

COORDINATION OF PRICING, SOURCING AND PRODUCT DESIGN DECISIONS
IN A SUPPLY CHAIN

by

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ABSTRACT

Supply chain management is more than a movement advocated by a few pioneers today. Supply chain performance can be affected by a large set of factors. This dissertation is intended to identify a number of typical application scenarios and focus on decision issues integrating pricing, sourcing, product attributes, evolving partnerships between firms. It consists of three manuscripts to be submitted for journal publication.

The first paper studies the coordination of pricing and sourcing in two scenarios where multiple suppliers and multiple retail channels are involved. This study investigates the cross price effects between retail channels on the overall profitability. In addition to the analytical analysis, a number of numerical experiments are conducted to investigate realistic issues decision makers may encounter.

The second paper studies a supply chain in which a manufacturer sells a configurable product through a retailer. We take the configurable product as a parameterized product and examine the impact of such a feature decision on equilibrium between the manufacturer and the retailer. The analysis considers three different cost functions: (1) linear cost function, (2) quadratic cost function and (3) exponential cost function and examines how the cost functions affect the optimal solutions.

The third paper presents a novel framework in which supply chain structure evolves from one stage to the next in terms of changing memberships and business partnerships between members. All of the research works so far on supply chain coordination assumes a *static* supply chain structure, which remains the same throughout the sequence of events. Among a large variety of possible evolving scenarios, this research focuses on a case in which a second manufacturer joins a supply chain initially established with one manufacturer and one retailer. The two partnerships are established one after another on two competing products. Compatibility between demand models in two stages are established. Based on the analytical non-closed form solution, a number of numerical experiments are developed to demonstrate the impact on the supply chain performance.

LIST OF ABBREVIATIONS AND SYMBOLS

α_j	base demand for channel
β_{jk}	cross-price elasticity for channel j, k
x_{ij}	retail channels j 's order quantity from supplier i
p_j	retail price at retail channel j
m	the number of suppliers
n	the number of retail channels
A	the market size or the total maximum demand;
K	the capacity ratio
C_i	the capacity of supplier i
\bar{c}	average unit cost
δ	cost differentiation factor
c_{ij}	unit cost for a unit by supplier i for retailer j
b	base factor for price elasticity in Chapter 2
β_{jj}	self-price elasticity of demand
I	strength of cross-price effect
u_{jk}	a random factor matrix
p	retail price
x	a product feature
d	deterministic demand
C	manufacturing cost
κ	feature sensitivity factor
β	price sensitivity factor
D	the stochastic demand
c	unit manufacturing cost
q	the mean value of the demand D
ξ	the random component of D
φ	a ratio parameter
w	the wholesale price

b	the buyback credit in Chapter 4
\bar{p}	the upper bound of p
z	safety stock factor
a	the market segment size
θ_i	coefficient on the quantity of product i in inverse demand.
γ	coefficient on the other product's quantity in inverse demand.
β_i	the price sensitivity factor for one-product case
β_{ij}	the price sensitivity factor for two-product case
τ	the cross price sensitivity factor
$\hat{?}$	the corresponding symbol "?" in stage 2 in Chapter 4

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Chapter 1

Introduction

Supply chain management is more than a movement advocated by a few pioneers today. It is a common strategic as well as operational revolution for business activities. Better understanding of business principles helps the widely acceptance that coordination along a supply chain can achieve a win-win situation. In the meanwhile, the penetration of Internet and information technology into every corner of our life enables a large scale of cooperation between multiple business units. Supply chain management has gone beyond the early stage when only basic issues being discussed. More realistic application scenarios are to be identified and investigated.

Supply chain management as an operational form to build partnerships and to coordinate business activities between firms has gained substantial attention in both industry and academia. A supply chain can be specified with a bunch of decision variables, parameters and the supply chain structure. Changes to any component above as well as customer behaviors can affect the supply chain performance. This dissertation addresses some of the pricing, sourcing and product design problems in a supply chain network where suppliers and retailers sell substitutable products into a common market.

Pricing decisions for either wholesalers or retailers are critical to demand fluctuation and profit variation. Given a price offered by a retailer for a product, a customer may choose to buy from this retailer, switch to another retailer, switch to another competing product, or not to buy at all. Thus, customer demand as well as the profit is affected. Price is also a lever for a firm to fight against competitions or take advantage of competitions. Anderson and Bao (2010) study the effect of varying the level of price competition on the profits of the

supply chain participants in two scenarios, distribution channel being vertically integrated and decision making process being decentralized. Besbes and Saure (2011) study equilibrium behavior for product assortment and price competition in a duopoly of retailers when consumers have full knowledge of the retailers' offerings when making purchase decisions. Price is considered as a contract term as well as a decision variable in a large collection of the literature. Cachon (2003) gives an overview of supply chain coordination with price only contracts. Coordination contracts under a variety of scenarios are investigated. In a number of cases, coordination can be achieved with many different contract forms. Cachon and Lariviere (2005) study revenue-sharing contract in a supply chain with a supplier and a retailer facing stochastic demand, which is a function of the retail price. Sajeesh and Raju (2010) study competitive positioning and pricing strategies in markets where consumers seek variety.

When multiple suppliers sell homogeneous products or partially substitutable products through the same retailer, sourcing decisions by the retailer are intended for cost-effective supplies or supply stability. The retailer can take advantage of the competition between the suppliers. Choi (1991) studies price competition between two manufacturers that produce substitutable products. He compares the situation where a single retailer distributes the products of both manufacturers to that where each manufacturer uses an exclusive retailer. Adida and DeMiguel (2011) study competition in a supply chain where multiple manufacturers compete in quantities to supply a set of products to multiple retailers who compete in quantities to satisfy the uncertain consumer demand. Cachon and Kök (2010) consider two manufacturers who compete to supply a single retailer and study the relative merits of the wholesale price, quantity discount, and two-part tariff contracts. They conclude that, in the presence of competing manufacturers, a retailer may favor the quantity discount and two-part tariff contracts to the wholesale price contract, a result that contrasts with the results obtained in the literature for the case with a single manufacturer.

To the end customers, a product is selected by matching the product attributes against

his/her expectation upon the product. The expectation can be materialized with a utility function, which is a function of not only the price but a set of product attributes that the customer cares most. For manufacturers, a product is designed and expected to capture the largest share of the targeted market segment. Products for the same market segment by different manufacturers can be fully substitutable or partially substitutable. Fully substitutable products are not distinguishable and can be considered as identical. Partially substitutable products are products with different feature sets targeted at the same market segment. A particular customer may hesitate on a choice between partially substitutable products. Substitutability between two products can be calculated as a function of products attributes and customer utilities. When it comes to partially substitutable products by different manufacturers, purchasing decisions are not only based on price but also likely on other product attributes. Ben-Akiva and Bierlaire (1999) and Anderson *et al.* (1992) provide mathematical tools of consumer choices and product differentiation. Manufacturers can work with retailers to figure out the optimal product attribute set for maximized profits because there is a cost related to different configurations of attribute set.

Hua *et al.* (2011) investigate the product design problem in a two echelon supply chain. The manufacturer has the options of designing two products for two market segments and sells through one retailer. They come up with strategies for the optimal product design decisions. Revenue sharing contract can perfectly coordinate the supply chain on product design decisions through reducing the difference of customers' utility in different market segments. Pero and Abdelkafi (2010) develop an alignment framework for new product design and supply chains, which is tested using a multiple case study design. It is shown that product innovativeness can have a critical impact on the supply chain and supply chain complexity must be adequately adapted depending on the product features. Schön (2010) studies a seller's decision on what products to offer and at what price. Each product is characterized by a unique combination of non-price related product attributes. Xiao *et al.* (2007) examines production and outsourcing decisions for two manufacturers that produce

partially substitutable products and play a strategic game with quantity competition.

The structure of a supply chain is specified with the firms involved and the partnerships between them, which is analogous to a graph in mathematics. The structure of a supply chain can change due to three events: 1) a firm leaves the supply chain due to the expiration of its partnerships with other firms in the chain; 2) another firm joins the supply chain with a new partnership; 3) the partnership between a pair of firms is revised or redefined. All the research works on supply chain coordination as far as we know assume a static supply chain structure without considering the events above. In reality, however, partnerships within a supply chain network are not all established at one time and supply chain structure is subject to evolution. New partnership decisions must be made conditional on existing partnerships.

This dissertation consists of three manuscripts to be submitted for publication. The research work in this dissertation attempts to address supply chain coordination in a number of application scenarios. In addition to analytical analysis, this research tries to discover managerial insights and suggestions for strategy implementation.

1. The first paper studies the coordination of pricing and sourcing in two scenarios where multiple suppliers and multiple retail channels are involved. This study investigates the cross price effects between retail channels on the overall profitability. Those business units have a common profit goal and decisions are made to maximize supply chain profit. When it comes to strategy execution, each individual unit has a certain degree of freedom to manage its business. So trade-offs between units may come in even if it is not encouraged by the top executive management.

In addition to the analytical analysis, a number of numerical experiments are conducted to investigate realistic issues decision makers may encounter. First, these experiments are used to study the relative impact of differentiated pricing and common pricing policies across retail channels. Second, this study considers the effect of ignoring cross price effects between retail channels.

2. The second paper studies a supply chain in which a manufacturer sells a configurable product through a retailer. We take the configurable product as a parameterized product and examine the impact of such a feature decision on equilibrium between the manufacturer and the retailer. The analysis considers three different cost functions: (1) linear cost function, (2) quadratic cost function and (3) exponential cost function and examines how the cost functions affect the optimal solutions.

3. The third paper presents a novel framework in which supply chain structure evolves from one stage to the next in terms of changing memberships and business partnerships between members. All of the research work so far on supply chain coordination assumes a *static* supply chain structure, which remains the same throughout the sequence of events. In reality, however, supply chain memberships and the partnerships between those member firms are not all established at one time and the supply chain structure is subject to evolution. Firms may join or leave a supply chain network and the partnerships between two firms may change as well. Among a large variety of possible evolving scenarios, this research focuses on a case in which a second manufacturer joins a supply chain initially established with one manufacturer and one retailer. The supply chain evolves from the stage with one partnership to the next stage with two partnerships. The two partnerships are established one after another on two competing and differentiated products. Compatibility between demand models in two stages are established. Based on the analytical non-closed form solution, a number of numerical experiments are developed to demonstrate the impact of the introduction of a competing product on the optimal solution of product one.

Chapter 2

Coordination of Pricing and Sourcing in a Multi-Channel Supply Chain

2.1 Introduction

Supply chain coordination has gained substantial attention recently in both academia and industry. Coordination is not only a mechanism for particular business strategies between firms but also brings into consideration the mutual effect between those strategies such as sourcing, pricing, routing, etc. Supply chain performance is primarily determined by supply chain structure, interactions between firms and coordination strategies. The advent of Internet-based e-commerce provides competitive opportunities to reach customers more efficiently. Multi-channel retailing plus interactions with upstream suppliers makes decision making a challenging problem. Conflicts between channel members tend to hurt the overall profitability.

This paper studies the coordination of pricing and sourcing in a two-echelon supply chain where multiple suppliers sell homogeneous products through multiple retail channels owned by one retail company. It is intended to investigate how cross-price effects between retail channels and sourcing decisions affect the overall profitability. In addition, with the guidance of analytical analysis, we conduct a set of numerical experiments to answer further realistic questions and discover managerial insights.

Two scenarios with different structures motivated with observation in real word applications are the research subject. Depicted in Figure 2.1(a) is a case where suppliers and retail channels are vertically integrated. Retail channels, RC_1, RC_2, RC_3 and RC_4 are owned by one retail company, which is integrated with suppliers S_1, S_2 and S_3 . These business units

involved have a common profit goal. A centralized decision maker is in the role to maximize the supply chain profit. A gas distributor that operates a number of distribution centers and a number of retail gas stations would be a good example of this case. Gas is shipped from its distribution centers to its retail gas stations. The retail gas stations have the choice to charge a common price or differentiated prices for one particular product. The distribution centers are not identical in terms of delivery frequency and service level. The management of this company has the motivation to find an optimal solution to maximize the overall profit of the whole company.

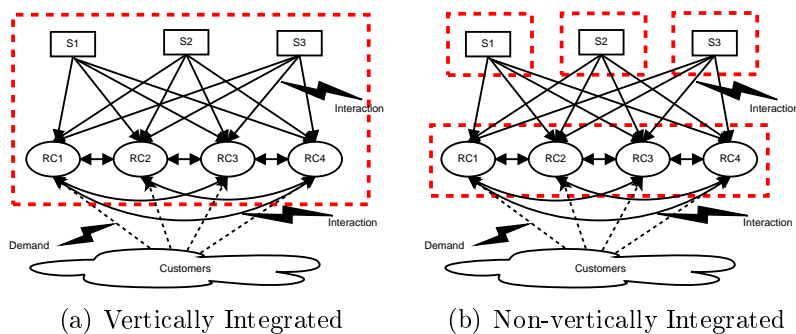


Figure 2.1 : Supply chain scenarios to be studied

Depicted in Figure 2.1(b) is another case where the suppliers and retail channels are not vertically integrated. Suppliers S_1, S_2 and S_3 are independent firms. They are not in the picture for profit maximization. Now the problem becomes the retail company's decision on how to maximize its profit by optimizing retail prices and sourcing decisions across retail channels. It is not a problem how independent suppliers and independent retailers work together to overcome the double marginalization but an optimization problem of the retailer considering sourcing decisions. Wal-mart can be viewed as an example of the later case. Wal-mart carries detergents of a number of brands, such as Tide, Downy, Gain, Purex, etc. Wal-mart makes decisions on retail price and order quantity of each product and strives for profit maximization without having those manufacturers integrated. We will show that these two cases are mathematically equivalent.

Pricing decisions for either wholesalers or retailers are critical to conquer demand fluctuation and assuage profit variation. Given a price offered by a retailer for a product, a customer may choose to buy from this retailer, switch to another retailer, switch to another competing product, or not to buy at all. Thus, customer demand as well as the profit is affected. Price is also a lever for a firm to fight against competitions or take advantage of competitions. When multiple suppliers sell homogeneous products or substitutable products through the same retailer, sourcing decisions by the retailer are intended for more cost-effective offers or supply stability. The retailer can take advantage of the competition between the suppliers.

The structure of a supply chain or more accurately, a supply chain network determines the partnerships between firms or business units. The most commonly studied structure consists of a single pair of firms of one supplier and one retailer. Research questions on such a structure typically focus on a variety of contracts to coordinate the chain such that the overall profit is as much as that can be achieved by an integrated business unit. When multiple independent retailers or multiple independent suppliers are involved, a solution of contract analysis can be very difficult.

This research identifies necessary conditions under which the overall profit is a concave function of the retail price vector with reasonable assumptions. Then, we investigate the cross-price effects between retail channels on pricing decisions, sourcing decisions and the overall profitability. The business units have a common profit goal and decisions are made to maximize supply chain profit. When it comes to strategy execution, each individual unit has a certain degree of freedom to manage its business. So trade-offs between units may come in even if it is not encouraged by the top executive management. In addition to the analytical analysis, two numerical experiments are conducted to investigate realistic issues decision makers may encounter. First, these experiments are used to study the relative impact of differentiated pricing and common pricing policies across retail channels. Second, this study considers the effect of ignoring cross-price effects between retail channels.

The rest of the paper is organized as follows. Section 2.2 presents a relevant literature

review. Section 2.3 defines the problem formally, establishes mathematical formulation and discuss assumptions. Section 2.4 shows an analytical analysis to investigate the concavity of the overall profit function and identify necessary conditions for the concavity. Section 2.5 reports two numerical experiments to show whether a common pricing strategy or a differentiate pricing strategy works better and the cost or benefit for the top executive management ignoring cross-price effects between firms. Section 2.6 concludes our findings and recommends potential extensions.

2.2 Literature Review

Pricing has been a topic dated back to two hundred years ago in economics. It naturally draws attention when more than one business units are considered in a supply chain. A variety of pricing strategies in supply chain coordination have been covered by many researchers. Lariviere and Porteus (2001) study how the wholesale price contract works between a manufacturer and a retailer on a single product. Anderson and Bao (2010) study the effect of varying the level of price competition on the profits of the supply chain participants in two scenarios, distribution channel being vertically integrated and decision making process being decentralized. Cachon (2003) gives an overview of supply chain coordination with price only contracts. Coordination contracts under a variety of scenarios are investigated. In a number of cases, coordination can be achieved with many different contract forms. Cachon and Lariviere (2005) study revenue-sharing contract in a supply chain with a supplier and a retailer facing stochastic demand, which is a function of the retail price. Sajeesh and Raju (2010) study competitive positioning and pricing strategies in markets where consumers seek variety. Tsay and Agrawal (2000) studied a distribution system in which a manufacturer supplies a common product to two independent retailers that compete on price and service. They concluded that a general wholesale pricing mechanism can coordinate the system only under some very special conditions. In another set of literature, pricing is studied together with

inventory, service, etc. Bernstein and Federgruen (2004) study retailer competitions on price and service in a single echelon.

Sourcing decisions are intended for cost-effective supplies and supply stability. With the scale of business cooperation reach every possible corner globally, sourcing is a more important factor for revenue than ever before. Choi (1991) studies price competition between two manufacturers that produce substitutable products. He compares the situation where a single retailer distributes the products of both manufacturers to that where each manufacturer uses an exclusive retailer. Adida and DeMiguel (2011) study competition in a supply chain where multiple manufacturers compete in quantities to supply a set of products to multiple retailers who compete in quantities to satisfy the uncertain consumer demand. Cachon and Kök (2010) consider two manufacturers who compete to supply a single retailer and study the relative merits of the wholesale price, quantity discount, and two-part tariff contracts. They conclude that, in the presence of competing manufacturers, a retailer may favor the quantity discount and two-part tariff contracts to the wholesale price contract, a result that contrasts with the results obtained in the literature for the case with a single manufacturer.

Coordination between a single supplier and a single retailer has been studied exhaustively. When multiple firms are considered at each level, interactions become much more complicated and it is harder to obtain a solution. Ingene and Parry (1995) explore channel coordination by a manufacturer sells through competing retailers. They found that the channel cannot be coordinated except in the trivial cases of identical or non-competing retailers. But an appropriately specified quantity discount schedule will enable the channel to earn the same profits generated by a vertically integrated system. Ha *et al.* (2003) study a supply chain with two suppliers competing for supply to a single customer based on price and delivery frequency. They showed that delivery frequency can be a source of competitive advantage and higher delivery frequencies lower the value of getting deliveries from the second supplier and there intensify the competition. When customer control the deliveries, she would strategically increase delivery frequencies to lower prices. The overall performance

is lower when the customer control the deliveries. Yao *et al.* (2008) investigate the case of a single manufacturer and two competing retailers. The manufacturer, as a Stackelberg leader, offers a revenue sharing contract to the two retailers who face stochastic demand. They found that a revenue-sharing contract can obtain better performance than a price-only contract.

Analysis for a centralized decision-making scenario is rarely being an individual and isolated research subject but a benchmark to a decentralized decision-making scenario. Bernstein and Federgruen (2003) study the coordination among a single supplier and multiple retailers competing on deterministic and price dependent demand. They compare the optimal performance of the centralized supply chain with that of the decentralized supply chains operating under given types of wholesale pricing schemes. Pekgun *et al.* (2006) study two firms that compete on the basis of price and lead-time decision in a common market. They conclude that a firm's preference for a centralized or decentralized decision making strategy may change depending on market and firm characteristics. Eppen (1979) shows that under demand uncertainty, a centralized inventory strategy provides risk-pooling benefits and reduces expected costs versus a decentralized strategy.

2.3 Model

The model presented here represents the scenario depicted in Figure 2.1(a) where a single retailer purchases a single product from multiple suppliers (m) and then sells that product through its multiple retail channels (n) over a single sales period. These retail channels may be individual retail stores or distribution centers that serve Internet sales or catalog sales. The production capacity, wholesale price, and transportation cost for each supplier are assumed to be known in advance. There are no intermediate distribution centers shared across the retail channels; the suppliers ship products directly to each retail channel. Each retail channel faces a deterministic price-sensitive market demand. Although the retail

channels are designed to serve different geographic or demographic markets, there is some overlap in those markets. Therefore, the price offered through a particular retail channel affects not only its own demand (price elasticity), but it can also affect the demand in other retail channels (cross-price elasticity). The retailer may choose to offer a different retail price in each channel, but it coordinates the order quantities from each retail channel to avoid exceeding the production capacity at any supplier. We will show that mathematically the scenario in Figure 2.1(b) is equivalent to that in Figure 2.1(a).

2.3.1 Price-sensitive demand model

The demand in each retail channel is sensitive to its own retail price as well as the prices offered through the other channels. Although the model can accommodate a more general price-demand function, a linear function is used here because there is a wide-spread precedence for its use in the academic literature: economics, marketing and operations management (Choi (1991); Cachon (2003); Huang *et al.* (2012), etc.). Let p_j be the retail price in channel j , $j = 1 \dots n$, and let $\bar{p} = (p_j)$ be the vector of retail channel prices. Let $d_j(\bar{p})$ be the demand in channel j under price vector \bar{p} . The linear price-sensitive demand function used in the model can be expressed as

$$d_j(\bar{p}) = \alpha_j + \sum_{k=1}^n \beta_{jk} p_k,$$

where

$$\begin{aligned} \alpha_j &= \text{base demand for channel } j = 1, \dots, n, \text{ and} \\ \beta_{jk} &= \text{cross-price elasticity for channels } j, k = 1, \dots, n. \end{aligned}$$

The base demand α_j represents the demand that accrues to channel j when all of the retail prices are zero and is assumed to be nonnegative ($\alpha_j > 0$). For $j = k$, β_{jj} is assumed to be negative ($\beta_{jj} < 0$) reflecting the price elasticity of the demand in channel j , *i.e.*, the demand in channel j decreases as p_j increases. For $j \neq k$, β_{jk} is assumed to be nonnegative ($\beta_{jk} \geq 0$) reflecting the cross-price elasticity effect from the price in another channel, *i.e.*,

the demand in channel j increases as the price in another channel k increases. If the price in channel k has no effect on the demand in channel j (no cross-price elasticity) then $\beta_{jk} = 0$.

Figure 2.2 illustrates the fundamental assumptions about consumer behavior that underly this model. A customer who would normally order through a particular retail channel may choose not to do so because of a price increase in that channel. If that customer chooses not to order through any of the other channels then the demand is lost and the total retailer demand decreases. However, the customer might choose to order through another channel because of its lower price. Such demand switching preserves the total retailer demand, but still results in lower total revenue for the retailer.

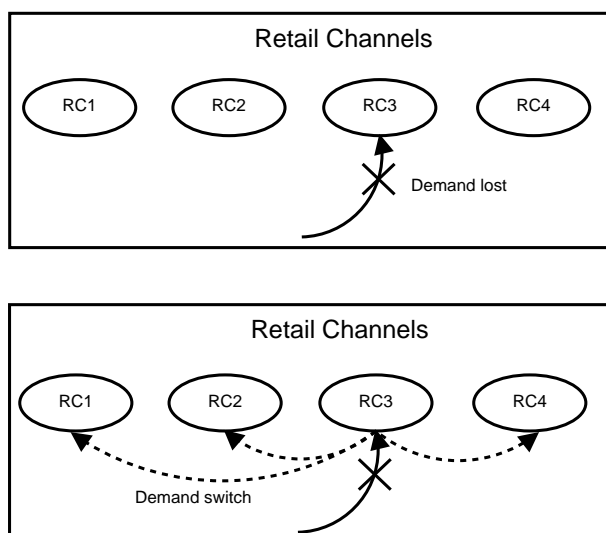


Figure 2.2 : Demand loss and demand switching

For structural reasons that will be explored later in the paper, some additional assumptions are needed on the matrix of price elasticity coefficients, $\mathbf{B} = (\beta_{jk})$. First of all, it is reasonable to assume that when the price in channel j increases, not all of the customers who leave channel j will switch to another retail channel, and those who do switch to another channel will not purchase any more than they would have in channel j . This is the first assumption on \mathbf{B} .

Assumption 2.1. $|\beta_{jj}| > \sum_{k \neq j} \beta_{kj}, \forall j = 1, \dots, n$: Each unit reduction in the channel j demand results in less than one unit of additional demand in the other channels $k \neq j$.

The second assumption on \mathbf{B} ensures that the demand in each of the retail channels is a finite, bounded function of \bar{p} .

Assumption 2.2. $|\beta_{jj}| > \sum_{k \neq j} \beta_{jk}, \forall j = 1, \dots, n$: A price increase of $\delta (\delta > 0)$ in every retail channel results in a decrease in demand in every retail channel.

To see how the inequality guarantees that the demand function is decreasing in δ , let $\bar{\delta}$ be a vector of dimension n with each component equal to δ . Then we see that

$$\begin{aligned} d_j(\bar{p} + \bar{\delta}) &= \alpha_j + \sum_{k=1}^n \beta_{jk}(p_k + \delta) \\ &= d_j(\bar{p}) + \delta(\beta_{jj} + \sum_{k \neq j} \beta_{jk}) \\ &< d_j(\bar{p}). \end{aligned}$$

If the cross-price elasticity effects are symmetric, *i.e.*, $\beta_{jk} = \beta_{kj}$, then these two assumptions are mathematically equivalent.

2.3.2 Profit Functions

The wholesale price w_i , the unit manufacturing cost c_i by supplier i , and the capacity of each supplier, C_i , are known in advance for $i = 1, \dots, m$ and $j = 1, \dots, n$. There are two sets of decision variables in this model. One is for sourcing decisions, x_{ij} and the other is for pricing decisions, p_j .

$$\begin{aligned} x_{ij} &= \text{retail channels } j\text{'s order quantity from supplier } i, \\ &\text{for } i = 1, \dots, m \text{ and } j = 1, \dots, n; \\ p_j &= \text{retail price at retail channel } j, \text{ for } j = 1, \dots, n. \end{aligned}$$

Assuming that the retail channels pay for the transportation costs, the profit for retail channel j can be expressed as

$$\pi_j^r(\bar{p}, \bar{x}) = p_j d_j(\bar{p}) - \sum_{i=1}^m (w_i + t_{ij}) x_{ij}. \quad (2.1)$$

Assuming that each supplier i produces exactly enough to cover its shipments, x_{ij} , the profit of supplier i is

$$\pi_i^s(\bar{p}, \bar{x}) = \sum_{j=1}^n (w_i - c_i) x_{ij}. \quad (2.2)$$

2.3.3 The Formulation

The mathematical formulation of this problem depends on the structure of the supply chain. Suppose the suppliers and retail channels are owned and managed by a single firm as illustrated in Figure 2.1(a), then a centralized decision model is appropriate. This is the scenario indicated by the example of a gas distributor, which operates a number of distribution centers and a number of retail gas stations. The retail gas stations may charge a common price or differentiated prices. If it is the case of differentiated prices, cross-price effect may occur between retail channels. The management of this company has the motivation to find an optimal solution to maximize the overall profit of the whole company. The profit can be calculated as

$$\sum_{i=1}^m \sum_{j=1}^n (w_i - c_i) x_{ij} + \sum_{j=1}^n \left[p_j d_j(\bar{p}) - \sum_{i=1}^m (w_i + t_{ij}) x_{ij} \right], \quad (2.3)$$

which include the profits of suppliers and the profits of retail channels. The two terms with coefficients of x_{ij} can be merged and this expression can be simplified as

$$\sum_{j=1}^n p_j d_j(\bar{p}) - \sum_{i=1}^m \sum_{j=1}^n (c_i + t_{ij}) x_{ij} \quad (2.4)$$

As such, this decision problem can be written as

$$\text{Maximize: } \sum_{j=1}^n p_j d_j(\bar{p}) - \sum_{i=1}^m \sum_{j=1}^n (c_i + t_{ij}) x_{ij} \quad (2.5)$$

$$\text{Subject to: } \sum_{i=1}^m x_{ij} = d_j(\bar{p}), \forall j = 1, \dots, n \quad (2.6)$$

$$\sum_{j=1}^n x_{ij} \leq C_i, \forall i = 1, \dots, m \quad (2.7)$$

$$d_j(\bar{p}) \geq 0, \forall j = 1, \dots, n \quad (2.8)$$

$$x_{ij} \geq 0, \forall i = 1, \dots, m \text{ and } \forall j = 1, \dots, n \quad (2.9)$$

$$p_j \geq 0, \forall j = 1, \dots, n. \quad (2.10)$$

Constraint set (2.6) means the total shipments from each supplier to retailer j must be equal to the demand at retailer j . Constraint set (2.7) is of capacity constraint of each supplier, i.e. the total shipments out of supplier i must be less than or equal to its capacity. The rest constraint sets in the list are just non-negative constraints.

In the model above, all constraints and the second item in the objective function are linear. The only non-linear item is the retail revenue, $\sum_{j=1}^n p_j d_j(\bar{p})$. It determines the concavity of the whole objective function.

Note: (2.6) and (2.7) together imply that $\sum_{j=1}^n d_j(\bar{p}) \leq \sum_{i=1}^m C_i$.

The profit for the retailer with multiple retail channels shown in Figure 2.1(b) is calculated as the sum of each channel's profit,

$$\sum_{j=1}^n p_j d_j(\bar{p}) - \sum_{i=1}^m \sum_{j=1}^n w_i x_{ij} \quad (2.11)$$

Obviously, the expression above is mathematically equivalent to the expression for the overall profit in (2.5). The only literal difference is the cost term for integrated supply

chain and the retailer. The wholesale price as an internal transferring cost disappears in the integrated supply chain. The unit manufacturing cost and transportation cost are constant terms and can be combined. The wholesale price in (2.11) paid by the retailer is also a constant term. As such, only one term comes as the coefficient of x_{ij} in either (2.5). and (2.11). All the constraints remain the same for the second scenario. So the models for the two scenarios introduced in section 2.1 are mathematically equivalent to each other.

2.4 Analysis

The model in section 2.3 shows a quadratic objective function and linear constraints. If the objective function is concave under a certain circumstances, it would be easy to obtain a global optimal solution. The section presents analytical analysis and tries to figure out necessary conditions for objective function to be concave. The only non-linear item in the objective function is, $\sum_{j=1}^n p_j d_j(\bar{p})$. Let $R_j(\bar{p})$ be the revenue at retail channel j , $R_j(\bar{p}) = p_j d_j(\bar{p})$ and $R(\bar{p})$ as the overall revenue of the whole supply chain, $R(\bar{p}) = \sum_{i=1}^n R_j(\bar{p})$. Before moving on to conclusions on the concavity, we need two lemmas on $R(\bar{p})$'s properties as shown below. The proofs are in appendix A.

Lemma 2.1. *The gradient of $R_j(\bar{p})$ on \bar{p} is*

$$\nabla R_j(\bar{p}) = \left\{ \frac{\partial R_j(\bar{p})}{\partial p_l} \right\} = \begin{cases} \alpha_j + 2\beta_{jj}p_j + \sum_{k \neq j} \beta_{jk}p_k, \forall l = j \\ \beta_{jl}p_j, \forall l \neq j \end{cases} \quad (2.12)$$

Lemma 2.2. *The Hessian of $R(\bar{p})$ is*

$$H = \begin{bmatrix} 2\beta_{11} & \beta_{12} + \beta_{21} & \dots & \beta_{1n} + \beta_{n1} \\ \beta_{12} + \beta_{21} & 2\beta_{22} & \dots & \beta_{2n} + \beta_{n2} \\ \dots & \dots & \dots & \dots \\ \beta_{1n} + \beta_{n1} & \beta_{2n} + \beta_{n2} & \dots & 2\beta_{nn} \end{bmatrix}$$

Two theorems about $R(\bar{p})$'s concavity are concluded as below under two different cases. The first one states the sufficient condition for its concavity when the cross-price effect matrix is symmetric and assumption 1 holds. The second one is for the case in which the cross-price effect matrix is asymmetric and both assumptions hold.

Theorem 2.1. *When the cross-price effect, \mathbf{B} , is symmetric and assumption 1 holds, $R(\bar{p})$ is concave.*

Theorem 2.2. *When the cross-price effect, \mathbf{B} , is asymmetric and assumptions 1 and 2 hold, $R(\bar{p})$ is concave.*

In addition to the assumptions above, one key property is needed to prove the two theorem that a symmetric matrix is positive definite if it is strictly (row) diagonally dominant with positive diagonal entries a_{ii} for all i (Engeln-Müllges and Uhlig (1996); Hackbusch (1992)), i.e.

1. $|a_{ii}| > \sum_{j \neq i} |a_{ij}|, \forall i;$
2. $a_{ii} > 0, \forall i.$

Once the concavity of $R(\bar{p})$ is determined, the concavity of the overall profit function in the retail price vector can be determined as well. Thus, we identify the sufficient conditions to ensure the existence of a global maximum. It provides a guidance for the numerical experiment design in next section.

2.5 Numerical experiments

With a set of numerical experiments, this section is intended to investigate a number of realistic issues decision makers may encounter in the scenarios introduced earlier. The first experiment examines how the supply chain parameters affect the pricing decisions,

the sourcing decisions and the profitability. The second experiment compares the common pricing strategy and the differentiate pricing strategy. The third experiment studies the extra cost or benefit by ignoring cross-price effects between retail channels.

The experiments are coded in AMPL and solved with MINOS. The code runs through settings with combinations of eight factors specified in Section 2.3. Ten replications are executed for each instance. So totally $3^8 \times 10 = 65610$ output records are generated for each experiment. We import the outputs into Minitab for statistical analysis.

2.5.1 Experiment design

A value set for each supply chain parameter is designed as below. A certain degree of randomness is introduced to offset bias but not to generate excessive noise.

- $m \in \{2, 3, 4\}$, the number of suppliers;
- $n \in \{2, 6, 10\}$, the number of retail channels;
- $A \in \{2500, 5000, 7500\}$, the market size or the total maximum demand;
- $\alpha_j = A/n$, the base demand of retailer j , $j = 1, \dots, n$;

- $K \in \{0.1, 0.5, 0.9\}$, the capacity ratio;
- $C_i = K \cdot A/m$, the capacity of supplier i , $i = 1, \dots, m$;

- $\bar{c} \in \{1.0, 3.0, 5.0\}$, average unit cost;
- $\delta \in \{0.05, 0.25, 0.45\}$, cost differentiation factor;
- $c_{ij} \sim \text{Uniform}[(1 - \delta)\bar{c}, (1 + \delta)\bar{c}]$, $i = 1, \dots, m$ and $j = 1, \dots, n$;
unit cost for a unit by supplier i for retailer j ;

- $b \in \{0.08, 0.10, 0.12\}$, base factor for price elasticity;
- $\beta_{jj} = -b \cdot A/n$, $j = 1, \dots, n$, self-price elasticity of demand;

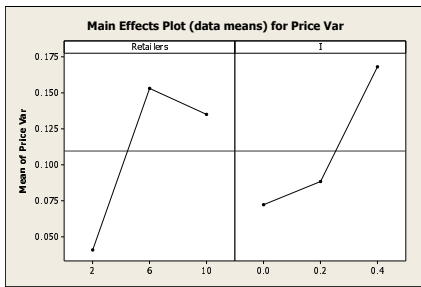
$$\begin{aligned}
I &\in \{0, 0.2, 0.4\}, \text{ strength of cross-price effect;} \\
u_{jk} &\sim \text{Uniform}[0,1], j = 1, \dots, n \text{ and } k = 1, \dots, n, \text{ a random factor matrix;} \\
\beta_{jk} &= I \cdot \min \left\{ \frac{u_{jk}}{\sum_{t \neq j} u_{jt}} |\beta_{jj}|, \frac{u_{jk}}{\sum_{t \neq k} u_{tj}} |\beta_{kk}| \right\}, j \neq k, \text{ cross-price elasticity.}
\end{aligned}$$

We assume that the market size or the maximum total demand, A and the total capacity, $\sum_i C_i$ are fixed regardless of the number of retail channels or the number of suppliers. Each retail market is equally assigned a market share, α_j and each supplier is equally assigned a capacity, C_i . K is the ratio between market size and total capacity such that $\sum_i C_i = K \cdot A$. δ is the differentiating factor for both suppliers and retail channels. With an instance of \bar{c} and δ , we generate c_{ij} using uniform distribution with a lower bound, $(1 - \delta)\bar{c}$ and a upper bound, $(1 + \delta)\bar{c}$.

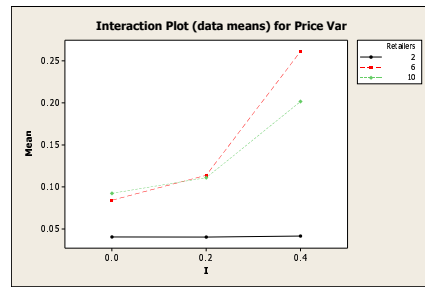
To build the matrix of cross-price effects, we should have the property of diagonal dominance in mind. With a base factor b , the diagonal elements or the elements of self-price elasticity of demand of each retailer, β_{jj} , is equally assigned as $-b \cdot C/n$. $\{u\}_{n \times n}$ is designed as a matrix of redistribution weights. $\frac{u_{jk}}{\sum_{t \neq j} u_{jt}} |\beta_{jj}|, k = 1, \dots, n$ is for the redistribution of β_{jj} across the row j . Likewise, $\frac{u_{jk}}{\sum_{t \neq k} u_{tj}} |\beta_{kk}|, j = 1, \dots, n$ is for the redistribution of β_{kk} across the column k . To ensure the both the row diagonal dominance and the column diagonal dominance, we take the minimum of those two candidate values. Multiplication with, $I \in [0, 1)$, the strength of cross-price effects, provides a lever to adjust the strength level between retail channels. The open upper bound of I ensures the *strict* diagonal dominance.

2.5.2 Pricing Decisions

When differentiated pricing strategy is considered, retail prices across different channels vary to accommodate their costs. With the retail price variance as a metric, this experiment is to investigate how the pricing decision is affected by supply chain parameters. Figure 2.3(a) shows the main effect plots of two parameters, the number of retail channels and the



(a)

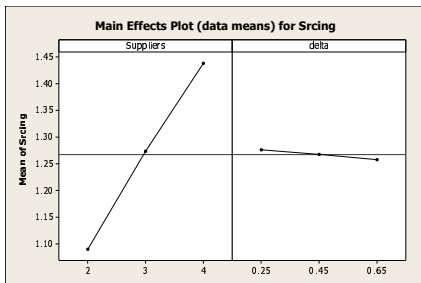


(b)

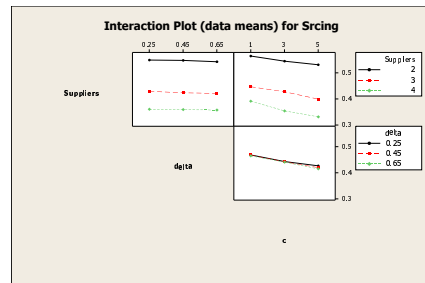
Figure 2.3 : Cross-price effect on pricing

interaction intensity. It appears that the price variance does not always increase with the number of retail channels. It increases sharply when the number of retail channels go up to a certain level, 6 for this example and decreases a little thereafter. The price variance, however, increases constantly when the interaction intensity grows. The interaction plot shown in Figure 2.3(b) indicates that the number of retail channels intensify the effect of the price interaction on the price variance when the number of is 6 or 10. It coincides the result by the main effect plots that the intensification by the number of retail channels decreases a little when it grows from 6 to 10.

2.5.3 Sourcing Decisions



(a)



(b)

Figure 2.4 : Sourcing decisions

Sourcing decisions in the scenarios under investigation are indicated with transshipments. To show the source diversity for a particular instance, a sourcing diversity factor is designed as,

$$\frac{\sum_{i=1}^M \sum_{j=1}^N I_{[x_{ij}>0]}}{N},$$

which is an average of the number of suppliers serving each retail channel.

As shown in Figure 2.4(a), the sourcing diversity factor increases as the number of suppliers grows. It follows the intuition that more alternatives are available with an increasing number of suppliers. An increase in the degree of sourcing differentiation reduces the sourcing diversity because suppliers offering cheaper prices are preferred. The interaction plot in Figure 2.4(b) shows there is no much interference between the number of suppliers and the differentiation between them.

2.5.4 Effect on the profitability

Indicated in Figure 2.5 is the main-effect plot to show how the overall profit varies with each of the eight factors. The strength of cross-price effects measures the ratio of cross-price elasticity over self-price elasticity. The overall profit increases along with the strength of cross-price effects. It confirms that more switching demands are observed when the cross-price elasticity increases. When the self-price elasticity increases, the overall profit drops as the market size increases, the overall profit goes up too due to the possibility of more demand being realized. When K goes up, the total capacity is getting close to the market size and more demand can be served by suppliers. So the overall profit increases with K . But we see diminishing effect of K on the overall profit when $K > 0.5$. δ is the factor to differentiate retail channels and suppliers. When the differentiation increases, the overall profit increases too due to the fact that a larger differentiation gives rise to a larger feasible region. But this experiment does not show us a strong effect of δ on the overall profit. The average unit cost, c , does impose a strong effect on the overall profit with a similar trend as b . When the

average unit cost increases, the overall profit drops. Compared to other factors, the number of retail channels and the number of suppliers do not have strong effect on the overall profit. The curve of overall profit is quite flat when demonstrated with the same scales.

There are a few noteworthy interactions in the interaction plots (Figure 2.7) When the strength of cross-price effects is getting stronger, it amplifies the effect on A , the market size on the overall profit. K , the capacity ratio, when less than 0.5, amplifies the effect of c . But when it is greater than 0.5, the interaction effect between K and c on the profit almost disappears.

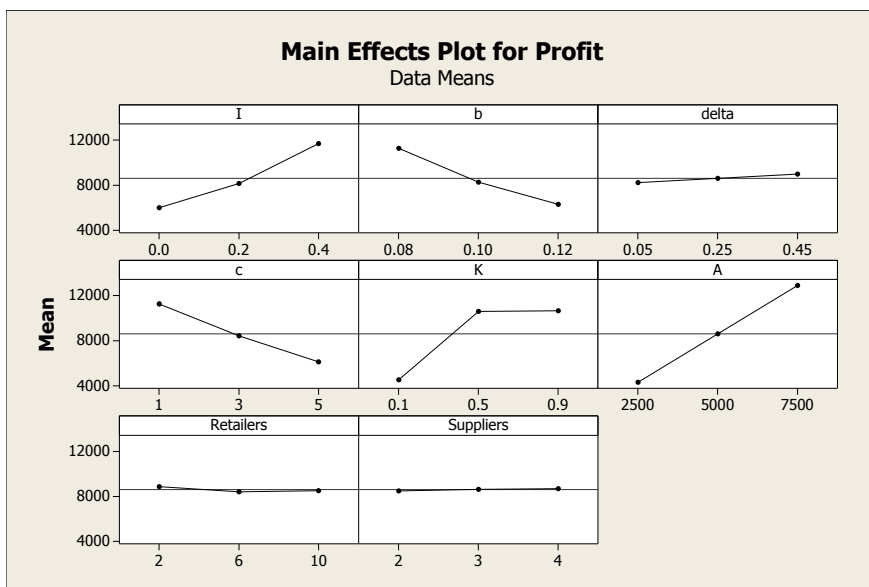


Figure 2.5 : Main effect plots of individual factors

2.5.5 Common price versus differentiated price

The key to designing an experiment to compare common pricing strategy against differentiated pricing strategy is that the set of parameter values must ensure the feasibility for each case. By calculating the upper bound and lower bound of the common single price, we can easily check the feasibility and adjust α or β to restore the feasibility if it is violated.

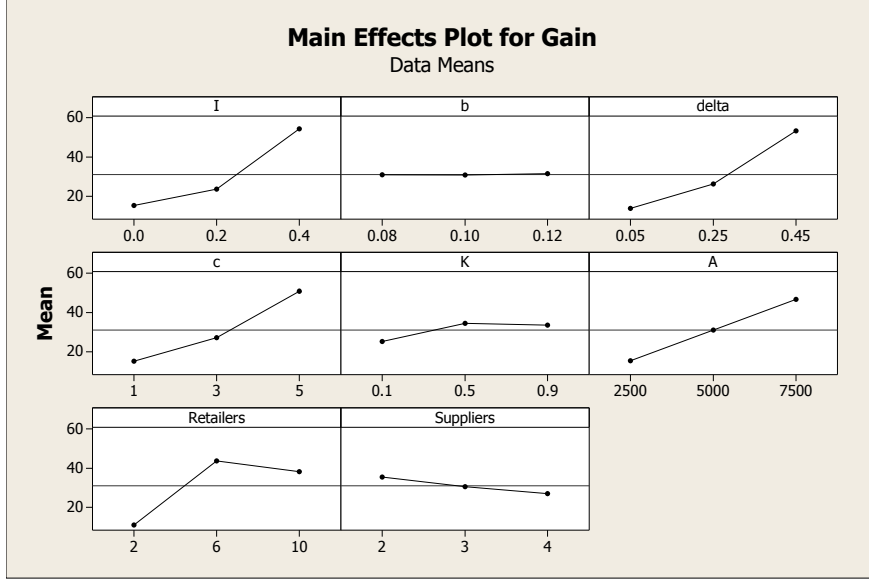


Figure 2.6 : Main effect: Common pricing vs. Differentiate Pricing

The lower bound can be obtained from the constraint that the total demand must be less than or equal to the total capacity, $\sum_j d_j(p) = \sum_j \alpha_j + p \sum_j \sum_k \beta_{jk} \leq \sum_i C_i$. Eventually, we have $p \geq \frac{\sum_i C_i - \sum_j \alpha_j}{\sum_j \sum_k \beta_{jk}}$

Similarly, we can get the upper bound. The demand at each retailer is assumed to be non-negative, $d_j(p) = \alpha_j + p \sum_k \beta_{jk} \geq 0, \forall j$. Then, we have $p \leq \frac{-\alpha_j}{\sum_k \beta_{jk}}, \forall j$ and $p \leq \min_j \left\{ \frac{-\alpha_j}{\sum_k \beta_{jk}} \right\}$.

Positive profit gain of differentiated pricing over common pricing is shown in Figure 2.6. The profit gain increases gradually along with I , δ , A and c . b almost does not have any effect on the profit gain. The effect of K on the profit gain increases gradually before K reaching 0.5 but becomes not obvious thereafter.

The profit gain drops slowly all the way along with the number of suppliers. But the profit gain shows a concave curve along with the number of retail channels. When the number of retail channels increases from 2 to 6, the overall profit increases. But it starts to drop when it is greater than 6. It can be explained that with more retail channels from the beginning, there is more room for optimal differentiate prices. So we can have higher prices and in turn

higher profit. When the number of retail channels surpass a threshold, the smaller demand share of each retailer starts to offset the gain from positive price difference.

There are a few noteworthy interactions on the profit gain. The value of δ magnifies the effect of c on the profit gain. The number of retail channels has a magnifying effect when it is less than 6 but a diminishing effect on I after it reaches 6.

2.5.6 Benefit or penalty by ignoring cross-price effects

With this experiment, we attempt to compare three cases, (1) considering cross-price effects, (2) ignoring cross-price effects completely and (3) ignoring cross-price effects partially.

In case one, cross-price effects are considered using randomly generated parameters in Section 2.5.1.

$$(\mathbf{p}^*, \mathbf{x}^*) = \underset{\mathbf{p}, \mathbf{x}}{\operatorname{argmax}} \pi(\mathbf{p}, \mathbf{x}, \mathcal{B}) \quad (2.13)$$

In case two, cross-price effects between retail channels are ignored completely, i.e. demand switching between retail channels is ignored. This is different from the case without demand switching at all in that the pricing effect on one particular retail channel itself exists but invisible to other retail channels. Generally, customers just stop buying due to a price increase. So in this case, we take the \mathcal{B} matrix for case one and modify it by keeping diagonal elements but zeroing the off-diagonal elements, denoted by $\tilde{\mathcal{B}}$.

$$\max_{\mathbf{p}, \mathbf{x}} \pi(\mathbf{p}, \mathbf{x}, \tilde{\mathcal{B}})$$

In case three, we assume that the optimal price vector is obtained by solving the original model with cross-price effects. Then we would like to see how the decision on transshipment affects the overall profit while the pricing strategy is determined and cross-price effects are actually ignored. We rerun the optimization on transshipments only by having the price

vector fixed.

$$\max_{\mathbf{x}} \pi(\mathbf{p}^*, \mathbf{x}, \hat{\mathcal{B}})$$

This experiment returns two profit differences which turn out to be the penalty by ignoring cross-price effects.. The first one, denoted by CC , called *cost of ignoring cross-price effects completely*, is the profit difference between case one case two. The second one, denoted by CP , called *cost of ignoring cross-price effects partially*, is the profit difference between case three and case two.

The main effect plots of eight factors on penalty CC is shown in Figure 2.9. The sharp increase of the cost with the market size, A means that a larger market size would help increase the penalty by ignoring cross-price effects or the benefit of considering cross-price effects. Similarly, stronger strength of cross-price effects, I would increase the penalty too. The self-pricing elasticity, b works the other way. When b is getting bigger, the penalty gradually drops. We do not see obvious effects of K, c, δ and the number of suppliers on the penalty.

The characteristics of penalty CP are very similar to those of CC . It is shown that optimization on transshipments can help improve the overall profit.

The only noteworthy interaction on cost CC is the one between A and I . The introduction of A magnifies the effect of I on the cost CC , as shown in Figure 2.10.

2.6 Conclusions

This research is motivated with an observation of two typical supply chain scenarios in which multiple suppliers sell homogeneous products to a retailer with multiple retail channels. The analytical analysis identifies the sufficient conditions for the concavity of the overall profit function with a few assumptions. With the guidance of the analytical results, we design a set of numerical experiment to study a number of realistic business issues that managers may encounter in real business operations.

With the numerical experiment and statistical analysis, we are able to find further managerial insights. When differentiated pricing strategy is considered, retail prices across different channels vary to accommodate their costs. The sourcing diversity increases as the number of suppliers grows. It follows the intuition that more alternatives are available with an increasing number of suppliers. An increase in the degree of sourcing differentiation reduces the sourcing diversity because suppliers offering cheaper prices are preferred. Differentiated pricing results in a higher profit than common pricing. Also, The overall profit is higher when cross-price effects are considered because the unsatisfied demand at one retailer is not lost but switching to other retail channels. Simply ignoring the cross-price effects between retail channels would incur penalty on the overall profitability.

Interaction Plot for Profit

Data Means

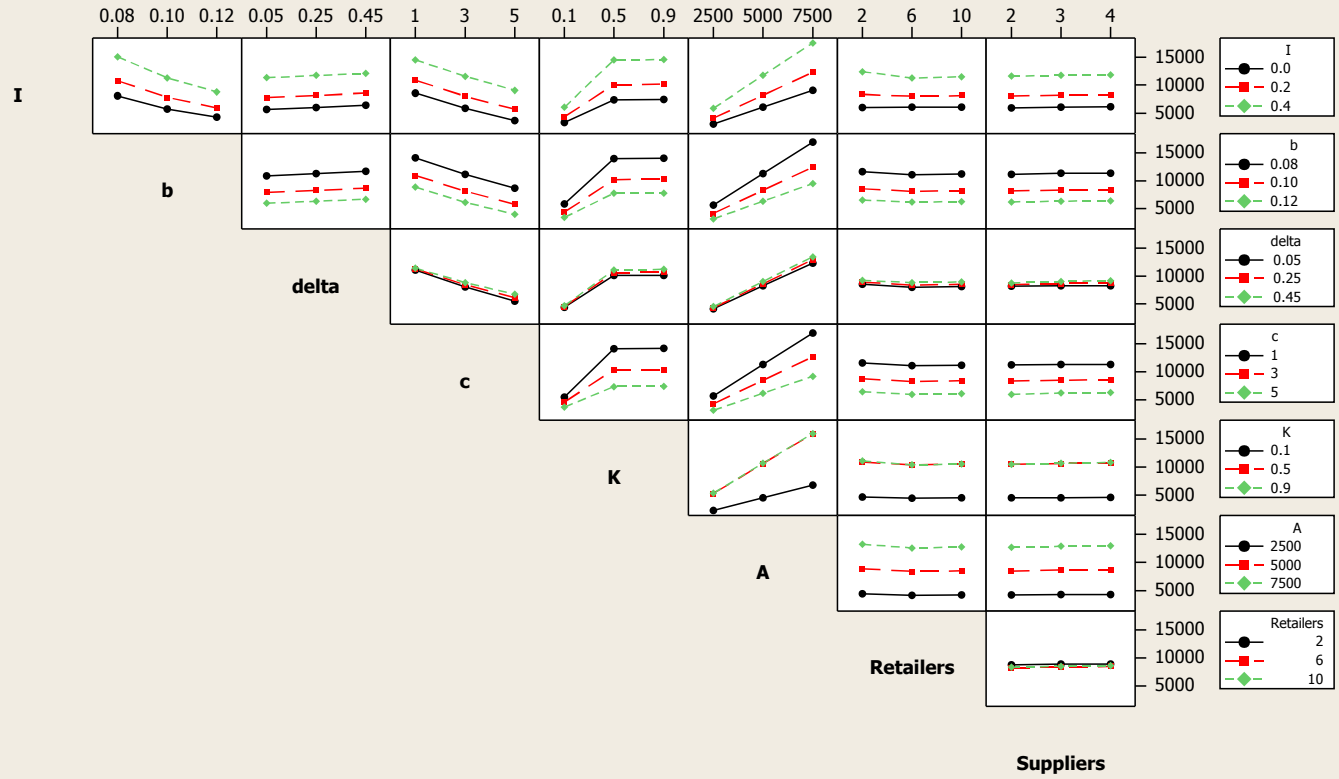
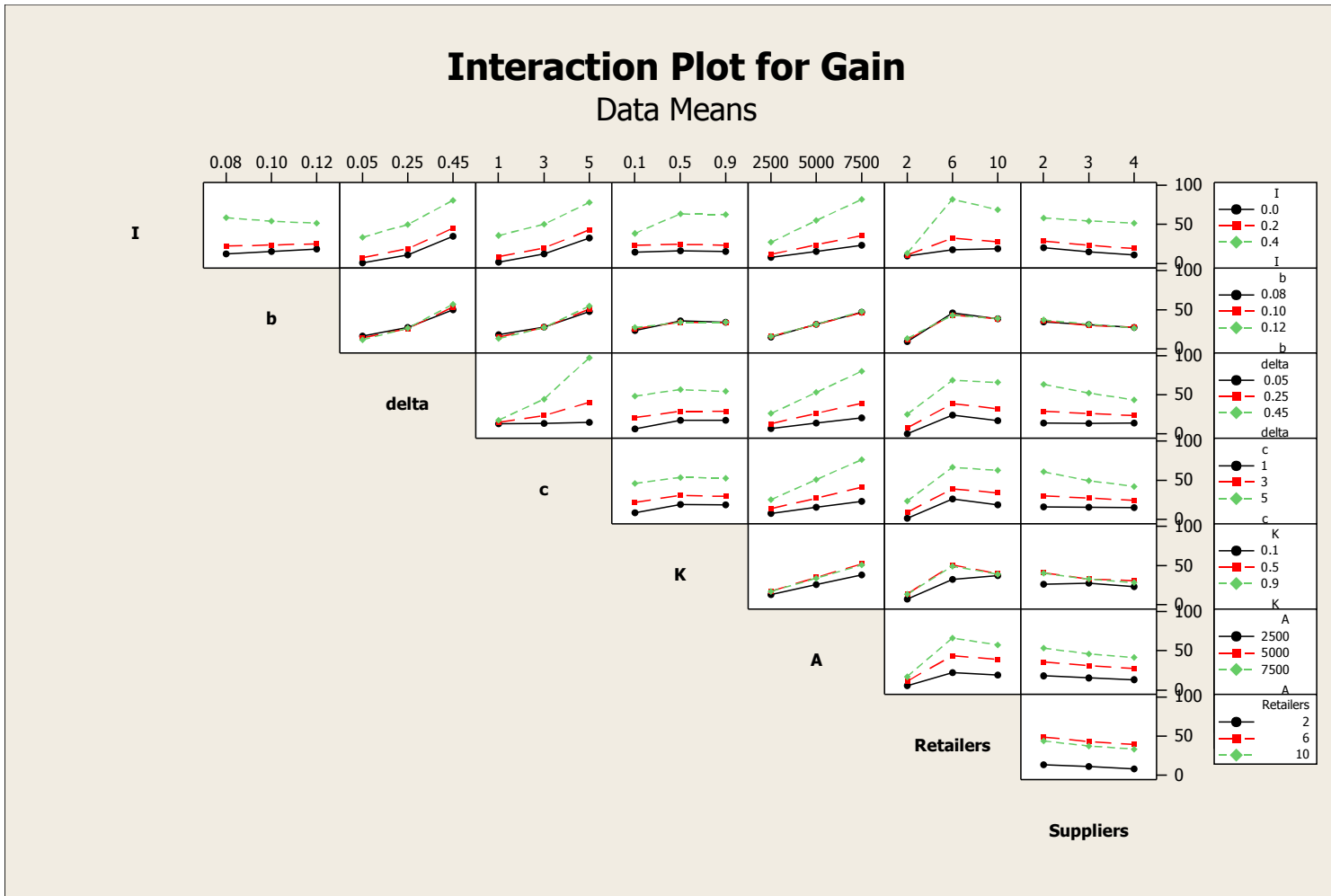


Figure 2.7 : Basic interaction

Figure 2.8 : Interaction: Common pricing vs. Differentiate Pricing



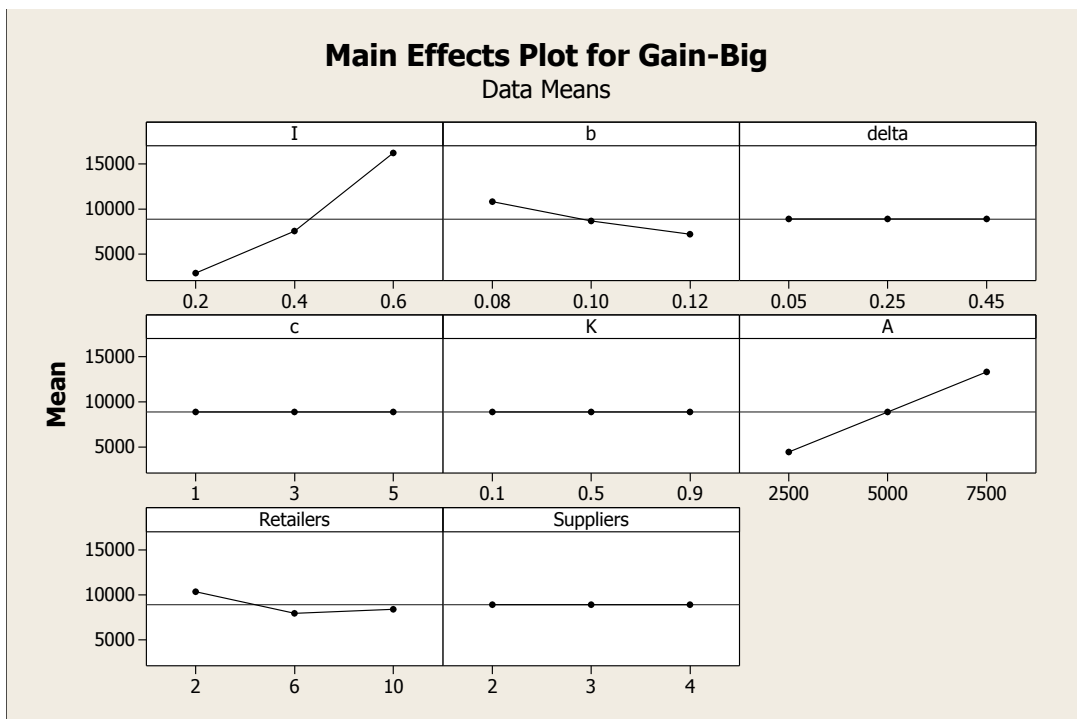


Figure 2.9 : Main Effect - Penalty by ignoring cross-price effects

Interaction Plot for Gain-Big

Data Means

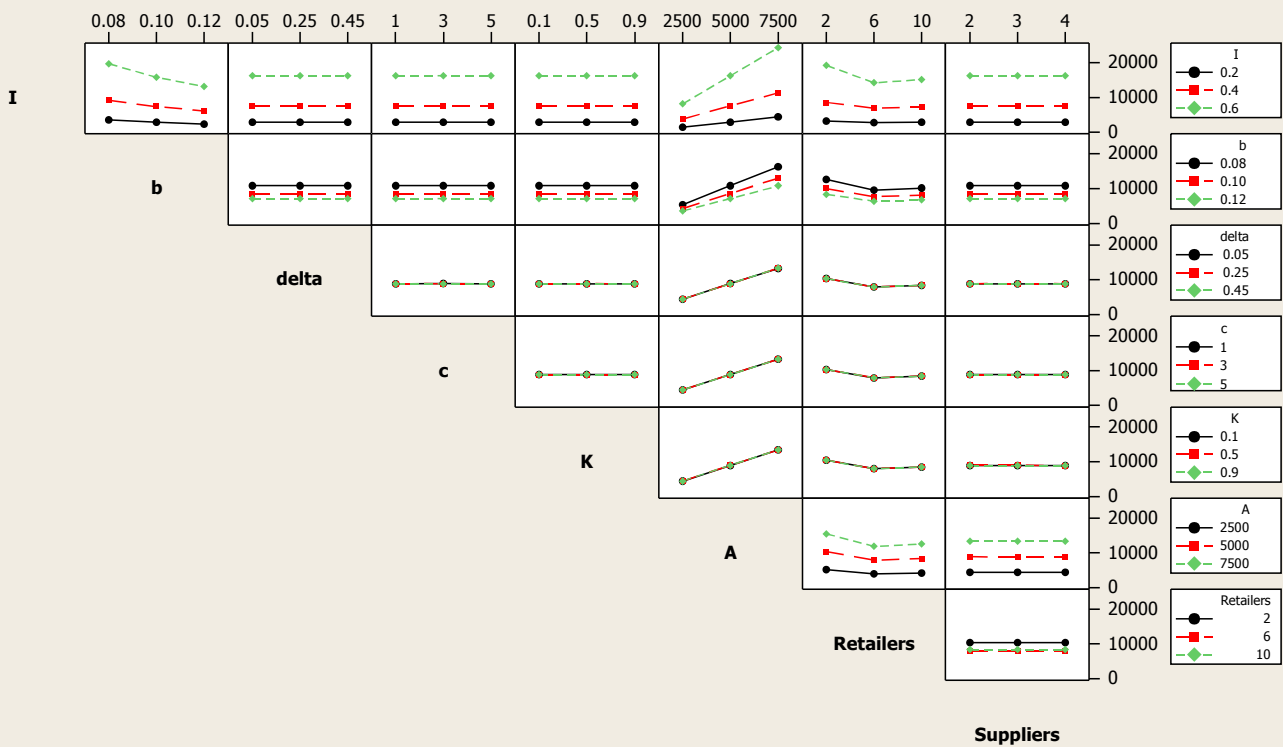


Figure 2.10 : Interaction - Penalty by ignoring cross-price effects

Chapter 3

Parametrized Equilibrium with Manufacturing Cost on Product Feature

3.1 Introduction

In the industry of electronics, it is common to see that a manufacturer develops a product prototype with a set of configurable product features or components. Then it is marketed as different models. Choices for a particular feature or component results in a variation in retail price. Accordingly, customers make a purchasing decision between competing products based not only on price but also on other product features and likely end up with either one. For example, a personal computer can be configured with different CPUs in terms of clock frequency and hard drives in terms of storage capacity. Or even an optional component like bluetooth module or web camera can be dropped

Such a product family with configurable features incurs a variation of manufacturing costs too. Cost for such a configuration on feature level follows different cost functions. This research studies the impact of cost functions on equilibria between a manufacturer and a retailer. The manufacturer sells a configurable product through a retailer. They compete for the revenue earned with the same set of demand realizations.

We investigate three types of cost functions, linear cost, quadratic cost and exponential cost. For the level of a particular feature value, we figure out how to determine the feasible range. We conduct an analysis on the concavity of the profit function on that range.

Decision issues on such a configurable feature faced by manufacturers and retailers are interesting and challenging. However, the supply chain coordination literature focuses only on price to differentiate products and influence purchasing decisions. It is expected that such

an analysis can help find reasonable values for configuration choices.

3.2 Literature Review

Supply chain games between a manufacturer and a retailer competing on supply chain profit have been widely investigated. The immediate problem to conquer is the double marginalization problem. It is first identified by Spengler (1950) that aggregate profit in a decentralized supply chain with one manufacturer and one retailer may be lower than in a vertically integrated chain where a central planner makes decisions for both the manufacturer and the retailer. It is the case whether both the players make a simultaneous decision or the supplier makes the decision first and acts as a Stackelberg leader (Kogan and Tapiero, 2007). Double marginalization with price factor only in a setting with one manufacturer and one retailer has been addressed thoroughly in the literature. Cachon (2003) conducts a survey on supply chain coordination with contracts between independent decision makers.

As important as that of price, consumer's perception of quality is considered another pivotal determinant of shopping behavior and product choice (Bishop, 1984; Jacoby and Olson, 1985; Sawyer, 1975). As pointed out by Brekke *et al.* (2010), quality is inherently difficult to measure. Quality is generally considered as an abstract product feature which vertically differentiates products by different manufacturers (Moorthy, 1988). Quality enhancement to a higher level incurs extra costs. are naturally characterized by spatial competition. Product configuration with a variety of choices offers flexibility for such differentiation. When a product prototype is developed, it holds a single specification. When it comes to market, it comes with a number of models for the same product. It can be considered as a parameterized product.

One stream of the literature in existence focuses on price and quality competition. Specifically, Hotelling (1929) introduces a spatial competition model, in which firms compete and differentiate their products with only one feature, geographic location. Transportation

cost over this distance can be considered as the inverse of the quality enhancement cost. D'Aspremont *et al.* (1979) and Neven (1985) extend this idea with non-linear transportation cost. Banker *et al.* (1998) investigate a price and quality competition under a duopolistic setting, where the demand is modeled as a linear function of price and quality levels, and the cost as a quadratic function of the quality level. In Moorthy (1988), the marginal cost of supplying a product of quality is also defined as a quadratic function of the quality level. He assumes homogeneous ideal points under equal prices. A higher quality product costs more to produce than a lower quality product. In contrast, Shaked and Sutton (1982) assume that each product costs the same to produce. Matsubayashi and Yamada (2008) study the impact of asymmetry between firms on the outcome of price and quality competition. The quality level affects not only the variable production cost linearly, but also the fixed cost quadratically. El Ouardighi and Kim (2010) study collaborations between a single supplier and two manufacturers on design quality improvements for their respective products. The manufacturers compete for market demand both on price and design quality. The authors develop a non-cooperative dynamic game and analyzes how each party should allocate resources for quality improvement over time.

There is another stream of literature which could help modeling supply chain coordination with cost aware attributes. Either manufacturers or retailers can take efforts to spur demand in addition to the product itself. Cachon (2003) also covers an introduction to coordination with effort dependent demand. A number of marketing papers under this survey adopt an effort dependent stochastic demand model where the distribution of demand is a function of the effort level e , $F(q|e)$. In contrast, this paper adopts a deterministic demand model in which the demand is dependent on both price and another product attribute. Retailer's service level is another common driver to affect customer demand. Tsay and Agrawal (2000) study a distribution system in which a manufacturer supplies a common product to two independent retailers, who in turn use service as well as retail price to directly compete for end customers. Bernstein and Federgruen (2004) study a number of competition scenarios

with price and service level. They investigate under which of a number of demand models a firm responds to a change by adjusting its service level and price in either directions.

Feature decision on product design can be considered as one of the manufacturer's efforts to capture a larger market share. Consumers perception of product features or attributes immediately affect retail pricing and the product's profitability. Schön (2010) studies a seller's decision on what products to offer and at what price. Each product is characterized by a unique combination of non-price related product attributes. Hilletofth and Eriksson (2011) conduct a case study based on empirical data to answer the question what linkages exist between new product development with supply chain management. Hua *et al.* (2011) investigate the product design problem in a two echelon supply chain. The manufacturer has the options of designing two products for two market segments and sells through one retailer. They come up with strategies for the optimal product design decisions. The revenue sharing contract can perfectly coordinate the supply chain on product design decisions through reducing the difference of customers' utility in different market segments. Pero and Abdelkafi (2010) develop an alignment framework for new product design and supply chains, which is tested using a multiple case study design. It is shown that product innovativeness can have a critical impact on the supply chain and supply chain complexity must be adequately adapted depending on the product features.

3.3 Model

Consider a two-echelon supply chain consisting of a single manufacturer selling a product through a single retailer. The product is specified with two attributes, price p and another positively correlated attribute x . The value of x is subject to the configuration of the corresponding component. Attribute x is a continuous decision variable for the manufacturer. Demand is considered as a function of (p, x) . Manufacturing cost increases as the value of attribute x increases. We assume that both the demand function, $d(p, x)$ and the cost

function, $C(x)$ are continuous, twice-differentiable.

We consider a Stackelberg game in which the manufacturer acts as a leader and the retailer as a follower. They compete for profit sharing earned with the same set of product units sold through this supply chain. It is a two-stage non-cooperative decision process in which the choice of attribute levels is prior to the decision on the price. The sequence of events are specified as follows.

1. The manufacturer makes a decision on attribute value x ;
2. The manufacturer makes a decision on wholesale price w ;
3. The retailer makes a decision on retail price p or retail margin m ($p = w + m$).

We assume that the retailer faces a deterministic demand, $d = d(p, x)$. It is convex endogenous in p , which decreases as the retail price increases. The gradually flatten slope of the demand curve shows that the marginal customer demand decreases or the demand is less price sensitive as the price grows.

Assumption 3.1. $\frac{\partial d(p, x)}{\partial p} < 0$ and $\frac{\partial^2 d(p, x)}{\partial p^2} \leq 0$, as shown in Figure 3.1(a).

It is assumed to be concave endogenous in x , which increases as the attribute x increases. The demand is less attribute sensitive to x when this attribute level grows.

Assumption 3.2. $\frac{\partial d(p, x)}{\partial x} > 0$ and $\frac{\partial^2 d(p, x)}{\partial x^2} \leq 0$, as shown in Figure 3.1(b).

In addition, we assume that customer demand becomes less price sensitive as product attribute x increases, and customer demand becomes less sensitive to product attribute x when the price grows. i.e.

Assumption 3.3. $\frac{\partial^2 d}{\partial x \partial p} < 0$ and $\frac{\partial^2 d}{\partial p \partial x} < 0$.

The manufacturing cost is a function of the attribute x , $c = c(x)$. We assume that it is convex endogenous in x , which increases as the attribute x increases, i.e.,

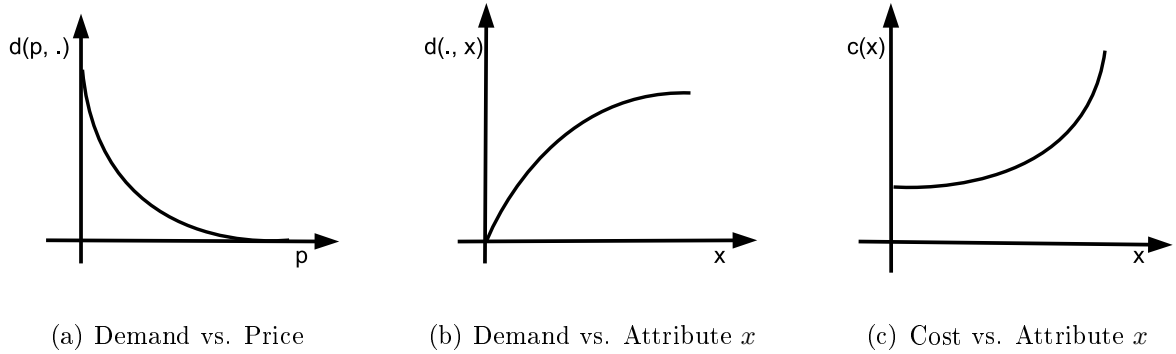


Figure 3.1 : Demand function and Cost function

Assumption 3.4. $C'(x) > 0$ and $C''(x) \geq 0$, as shown in Figure 3.1(c).

The manufacturer's problem is to maximize its profit with a decision on the wholesale price w given a certain value of x . The manufacturing cost related to x , $C(x)$ is involved. specifically

$$\begin{aligned} \max_w \quad & \Pi_M(w|x) = (w - C(x)) \cdot d(m + w, x) \\ \text{s.t.} \quad & w \geq C(x), d(m + w, x) > 0, x \geq 0. \end{aligned} \quad (3.1)$$

where $\Pi_M(w|x)$ is the manufacturer's profit function given a certain value of x .

The retailer's problem is to maximize its profit with a decision on the retail margin m .

$$\begin{aligned} \max_m \quad & \Pi_R(m|w^*, x) = m \cdot d(m + w^*, x) \\ \text{s.t.} \quad & m \geq 0, d(m + w^*, x) \geq 0. \end{aligned} \quad (3.2)$$

where $\Pi_R(m|w^*, x)$ is the retailer's profit.

The product feature value x is not considered as a game decision variable for the manufacturer's profit because as the retailer's decision on the retail margin m is only immediately in response to its wholesale price. From the perspective of the manufacturer, its decision

on the wholesale price w depends on the level of feature x . It is virtually a function of x . But we do not have any idea about the function at the beginning of the game. We find the specific function of $w(x)$ for optimum w with equilibrium analysis.

3.4 Analysis

The analysis on a classic version of the game between a manufacturer and a retailer is well documented in Kogan and Tapiero (2007). It considers a model with constant manufacturing cost without awareness of a particular product feature. We extend the model with a manufacturing cost function in a product feature value, $C(x)$. We specify the demand function with the commonly used linear deterministic demand model before proceeding to the analytical analysis on parameterized equilibria.

$$d(p, x) = a - \beta(w + m) + \kappa x. \quad (3.3)$$

We have $p = w + m$, $a > 0$, $\beta > 0$, $\kappa > 0$, where w is the wholesale price by the manufacturer and m is the retailer's margin.

3.4.1 Parameterized equilibrium of w and m

Following the backward induction for Stackelberg games, we start with a quest for the retailer's best response function in response to the manufacturer's decision on w given a certain value of x . For the retailer's profit function with (3.2), it is straight forward to verify the concavity in m ,

$$\frac{\partial^2 \Pi_R}{\partial m^2} = 2 \frac{\partial d(m + w^*, x)}{\partial p} + m \frac{\partial^2 d(m + w^*, x)}{\partial p^2} < 0. \quad (3.4)$$

So the retailer's best response function, $m(w|x)$, is determined by the first-order condition with backward induction.

$$\frac{\partial \Pi_R}{\partial m} = d(m + w^*, x) + m \frac{\partial d(m + w^*, x)}{\partial p} = 0 \quad (3.5)$$

$$\Rightarrow m(w|x) = \frac{\kappa x + a - \beta w}{2\beta} \quad (3.6)$$

By substituting (3.6) back into (3.1), we are able to find the stationary point by first-order condition for the manufacturer's problem.

$$\frac{\partial \Pi_M(w|x)}{\partial w} = \frac{1}{2}a - \beta w + \frac{1}{2}\kappa x + \frac{1}{2}\beta C(x) = 0 \quad (3.7)$$

By the second order condition, it is straight forward to show the concavity in w .

$$\frac{\partial^2 \Pi_M(w|x)}{\partial w^2} = -\beta < 0 \quad (3.8)$$

By solving the first order condition in (3.7), we obtain the optimum wholesale price as a function of x and then the retail margin as a function of x as well.

$$w^*(x) = \frac{a + \kappa x + \beta C(x)}{2\beta} \quad (3.9)$$

$$m^*(x) = \frac{a + \kappa x - \beta C(x)}{4\beta} \quad (3.10)$$

Accordingly, the parameterized optimum profits of the manufacturer and the retailer are obtained as below.

$$\Pi_M^*(x) = \frac{(a + \kappa x - \beta C(x))^2}{8\beta} \quad (3.11)$$

$$\Pi_R^*(x) = \frac{(a + \kappa x - \beta C(x))^2}{16\beta} \quad (3.12)$$

	Linear cost	Quadratic cost	Exponential cost
$w^*(x)$	$\frac{a+\kappa x+\beta cx}{2\beta}$	$\frac{a+\kappa x+\beta cx^2}{2\beta}$	$\frac{a+\kappa x+\beta e^{(cx)}}{2\beta}$
$m^*(x)$	$\frac{a+\kappa x-\beta cx}{4\beta}$	$\frac{a+\kappa x-\beta cx}{4\beta}$	$\frac{a+\kappa x-\beta e^{(cx)}}{4\beta}$
$\Pi_M^*(x)$	$\frac{(a+\kappa x-\beta cx^2)^2}{8\beta}$	$\frac{(a+\kappa x-\beta cx^2)^2}{8\beta}$	$\frac{(a+\kappa x-\beta e^{(cx)})^2}{8\beta}$
$\Pi_R^*(x)$	$\frac{(a+\kappa x-\beta cx)^2}{16\beta}$	$\frac{(a+\kappa x-\beta cx^2)^2}{16\beta}$	$\frac{(a+\kappa x-\beta e^{(cx)})^2}{16\beta}$

Table 3.1 : Optimum values with different cost functions

Next we investigate how the cost functions affect the profit equilibrium. As the retailer's optimum profit indicates an identical structure to that of the manufacturer, we show the analysis on the manufacturer only. A summary is recorded in table (3.1) and (3.2). In addition, we try to use the constraint, $w(x) > C(x)$ to find the upper bound of x and in turn the feasible region of x . The constraint by non-negative demand and the one by $w(x) > C(x)$ turn out to be equivalent.

With $w - C(x) > 0$, we assume that the manufacturer is profitable. Otherwise, the manufacturer must set an agreement with the retailer on sharing the revenue earned by the whole supply chain based on its bargaining power or outside alternatives. In the rest of this section, we investigate three cost functions, linear cost function, quadratic cost function and exponential cost function.

3.4.2 with linear cost

With a linear cost function, $C(x) = cx, c > 0$, we have the parameterized optimal solution,

$$w^*(x) = \frac{a + \kappa x + \beta cx}{2\beta} \quad (3.13)$$

$$m^*(x) = \frac{a + \kappa x - \beta cx}{4\beta} \quad (3.14)$$

$$\Pi_M^*(x) = \frac{(a + \kappa x - \beta cx)^2}{8\beta} \quad (3.15)$$

	linear cost	quadratic	exponential
x	$[0, \frac{a}{\beta c - \kappa}]$, if $\beta c > k$ $[\frac{-a}{\beta c - \kappa}, +\infty)$, otherwise	$[0, \frac{\kappa + \sqrt{\kappa^2 + 4c\beta a}}{2c\beta})$	$[0, x_0)$ such that x_0 solves $a + \kappa x > \beta e^{(cx)}$

Table 3.2 : Feasible range of x

The second order derivative turns out to be greater than or equal to zero, which indicates that the profit function is convex. So the optimum is x is at its boundary, which is illustrated in the Figure 3.2.

$$\frac{\partial^2 \Pi_M^*(x)}{\partial x^2} = \frac{(\kappa - \beta c)^2}{4\beta} \geq 0 \quad (3.16)$$

With $w > C(x)$, we obtain the upper bound for x and then the feasible range.

Proposition 3.1.

(1). $x \in [0, \frac{a}{\beta c - \kappa}]$, if $\beta c > k$.

$\Pi_M^*(x)$ is monotonically decreasing on this interval. It makes no sense to have this feature in this case. The marginal gain from this feature is canceled out by the cost and decreasing demand.

(2). $x \in [\frac{-a}{\kappa - \beta c}, +\infty)$, otherwise.

$\Pi_M^*(x)$ is monotonically increasing on this interval. The feature sensitivity factor is stronger than the price sensitivity and the cost coefficient. It means that the gain by increasing the feature value surpasses the loss by increasing price and rising cost. It is always beneficial to increase the feature value.

3.4.3 with quadratic cost

With quadratic cost function, $C(x) = cx^2$, we have the parameterized optima as below.

$$w^*(x) = \frac{a + \kappa x + \beta cx^2}{2\beta} \quad (3.17)$$

$$m^*(x) = \frac{a + \kappa x - \beta cx^2}{4\beta} \quad (3.18)$$

$$\Pi_M^*(x) = \frac{(a + \kappa x - \beta cx^2)^2}{8\beta} \quad (3.19)$$

With $w > C(x)$, we have the upper bound for x , $x < \frac{\kappa + \sqrt{\kappa^2 + 4c\beta a}}{2c\beta}$. So, the feasible range for x is $[0, \frac{\kappa + \sqrt{\kappa^2 + 4c\beta a}}{2c\beta})$. By first order condition, $\Pi_M^*(x)$ has three stationary points in the domain of x .

$$x_1 = \frac{\kappa - \sqrt{\kappa^2 + 4c\beta a}}{2c\beta}, \quad x_2 = \frac{\kappa}{2\beta c}, \quad x_3 = \frac{\kappa + \sqrt{\kappa^2 + 4c\beta a}}{2c\beta}.$$

and $f(x_1) = f(x_3) = 0$. It is obvious that $\frac{\kappa - \sqrt{\kappa^2 + 4c\beta a}}{2c\beta} \leq 0$ and x_2 is supposed to be greater than or equal to 0. So in our feasible range of x , x_3 is right at the upper bound and 0 is at the lower bound. x_2 is in between. It is easy to verify that $0 < f(0) < f(x_2)$ and $f(x_3) = 0$.

Proposition 3.2. $\Pi_M^*(x)$ is strictly quasiconcave in x on the set $S = \{x : 0 < x < \frac{\kappa + \sqrt{\kappa^2 + 4c\beta a}}{2c\beta}\}$. There exists a global maximum at $x = \frac{\kappa}{2\beta c}$.

We are able to determine the global optimum on set S, which is optimal for either manufacturer or the retailer individually. If we do not require that $w > C(x)$, the manufacturer charges $w = C(x)$ and earns no immediate profit before the demand is realized eventually. It can be coordinated with a revenue sharing a contract (Cachon, 2003) in which the manufacturer earns a fraction of the supply chain revenue.

3.4.4 with exponential cost

With exponential cost function, $C(x) = e^{(cx)}$, $c > 0$, we have the parameterized optimal solution as below.

$$w^*(x) = \frac{a + \kappa x + \beta e^{(cx)}}{2\beta} \quad (3.20)$$

$$m^*(x) = \frac{a + \kappa x - \beta e^{(cx)}}{4\beta} \quad (3.21)$$

$$\Pi_M^*(x) = \frac{(-a - \kappa x + \beta e^{(cx)})^2}{8\beta} \quad (3.22)$$

With $w > C(x)$, the upper bound is determined by solving

$$a + \kappa x - \beta e^{(cx)} > 0 \quad (3.23)$$

In this case, the upper bound of x does not have a close-form solution.

By the first order condition of (3.22), its stationary points must satisfy that $(a + \kappa x - \beta e^{(cx)})(\kappa - \beta c e^{(cx)}) = 0$. With the constraint by (3.23), the only stationary point is determined by $\kappa - \beta c e^{(cx)} = 0$, e.g.

$$x_0 = c^{-1} \ln\left(\frac{\kappa}{\beta c}\right) \quad (3.24)$$

Even this stationary point must satisfy the constraint by (3.23), which is summarized in the following proposition and illustrated in a numerical example (Figure 3.3(b)). The proof is straightforward by substituting (3.24) back into (3.23).

Proposition 3.3. *For $\Pi_M^*(x)$ to have a valid stationary point in x 's feasible range, it must satisfy the condition $k \cdot \ln\left(\frac{k}{\beta c}\right) + ca - k > 0$.*

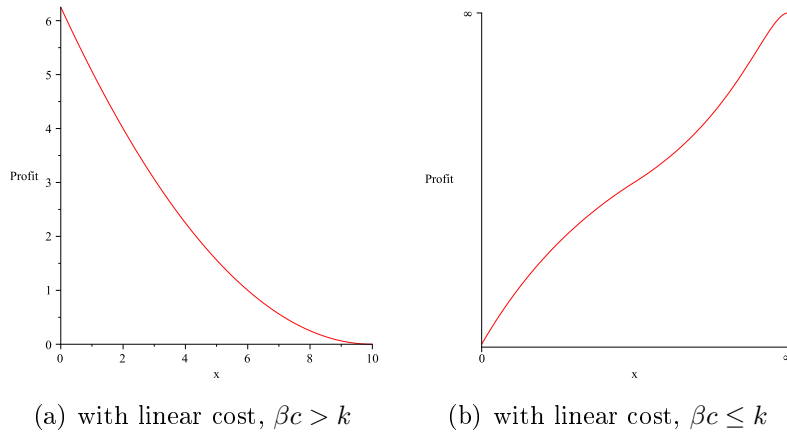


Figure 3.2 : Profit curves with linear cost function

3.5 Numerical illustration

In this section, we build a number of numerical examples to help demonstrate the findings in section 3.4. Especially for the case with an exponential cost function which does not result in a closed-form solution, a numerical illustration is necessary.

3.5.1 with linear cost

- When $c\beta > \kappa$, let $a = 10, \beta = 2, \kappa = 3$ and $c = 2$. The graph for $\Pi_M^*(x)$ is shown in Figure 3.2(a). The profit is monotonically decreasing in this range. It makes no sense to have this feature or component included for this case.
- When $c\beta \leq \kappa$, let $a = 10, \beta = 2, \kappa = 5$ and $c = 2$. The manufacturer's profit keeps increasing in x , as shown in Figure 3.2(b).

3.5.2 with quadratic cost

Let $a = 10, \beta = 2, \kappa = 3$ and $c = 2$. We have the graph for $\Pi_M^*(x)$ shown in Figure 3.3(a). The reasonable range for x is $S = [0, 2)$, which satisfies the constraint $w > C(x)$. The global maximum in S is at 0.9275.

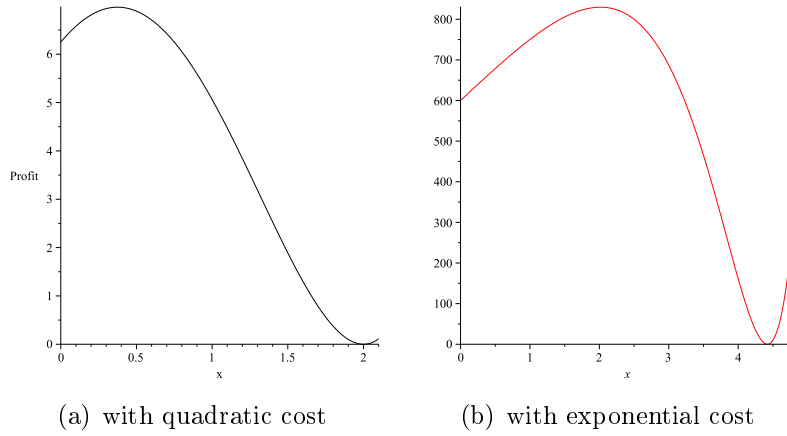


Figure 3.3 : Profit curves with quadratic cost and exponential cost

3.5.3 with exponential cost

Let $a = 100, \beta = 2, \kappa = 15$ and $c = 1$. We have the graph for $\Pi_M^*(x)$ shown in Figure 3.3(b). The reasonable range for x is $S = [0, 4.4207)$, which satisfies the constraint $w > C(x)$. The only stationary point, i.e. the global maximum in S is at $x = c^{-1} \ln\left(\frac{\kappa}{\beta c}\right) = 2.0149$.

3.6 Conclusion

This research studies a supply chain game between a manufacturer and a retailer on a configurable product. A purchasing decision is always a trade-off between price and features. From the manufacturer’s perspective, choices allowed for a configuration incurs cost variation. The value of the configurable feature or component and its ensuing manufacturing cost affect equilibrium decisions. With three different cost functions, we find the feasible range for the configurable feature and examine how cost functions affect optimal solutions.

We assume that manufacturer set a wholesale price higher than the unit manufacturing cost. Thus with linear cost function, we need to differentiate two cases. One is that the marginal gain from the feature value is weaker than the interaction of the cost coefficient and the price sensitivity factor. In this case, the profit drops monotonically in feature value. It makes no sense to have this feature or component. The other one is that the marginal

gain from the feature value is stronger. In this case, the profit keeps increasing all the way. It seems so optimistic that we need to take a second thought on the rationality of the cost function itself. With quadratic cost function, the profit function is quasiconcave on the feature variable's feasible range. There exists a global optimum. This is an ideal case. With exponential cost function, a constraints composed of the parameters must be satisfied for the profit function to have a valid stationary point in the feasible range of feature variable. When it is true, this is the only stationary point. There is no closed-form solution for the feasible range. It is illustrated with a numerical example.

Chapter 4

Competing Products in an Evolving Supply Chain Network

4.1 Introduction

A supply chain is created with partnerships being established between firms involved in this chain. The partnership between firms is subject to change over time. A firm could get involved in more than one supply chain at the same time. The structure of a supply chain or a supply chain network changes when firms join or leave. Suppose there exists a partnership between AT&T and Motorola on a particular model as AT&T sells phones manufactured by Motorola to end customers. A similar partnership exists between AT&T and Samsung. AT&T as a retailer in the supply chain is a downstream firm. Motorola and Samsung as manufacturers are the upper stream firms. Quite a few studies on supply chain coordination with such structures have been reported in literature in which the structure remains the same throughout the sequence of events once it has been established. In contrast to the *static* structure assumed in the literature, this paper takes an evolving supply chain as the research subject.

In an evolving supply chain network, partnerships between firms are not all established at one time. We assume these partnerships are managed with contracts in this paper. As we know, Motorola Atrix 4G and Samsung Infuse 4G are two Android smart phone with similar specification carried by AT&T. There should not be much objection if they are considered as competing products or substitutable products for the same market segments. Suppose initially AT&T signs a contract with Motorola for its 4G smart phone, Atrix. When the contract on Atrix phone between Motorola and AT&T expires, they may decide to renew,

revise the contract or pull this product out of the market. In the meanwhile, Samsung joins AT&T with its own 4G smart phone, Infuse, before or after the contract for Atrix expires. Before the contract between AT&T and Motorola is revised, AT&T has the option of adjusting Atrix's retail price. Upon the expiration of this contract, AT&T and Motorola have the option to revise the existing contract. The introduction of Samsung Infuse may affect Atrix's profitability. In this scenario, the supply chain starts with the partnership between AT&T and Motorola. Another partnership between AT&T and Samsung is introduced later and competition between Atrix and Infuse comes to the scene.

A supply chain structure is called *static* if the number of firms and the partnerships between those firms remain same over the life cycle of transactions under investigation. In contrast, the structure of a supply chain network evolves when the following events occur: (1) an existing partnership expires and members leave; (2) an existing partnership is renewed or revised; (3) a new partnership is established between a new member and an existing member. Given the existence of a number of partnerships, the decision on a new partnership or contract is conditional on the existing partnerships and other decisions outside of the contract terms may be affected by this new partnership. We assume that the contract structure for the new partnership is identical to that of the existing partnership. There could be numerous scenarios in reality under this novel framework of evolving supply chain. Evolving events can occur to memberships and/or partnerships.

Among a large variety of possible evolving scenarios, this paper studies a case with two stages in a two-echelon supply chain. In stage one, it consists of a single manufacturer and a single retailer coordinated with a specific revenue sharing contract on a product by the manufacturer. It is a decentralized version of the price-setting newsvendor problem. In stage two, another manufacturer joins the supply chain with its competing and differentiated product before the current contract expires. We assume the same contract structure for the second product. The profitability of the first product may be affected by the newly introduced product. The retail price of the product from the first supplier may have to

be adjusted if it is not a contract term between the retailer and the manufacturer. This research investigates the impact by the new product on the current product's optimal price and optimal profit. The product differentiation factor or product substitution factor works as an index of competition between the two products. Please be noted that the concept *stage* in this paper is different from the concept *period* usually used in operations management for ordering policy or inventory policy. We assume that the activities above occur within one single period.

The current literature in operations management and marketing holds a large collection of publications on supply chain coordination with structures indicated in each stage above respectively and separately. There has never been a study linking them together in a sequence where the dependence between partnerships are taken into consideration.

The rest of this paper is organized as follows. Section 4.2 presents a relevant literature review. Section 4.4 defines the model and the sequence of events occurred in the scenario introduced above. Section 4.5 conducts analysis of the coordination in each stage and discover the link between them. In addition, we study the performance impact on the first product profitability by introducing the second product. The final section discusses our results and conclude.

4.2 Literature Review

Even though quite a few scholars in economics, marketing and operations management have conducted researches on channel coordination or supply chain coordination in the past decades, all their research subjects are in a context of *static* supply chain structure. No any research paper on supply chain coordination in the literature has been found as far as we know focusing on such an evolving supply chain as introduced in section 4.1. As each stage of the evolving scenario carries a classic supply chain structure which has been studied extensively, related results obtained in the literature on general supply chain coordination

can be used as building blocks for solutions to research questions in this new framework.

At the first stage, the supply chain consists of one manufacturer and one retailer. The issue of suboptimal overall performance is first identified by Spengler (1950) that aggregate profit in a decentralized supply chain with one manufacturer and one retailer may be lower than that in a vertically integrated chain where a central planner makes decisions for both the manufacturer and the retailer. It is the well known double marginalization problem. Cachon (2003) conducts a survey of supply chain coordination with contracts. Coordination contracts under a variety of scenarios are investigated. In a number of cases, coordination can be achieved with many different contract forms. Cachon and Lariviere (2005) investigate strengths and limitations supply chain coordination with revenue-sharing contracts by comparing it to a number of other supply chain contracts. They explain why it is not prevalent in industry. Cachon and Netessine (2006) gives a tutorial on game modeling with supply chain analysis which is a basic methodology for analysis on supply chain coordination. A game version of the classic newsvendor problem is a basic model for this problem (Chen and Simchi-Levi, 2004; Parthasarathi *et al.*, 2010; Serin, 2007; Qin and Yang, 2008; Qin *et al.*, 2011; Zhou and Chen, 2011).

At the second stage, the supply chain consists of two competing products and one retailer. While the first manufacturer is not in the game, the pricing decision on its product is a decision variable of the retailer. It is different from the case that two manufacturers and one retailer are involved in a strategic game. Ha *et al.* (2003) study a supply chain with two suppliers competing for supply to a single customer based on price and delivery frequency. They showed that delivery frequency can be a source of competitive advantage and higher delivery frequencies lower the value of getting deliveries from the second supplier and there intensify the competition. When customer control the deliveries, she would strategically increase delivery frequencies to lower prices. The overall performance is lower when the customer control the deliveries. Cachon and K ok (2010) conduct a research on coordination with price for a setting with two competing manufacturers selling through a single retailer.

The retailer faces price sensitive customer demands. The linear demand function is employed for the analysis. They investigate the performance of a few payment schemes, such as wholesale-price contract, quantity-discount contract and two-part tariff contract. When a competing manufacturer joins, it may lead to an equilibrium in which each party is worse off than the case without competition. Either the retailer or the manufacturers may offer sophisticated contracts to induce the other side to coordinate.

Analysis of the single period newsboy problem with two substitutable products appears in (McGillivray and Silver, 1978; Parlar and Goyal, 1984; Parlar, 1988; Pasternack and Drezner, 1991; Khouja, 1999; Nagarajan and Rajagopalan, 2008). Parlar and Goyal (1984) study the decision problem on order quantity only for two substitutable products with stochastic demand. Parlar (1988) is the first analysis of game version of the newsvendor problem with two retailers competing on the availability of substitutable product inventory. Huang *et al.* (2011) studies a multi-product competitive newsboy problem with partial product substitution. We characterize the unique Nash equilibrium of the competitive model and analyze some properties of the equilibrium. An iterative algorithm is developed on the basis of approximating the effective demand as well as the expected profit function for each product. Numerical experiments are conducted to illustrate the impacts of product substitution, demand correlation and demand variation on the optimal order quantities and the corresponding expected profits, and to compare the total optimal inventory level of the competitive case with that of the centralized case. The conclusion that competition always results in a higher total inventory level, even under the effect of product substitution is drawn in the symmetric case.

Customers either choose to buy the product by the first manufacturer or not to buy at the first stage. At the second stage two competing choices from the retailer at the second stage. Demand substitution has been modeled and investigated in areas of assortment planning and inventory management. Kök *et al.* (2009) gives a review on assortment planning in which demand models with substitution are discussed in detail. Kök and Fisher (2007) specifies

substitution models in order for demand estimation in assortment planning scenarios. Parlar (1988) analyzes the inventory problems with two substitutable products which fit into the second stage problem in our research. A substitution rate is known in advance. Serin (2007) studies competitive newsvendor problem with two random demands. When there is excess demand for one product, it is reallocated to the other product with a known substitutable rate. In this case, customers are not choosing the products on their own.

The attempt to integrate inventory control and pricing strategies was first fulfilled by Whitin (1955). Closed-form solution for a style goods model with a rectangular demand distribution is presented. Petruzzi and Dada (1999) provides a review on dynamic inventory models with pricing and stochastic demand. Qin *et al.* (2011) conduct a review on newsvendor problem with specific extensions in the context of modeling customer demand, supplier costs, and the buyer risk profile. Li and Huh (2011) investigates pricing decisions during inter-generational product transition. Even the subject in this paper is a line of product by the same vendor, the idea can be used for competing products from different manufacturers. Shi *et al.* (2013) study the effect of supply chain power structure on supply chain performance. It is based on equilibrium analyses in the framework of manufacturer Stackelberg, retailer Stackelberg and Nash games assuming stochastic additive demand and stochastic multiplicative demand. The manufacturer makes decision on wholesale price. The retailer makes decision on retail price and order quantity or equivalently safety stock level after being transformed.

4.3 Product differentiation and substitution

The supply chain structure under investigation evolves when a second competing and differentiated product is introduced to the retailer by the second manufacturer. Demand switching occurs between the two competing products. The concept of product substitution here is a little from different from typical mechanism assumed in the literature of operations

management. Let us discuss the issue of product differentiation and substitution before moving ahead to formulate the supply chain games.

It is very common in the operations literature that demand substitutions are classified as two categories, *stockout-based substitution* and *assortment-based substitution* (Agrawal and Smith, 2003; K ok and Fisher, 2007; Dong *et al.*, 2009; Zhao and Atkins, 2009). Substitutions in both of these categories occur when the target purchase is not fulfilled. In this paper, products are considered as substitute to each other not necessarily when one is unavailable.

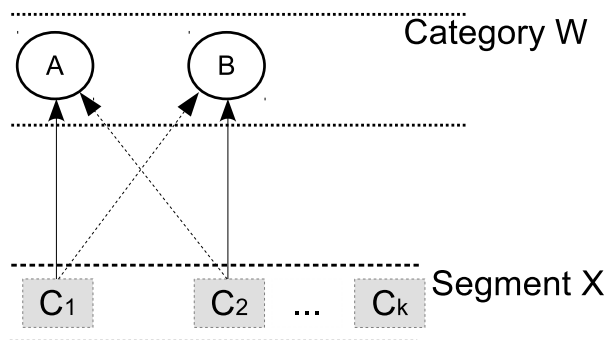


Figure 4.1 : Product substitution and demand switching

Suppose that product A and product B are in the same category or in the same assortment. Customer C_1, C_2, \dots, C_k in market segment X hold the same budget for either product A or product B. As shown in figure 4.1, customers could choose either product A or product B. If the percentage difference of segment X between choosing A and choosing B is less than a threshold, we consider product A and product B are substitutable for this market segment. Demand switching in the scenario under investigation occurs in only one direction from A to B.

Product differentiation and substitution have been research topics on product design, retail assortment, etc. for many years. Firms differentiate their products in order to capture a larger share of the target market. For customers, products in the same market by different

firms may serve as substitutes of each other. It is subject to each customer's utility function to determine how his/her choice responds to the differentiation and/or the substitution. The definition and the degree of products to be differentiated or substitutable affects the model formulation for a particular decision problem. In Adida and Perakis (2009), product differentiation is captured with the structure of the matrix of inverse demand sensitivities. In Alexandrov (2011), two manufacturers compete through a retailer and products are differentiated with advertising. The demand formulation is based on Hotelling model.

With is linear demand function, cross-price elasticity interprets not only the demand variation in prices but also the degree of differentiation or the substitutability between the two products (McGuire and Staelin, 1983; Gibbons, 1992; Anderson and Bao, 2010; Zhu and Thonemann, 2009; Tremblay and Tremblay, 2011). The greater the difference in the cross-price elasticity in the demand function, the less substitutable are the products.

4.4 Model

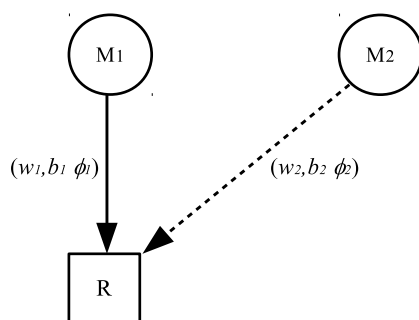


Figure 4.2 : Manufacturer M_2 joins

The supply chain under investigation in this research evolves through two stages (Figure 4.2). At the first stage, two risk-neutral firms, manufacturer M_1 and retailer R are in a partnership with a profit sharing contract $\{w_1^*, b_1^*, \phi_1\}$ on M_1 's product P_1 , where w_1^* is the optimal wholesale price, b_1^* is the buyback credit and ϕ_1 is the fraction of revenue allocated

to the retailer. Then at the second stage, another risk-neutral manufacturer, M_2 joins the retailer for a partnership on its product P_2 , which is competing and substitutable with P_1 for the same market segment. The firms have the same knowledge about the market size potentially for P_1 and P_2 . A percentage of customers in this market segment who initially choose not to buy P_1 now turns to P_2 . For the retailer, demand in this segment increases with the introduction of P_2 .

We assume the linear stochastic demand model with additive uncertainty in favor of expressiveness and tractability.

$$D(p) = q(p) + \xi, \text{ assumed } > 0, \quad (4.1)$$

where $q(p) = a - \beta p$, is the deterministic component and ξ is a nonnegative price independent random variable. Let $F(x)$ and $f(x)$ be the distribution and density function of ξ respectively and $\bar{F}(\cdot) = 1 - F(\cdot)$. Define the failure rare function of ξ as

$$r(\xi) = \frac{f(\xi)}{1 - F(\xi)} \quad (4.2)$$

We assume that $r'(\xi) > 0$, i.e. ξ has increasing failure rate (IFR). For a detailed discussion about the properties of IFR, please refer to (Barlow *et al.*, 1987; Lariviere, 1999; Lariviere and Porteus, 2001; Lariviere, 2006),

In this model, the sales period is exogenously determined and only the retailer generates revenue. The manufacturer makes a decision on the wholesale price, w for its product. The retailer makes two decisions for a particular product: the purchasing quantity, $q \geq 0$, and the retail price, p . Let p_i denote the unit retail price and c_i the unit manufacturing cost of product i , $i=1,2$. The manufacturer offers a buyback credit k for each unsold unit. Unsatisfied demand is lost, either switching to another product or choosing not to buy. We assume $b < w$.

Notations to be used in the rest of this paper is listed in the following table. In addition, we use superscript $*$ for the optimal solutions and use subscripts M and R to distinguish between items for manufacturer and retailer. When a discussion about counterpart variables in two stages is going on, a symbol in its original format is for it in the first stage and the one with hat, ' $\hat{\cdot}$ ' for it in the second stage. The two manufacturers are denoted by M_i or $M_j, i, j = 1, 2$.

D : the stochastic demand	c : unit manufacturing cost
$q(p)$: the mean value of the demand D	q : the order quantity
ξ : the random component of D	φ : a ratio parameter
w : the wholesale price	b : the buyback credit
p : the retail price	γ : product substitutability
z : safety stock factor	δ : a difference parameter
a : the market segment size	β_i : the price sensitivity factor
θ_i : coefficient on the quantity of product i in inverse demand.	
γ : coefficient on the other product's quantity in inverse demand.	
τ : the cross price sensitivity factor	
\bar{p} : the upper bound of p , $D(p) = 0, \forall p \in [\bar{p}, +\infty]$	
$\hat{\cdot}$: the corresponding symbol "?" in stage 2	

4.4.1 Stage 1

The first stage observes a decentralized version of price-setting newsvendor problem between one manufacturer and one retailer, which has been extensively investigated (Pasternack, 1985; Petruzzi and Dada, 1999; Agrawal and Seshadri, 2000; Cachon, 2003; Chen and Simchi-Levi, 2004; Cachon and Netessine, 2006; Zhao, 2008; Özen *et al.*, 2011; Shi *et al.*, 2013). It runs as a Stackelberg game with the manufacturer as the leader. The manufac-

turer makes decisions on the wholesale price and the retailer makes decisions on its order quantity and retail price.

We assume that the retailer and the first manufacturer agree on a contract proposed by (Yao, 2002; Yao *et al.*, 2006) in order to achieve system wide optimum. The contract offered by the manufacturer specifies a wholesale price and a buyback price which together implements a profit sharing policy. The retailer chooses retail price and order quantity as its decision variables. With such a contract, the optimal solution leading to system optimization of overall profit also optimizes the retailer's payoff and the manufacturer's payoff, e.g. $q_S^* = q_R^* = q_{M_1}^*$. We have the payoff functions listed as below, Π_S for system wide supply chain profit, Π_R for the retailer's profit and Π_{M_1} for the current manufacturer's profit.

$$\Pi_S(p_1, q_1) = p_1 E[\min(q_1, D_1)] - c_1 q_1 \quad (4.3)$$

$$\Pi_R(p_1, q_1) = p_1 E[\min(q_1, D_1)] - w_1 q_1 + b_1 E[q_1 - D_1]^+ \quad (4.4)$$

$$\Pi_{M_1}(w_1, b_1) = (w_1 - c_1) q_1 - b_1 E[q_1 - D_1]^+ \quad (4.5)$$

The profit sharing contract is specified with $w_1(p_1^*) = \phi_1(p_1^* - c_1) + c_1$ and $b_1(p_1^*) = \phi_1 p_1^*$, where $0 \leq \phi_1 \leq 1$. It can be shown that the retailer's profit accounts for a fraction ϕ_1 of the system optimum profit and the manufacturer's accounts for $1 - \phi_1$. With such a coordination mechanism, as long as the optimal solution to the system-wide optimization problem is solved, solutions for the retailer and the manufacturer are also available. We examine the optimality of the system side optimization problem only.

4.4.2 Stage 2

The second stage observes an evolving step of the supply chain structure in which another manufacturer, M_2 joins with its competing product. M_2 offers a wholesale price w_2 and a revenue share ϕ_2 to the retailer. Then the retailer makes a decision on the retailer price p_2

and p_1 .

Now the supply chain consists of two manufacturers and one retailer. But it is different from the subjects investigated in (Choi, 1991, 1996; Hsieh and Wu, 2009; Cachon and K ok, 2010) where two manufacturers make decisions simultaneously. Aydin and Porteus (2008) study a multiple product case on joint inventory and pricing problem, which assumes that the demand of different products are statistically independent from each other for a given retail price. Zhao and Atkins (2008) study newsvendor games competing on price and stock availability. In the scenario investigated in this paper, the partnership between manufacturer one and the retailer has already established with their contracts. Manufacturer one is not supposed to change its wholesale price before the contract expires. But the retailer is free to adjust the retail price of product one. Now the retailer aims to maximize its profit based on three decision variables, the retail price of product one, p_1 , the retail price of product two, p_2 , and the order quantity of product two, q_2 . The optimal order quantity of product one is already determined in stage 1.

With the introduction of product two, demand is redistributed between the two competing products either on price or features. A percentage of customers who initially choose to buy product one now switch to product two. Another percentage of customers who initially decide not to buy product one now are attracted by product two. The purchasing decision is made based on a trade-off between price and other product features, such as processor frequency, battery life and so on for a smart phone.

4.4.2.1 Demand compatibility

As the supply chain evolves through stages, the demand model extends from the version for one single product to the one for two competing products. In both stages, the supply chain is targeted at the same market segment, the demand models for each stage must be

compatible to each other. Let us start with the demand model for two products

$$D_i = q_i(p) + \xi_i, i = 1, 2. \quad (4.6)$$

As assumed that the random component is independent across stages and between products, it is acceptable to establish the compatibility on the deterministic component, $q_i(p)$ only. Linear demand model is immediately a result of quadratic utility maximization in its inverse format. For two competing products (Singh and Vives, 1984; Zhang and Bell, 2007; Kim and Bell, 2011; Clarke and Collie, 2003; Theilen, 2012; Symeonidis, 2003; Ledvina and Sircar, 2011)

$$p_i(q) = \alpha_i - \theta_i q_i - \gamma q_j, \quad (4.7)$$

$$q_i(p) = a_i - \beta_i p_i + \tau p_j \quad (4.8)$$

$$\text{where } a_i = \frac{\alpha_i \theta_j - \alpha_j \gamma}{\theta_i \theta_j - \gamma^2}, \beta_i = \frac{\theta_j}{\theta_i \theta_j - \gamma^2}, \tau = \frac{\gamma}{\theta_i \theta_j - \gamma^2}$$

The demand function reduces to $q_i(p_i) = a_i - \beta_i p_i$ for a single product, where $a_i = \frac{\alpha_i}{\theta_i}$ and $\beta_i = \frac{1}{\theta_i}$.

Let $d = \frac{\gamma^2}{\theta_i \theta_j}$, considered as the degree of product differentiation when $\alpha_i = \alpha_j$. When d is approaching 0, the two products are closer to be heterogeneous. When d is approaching 1, they are closer to be homogeneous and more substitutable to each other. To express the demand redistribution discussed above, two properties summarized in the ensuing propositions must be satisfied.

Proposition 4.1. $\hat{q}_1(p_1, p_2) \leq q_1(p_1)$. *The demand of product one in stage two is no more than it in stage one.*

Proof.

$$q_1 - \hat{q}_1(p_1, p_2) = \frac{\alpha_1}{\theta_1} - \frac{1}{\theta_1} p_1 - \left[\frac{\alpha_1 \theta_2 - \alpha_2 \gamma}{\theta_1 \theta_2 - \gamma^2} - \frac{\theta_2}{\theta_1 \theta_2 - \gamma^2} p_1 + \frac{\gamma}{\theta_1 \theta_2 - \gamma^2} p_2 \right] \quad (4.9)$$

$$= \frac{\gamma}{\theta_1} q_2 \geq 0 \quad (4.10)$$

□

Proposition 4.2. $q_1(p_1) \leq \hat{q}_1(p_1, p_2) + \hat{q}_2(p_1, p_2)$. *The demand of product one in stage one is no more than the total demand of two products in stage two.*

Proof.

$$\hat{q}_1(p_1, p_2) + \hat{q}_2(p_1, p_2) - q_1(p_1) = \frac{\theta_1 - \gamma}{\theta_1} q_2 \quad (4.11)$$

It is widely accepted in economics and verified by Patrick and Wolak (2001) that the cross-price elasticity is greater than self-price elasticity, e.g. $\beta_1 \geq \tau$ for the model specified in (4.8). Subsequently, it is straightforward that $\beta_1 \geq \tau \Rightarrow \theta_1 \geq \gamma$. Thus, (4.11) is non-negative.

□

4.4.2.2 Payoff functions at this stage

The second manufacturer works as the Stackelberg leader at this stage and followed by the retailer. For product one, its wholesale price, order quantity and buyback price are already determined at stage one with the first-order-condition solution and the contract. The retailer has the freedom to adjust its retail price to counteract the potential demand loss due to forthcoming product competition. Thus the retailer needs to make decisions on three variables, (p_1, p_2, q_2) . We have the payoff functions listed below, Π_S for system wide

supply chain profit, Π_R for the retailer's profit and Π_{M_2} for the current manufacturer's profit.

$$\Pi_S(p_1, p_2, q_2) = p_1 E[\min(q_1, \hat{D}_1)] - c_1 q_1 + p_2 E[\min(q_2, \hat{D}_2)] - c_2 q_2 \quad (4.12)$$

$$\begin{aligned} \Pi_R(p_1, p_2, q_2) &= p_1 E[\min(q_1, \hat{D}_1)] - w_1 q_1 + b_1 E[q_1 - \hat{D}_1]^+ \\ &\quad + p_2 E[\min(q_2, \hat{D}_2)] - w_2 q_2 + b_2 E[q_2 - \hat{D}_2]^+ \end{aligned} \quad (4.13)$$

$$\Pi_{M_2}(w_2, b_2) = (w_2 - c_2) q_2(w_2) - b_2 E[q_2 - D_2] \quad (4.14)$$

We use the same type of contract as that for the game at stage one for this stage. We are able to come up with $w_2(p_2^*) = \phi_2(p_2^* - c_2) + c_2$ and $b_2(p_2^*) = \phi_2 p_2^*$, where $0 \leq \phi_2 \leq 1$. With such a mechanism, the solution to the retailer's problem solves the manufacturer's problem optimally as well. With such a contract with manufacturer two, the partnership for the second product is able to achieve supply chain optimization.

4.5 Analysis

We have the problem for each stage defined in section 4.4. A contract proposed by Yao (2002) is employed to coordinate the partners in each game. With such a mechanism, the partners have a uniform goal toward an optimal solution. In this section, we show the optimality can be achieved for each stage and examine how the introduction of a differentiated product affect profitability of both products. The optimality is not our goal with this research but a building block for further comparison. As the optimal solution is non-closed form, we conduct numerical experiments to show how the optimal solution is affected by the product differentiation factor.

The numerical experiments assume that the demand of each product follows a normal distribution. Specifically, the uncertainty term in (4.6), $\xi_i \sim N(0, 1)$. The numerical experiments report a group of comparisons, each of which is under three conditions, $\beta_1 > \beta_2$, $\beta_2 > \beta_1$ and $\beta_1 = \beta_2$.

4.5.1 Optimality analysis for Stage 1

By utilizing the relationship between safety stock factor z and the order quantity q that $q = a - \beta p + z$, the profit function specified with (4.3) for the coordinated supply chain can be transformed as follows:

$$\Pi_S(p_1, z_1) = (p_1 - c_1)(a - \beta_1 p_1 + z_1) - p_1 \int_{\underline{\xi}_1}^{z_1} F(x) dx. \quad (4.15)$$

Take derivative on z_1 and solve the first-order-condition, we obtain the optimum for z_1 .

$$z_1^*(p_1) = F^{-1}(\varphi_1), \text{ where } \varphi_1 = \frac{p_1 - c_1}{p_1}. \quad (4.16)$$

Substitute the above into (4.15) and we have the profit function transformed into a function of p_1 only.

$$\Pi_S(p_1) = (p_1 - c_1)(a - \beta_1 p_1 + F^{-1}(\varphi_1)) - p_1 \int_{\underline{\xi}_1}^{F^{-1}(\varphi_1)} F(t) dt. \quad (4.17)$$

The following theorem shows the concavity of the profit function and we are able to determine the optimal solution by solving the first-order-condition.

Theorem 4.1. *(Yao et al., 2006) With a demand function having an increasing price elasticity (IPE) deterministic component and the random component follows an increasing failure rate (IFR) distribution, the profit function is quasi-concave in p_1 on $[c_1, \bar{p}_1]$. Thus, the profit function has a unique optimum solution that maximizes it.*

Next we keep working on the first order condition. The optimal solution p_1^* is implicitly

embedded in the non-closed form expression below.

$$\frac{\partial \Pi_S}{\partial p_1} = -\beta_1(p_1 - c_1) + (a - \beta_1 p_1) + \int_{\underline{\xi}_1}^{F^{-1}(\varphi_1)} t f(t) dt + (1 - \varphi_1) F^{-1}(\varphi_1) = 0 \quad (4.18)$$

$$\Rightarrow a - 2\beta_1 p_1 + \beta_1 c_1 + \int_{\underline{\xi}_1}^{F^{-1}(\varphi_1)} t f(t) dt + (1 - \varphi_1) F^{-1}(\varphi_1) = 0 \quad (4.19)$$

4.5.2 Optimality analysis for Stage 2

Using the similar technique as in last section, we are able to transform the system profit function at stage two into a function of (p_1, p_2, z_2) . Similarly, we obtain the optimum z_2^* as a function of (p_1, p_2) .

$$\begin{aligned} \Pi_S(p_1, p_2, z_2) &= (p_1 - c_1)(a_1 - \beta_{11}p_1 + \beta_{12}p_2 + z_1) - p_1 \int_{\underline{\xi}_1}^{z_1} F(x) dx \\ &\quad + (p_2 - c_2)(a_2 + \beta_{21}p_1 - \beta_{22}p_2 + z_2) - p_2 \int_{\underline{\xi}_2}^{z_2} F(x) dx \end{aligned} \quad (4.20)$$

$$z_2^*(p_2) = F^{-1}(\varphi_2), \text{ let } \varphi_2 = \frac{p_2 - c_2}{p_2} \quad (4.21)$$

Substitute z_2^* back into (4.22), we have the profit function transformed into a function of (p_1, p_2) only.

$$\begin{aligned} \Pi_S(p_1, p_2) &= (p_1 - c_1)(a_1 - \beta_{11}p_1 + \beta_{12}p_2 + F^{-1}(\varphi_1)) - p_1 \int_{\underline{\xi}_1}^{z_1} F(x) dx \\ &\quad + (p_2 - c_2)(a_2 + \beta_{21}p_1 - \beta_{22}p_2 + F^{-1}(\varphi_2)) - p_2 \int_{\underline{\xi}_2}^{z_2} F(x) dx \end{aligned} \quad (4.22)$$

In order to show the joint concavity, we need an assumption on the matrix of product substitution factors β_{ij} . It is commonly recognized as a property of matrix, *diagonal dominance*. A strictly diagonal dominant Hessian matrix ensures a function's unimodality.

Assumption 4.1. $|\beta_{ii}| > \beta_{ij}$, Each unit reduction in product i 's demand results in less than one unit of additional demand in the other product, $i, j = 1, 2$.

Theorem 4.2. *With a demand function having a increasing price elasticity (IPE) deterministic component and the random component follows an increasing failure rate (IFR) distribution, the profit function in (4.22) is joint concave in (p_1, p_2) is quasi-concave in p_1 on $[c_1, \bar{p}_1]$. Thus, the profit function has a unique optimum solution that maximizes it.*

The optimal solution, (p_1^*, p_2^*) , solves the first order condition as a group of two equations below.

$$\begin{cases} a_1 - 2\beta_{11}p_1 + (\beta_{12} + \beta_{21})p_2 + c_1\beta_{11} - c_2\beta_{21} + \int_{\underline{\xi}_1}^{F^{-1}(\varphi_1)} tf(t)dt + (1 - \varphi)F^{-1}(\varphi_1) = 0, \\ a_2 - 2\beta_{22}p_2 + (\beta_{12} + \beta_{21})p_1 + c_2\beta_{22} - c_1\beta_{12} + \int_{\underline{\xi}_2}^{F^{-1}(\varphi_2)} tf(t)dt + (1 - \varphi_2)F^{-1}(\varphi_2) = 0. \end{cases} \quad (4.23)$$

4.5.3 Impact on retail price p_1

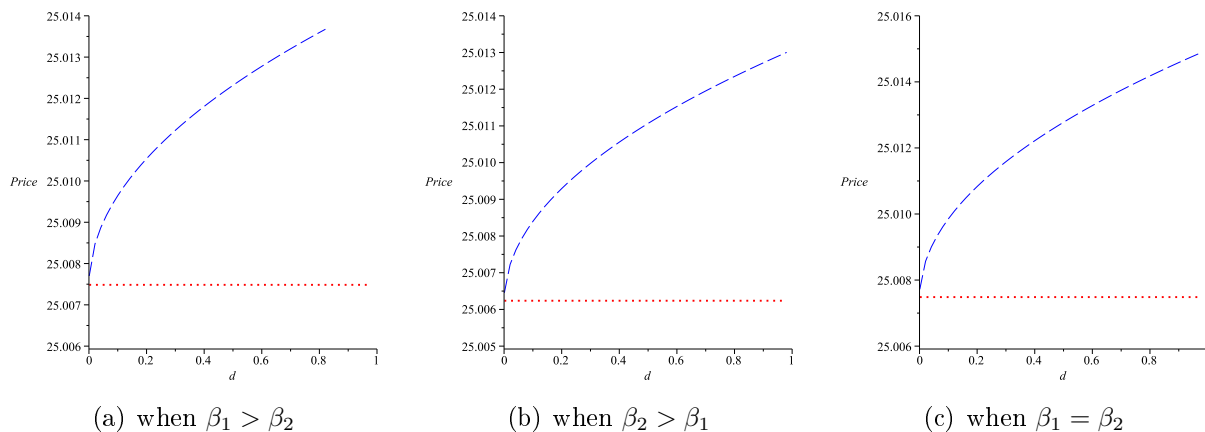


Figure 4.3 : $p_1^*(d)$, ... for stage 1 and - - - for stage 2

As a second product competing for the same market segment joins, a percentage of the demand initially in favor of product one now turns to product two. The retailer has freedom to adjust the retail price of the first product, p_1 , for optimal profit with reallocated demand.

Figure 4.3 demonstrates a comparison of the optimal retail prices of product one, $p_1^*(d)$ in two stages as a function of product differentiation factor d . The three cases under conditions,

$\beta_1 > \beta_2$, $\beta_2 > \beta_1$ and $\beta_1 = \beta_2$ follow a similar trend. d has nothing to do with p_1^* in stage one and p_1^* remains constant in the graph. With d increasing gradually, the degree of substitution between the two products increases in stage two. $p_1^*(d)$ for stage two shows an increasing convex curve with an initial value equal to the constant for stage one. It seems counterintuitive with an increasing optimal retail price when its demand is decreasing. But it is reasonable that a price increase counterbalances the effect by demand loss for a higher profit.

4.5.4 Potential impact on wholesale price w_1 and b_1

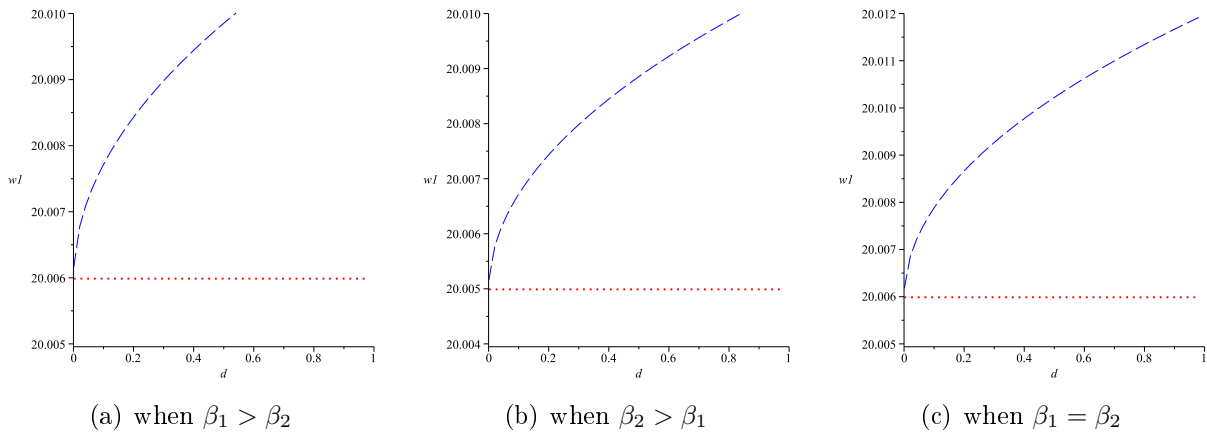


Figure 4.4 : $w_1^*(d)$, ... for stage 1 and - - - for stage 2

As discussed in section 4.4.1, w_1 and b_1 are linear functions of p_1^* . Thus, the curves of $w_1^*(d)$ and $b_1^*(d)$ show properties similar as that of $p_1^*(d)$. Figure 4.4 shows curves of $w_1^*(d)$ under three conditions. It is assumed that the wholesale price w_1 and buyback price b_1 for product one are fixed in the contract offered by the first manufacturer. Thus, the graph just presents an indication of potential impact on w_1^* and b_1^* , which can be considered for contract revision.

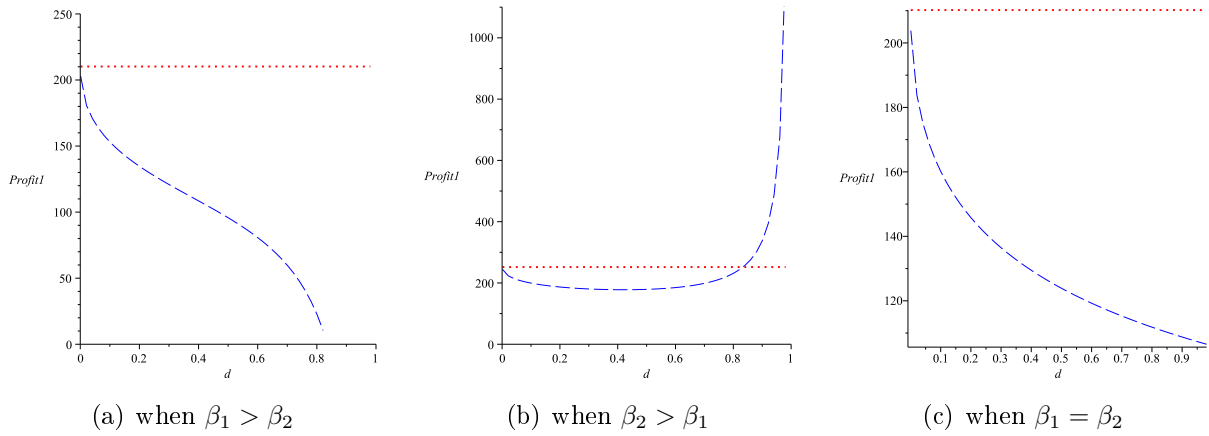


Figure 4.5 : $\Pi_1^*(d)$, ... for stage 1 and - - - for stage 2

4.5.5 Impact on the profitability of product one

In addition to the impact on optimal prices, we examine the impact on the profitability of product one. Curves of $\Pi_1^*(d)$ under three cases shown in Figure 4.5 are quite different from one another. Both curves in Figure 4.5(a) and Figure 4.5(c) drop gradually with increasing d . But there is a reverse of convexity in Figure 4.5(a) and it reaches to zero before d increases to its upper bound. In contrast, it remains concave in Figure 4.5(c) up to the upper bound of d . In Figure 4.5(b), the curve is concave and there exist a local minimum within the feasible region.

4.5.6 Impact on the profitability of product two

The existence of product one affects the profitability of product two as well. Figure 4.6 reports a comparison of two products on profitability in stage two in the range of d . In Figure 4.6(a) for the case that $\beta_1 > \beta_2$, the curve of $\Pi_2^*(d)$ is convex in the feasible region of d and there is a local minimum. In contrast, the curve of $\Pi_1^*(d)$ drops monotonically and reaches zero before d increases to its upper bound. Thereafter, while d is getting close to its upper bound, $\Pi_2^*(d)$ has a sharp increase. This observation interprets the fact that product two seizes all the demand after d reaches a certain point, which means that product two has

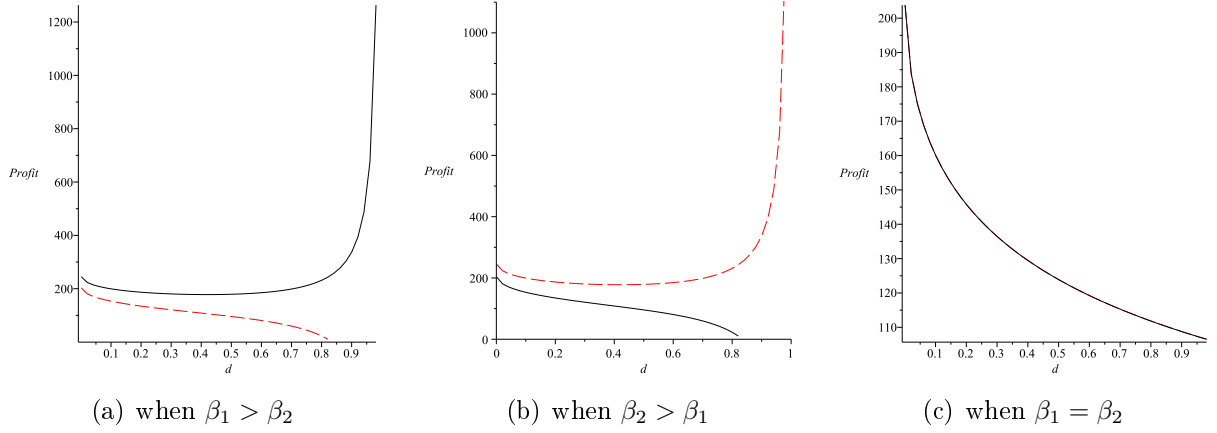


Figure 4.6 : Profits in stage two, - - - for $\Pi_1^*(d)$ and — for $\Pi_2^*(d)$

a higher degree of substitution for product one. Please be noticed that this is the case that $\beta_1 > \beta_2$, i.e. product two has price advantage. The case shown in Figure 4.6(b) is just a reflection of the one in Figure 4.6(a). The gap between the initial values of $\Pi_1^*(d)$ and $\Pi_2^*(d)$ at $d = 0$ comes from equation (4.8).

The third case shown in Figure 4.6(c) is a trivial one in which $\beta_1 = \beta_2$. The two curves overlap each other and show downslope convexity.

4.6 Conclusion

This paper proposes a novel framework of evolving supply chain structure, which allow firm partnerships to change over the period of the scenarios under investigated. With this framework, a large set of research questions addresses in the literature deserve a review. Among the possible evolving scenarios, this paper specifically studies a case that begins with one partnership between two firms on one product and ends with two partnerships among three firms on two differentiated products.

With the optimality analysis, we show it is feasible to coordinate the supply chain with the profit-sharing contract proposed by Yao (2002). We study how the introduction of a competing and differentiated product affects the optimal solution of product one with the

differentiation factor as an index. Based on the non-closed form solution, we develop a set of numerical experiments to demonstrate the impacts following a decrease of product differentiation.

The curves observed with the numerical experiments indicate a number of interesting results. The optimal retail price p_1^* shows a monotonic increasing concave curve when the products are getting less differentiated. The contract parameters w_1 and b_1 which are based on p_1^* show similar trends. The optimal profit of product one shows quite different curves under three cases of $\beta_1 > \beta_2$, $\beta_2 > \beta_1$ and $\beta_1 = \beta_2$. It is an indication that β_i and d may work together in the same way or opposite on profitability. Also interesting is a comparison between two products on profitability in stage two. The product with less price advantage gradually loses its demand when they are getting less differentiated or highly substitutable.

Chapter 5

Conclusion

Motivated by observations of business activities, this dissertation is intended to address some integrated research questions on supply chain operation. It covers pricing, sourcing and manufacturing cost in a number of realistic and significant scenarios. We propose a novel framework for research questions on supply chain management, which considers evolving supply chain structures. This dissertation is expected to identify more realistic and significant questions, provide solutions and deliver business insights to supply chain managers.

The analysis and experiments in the first paper examines the relative impact of differentiated pricing and common pricing policies across retail channels in the suggested scenarios. Also, this study considers the effect of ignoring cross price effects between retail channels.

The second paper studies a supply chain in which a manufacturer sells a configurable product through a retailer. We take the configurable product as a parameterized product and examine the impact of such a feature decision on equilibrium between the manufacturer and the retailer. The analysis considers three different cost functions: (1) linear cost function, (2) quadratic cost function and (3) exponential cost function and examines how the cost functions affect the optimal solutions.

The third paper proposes a novel framework of evolving supply chain structure, which allow firm partnerships to change over the period of the scenarios under investigated. This paper specifically studies a case that begins with one partnership between two firms on one product and ends with two partnerships among three firms on two differentiated products. We study how the introduction of a competing and differentiated product affects the optimal solution of product one with the differentiation factor as an index. Compatibility between

demand models in two stages are established. Based on the analytical non-closed form solution, a number of numerical experiments are developed to demonstrate the impact of the introduction of a competing product on the optimal solution of product one.

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Appendix A

Proofs of Chapter 2

1. Proof of Lemma 2.1

Proof. First, let's expand the expression of $R_j(\bar{p})$,

$$R_j(\bar{p}) = p_j d_j(\bar{p}) \quad (\text{A.1})$$

$$= \alpha_j p_j + \sum_{k=1}^n \beta_{jk} p_j p_k \quad (\text{A.2})$$

$$= \alpha_j p_j + \beta_{jj} p_j^2 + \sum_{k \neq j} \beta_{jk} p_j p_k \quad (\text{A.3})$$

Then, take first derivative on each individual component of the price vector \bar{p} , $p_l, l = 1 \dots n$. The gradient of $R_j(\bar{p})$ on \bar{p} is

$$\nabla R_j(\bar{p}) = \left\{ \frac{\partial R_j(\bar{p})}{\partial p_l} \right\} = \begin{cases} \alpha_j + 2\beta_{jj} p_j + \sum_{k \neq j} \beta_{jk} p_k, \forall l = j \\ \beta_{jl} p_j, \forall l \neq j \end{cases} \quad (\text{A.4})$$

□

2. Proof of Lemma 2.2

Proof. The second order derivatives of R_j

$$\frac{\partial^2}{\partial p_j^2} R_j(\bar{p}) = 2\beta_{jj}, \quad (\text{A.5})$$

$$\frac{\partial^2}{\partial p_l^2} R_j(\bar{p}) = 0, l \neq j \quad (\text{A.6})$$

$$\frac{\partial^2}{\partial p_j \partial p_l} R_j(\bar{p}) = \beta_{jl}, \forall l \neq j \quad (\text{A.7})$$

$$\frac{\partial^2}{\partial p_l \partial p_j} R_j(\bar{p}) = \beta_{lj}, \forall l \neq j \quad (\text{A.8})$$

$$\frac{\partial^2}{\partial p_l \partial p_k} R_j(\bar{p}) = 0, \forall l \neq j, \forall k \neq j \quad (\text{A.9})$$

Let H_i be the Hessian of $R_j(\bar{p})$.

$$H_1 = \begin{bmatrix} 2\beta_{11} & \beta_{12} & \dots & \beta_{1n} \\ \beta_{21} & 0 & \dots & 0 \\ \dots & 0 & \dots & 0 \\ \beta_{n1} & 0 & \dots & 0 \end{bmatrix} H_2 = \begin{bmatrix} 0 & \beta_{12} & \dots & 0 \\ \beta_{21} & 2\beta_{11} & \dots & \beta_{2n} \\ 0 & \dots & \dots & 0 \\ 0 & \beta_{n2} & \dots & 0 \end{bmatrix} \dots H_n = \begin{bmatrix} 0 & 0 & \dots & \beta_{1n} \\ 0 & 0 & \dots & \beta_{2n} \\ 0 & 0 & \dots & \dots \\ \beta_{n1} & \beta_{n2} & \dots & 2\beta_{nn} \end{bmatrix}$$

$$H = \sum_{i=1}^n H_i = \begin{bmatrix} 2\beta_{11} & \beta_{12} + \beta_{21} & \dots & \beta_{1n} + \beta_{n1} \\ \beta_{12} + \beta_{21} & 2\beta_{22} & \dots & \beta_{2n} + \beta_{n2} \\ \dots & \dots & \dots & \dots \\ \beta_{1n} + \beta_{n1} & \beta_{2n} + \beta_{n2} & \dots & 2\beta_{nn} \end{bmatrix}$$

□

3. Proof of Theorem 2.1

Proof. When \mathbf{B} is symmetric and assumption 1 holds, the Hessian matrix ${}^{-}H$ is strictly diagonally dominant with positive diagonal entries, where ${}^{-}H = (-1)H$. Thus, ${}^{-}H$ is positive definite and H is negative definite. So $R(\bar{p})$ is concave. □

4. Proof of Theorem 2.2

Proof. When \mathbf{B} is asymmetric and assumption 1 and 2 hold, the Hessian matrix ${}^{-}H$ is strictly diagonally dominant with positive diagonal entries, where ${}^{-}H = (-1)H$. Thus, ${}^{-}H$ is positive definite and H is negative definite. So $R(\bar{p})$ is concave. □

Appendix B

Proofs of Chapter 3

1. Proof of Proposition 3.2

$$\textit{Proof. } \forall x_1, x_2 \in S = \left\{ x : 0 < x < \frac{\kappa + \sqrt{\kappa^2 + 4c\beta a}}{2c\beta} \right\},$$

$$(1) \quad \Pi_M^*(x_1) < \Pi_M^*(x_2) \Rightarrow \kappa < \beta c(x_1 + x_2) \quad (\text{B.1})$$

$$(2) \quad w^*(x) > C(x) \Rightarrow a + \kappa x > \beta c x^2 \quad (\text{B.2})$$

$$(3) \quad x^* = \frac{\kappa}{2c\beta} \text{ is the only stationary point in } \left[0, \frac{\kappa + \sqrt{\kappa^2 + 4c\beta a}}{2c\beta} \right]. \quad (\text{B.3})$$

By theorem 3.5.4 in Bazaraa *et al.* (2006), the following is always true.

$$(a + \kappa x_2 - \beta c x_2^2)(\kappa - 2\beta c x_2)(x_1 - x_2) > 0, \quad (\text{B.4})$$

So, we claim that $\Pi_M^*(x)$ is strictly quasiconcave in x on the set S . As $x = \frac{\kappa}{2\beta c}$ is the only stationary point, it is a global maximum.

□

Appendix C

Proofs of Chapter 4

Proof. 1. Proof of Theorem 4.2

$$\begin{aligned} \Pi_R(p_1, p_2) &= (p_1 - w_1^*)y_1(p_1, p_2) \\ &\quad - (p_1 - b_1) \int_{\underline{\xi}_1}^{z_1^*} t f_1(t) dt + (p_2 - w_2)y_2(p_1, p_2) - (p_2 - b_2) \int_{\underline{\xi}_2}^{z_2^*} t f_2(t) dt, \end{aligned} \quad (\text{C.1})$$

$$z_2^* = F^{-1}(\varphi), \varphi = \frac{p_2 - w_2}{p_2 - b_2} \quad (\text{C.2})$$

$$\frac{\partial \Pi_R}{\partial p_1} = a_1 - \beta_{11}(2p_1 - w_1) + (\beta_{12} + \beta_{21})p_2 - w_2\beta_{21} + \int_{\underline{\xi}_1}^{F^{-1}(\varphi_1)} t f(t) dt + (1 - \varphi)F^{-1}(\varphi_1) \quad (\text{C.3})$$

$$\frac{\partial^2 \Pi_R}{\partial p_1^2} = -2\beta_{11} + \frac{1}{f_1(F_1^{-1}(\varphi))} \frac{(w_1 - b_1)^2}{(p_1 - b_1)^2} \quad (\text{C.4})$$

$$\frac{\partial^2 \Pi_R}{\partial p_1 \partial p_2} = \beta_{12} + \beta_{21} \quad (\text{C.5})$$

$$\frac{\partial \Pi_R}{\partial p_2} = a_2 - \beta_{22}(2p_2 - w_2) + (\beta_{12} + \beta_{21})p_1 - w_1\beta_{12} + \int_{\underline{\xi}_2}^{F^{-1}(\varphi_2)} t f(t) dt + (1 - \varphi)F^{-1}(\varphi_2) \quad (\text{C.6})$$

$$\frac{\partial^2 \Pi_R}{\partial p_2^2} = -2\beta_{22} + \frac{1}{f_2(F_2^{-1}(\varphi))} \frac{(w_2 - b_2)^2}{(p_2 - b_2)^2} \quad (\text{C.7})$$

With the approach in the proof of theorem 4.1, we are able to show that $\frac{\partial^2 \Pi_R}{\partial p_1^2} < 0$ and $\frac{\partial^2 \Pi_R}{\partial p_2^2} < 0$. To make sure that $\Pi_R(p_1, p_2)$ is jointly concave in (p_1, p_2) , we need to show that the determinant of its Hessian matrix is positive, i.e.

$$\left(-2\beta_{11} + \frac{1}{f_1(F_1^{-1}(\varphi))} \frac{(w_1 - b_1)^2}{(p_1 - b_1)^2} \right) \times \left(-2\beta_{22} + \frac{1}{f_2(F_2^{-1}(\varphi))} \frac{(w_2 - b_2)^2}{(p_2 - b_2)^2} \right) > (\beta_{12} + \beta_{21})^2 \quad (\text{C.8})$$

With the assumption of diagonal dominance, $\beta_{ii} > \beta_{ij}, i, j = 1, 2$, the above is true. \square