure 2.1 Relative Primary Commodity Prices

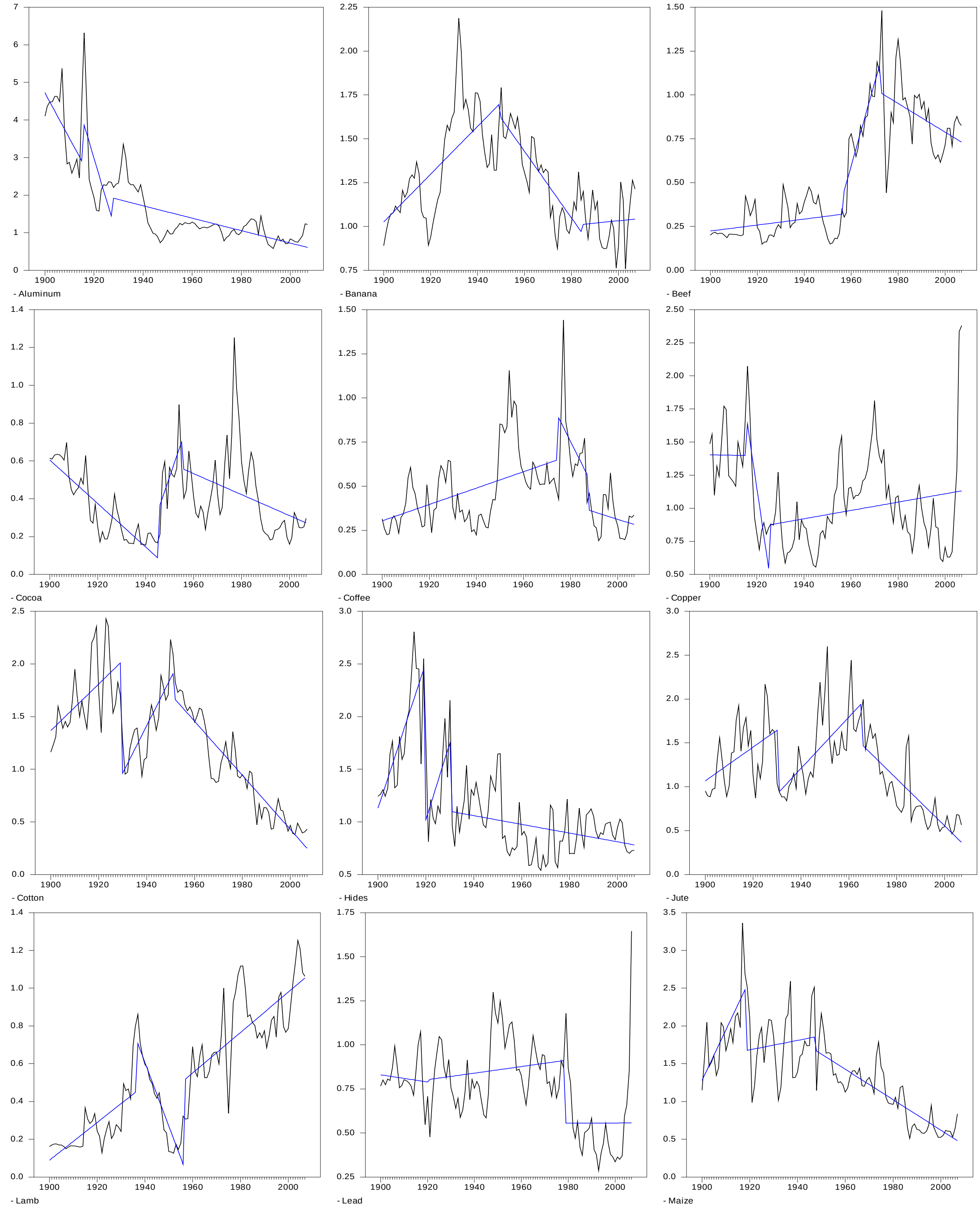
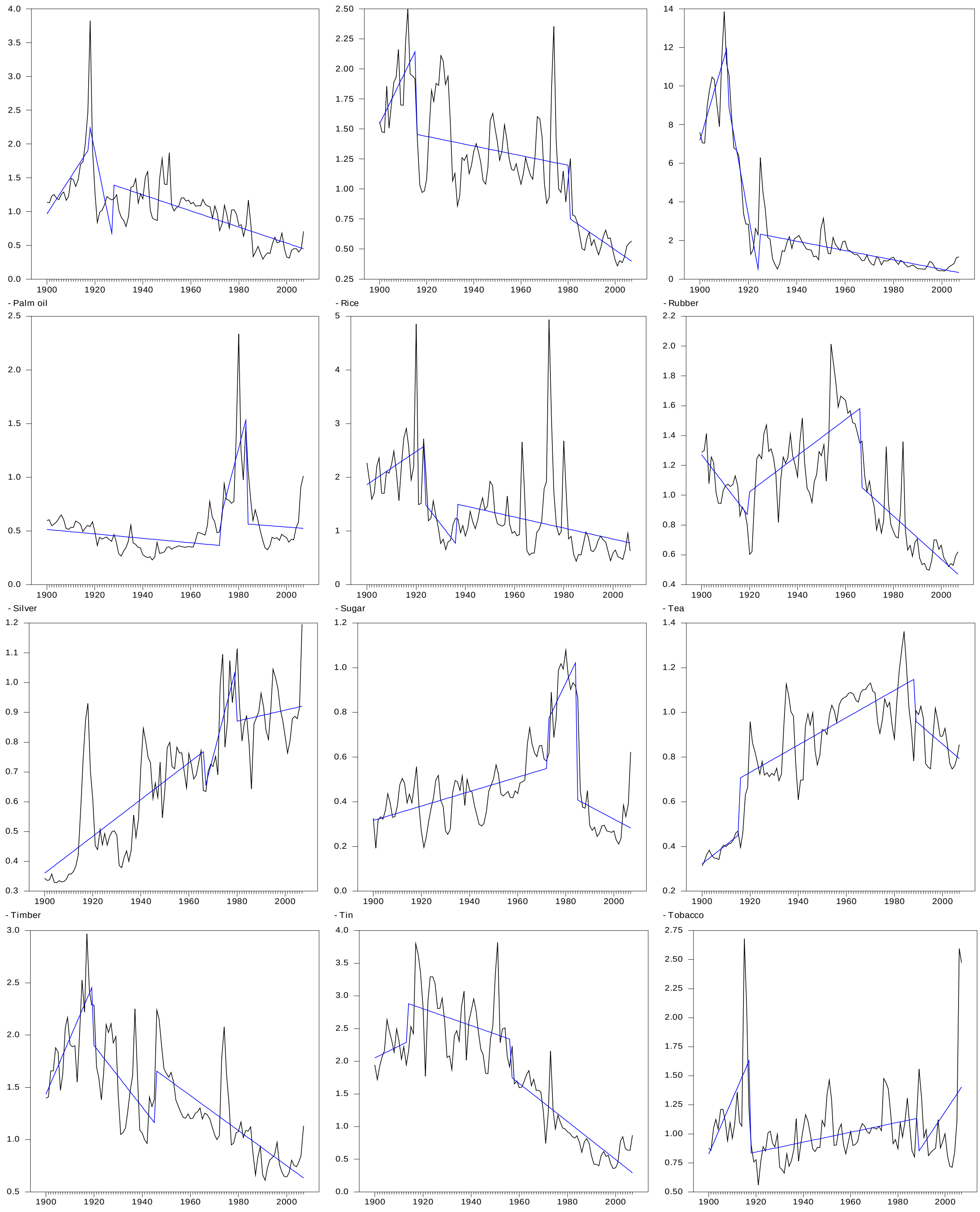


Figure 2.1 Relative Primary Commodity Prices (continued)



CHAPTER 3

MORE POWERFUL FOURIER LM UNIT ROOT TESTS WITH NON-NORMAL ERRORS

The issue of whether an economic time series is best characterized by either a unit root or a stationary process has assumed great importance in both the theoretical and the applied time series econometrics literature. As a consequence, tests of the null hypothesis that a series is integrated of order one, I(1), against the alternative hypothesis that it is integrated of order zero, I(0), have received much attention. Now it has been well documented that traditional unit root tests tend to have low power problem when the deterministic components of the testing time series are mis-specified. In a seminal paper, Perron (1989) first documented that failure to account for a structural break can result in the standard Dickey-Fuller (DF) test lose power significantly. To provide a remedy, Perron suggested a modified DF unit root test that includes dummy variables to control for known structural breaks in the level and slopes. Perron's paper has three assumptions that are hardly satisfied in empirical researches, which includes:(i) the structural breaks are assumed to be known a priori, (ii) the structural breaks are assumed to be steep or abrupt, and (iii) the error term is assumed to follow a standard normal distribution, respectively. Subsequent papers further modified unit root tests have been developed to relax one or more of these assumptions.

In empirical study, the information on breaks is generally unknown. Various tests are developed to allow for one or more unknown structural breaks that are determined endogenously from the data. The most popular endogenous break unit root tests include the tests developed in Zivot and Andrews (1992), Lumsdaine and Papell (1997), Perron (1997), Vogelsang and Perron (1998), and Lee and Strazicich (2003, 2004). These tests are featured with searching for possible structural breaks using a grid search, which the breakpoint is determined where the t-statistic on the unit root hypothesis is minimized or the t-statistic/f-statistic for the break coefficient is maximized. However, most of these tests are subject to the so-called spurious rejection problem when they assume no break under the null; see Lee and Strazicich (2001). Lee and Strazicich (2003) allow for breaks both under the null and alternative hypotheses, and provide a remedy for this problem. Lee, Strazicich and Meng (2012) compared the performance of these endogenous break unit root tests, and they found that all the above endogenous tests have some drawbacks, some of which are serious and others are minor. In a following paper, Lee, Meng and Strazicich (2012) suggested a test based on a two-step procedure, which searches for a structural break in the first step and test for the unit root in the second step. The Monte Carlo experiments show the two-step test does not exhibit size distortions, has power similar to that of the exogenous tests, and accurately identifies the breaks.

However, there are several minor problems for the two-step unit root tests. Firstly, the maximum number of structural breaks need to be specified in advance; secondly, the searching procedure works poorly if breaks are located at the beginning or the end of the testing series; lastly, the searching procedure works poorly if breaks are adjacent to each other.

Typically structural breaks in the deterministic terms are assumed to be steep and abrupt, yet a number of researchers argue that the structural changes can be gradual and smooth. To allow for such structural changes, Leybourne *et al*. (1998) and Kapetanios *et al*. (2003) develop unit-root tests such that the deterministic component of the series is a smooth transition process.

Similar to the exogenous break unit root test of Perron (1989), the information about the break date and the specific functional form of the nonlinearity needs to be specified as a priori. Another problem of this type of nonlinear unit root tests is the spurious rejection problem. Meng (2012) provide evidence that assuming no break under the null in these nonlinear unit root tests causes the test statistic to diverge and lead to significant rejections of the unit root null when the true data-generating process (DGP) is a unit root with nonlinear deterministic term(s). A third problem is although it is possible to allow for more smooth breaks, these tests are not powerful since too many parameters need to be estimated.

Recently, Enders and Lee (2012a, hereafter EL) suggest a different approach to address structural breaks. They suggest a LM based unit root test which uses a variant of Gallant's (1981) flexible Fourier form (hereafter FLM) to control for the unknown nature of the structural break(s).[1] They follow Becker *et al*. (2004, 2006) and illustrate that the essential characteristic of a series with one or more structural breaks can often be captured using a small number of low-frequency components from a Fourier approximation. First of all, their tests allow for structural break under both the null hypothesis and the alternative hypothesis, thus they are free of the spurious rejection problem. Secondly, these tests use Fourier approximation to circumvent the needs to assume the number of breaks, the break date, and the specific form of the break(s). Their Monte Carlo simulations prove their test has good size and power properties for the various types of break forms often used in empirical economic analysis.

My essay 1 develops a Residual Augmented Least Squares Lagrange Multiplier (RALS-LM) test which utilizes the information that exists when the errors in the testing equation exhibit

[1] Similar Dickey-Fuller Generalized Least Squares (DF-GLS) based Fourier unit root test and DF based Fourier unit root test are provided in Rodrigues and Taylor (2012) and Enders and Lee (2012b).

any departure from normality, such as non-linearity, asymmetry, or fat-tailed distributions.[2] The underlying idea of the RALS procedure is appealing because it is intuitive and easy to implement. If the errors are non-normal, the higher moments of the residuals contain the information on the nature of the non-normality. The RALS procedure conveniently utilizes these moments in a linear testing equation without the need for a priori information on the nature of the non-normality, such as the density function or the precise functional form of any non-linearity. The simulation result showsthe power gain of the RALS-LM tests over the usual LM tests is considerable when the error term is asymmetric or has a fat-tailed distribution.

Non-normality is common to find when dealing with real-world data (Im, Lee and Tieslau, 2012).In many cases, however, it is hard to distinguish between non-normal distributions and some forms of non-linearity. In addition, certain economic time series might contain a mixture of both non-linearity and non-normality. If a specific nonlinear form is known, it would be proper to utilize non-linear unit root tests with the specific form;[3] if the true density of the non-normal error is known, it is also possible to use maximum likelihood estimators (MLE) to get a more efficient test. Unfortunately, the information about the non-linear forms and non-normal errors is generally unknown a priori. In this regards, we explore a joint treatment of both unknown forms of structural breaks and unknown forms of non-normal errors by extending the FLM unit root tests of EL using the RALS estimation procedure of Im and Schmidt (2008). We refer to this test as RALS-FLM unit root tests. The FLM type of tests could be used to control for smooth structural breaks of an unknown functional form and the RALS procedure could

[2] Im, Lee and Tieslau (2012) suggests a similar test based on the traditional DF unit root tests; Im, Lee and Lee (2012) suggests applying the RALS procedure to the cointegration tests. Both papers confirm power gain if the error term follows a non-normal distribution.

[3] There is another problem for those nonlinear unit root tests. Lee, Meng and Lee (2011) found the presence of non-normal errors in nonlinear unit root tests will lead to a significant loss of power, although non-normal errors do not pose a problem in the usual unit root tests since the least square estimator will still be the most efficient under certain ideal conditions regardless of normal or non-normal errors.

utilize information contained in non-normal errors. With our new proposed tests, it is possible to test for a unit root without having to model the precise form of the structural break, and it is also possible to improve the testing power when the error term follows a non-normal distribution without the need to specify a clear form of the non-normality. Our simulation results confirm significant power gains over the FLM tests.

This essay proceeds as follows. In Section 3.1, we discuss the FLM Tests and propose the RALS-FLM tests. In Section 3.2, we provide simulation results to examine the size and power properties and compare them with those of the FLM tests. Section 3.3 provides concluding remarks.

3.1. The RALS-FLM Test

To begin with our analysis, we consider data generated according to the following DGP:

$$y_t = a + b \cdot t + d(t) + e_t, t = 1, 2, ..., T, \tag{3.1.1}$$

$$e_t = \theta e_{t-1} + \varepsilon_t, \tag{3.1.2}$$

where we the initial condition, e_0, to be an $O_p(1)$ random variable. We are interested in testing the null hypothesis of a unit root ($H_0 : \theta = 1$) against the alternative hypothesis of a stationary ($H_1 : \theta < 1$) in equation (3.1.2).If the functional form of $d(t)$ is known, it is possible to test the unit root directly. If the form of $d(t)$ is unknown, we follow EL and approximate $d(t)$ using the Fourier expansion:

$$d(t) \cong \sum_{k=1}^{n} \alpha_k \sin(2\pi kt/T) + \sum_{k=1}^{n} \beta_k \cos(2\pi kt/T)$$

EL point out that the deterministic components in the above equation could be used to approximate nonlinearity for any desired level of accuracy. However, as a practice matter, EL recommends using a small number of frequencies to avoid losing too many degrees of freedom and an overfitting problem. In light of the observation made in EL and Becker *et al*. (2006) that a single Fourier frequency generally approximates nonlinear structural break well, for the time being, we consider only a single frequency k. This enables our DGP to be rewritten as follows:

$$y_t = a_0 + b \cdot t + \alpha_k \sin(2\pi kt/T) + \beta_k \cos(2\pi kt/T) + e_t,$$

$$e_t = \theta e_{t-1} + \varepsilon_t. \tag{3.3.3}$$

Following the LM (score) principle, we impose the null restriction and consider the following regression in first-differences:

$$\Delta y_t = \delta_0 + \delta_1 \Delta \sin(2\pi kt/T) + \delta_2 \Delta \cos(2\pi kt/T) + u_t. \tag{3.1.4}$$

The estimated coefficients are denote as $\widetilde{\delta}_0$, $\widetilde{\delta}_1$, and $\widetilde{\delta}_2$, respectively. The unit root test statistics are then obtained from the following regression:

$$\Delta y_t = \phi \widetilde{S}_{t-1} + d_0 + d_1 \Delta \sin(2\pi kt/T) + d_2 \Delta \cos(2\pi kt/T) + \varepsilon_t. \tag{3.1.5}$$

Here, $\widetilde{S}_t$denotes the LM detrended series, which defined as:

$$\widetilde{S}_t = y_t - \widetilde{\psi} - \widetilde{\delta}_0 t - \widetilde{\delta}_1 \sin(2\pi kt/T) - \widetilde{\delta}_2 \cos(2\pi kt/T), \tag{3.1.6}$$

where $\widetilde{\psi} = y_1 - \widetilde{\delta}_0 - \widetilde{\delta}_1 \sin(2\pi k/T) - \widetilde{\delta}_2 \cos(2\pi k/T)$, and y_1 is the first observation of y_t. Subtracting $\widetilde{\psi}$ in (3.1.6) makes the initial value and the ending value of the detrended series to be zero with $\widetilde{S}_0 = 0$ and $\widetilde{S}_T = 0$. We let τ_{FLM} denote the t-statistic for the null hypothesis $\phi = 0$ in (3.1.5). The asymptotic distribution of τ_{FLM} was shown in EL Theorem 1.

We next explain how to utilize the information in non-normal errors to improve the testing power for the FLM test. We adopt the RALS estimation procedure as suggested in Im and Schmidt (2008). The RALS procedure is to augment the following term $\hat{w}_t$ to the Fourier-LM testing regression (3.1.5):

$$\hat{w}_t = h(\hat{\varepsilon}_t) - \hat{K} - \hat{\varepsilon}_t \hat{D}_2, \tag{3.1.7}$$

where $\hat{\varepsilon}_t$ denote the residuals from the FLM regression (3.1.5), $h(\hat{\varepsilon}_t) = [\hat{\varepsilon}_t^2, \hat{\varepsilon}_t^3]'$, $\hat{K} = 1/T\sum_{t=1}^{T} h(\hat{\varepsilon}_t)$, and $\hat{D}_2 = 1/T\sum_{t=1}^{T} h'(\hat{\varepsilon}_t)$. Specifically, we let $h(\hat{\varepsilon}_t) = [\hat{\varepsilon}_t^2, \hat{\varepsilon}_t^3]'$, which involves the second and third moments of the residual $\hat{\varepsilon}_t$. Then, letting $\hat{m}_j = T^{-1}\sum_{t=1}^{T}\hat{\varepsilon}_t^j$, the augmented term can be given as:

$$\hat{w}_t = [\hat{\varepsilon}_t^2 - \hat{m}_2, \hat{\varepsilon}_t^3 - \hat{m}_3 - 3\hat{m}_2\hat{\varepsilon}_t]'. \tag{3.1.8}$$

The first term in $\hat{w}_t$ is associated with the moment condition $E[(\varepsilon_t^2 - \sigma_\varepsilon^2)\widetilde{S}_{t-1}] = 0$, which is the condition of no heteroskedasticity. This condition improves the efficiency of the estimator of ϕ when the error terms are not symmetric. The second term in $\hat{w}_t$ improves efficiency unless $m_4 = 3\sigma^4$. In general, knowledge of higher moments m_{j+1} are uninformative if $m_{j+1} = j\sigma^2 m_{j-1}$. This is the redundancy condition. The normal distribution is the only distribution that satisfies the redundancy condition. However, if the distribution of the error term is not normal, the condition is not satisfied. In such cases, one may increase efficiency by augmenting the testing regression (3.1.5) with $\hat{w}_t$. Then the RALS based FLM test statistic is obtained from the regression:

$$\Delta y_t = \phi\widetilde{S}_{t-1} + d_0 + d_1\Delta\sin(2\pi kt/T) + d_2\Delta\cos(2\pi kt/T) + \hat{w}_t'\gamma + u_t. \tag{3.1.9}$$

We denote this test as RALS-FLM test and the corresponding t-statistic for $\phi = 0$ from regression (3.1.9) as $\tau_{RALS-FLM}$. Then it can be shown that the asymptotic distribution of $\tau_{RALS-LM}$ is given as follows:

$$\tau_{RALS-FLM} \to \rho\tau_{FLM} + \sqrt{1-\rho^2}Z, \tag{3.1.10}$$

where τ_{FLM} denotes the FLM distribution as defined in (3.1.5), Z indicates the standard normal distribution, and ρ reflects the relative ratio of the variances of two error terms:

$$\rho = \sigma_{\varepsilon u}/\sigma_\varepsilon\sigma_u, \tag{3.1.11}$$

ε_t and u_t are the error term in regression (3.1.5) and (3.1.9), respectively.

The critical values for the null hypothesis of a unit root of the single frequency FLM test in EL depend on the frequency k, however, our RALS-FLM test statistic depend on both the frequency k and the correlation ρ. From equation (3.1.11) we can see that ρcould takes any value between 0 and 1. In particular, if ρ = 1, which is the case when there is no extra information in the second and third moment of the error term, the distribution for the RALS-FLM statistic is the same as the distribution of FLM statistic. If ρ = 0, which is the case when the two error terms are orthogonal to each other, the distribution for RALS-FLM statistic is the same as the standard normal distribution. If ρ takes any value between 0 and 1, the RALS-FLM statistic distribution is a weighted mix of the FLM statistic distribution and a standard normal distribution. Since the critical values for the RALS-FLM test under the null hypothesis do not depend on the coefficients of the Fourier terms or other deterministic terms, we can tabulate critical values using simulations. The critical values for single Fourier frequency RALS-FLM test are reported in Table 3.1.a.

In practice, the true value of ρ is not available, we could estimate it using the residuals from regression (3.1.5) and (3.1.9):

$$\hat{\rho}^2 = \hat{\sigma}_A^2/\hat{\sigma}^2,$$

where$\hat{\sigma}^2$ is the usual estimate of the error variance from FLM regression (3.1.5), and $\hat{\sigma}_A^2$ is the estimate of the error variance in the RALS-FLM regression (3.1.9).

We next consider the RALS-FLM test with unknown deterministic trend. In this case, two issues need to be taken care of. The first issue is pretesting for a nonlinear trend. If nonlinear trend is absent from the DGP, the RALS-LM test suggested in my Essay 1will be more powerful than our RALS-FLM test. Following EL, a F test is applied to test the null hypothesis of linearity against the alternative hypothesis of a Fourier trend with given frequency k. EL denote this test as $F(k)$ and the critical values are reported in Panel b of Table 1 in EL. The second issue is selecting the frequency k that provides the best fit to the unknown nonlinear trend. Again, we can apply the data-driven grid search method suggested in EL that select the frequency minimizes the sum of squared residuals from equation (3.1.5). Logically, we need to pretest for a nonlinear trend before we model the nonlinear trend using a Fourier expansion with a frequency k. In empirical study, we need to do this reversely. To summarize, the data-driven RALS-FLM testing procedure with unknown deterministic trend can be summarized as follows:

Step 1: Estimate (3.1.5) for all integer values of k such that $1 \leq k \leq k_{max}$.[4] For each integer value of k, we simultaneously determine the optimal lags $p$using a general to specific method suggested by Ng and Perron (1995).[5] The regression with the smallest sum of squared residuals (SSR) yields $\hat{k}$ and the corresponding $\hat{p}$.The resulting test is denoted as $F(\hat{k})$.

Step 2: For the optimal $\hat{k}$ and $\hat{p}$ identified in the first step, we test for the existence of a nonlinear trend. For this, we perform the usual$F$-test for the null hypothesis of linearity (i.e. $\alpha_k = \beta_k = 0$ in equation (3.1.3)), and compare the F-statistic with the critical value for $F(\hat{k})$. If the sample $F(\hat{k})$ is sufficiently large, we can employ the RALS-FLM test with frequency $\hat{k}$ and number of lags $\hat{p}$. If the null is not rejected, then we can employ the RALS-LM unit root proposed by Essay 1 without a nonlinear trend.

The critical values of $F(\hat{k}) = MaxF(k)$ are reported in Table 3.1.b.

[4] EL suggest choosing $\hat{k}$ from the integer values 1 through 5. However, this imposes a potentially important restriction on the trigonometric component of the trend function in that it must start and end at the same position relative to the linear trend component of the DGP. To avoid this restriction, it is also possible to consider frictional frequencies.

[5] For each frequency k, we set the maximum lags to be p and augment the regression with p lags. If the t-statistic for the pth lag is significant, we set the optimal lag for frequency k equal to p. If not, we continue with setting the number of lags equal to $p-1$. The searching procedure continues until the number of lags equal to zero or the t-statistic for the mth lag is significant.

We also consider the RALS-FLM tests which allow for cumulative Fourier frequencies. The motivation of using cumulative frequencies is that they can provide a more precise approximation. We denote n as the number of frequencies used in the test. If n = 1, the test is the same as the single frequency test using k = 1. If $n \geq 2$, all the frequencies from 1 to n are used in the regression. Since the individual frequency components are orthogonal to each other, it can be easily shown that the critical values depend only on n and ρ^2. We denote the RALS-FLM test with cumulative frequencies as $\tau_{RALS-FLM}(n)$ to distinguish it from the test using a single frequency. The appropriate critical values for $\tau_{RALS-FLM}(n)$ for n = 2, 3, ρ^2 = 0, 0.1,…, 1.0, and T = 100, 200, and 2500 are reported in Table 3.2. Although using cumulative frequencies can offer more precise estimate, there is a caveat that they can yield significant loss of power due to over-fitting of the data.

3.2. The Monte Carlo Experiments

In this section, we investigate the performance of the suggested RALS-FLM test using Monte Carlo simulation experiments. All simulations are performed using 20, 000 replications with WinRATS 7.2.The DGP was given in equation (3.1.3) with the initial value e_0 follows a N(0,1) distribution. To examine the impact of the distribution of the error term on the performance of RALS-FLM test, we allow the error term to follow seven types of non-normal distribution: χ^2distribution with $df = 1, 2, 3, 4$, and t-distribution with $df = 2, 3, 4$. We also allow the error term following a standard normal distribution for comparison purpose.To conserve space, we report only results using 5% nominal critical values with sample size equal to 100 and 200.[6]

[6] The results for T = 500 and 2500 are available from the author.

To begin with, we show that the $\tau_{RALS-FLM}$ test will depend on k, but will be invariant to the magnitude of the α_k and β_k. To do this, we assume that the value of k is known a priori, and we change the combinations of α_k and β_k in the DGP. The results in Table 3.3 show that the size of our tests is close to 5% in all cases with different values of k, α_k and β_k, regardless of the distribution of the error term. The $\tau_{RALS-FLM}$ test shows a mild over rejection when the error term follows a standard normal distribution. This is expected because the RALS procedure barely gets any extra information when the error term follows a normal distribution; nevertheless, the $\tau_{RALS-FLM}$ test still augments the higher moment condition terms, which cause a loss in the testing efficiency. When the sample size is small, the power of τ_{FLM} is low. The $\tau_{RALS-FLM}$ tests show significantly improved power over the τ_{FLM} tests when the errors follow non-normal distribution with either a χ^2- or t-distribution. The power gain of the $\tau_{RALS-FLM}$ test is greater when the degrees of freedom of the χ^2 or the t-distribution are smaller, implying more asymmetric patterns or fatter tails of the error distribution. The power gain falls when the distribution of the error term towards a normal distribution. When the error term follows a normal distribution, the adjusted power of $\tau_{RALS-FLM}$ is slightly lower than the adjust power of τ_{FLM} test. The power for the $\tau_{RALS-FLM}$ test improves as $k$gets bigger; this finding is consistent with the finding in EL for the τ_{FLM} tests. When sample size increase, the size distortion becomes smaller and the power increase greatly, implying that the test is consistent. When sample size is greater than 500, the testing power for both tests exceeds 99% when θ = .9.

Next, we examine the cases where k is treated as unknown and estimated from the data. We follow the two-step procedure as discussed in section two. We test the null hypothesis that the trend in the DGP is linear by using the critical values of $F(\hat{k})$ reported in Table 3.1.b. If the null hypothesis of a linear trend is rejected, we apply the $\tau_{RALS-FLM}$ test; otherwise, we apply the τ_{FLM} test. The simulation results are reported in Table 3.4. Similar to what we find in Table 3.3, both tests display correct size for different combinations of k, α_k, and β_k. While the power of τ_{FLM} tests is robust to different distributions in the error term, $\tau_{RALS-FLM}$ tests are more powerful when the error term follows a non-normal distribution. To our surprise, tests using estimated value of k are more powerful than tests using known value of $k$when α_k, and β_k both equal to zero and the error term follows a severe non-normal distribution.

The results for tests using cumulative frequencies are reported in Table 3.5. The DGP is specified as equation (3.1.1 and 3.1.2). Here we choose $\alpha_k = \beta_k = 0$ for n = 1, 2, and 3, but the results would be invariant by using other values of α_k and β_k. The results indicate that the $\tau_{RALS-FLM}$ test shows a mild size distortion, which becomes serious as n becomes larger. While the $\tau_{RALS-FLM}$ test still shows a significant power gain when the error term follows a non-normal distribution compared with τ_{FLM} test, the power for both tests diminishes quickly as additional frequencies are added to the estimation equation. For example, when sample size is 100 and the error term follows a χ^2_1 distribution, the power for $\tau_{RALS-FLM}$ with n = 1 is 0.896, and the power drops to 0.663 if n = 2, and 0.375 if n = 3.As such, we suggest researchers use a small value of n if cumulative frequencies are needed in the estimation. In a former paper, Bierens (1997) developed a unit root test using a linear function of Chebishev polynomials to approximate the function of time. Contrary to our finding, Bierens found the testing power of his test appears to increase as n increases. We attribute this power increase to spurious rejection problem. In his test, Bierens assumes no nonlinearity under the null hypothesis. Thus rejection of the null hypothesis doesn't necessarily mean that the testing series is stationary, because there is still possibility that the series is a unit root process with nonlinear trend. Our tests allow for nonlinear trend both under the null and alternative hypotheses, and are free of this problem.

3.3. Concluding Remarks

In this essay, we develops a RALS based Fourier-LM test. While the Fourier-LM type of tests can be used to control for smooth breaks or multiple breaks of an unknown functional form, the RALS procedure can utilize additional information contained in non-normal errors. One important feature of the new test is that the knowledge of the underlying type of non-normal distribution or the precise functional form of the structural breaks is not required. By using parsimonious number of parameters to control for possible breaks in the deterministic term and use the additional information lied in nonlinear moments of the error term, the power of RALS-Fourier-LM test increase dramatically when the error term follows a non-normal distribution.

3.4. Appendix

Appendix Table 3.1 Critical Values of Transformed RALS-LM Test with Trend Break

T	R	%	ρ^2										
			0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
50	1	1	-2.333	-3.101	-3.390	-3.590	-3.745	-3.878	-3.996	-4.108	-4.222	-4.324	-4.421
		5	-1.640	-2.432	-2.736	-2.955	-3.131	-3.274	-3.404	-3.514	-3.613	-3.698	-3.790
		10	-1.286	-2.090	-2.396	-2.619	-2.799	-2.958	-3.091	-3.207	-3.309	-3.405	-3.487
	2	1	-2.333	-3.283	-3.639	-3.896	-4.113	-4.301	-4.459	-4.606	-4.752	-4.883	-5.004
		5	-1.640	-2.619	-2.995	-3.270	-3.494	-3.682	-3.848	-3.999	-4.133	-4.259	-4.377
		10	-1.286	-2.268	-2.651	-2.933	-3.162	-3.358	-3.532	-3.686	-3.824	-3.952	-4.074
100	1	1	-2.322	-3.086	-3.365	-3.564	-3.713	-3.840	-3.948	-4.046	-4.127	-4.207	-4.282
		5	-1.646	-2.427	-2.716	-2.923	-3.090	-3.233	-3.352	-3.458	-3.554	-3.639	-3.712
		10	-1.283	-2.078	-2.377	-2.598	-2.775	-2.924	-3.051	-3.165	-3.262	-3.350	-3.434
	2	1	-2.322	-3.264	-3.597	-3.843	-4.046	-4.212	-4.371	-4.498	-4.610	-4.717	-4.804
		5	-1.646	-2.604	-2.965	-3.229	-3.443	-3.622	-3.778	-3.918	-4.040	-4.146	-4.252
		10	-1.283	-2.247	-2.622	-2.899	-3.121	-3.309	-3.473	-3.621	-3.748	-3.865	-3.971
300	1	1	-2.332	-3.079	-3.337	-3.525	-3.672	-3.788	-3.894	-3.977	-4.064	-4.128	-4.188
		5	-1.642	-2.419	-2.705	-2.911	-3.077	-3.213	-3.327	-3.430	-3.518	-3.600	-3.669
		10	-1.276	-2.066	-2.369	-2.588	-2.766	-2.915	-3.039	-3.144	-3.240	-3.326	-3.400
	2	1	-2.332	-3.245	-3.582	-3.822	-4.007	-4.158	-4.288	-4.405	-4.513	-4.606	-4.683
		5	-1.642	-2.587	-2.941	-3.200	-3.405	-3.579	-3.730	-3.861	-3.977	-4.080	-4.173
		10	-1.276	-2.234	-2.604	-2.873	-3.091	-3.282	-3.440	-3.580	-3.703	-3.816	-3.913
1000	1	1	-2.345	-3.102	-3.368	-3.548	-3.688	-3.798	-3.894	-3.983	-4.049	-4.114	-4.175
		5	-1.652	-2.433	-2.725	-2.927	-3.091	-3.222	-3.333	-3.429	-3.516	-3.594	-3.654
		10	-1.291	-2.080	-2.379	-2.599	-2.771	-2.919	-3.046	-3.153	-3.245	-3.325	-3.395
	2	1	-2.345	-3.258	-3.586	-3.821	-4.006	-4.153	-4.281	-4.398	-4.496	-4.592	-4.672
		5	-1.652	-2.599	-2.953	-3.212	-3.419	-3.591	-3.741	-3.867	-3.982	-4.079	-4.158
		10	-1.291	-2.242	-2.610	-2.881	-3.100	-3.286	-3.444	-3.579	-3.703	-3.811	-3.907

Notes: T denotes the sample size; R denotes the break number; ρ^2 denotes the coefficient in equation (12). The transformation does not influence the critical value, so this table can be used on non-transformed RALS-LM test when the structural breaks are evenly distributed. When $\rho^2 = 0$, the critical values are the same with those of the standard normal distribution; when $\rho^2 = 1$, the critical values are the same with transformed LM test or non-transformed LM test with structural breaks evenly distributed in the data.

Appendix Table 3.2 Size and Power Property ($T = 100$, $R = 1$)

β	λ^*	τ	ϵ_t							
			χ^2(1)	χ^2(2)	χ^2(3)	χ^2(4)	t(2)	t(3)	t(4)	N (0,1)
Size Property										
1	.25	$\tau^*_{RALS-LM}$	0.064	0.072	0.075	0.071	0.051	0.053	0.059	0.087
		$\tau_{RALS-LM}$	0.040	0.050	0.044	0.047	0.033	0.037	0.040	0.072
		τ^*_{LM}	0.041	0.043	0.044	0.045	0.043	0.050	0.047	0.047
		τ_{LM}	0.030	0.033	0.032	0.035	0.029	0.034	0.036	0.040
		τ_{DF}	0.036	0.033	0.031	0.029	0.046	0.038	0.032	0.036
	.50	$\tau^{(*)}_{RALS-LM}$	0.045	0.050	0.058	0.063	0.042	0.048	0.057	0.092
		$\tau^{(*)}_{LM}$	0.044	0.044	0.045	0.048	0.037	0.045	0.050	0.051
		τ_{DF}	0.049	0.052	0.050	0.053	0.060	0.057	0.058	0.051
Power Property										
0.8	.25	$\tau^*_{RALS-LM}$	0.975	0.954	0.923	0.881	0.771	0.616	0.530	0.442
		$\tau_{RALS-LM}$	0.988	0.973	0.954	0.927	0.832	0.714	0.631	0.518
		τ^*_{LM}	0.363	0.365	0.365	0.377	0.366	0.365	0.371	0.370
		τ_{LM}	0.462	0.458	0.460	0.459	0.450	0.452	0.455	0.451
		τ_{DF}	0.341	0.342	0.345	0.352	0.319	0.334	0.342	0.335
	.50	$\tau^{(*)}_{RALS-LM}$	0.991	0.978	0.961	0.936	0.853	0.723	0.655	0.546
		$\tau^{(*)}_{LM}$	0.487	0.476	0.485	0.489	0.478	0.470	0.482	0.478
		τ_{DF}	0.364	0.362	0.372	0.380	0.339	0.364	0.373	0.362
.9	.25	$\tau^*_{RALS-LM}$	0.880	0.771	0.679	0.600	0.463	0.293	0.224	0.205
		$\tau_{RALS-LM}$	0.913	0.802	0.692	0.598	0.460	0.281	0.219	0.197
		τ^*_{LM}	0.112	0.120	0.123	0.129	0.119	0.127	0.128	0.138
		τ_{LM}	0.106	0.126	0.126	0.124	0.113	0.122	0.123	0.133
		τ_{DF}	0.087	0.089	0.089	0.093	0.097	0.095	0.091	0.096
	.5	$\tau^{(*)}_{RALS-LM}$	0.909	0.811	0.697	0.613	0.474	0.306	0.244	0.230
		$\tau^{(*)}_{LM}$	0.131	0.140	0.142	0.140	0.130	0.143	0.141	0.154
		τ_{DF}	0.109	0.114	0.116	0.111	0.119	0.122	0.111	0.123

Notes: β denotes the coefficient for the DGP; λ^* denotes the break location which defined as $\lambda^* = T_B/T$, where T_B is the break location; τ denotes the test statistics. $\tau_{RALS-LM}$, τ_{LM}, τ_{DF} denote the test statistic for RALS-LM test, LM test and DF test, respectively; * means transformed test. When $\lambda^* = 0.5$, the size and power for the transformed tests and untransformed tests are the same, we report them together to save space.

Table 3.1.a Critical Values of $\tau_{RALS-FLM}$ Tests

T	k	%	ρ^2										
			.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.
100	1	1	-2.332	-3.209	-3.529	-3.764	-3.955	-4.11	-4.246	-4.373	-4.497	-4.605	-4.692
		5	-1.652	-2.544	-2.886	-3.134	-3.335	-3.503	-3.652	-3.782	-3.902	-4.008	-4.112
		10	-1.287	-2.189	-2.539	-2.793	-3.003	-3.183	-3.34	-3.48	-3.605	-3.717	-3.822
	2	1	-2.332	-2.974	-3.233	-3.413	-3.567	-3.698	-3.819	-3.935	-4.032	-4.135	-4.221
		5	-1.652	-2.307	-2.561	-2.749	-2.905	-3.038	-3.16	-3.271	-3.377	-3.48	-3.573
		10	-1.287	-1.953	-2.211	-2.4	-2.557	-2.694	-2.814	-2.926	-3.034	-3.134	-3.224
	3	1	-2.332	-2.923	-3.138	-3.286	-3.408	-3.518	-3.618	-3.713	-3.799	-3.887	-3.959
		5	-1.652	-2.251	-2.475	-2.639	-2.771	-2.88	-2.974	-3.067	-3.145	-3.22	-3.303
		10	-1.287	-1.899	-2.13	-2.298	-2.431	-2.543	-2.642	-2.73	-2.812	-2.889	-2.961
200	1	1	-2.307	-3.191	-3.513	-3.748	-3.928	-4.078	-4.206	-4.325	-4.431	-4.527	-4.623
		5	-1.644	-2.527	-2.87	-3.115	-3.315	-3.487	-3.634	-3.759	-3.878	-3.978	-4.077
		10	-1.283	-2.182	-2.529	-2.783	-2.992	-3.167	-3.323	-3.462	-3.583	-3.699	-3.798
	2	1	-2.307	-2.953	-3.201	-3.38	-3.53	-3.663	-3.781	-3.896	-3.996	-4.098	-4.195
		5	-1.644	-2.304	-2.559	-2.748	-2.904	-3.04	-3.16	-3.273	-3.374	-3.472	-3.56
		10	-1.283	-1.948	-2.208	-2.403	-2.56	-2.697	-2.819	-2.93	-3.037	-3.132	-3.22
	3	1	-2.307	-2.896	-3.119	-3.28	-3.404	-3.512	-3.61	-3.708	-3.802	-3.888	-3.973
		5	-1.644	-2.251	-2.479	-2.64	-2.774	-2.885	-2.984	-3.075	-3.155	-3.232	-3.3
		10	-1.283	-1.897	-2.131	-2.303	-2.441	-2.558	-2.659	-2.748	-2.827	-2.901	-2.974
2500	1	1	-2.32	-3.185	-3.493	-3.727	-3.905	-4.051	-4.183	-4.29	-4.392	-4.48	-4.541
		5	-1.641	-2.52	-2.854	-3.096	-3.295	-3.461	-3.609	-3.737	-3.846	-3.946	-4.032
		10	-1.278	-2.175	-2.523	-2.779	-2.987	-3.162	-3.315	-3.446	-3.565	-3.675	-3.77
	2	1	-2.32	-2.974	-3.215	-3.4	-3.547	-3.672	-3.782	-3.886	-3.974	-4.055	-4.141
		5	-1.641	-2.309	-2.557	-2.741	-2.895	-3.026	-3.147	-3.257	-3.353	-3.452	-3.542
		10	-1.278	-1.946	-2.205	-2.396	-2.554	-2.692	-2.814	-2.924	-3.027	-3.125	-3.218
	3	1	-2.32	-2.918	-3.127	-3.279	-3.399	-3.515	-3.623	-3.719	-3.801	-3.867	-3.935
		5	-1.641	-2.26	-2.484	-2.646	-2.778	-2.89	-2.985	-3.07	-3.154	-3.231	-3.305
		10	-1.278	-1.897	-2.133	-2.303	-2.441	-2.56	-2.663	-2.754	-2.834	-2.909	-2.987

Note: To save space, we only report the critical values for T = 100, 200, and 2500, and k = 1, 2, and 3.

Table 3.1.b Critical Values of $F(\hat{k}) = MaxF(k)$

T = 100			T = 200			T = 500			T = 2500		
1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
6.940	5.037	4.227	6.458	4.808	4.069	6.399	4.646	3.907	6.194	4.615	3.891

Table 3.2 Critical Values of $\tau_{RALS-FLM}(n)$

T	n	%	ρ^2										
			.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.
100	2	1	-2.332	-3.456	-3.887	-4.193	-4.449	-4.669	-4.866	-5.035	-5.188	-5.334	-5.498
		5	-1.652	-2.784	-3.231	-3.559	-3.826	-4.056	-4.26	-4.444	-4.611	-4.763	-4.907
		10	-1.287	-2.43	-2.882	-3.216	-3.493	-3.73	-3.941	-4.13	-4.303	-4.463	-4.617
	3	1	-2.332	-3.663	-4.176	-4.556	-4.866	-5.139	-5.383	-5.609	-5.818	-6.001	-6.181
		5	-1.652	-2.99	-3.521	-3.912	-4.244	-4.527	-4.777	-5.002	-5.209	-5.401	-5.585
		10	-1.287	-2.642	-3.177	-3.576	-3.909	-4.201	-4.457	-4.695	-4.907	-5.103	-5.286
200	2	1	-2.307	-3.416	-3.832	-4.137	-4.384	-4.598	-4.783	-4.952	-5.109	-5.249	-5.391
		5	-1.644	-2.769	-3.209	-3.538	-3.802	-4.025	-4.221	-4.397	-4.56	-4.705	-4.839
		10	-1.283	-2.422	-2.868	-3.201	-3.474	-3.71	-3.92	-4.105	-4.271	-4.424	-4.563
	3	1	-2.307	-3.616	-4.129	-4.503	-4.807	-5.075	-5.301	-5.506	-5.69	-5.869	-6.037
		5	-1.644	-2.967	-3.492	-3.879	-4.201	-4.475	-4.719	-4.939	-5.137	-5.321	-5.483
		10	-1.283	-2.619	-3.148	-3.548	-3.875	-4.159	-4.407	-4.634	-4.839	-5.032	-5.207
2500	2	1	-2.32	-3.422	-3.841	-4.138	-4.371	-4.576	-4.755	-4.906	-5.047	-5.162	-5.271
		5	-1.641	-2.76	-3.19	-3.51	-3.766	-3.985	-4.181	-4.352	-4.511	-4.648	-4.774
		10	-1.278	-2.408	-2.85	-3.179	-3.447	-3.678	-3.882	-4.064	-4.226	-4.373	-4.508
	3	1	-2.32	-3.614	-4.109	-4.463	-4.759	-5.008	-5.226	-5.421	-5.6	-5.747	-5.88
		5	-1.641	-2.957	-3.466	-3.85	-4.16	-4.425	-4.663	-4.869	-5.059	-5.229	-5.388
		10	-1.278	-2.602	-3.128	-3.519	-3.84	-4.115	-4.361	-4.58	-4.783	-4.963	-5.131

Note: Critical values for n = 1 are omitted because they are the same with $\tau_{RALS-FLM}$ test using single frequency with k = 1.

Table 3.3 Finite Sample Performance with Known Frequencies (T = 100)

DGP				TESTS	Distribution of Error							
k	α_k	β_k	θ		χ_1^2	χ_2^2	χ_3^2	χ_4^2	t_2	t_3	t_4	$N(0,1)$
1	0	0	1.0	$\tau_{RALS-FLM}$	0.036	0.047	0.059	0.065	0.042	0.049	0.053	0.100
				τ_{FLM}	0.041	0.043	0.045	0.047	0.038	0.045	0.047	0.051
			0.9(A)	$\tau_{RALS-FLM}$	0.910	0.757	0.596	0.473	0.428	0.221	0.169	0.106
				τ_{FLM}	0.125	0.121	0.118	0.120	0.120	0.119	0.119	0.113
	0	5	1.0	$\tau_{RALS-FLM}$	0.036	0.047	0.059	0.065	0.042	0.049	0.053	0.100
				τ_{FLM}	0.041	0.043	0.045	0.047	0.038	0.045	0.047	0.051
			0.9(A)	$\tau_{RALS-FLM}$	0.910	0.757	0.596	0.473	0.428	0.221	0.169	0.106
				τ_{FLM}	0.125	0.121	0.118	0.120	0.120	0.119	0.119	0.113
	3	0	1.0	$\tau_{RALS-FLM}$	0.036	0.047	0.059	0.065	0.042	0.049	0.053	0.100
				τ_{FLM}	0.041	0.043	0.045	0.047	0.038	0.045	0.047	0.051
			0.9(A)	$\tau_{RALS-FLM}$	0.910	0.757	0.596	0.473	0.428	0.221	0.169	0.106
				τ_{FLM}	0.125	0.121	0.118	0.120	0.120	0.119	0.119	0.113
	3	5	1.0	$\tau_{RALS-FLM}$	0.036	0.047	0.059	0.065	0.042	0.049	0.053	0.100
				τ_{FLM}	0.041	0.043	0.045	0.047	0.038	0.045	0.047	0.051
			0.9(A)	$\tau_{RALS-FLM}$	0.910	0.757	0.596	0.473	0.428	0.221	0.169	0.106
				τ_{FLM}	0.125	0.121	0.118	0.120	0.120	0.119	0.119	0.113
2	0	0	1.0	$\tau_{RALS-FLM}$	0.044	0.051	0.059	0.063	0.043	0.044	0.047	0.082
				τ_{FLM}	0.045	0.045	0.047	0.046	0.040	0.045	0.047	0.050
			0.9(A)	$\tau_{RALS-FLM}$	0.941	0.849	0.736	0.645	0.598	0.398	0.302	0.196
				τ_{FLM}	0.219	0.223	0.215	0.215	0.224	0.222	0.220	0.210
	0	5	1.0	$\tau_{RALS-FLM}$	0.044	0.051	0.059	0.063	0.043	0.044	0.047	0.082
				τ_{FLM}	0.045	0.045	0.047	0.046	0.040	0.045	0.047	0.050
			0.9(A)	$\tau_{RALS-FLM}$	0.941	0.849	0.736	0.645	0.598	0.398	0.302	0.196
				τ_{FLM}	0.219	0.223	0.215	0.215	0.224	0.222	0.220	0.210
	3	0	1.0	$\tau_{RALS-FLM}$	0.044	0.051	0.059	0.063	0.043	0.044	0.047	0.082
				τ_{FLM}	0.045	0.045	0.047	0.046	0.040	0.045	0.047	0.050
			0.9(A)	$\tau_{RALS-FLM}$	0.941	0.849	0.736	0.645	0.598	0.398	0.302	0.196
				τ_{FLM}	0.219	0.223	0.215	0.215	0.224	0.222	0.220	0.210
	3	5	1.0	$\tau_{RALS-FLM}$	0.044	0.051	0.059	0.063	0.043	0.044	0.047	0.082
				τ_{FLM}	0.045	0.045	0.047	0.046	0.040	0.045	0.047	0.050
			0.9(A)	$\tau_{RALS-FLM}$	0.941	0.849	0.736	0.645	0.598	0.398	0.302	0.196
				τ_{FLM}	0.219	0.223	0.215	0.215	0.224	0.222	0.220	0.210

Note: 0.9 (A) means size adjusted power.

Table 3.3 (Continued), T = 200

DGP				TESTS	Distribution of Error							
k	α_k	β_k	θ		χ_1^2	χ_2^2	χ_3^2	χ_4^2	t_2	t_3	t_4	$N(0,1)$
1	0	0	1.0	$\tau_{RALS-FLM}$	0.042	0.051	0.051	0.055	0.042	0.042	0.046	0.069
				τ_{FLM}	0.042	0.048	0.046	0.046	0.039	0.043	0.048	0.050
			0.9(A)	$\tau_{RALS-FLM}$	0.999	0.994	0.981	0.953	0.905	0.732	0.600	0.350
				τ_{FLM}	0.406	0.391	0.401	0.399	0.412	0.402	0.386	0.380
	0	5	1.0	$\tau_{RALS-FLM}$	0.042	0.051	0.051	0.055	0.042	0.042	0.046	0.069
				τ_{FLM}	0.042	0.048	0.046	0.046	0.039	0.043	0.048	0.050
			0.9(A)	$\tau_{RALS-FLM}$	0.999	0.994	0.981	0.953	0.905	0.732	0.600	0.350
				τ_{FLM}	0.406	0.391	0.401	0.399	0.412	0.402	0.386	0.380
	3	0	1.0	$\tau_{RALS-FLM}$	0.042	0.051	0.051	0.055	0.042	0.042	0.046	0.069
				τ_{FLM}	0.042	0.048	0.046	0.046	0.039	0.043	0.048	0.050
			0.9(A)	$\tau_{RALS-FLM}$	0.999	0.994	0.981	0.953	0.905	0.732	0.600	0.350
				τ_{FLM}	0.406	0.391	0.401	0.399	0.412	0.402	0.386	0.380
	3	5	1.0	$\tau_{RALS-FLM}$	0.042	0.051	0.051	0.055	0.042	0.042	0.046	0.069
				τ_{FLM}	0.042	0.048	0.046	0.046	0.039	0.043	0.048	0.050
			0.9(A)	$\tau_{RALS-FLM}$	0.999	0.994	0.981	0.953	0.905	0.732	0.600	0.350
				τ_{FLM}	0.406	0.391	0.401	0.399	0.412	0.402	0.386	0.380
2	0	0	1.0	$\tau_{RALS-FLM}$	0.045	0.053	0.051	0.057	0.041	0.043	0.043	0.063
				τ_{FLM}	0.045	0.048	0.048	0.049	0.040	0.046	0.047	0.050
			0.9(A)	$\tau_{RALS-FLM}$	0.998	0.995	0.988	0.974	0.954	0.864	0.804	0.581
				τ_{FLM}	0.628	0.619	0.611	0.612	0.682	0.626	0.626	0.609
	0	5	1.0	$\tau_{RALS-FLM}$	0.045	0.053	0.051	0.057	0.041	0.043	0.043	0.063
				τ_{FLM}	0.045	0.048	0.048	0.049	0.040	0.046	0.047	0.050
			0.9(A)	$\tau_{RALS-FLM}$	0.998	0.995	0.988	0.974	0.954	0.864	0.804	0.581
				τ_{FLM}	0.628	0.619	0.611	0.612	0.682	0.626	0.626	0.609
	3	0	1.0	$\tau_{RALS-FLM}$	0.045	0.053	0.051	0.057	0.041	0.043	0.043	0.063
				τ_{FLM}	0.045	0.048	0.048	0.049	0.040	0.046	0.047	0.050
			0.9(A)	$\tau_{RALS-FLM}$	0.998	0.995	0.988	0.974	0.954	0.864	0.804	0.581
				τ_{FLM}	0.628	0.619	0.611	0.612	0.682	0.626	0.626	0.609
	3	5	1.0	$\tau_{RALS-FLM}$	0.045	0.053	0.051	0.057	0.041	0.043	0.043	0.063
				τ_{FLM}	0.045	0.048	0.048	0.049	0.040	0.046	0.047	0.050
			0.9(A)	$\tau_{RALS-FLM}$	0.998	0.995	0.988	0.974	0.954	0.864	0.804	0.581
				τ_{FLM}	0.628	0.619	0.611	0.612	0.682	0.626	0.626	0.609

Table 3.4 Finite Sample Performance of Tests using $F(\hat{k})$-test, T = 100

DGP				TESTS	Distribution of Error							
k	α_k	β_k	θ		χ_1^2	χ_2^2	χ_3^2	χ_4^2	t_2	t_3	t_4	$N(0,1)$
1	0	0	1.0	$\tau_{RALS-FLM}$	0.054	0.067	0.078	0.083	0.055	0.067	0.076	0.121
				τ_{FLM}	0.076	0.077	0.078	0.081	0.068	0.074	0.077	0.084
			0.9(A)	$\tau_{RALS-FLM}$	0.969	0.887	0.747	0.613	0.559	0.263	0.151	0.072
				τ_{FLM}	0.064	0.068	0.064	0.065	0.070	0.069	0.071	0.064
	0	5	1.0	$\tau_{RALS-FLM}$	0.111	0.091	0.083	0.077	0.058	0.048	0.054	0.091
				τ_{FLM}	0.044	0.047	0.049	0.046	0.058	0.044	0.048	0.050
			0.9(A)	$\tau_{RALS-FLM}$	0.489	0.385	0.323	0.279	0.290	0.196	0.151	0.113
				τ_{FLM}	0.130	0.129	0.123	0.130	0.071	0.127	0.129	0.124
	3	0	1.0	$\tau_{RALS-FLM}$	0.139	0.105	0.091	0.086	0.054	0.054	0.060	0.098
				τ_{FLM}	0.055	0.056	0.059	0.060	0.065	0.055	0.060	0.062
			0.9(A)	$\tau_{RALS-FLM}$	0.509	0.393	0.301	0.225	0.435	0.147	0.107	0.084
				τ_{FLM}	0.097	0.096	0.093	0.094	0.065	0.100	0.094	0.093
	3	5	1.0	$\tau_{RALS-FLM}$	0.094	0.082	0.079	0.074	0.057	0.048	0.054	0.090
				τ_{FLM}	0.043	0.046	0.048	0.046	0.053	0.043	0.047	0.049
			0.9(A)	$\tau_{RALS-FLM}$	0.580	0.467	0.392	0.339	0.278	0.204	0.157	0.113
				τ_{FLM}	0.126	0.127	0.122	0.127	0.085	0.122	0.124	0.124
2	0	0	1.0	$\tau_{RALS-FLM}$	0.054	0.067	0.078	0.083	0.055	0.067	0.076	0.121
				τ_{FLM}	0.076	0.077	0.078	0.081	0.068	0.074	0.077	0.084
			0.9(A)	$\tau_{RALS-FLM}$	0.969	0.887	0.747	0.613	0.559	0.263	0.151	0.072
				τ_{FLM}	0.064	0.068	0.064	0.065	0.070	0.069	0.071	0.064
	0	5	1.0	$\tau_{RALS-FLM}$	0.050	0.053	0.059	0.063	0.046	0.046	0.050	0.077
				τ_{FLM}	0.045	0.046	0.046	0.048	0.038	0.044	0.047	0.048
			0.9(A)	$\tau_{RALS-FLM}$	0.886	0.792	0.703	0.609	0.435	0.360	0.285	0.195
				τ_{FLM}	0.224	0.220	0.223	0.211	0.133	0.222	0.213	0.215
	3	0	1.0	$\tau_{RALS-FLM}$	0.089	0.072	0.068	0.066	0.047	0.043	0.045	0.065
				τ_{FLM}	0.039	0.039	0.039	0.043	0.048	0.039	0.041	0.042
			0.9(A)	$\tau_{RALS-FLM}$	0.659	0.537	0.452	0.388	0.502	0.280	0.226	0.164
				τ_{FLM}	0.185	0.186	0.184	0.179	0.088	0.190	0.187	0.174
	3	5	1.0	$\tau_{RALS-FLM}$	0.044	0.052	0.057	0.063	0.046	0.046	0.051	0.079
				τ_{FLM}	0.046	0.046	0.047	0.048	0.037	0.045	0.048	0.048
			0.9(A)	$\tau_{RALS-FLM}$	0.919	0.832	0.729	0.629	0.439	0.361	0.286	0.193
				τ_{FLM}	0.218	0.221	0.217	0.213	0.161	0.222	0.213	0.213

Table 3.4 (Continued), T = 200

DGP				TESTS	Distribution of Error							
k	α_k	β_k	θ		χ_1^2	χ_2^2	χ_3^2	χ_4^2	t_2	t_3	t_4	$N(0,1)$
1	0	0	1.0	$\tau_{RALS-FLM}$	0.053	0.066	0.072	0.073	0.057	0.062	0.068	0.103
				τ_{FLM}	0.078	0.078	0.079	0.078	0.067	0.076	0.082	0.086
			0.9(A)	$\tau_{RALS-FLM}$	0.999	0.995	0.987	0.972	0.936	0.794	0.630	0.275
				τ_{FLM}	0.286	0.291	0.273	0.284	0.322	0.308	0.282	0.278
	0	5	1.0	$\tau_{RALS-FLM}$	0.141	0.103	0.086	0.078	0.057	0.047	0.049	0.074
				τ_{FLM}	0.053	0.054	0.057	0.054	0.060	0.052	0.056	0.058
			0.9(A)	$\tau_{RALS-FLM}$	0.755	0.666	0.597	0.533	0.726	0.497	0.436	0.358
				τ_{FLM}	0.395	0.394	0.391	0.389	0.139	0.399	0.390	0.382
	3	0	1.0	$\tau_{RALS-FLM}$	0.112	0.090	0.079	0.080	0.056	0.054	0.061	0.087
				τ_{FLM}	0.065	0.067	0.065	0.070	0.064	0.064	0.070	0.069
			0.9(A)	$\tau_{RALS-FLM}$	0.928	0.856	0.770	0.680	0.868	0.369	0.259	0.178
				τ_{FLM}	0.187	0.180	0.182	0.178	0.237	0.199	0.185	0.179
	3	5	1.0	$\tau_{RALS-FLM}$	0.131	0.102	0.088	0.078	0.056	0.045	0.047	0.070
				τ_{FLM}	0.050	0.051	0.053	0.053	0.059	0.049	0.052	0.055
			0.9(A)	$\tau_{RALS-FLM}$	0.765	0.695	0.651	0.614	0.676	0.577	0.507	0.379
				τ_{FLM}	0.418	0.418	0.413	0.407	0.156	0.412	0.409	0.403
2	0	0	1.0	$\tau_{RALS-FLM}$	0.053	0.066	0.072	0.073	0.057	0.062	0.068	0.103
				τ_{FLM}	0.078	0.078	0.079	0.078	0.067	0.076	0.082	0.086
			0.9(A)	$\tau_{RALS-FLM}$	0.999	0.995	0.987	0.972	0.936	0.794	0.630	0.275
				τ_{FLM}	0.286	0.291	0.273	0.284	0.322	0.308	0.282	0.278
	0	5	1.0	$\tau_{RALS-FLM}$	0.084	0.071	0.066	0.058	0.050	0.040	0.040	0.057
				τ_{FLM}	0.043	0.040	0.042	0.043	0.046	0.043	0.043	0.046
			0.9(A)	$\tau_{RALS-FLM}$	0.942	0.918	0.891	0.863	0.797	0.810	0.743	0.592
				τ_{FLM}	0.618	0.643	0.623	0.621	0.251	0.622	0.624	0.611
	3	0	1.0	$\tau_{RALS-FLM}$	0.105	0.082	0.072	0.065	0.054	0.040	0.041	0.055
				τ_{FLM}	0.043	0.042	0.044	0.041	0.056	0.042	0.041	0.046
			0.9(A)	$\tau_{RALS-FLM}$	0.927	0.872	0.809	0.766	0.887	0.576	0.446	0.303
				τ_{FLM}	0.314	0.318	0.302	0.311	0.294	0.357	0.322	0.304
	3	5	1.0	$\tau_{RALS-FLM}$	0.069	0.064	0.062	0.056	0.050	0.043	0.041	0.061
				τ_{FLM}	0.044	0.042	0.043	0.045	0.041	0.045	0.045	0.049
			0.9(A)	$\tau_{RALS-FLM}$	0.973	0.962	0.950	0.936	0.776	0.844	0.784	0.590
				τ_{FLM}	0.631	0.653	0.637	0.629	0.282	0.623	0.631	0.612

Table 3.5 Finite Sample Performance using Cumulative Frequencies

T	n	θ	TESTS	Distribution of Error							
				χ^2_1	χ^2_2	χ^2_3	χ^2_4	t_2	t_3	t_4	N(0,1)
100	1	1.0	$\tau_{RALS-FLM}(n)$	0.036	0.047	0.059	0.065	0.042	0.049	0.053	0.100
			$\tau_{FLM}(n)$	0.041	0.043	0.045	0.047	0.038	0.045	0.047	0.051
		0.9(A)	$\tau_{RALS-FLM}(n)$	0.910	0.757	0.596	0.473	0.428	0.221	0.169	0.106
			$\tau_{FLM}(n)$	0.125	0.121	0.118	0.120	0.120	0.119	0.119	0.113
	2	1.0	$\tau_{RALS-FLM}(n)$	0.021	0.035	0.048	0.055	0.044	0.052	0.058	0.120
			$\tau_{FLM}(n)$	0.041	0.043	0.044	0.043	0.037	0.043	0.046	0.051
		0.9(A)	$\tau_{RALS-FLM}(n)$	0.728	0.508	0.347	0.280	0.240	0.119	0.100	0.073
			$\tau_{FLM}(n)$	0.082	0.090	0.087	0.093	0.087	0.084	0.087	0.081
	3	1.0	$\tau_{RALS-FLM}(n)$	0.011	0.024	0.037	0.049	0.050	0.057	0.065	0.133
			$\tau_{FLM}(n)$	0.039	0.046	0.046	0.047	0.037	0.044	0.047	0.051
		0.9(A)	$\tau_{RALS-FLM}(n)$	0.479	0.289	0.206	0.164	0.136	0.084	0.075	0.063
			$\tau_{FLM}(n)$	0.075	0.074	0.072	0.072	0.073	0.076	0.075	0.068
200	1	1.0	$\tau_{RALS-FLM}(n)$	0.042	0.051	0.051	0.055	0.042	0.042	0.046	0.069
			$\tau_{FLM}(n)$	0.042	0.048	0.046	0.046	0.039	0.043	0.048	0.050
		0.9(A)	$\tau_{RALS-FLM}(n)$	0.999	0.994	0.981	0.953	0.905	0.732	0.600	0.350
			$\tau_{FLM}(n)$	0.406	0.391	0.401	0.399	0.412	0.402	0.386	0.380
	2	1.0	$\tau_{RALS-FLM}(n)$	0.029	0.040	0.042	0.050	0.040	0.042	0.045	0.073
			$\tau_{FLM}(n)$	0.041	0.042	0.049	0.048	0.040	0.046	0.047	0.048
		0.9(A)	$\tau_{RALS-FLM}(n)$	0.995	0.972	0.927	0.853	0.784	0.513	0.386	0.210
			$\tau_{FLM}(n)$	0.241	0.238	0.232	0.224	0.231	0.226	0.231	0.226
	3	1.0	$\tau_{RALS-FLM}(n)$	0.019	0.031	0.037	0.044	0.044	0.043	0.045	0.075
			$\tau_{FLM}(n)$	0.043	0.045	0.046	0.049	0.041	0.045	0.047	0.047
		0.9(A)	$\tau_{RALS-FLM}(n)$	0.982	0.917	0.815	0.704	0.628	0.350	0.246	0.151
			$\tau_{FLM}(n)$	0.164	0.164	0.159	0.151	0.149	0.155	0.157	0.156

CONCLUSION

In this dissertation, we have tried to find ways to improve the power of unit root tests as well as deal with unknown structural breaks. Researchers dealing with empirical data usually have difficulties distinguish the time series they are dealing with is best characterized by a unit root or a stationary process due to the low power problem of current unit root tests. As such, the seeking of more powerful unit root tests is not a trivial concern.

In the first essay, we have suggested new unit root tests that are more powerful in the presence of non-normal errors. Although usual linear ordinary least square estimation is still the most efficient estimation under certain assumptions that do not include the normality assumption, this does not necessarily mean that the non-normal information in the error term is useless. The information of non-normal errors is utilized in a linearized testing regression. The suggested tests have the same property as the tests based on the generalized methods of moments where nonlinear moment conditions of the residuals are employed to capture unknown forms of non-normal distributions. To do this, we adopt a two-step procedure following the RALS method of Im and Schmidt (2008), which can make use of nonlinear moment conditions driven by non-normal errors. The idea is simple and intuitive. If the errors follows a non-normal distribution, the higher moments of the residual will contain information on the nature of the non-normal errors. If we can utilize this information, we can potentially obtain more powerful unit root tests. One advantage of RALS-LM test is that it is quite general. The knowledge of the non-normal errors is not required, and the nonlinear moment conditions associated with the non-normal distribution of the error term are generated automatically from the regression. Then these nonlinear moment terms are augmented to the testing regression to reduce the variance of the residual in the original regression. Since these nonlinear condition moments terms are orthogonal to the regressors in the original regression, the estimation of the coefficients is not changed. However, the smaller variance in the new residuals leads to a smaller confidence interval of the coefficients and thus makes the estimator more efficient. Another nontrivial advantage of the proposed tests is that the testing procedure is linear although we utilize the information embedded in the nonlinear moments of the error term. Lee, Meng and Lee (2011) found by Monte Carlo simulation that popular nonlinear unit root tests lost power significantly when the error term follows a non-normal distribution. However, this is not a problem in the linear models due to central limit theorem. Last but not the least important, the RALS-LM unit root tests inherited the excellent property from LM tests that they are invariant to the nuisance parameter denoting the location of level-break. This feature is especially important for practitioners when multiple level breaks are found. One set of critical value is good for various number and/or various locations of level-breaks. One caveat of RALS-LM test is that it is less powerful than usual LM test when the error term does follow a normal distribution. But the power loss is marginal and can be ignored without causing too much concern. Another caveat of the RALS-LM unit root tests inherits from LM unit root tests. While DF type unit root tests become more powerful as the initial value gets large, LM and DF-GLS type tests lost power completely as the initial gets large. Unreported Monte Carlo simulation results show this property and provide a warning to uses of RALS_LM tests. As such, we believe that it will be desirable to consider a strategy to exploit the desirable properties of both RALS-LM tests and

RALS-DF tests. At minimum, a strategy of a union of rejection decision rules can be considered. Such as strategy was advocated by Harvey *et al*. (2009) and Harvey and Leybourne (2005) when choosing between DF-GLS and DF tests. This topic remains for future research. Moreover, it is also interesting to extend the RALS-LM test to panel unit root tests.

The second essay tests the Prebisch-Singer hypothesis by re-examining the paths of relative primary commodity prices that are known to exhibit multiple trend breaks. To address the issue more properly, we improved the RALS-LM unit root tests proposed in first essay by allowing for multiple trend breaks using a simple transformation to solve the nuisance parameter problem. The suggested new tests have good size property and become more powerful when the error terms follows a non-normal distribution. When applying the RALS-LM unit root tests with multiple trend breaks to the newly extended Grilli and Yang index of 24 commodity series from 1900 to 2007, we found clear evidence of non-normal errors. We find more price series are characterized as trend stationary with one or two trend breaks, that 21 out of the 24 commodity prices are found to be stationary around a broken trend, implying that shocks to these commodities tend to be transitory. However, we find less commodity price series display significant negative trend more than 50% of the examined time period, yet, more price series display a significant positive trend or no significant trend. Our findings provide little evidence to support the Prebisch-Singer hypothesis. Our finding provide significant policy implications to the policy makers, especially those in the developing countries. For those stationary commodities, fiscal policies and/or monetary policies aimed to impact the price of these commodities will take effect only for a short time period, in the long run, the price will go back to it is long run mean or trend. For those countries which produce commodities have a significant decline in the relative price, it is beneficial to switch to the production and export of commodities with a significant positive trend or manufactured goods, and import these commodities with significant negative trend. These countries will benefit from these policies in the long run.

The third essay focus on the modeling of unknown forms of structural breaks. This essay extends the Fourier Lagrange Multiplier (FLM) unit root tests of Enders and Lee (2012a) by using the RALS estimation procedure of Im and Schmidt (2008). While the FLM type of tests can be used to control for smooth structural breaks of an unknown functional form, the RALS procedure can utilize additional higher-moment information contained in non-normal errors. For these new tests, knowledge of the underlying type of non-normal distribution of the error term or the precise functional form of the structure breaks is not required. By using parsimonious number of parameters to control for possible breaks in the deterministic term and use the additional information lied in nonlinear moments of the error term, the power of RALS-FLM test increase dramatically when the error term follows a non-normal distribution.

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