REFUELING STATION LOCATION PROBLEM FOR RENEWABLE ENERGY IN A TRANSPORTATION NETWORK WITH MULTIPLE TIME PERIODS, NONLINEAR DELAY, AND AUTONOMOUS USERS

by

SEYEDMOHAMMAD MIRALINAGHI

YINGYAN LOU, COMMITTEE CHAIR

BURCU B KESKIN

ROBERT G. BATSON

STEVEN L. JONES

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ABSTRACT

Construction of renewable energy refueling stations in a transportation network is a major step toward the promotion of alternative fuel vehicles among the customers. In this thesis, we aim to determine the locations of refueling stations in a transportation network through mathematical modeling.

Two optimization models are established with a centralized planner. The objective of the models is to minimize the sum of total cost of constructions, system travel time, and delay at refueling stations. The main difference between these models lies in the modeling of traveler's behaviors. In the first model, we assume that travelers fully comply with the planner's guidance about the routes and stations selection, while the second model is formulated based on independent behaviors of travelers. The first model is a mixed-integer model with constant link travel time and staircase delay at refueling stations. Two well-known solution algorithms, branch-and-bound and Lagrangian relaxation, are employed to solve the model. The second model is a mixed-integer nonlinear model with link travel time and refueling delay both as functions of link flow. To capture users' independent travel behaviors, this model is formulated as a bi-level program, where the lower level problem describes the user equilibrium traffic distribution parameterized by the locations of refueling stations. These models are tested on networks of different sizes to evaluate their effectiveness. Computational studies indicate that these two models can lead to different results. Refueling station location pattern can change completely by using the second model, which considers the autonomous drivers in selecting the

stations and paths. It is shown that optimal location pattern for the first model leads to higher overall cost of construction and total system travel time in comparison to the determined location pattern by the second model.

DEDICATION

This thesis is dedicated to my parents for their love, endless support and encouragement.

LIST OF ABBREVIATIONS AND SYMBOLS

$\mathcal{T}$	Set of time periods
Φ_1	Set of commodities and time periods with unmet demand
Φ_2	Set of commodities and time periods with overmet demand
β	Value of travel time
δ_{ijk}^{w}	$= \begin{cases} 1 & \text{If station } z = (i,j) \in Z \text{ is on path } k \text{ serving refueling commodity } w \\ 0 & \text{otherwise} \end{cases}$
ε	Desired accuracy level of complementary constraints
ζ	Parameter of Lagrangian relxation algorithm
η_r	Demand of customer r
θ	Parameter of Lagrangian relxation algorithm
λ	Lagrangian multipliers
π_i^w	Node potential of node i for non-refueling commodity $w \in W_1$
ρ_{ij}	Capacity of link (i,j)
σ_{rj}	Fraction of total demand η_r of customer site r fulfilled by facility j
τ	Parameter of Lagrangian relxation algorithm
ν_{ij}	Flow of link (i,j)
ν_{ij}^w	Flow of link (i,j) for commodity $w \in (W_1 \cup W_2)$
A	Set of actual links

J	Set of potential facility sites
K^w	Set of potential refueling paths for commodity w
$L_{z,p}$	Upper bound for delay level p at station z
N	Set of nodes
P	Set of possible intervals for staircase delay function
R	Set of customers
$T_z^{w,t}$	Shortest path travel time for any commodity w with a potential station z at time period t
$U_{z,p}^t$	$= \begin{cases} 1 & \text{if station at node } z \text{ operates at level } p \text{ during time period } t \\ 0 & \text{otherwise} \end{cases}$
$Ucap_z$	Capacity of refueling station z
V	Objective value of Lagrangian relaxation
W	Set of commodities (user groups by origin-destination pairs and refueling needs)
W_1	Set of refueling commodities
W_2	Set of non-refueling commodities
Z	Set of potential refueling station locations
c_k^w	Travel time of path k serving commodity w
cc_j	Construction cost of site j
$d_{z,p}$	Delay of refueling drivers in time period t at station z
f_k^w	Flow of path k serving commodity w
h_i^w	Net total demand for commodity $w$at node $i \in N$
$l_{z,p}$	Length of the p^{th} interval of the staircase delay function at station z
$m_{z,p}^t$	Amount of refueling vehicles at station z that are subject to a delay of $d_{z,p}$ at time period t
$q^{w,t}$	Travel demand for each commodity w at time period t

s_j	Capacity of facility j
t_{ij}^0	Free flow travel time of link (i,j)
t_{ij}	Travel time of link (i,j)
tc_{rj}	Unit cost of assigning a unit demand from customer site r to facility j
u^w	Shortest O-D travel time for commodity w
$x_z^{w,t}$	Percentage of travelers of commodity w to be assigned to refueling station z in time period t
y_j	$= \begin{cases} 1 & \text{If the facility is constructed at location } j \\ 0 & \text{Otherwise} \end{cases}$
BPR	Bureau of public roads
CPM	Centralized planning model
CPMAU	Centralized planning model with autonomous users
CFL	Capacitated facility location problem
LB	Lower bound
MIP	Mixed-integer program
MINLP	Mixed-integer nonlinear program
NP-hard	Non-deterministic polynomial-time hard
O-D	Origin-Destination

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Chapter 1. Introduction

"Earth provides enough to satisfy every man's needs, but not every man's greed."

Gandhi.

1.1. Background

Energy is a substantial foundation for economic growth and technology development. Existing energy resources fall into two main categories, renewable and non-renewable energy. Fossil fuels are the majority of non-renewable energy, which are broadly used in industries all over the world. Coal, oil and gas are called "fossil fuels" because they are formed from the organic remains of prehistoric plants and animals. Fossil fuel is an extremely powerful energy resource, easily combustible, and widely distributed with pipe lines and sea tankers. Along with these benefits, there are a number of concerns that motivate researchers to approach healthier fuels. Global warming is one of the most controversial issues during the last decades. It is highly related to concentrations of greenhouse gases produced by human activities such as burning fossil fuels especially in transportation sector. Studies show that a large share of global CO_2 emission is caused by the dependence of transportation on fossil fuels (Simões & Schaeffer, 2005). Besides, oil mining leads to irreversible damage to the adjoining environment, which sometimes results in massive environmental disaster. Such development is not sustainable due to its impacts on the environment and damages to natural resources. Furthermore, oil resources are running out. Although new oil deposits are being discovered each year, the amount does not equal to the level of annual consumption (Owen et al, 2010). This causes the increasing trend of oil prices during recent years. One study in Iran, one of the major exporters of natural resources, reveals that oil and natural gas reserves in Iran will end in the next 84 and 166 years (Ghorashi & Rahimi, 2011).

Due to these issues, demand for renewable energy resources and commercial applications is increasing rapidly. Renewable energy market becomes attractive for large corporations to provide plenty of products using renewable energy technologies. There are many types of renewable energy. Solar energy, hydrogen, and wind are the most well-known sources of renewable energy. The International Energy Agency reports that renewable energies would be capable of serving as the source for 29% of the total power and 7% of transporting fuels for the entire world in 2030 (International Energy Agency, 2007).

In the automobile industry, petroleum-based vehicles are currently the most prevalent with a share of 97% of all vehicles. Increasing the proportion of clean and renewable fuel is a part of government's effort to protect the environment in U.S. (Romm, 2006). For this purpose, hybrid vehicles, as a type of alternative fuel vehicles (AFV), are manufactured. Hybrid vehicle is the term used for vehicles that combine a conventional gasoline or diesel propulsion system with an on-board renewable energy storage system. While hybrid vehicles are able to reduce the emissions by 30% to 50% (Romm, 2006), their short range of driving distances and safety issues due to on-board fuel storage systems are two barriers against the wide use of hybrid vehicles. On the way to solving these issues, Toyota and Ford introduced two new products, namely Prius and Escape, that increase the hope for developing AFVs in the future. These vehicle models do not have the aforementioned issues. Proven safety records and greater driving range in comparison with fossil-fuel-based vehicles lead to the success of these vehicles (Romm, 2006). Hydrogen is another candidate for replacement of gasoline in diesel engine. Studies show that hydrogen could play a major role in transportation industry with an estimated share of 70% of the alternative fuel market in 2050 (Meibom, 2010). It can be produced with either electrolysis or by converting biomass. E-hybrid, also known as plug-in hybrid, is another type of alternative fuel vehicles with lower gas emissions. E-hybrid vehicles refer to vehicles that can utilize both electricity and gasoline. Plug-in hybrid vehicles have the ability to refuel at home, and can cover more range than regular AFVs and pure electricity vehicles.

One important step toward the promotion of hybrid vehicles is the preparation of the transportation infrastructure for vehicles to refuel en route. First of all, traditional single-fuel vehicles as part of the infrastructure should be modified in order to be able to use alternative fuels. The range of capital costs for this alteration is about $440 – 870 million or $310 – 620 per car, which can be viewed as a serious impediment to the development of an alternative-fuel transportation infrastructure (Ogden, 1999). Transportation fuel cells currently cost about $4000/kW, 100 times greater than the cost of internal combustion engines (Wald, 2004). On the other hand, according to the alternative-fuel literature and a survey conducted in 2006 among three groups of US National Renewable Energy Laboratory scientists and local coordinators of US Department of Energy Clean Cities coalitions, the lack of refueling stations is recognized as the biggest barrier against the development of hybrid vehicles (Melendez, 2006). If there is no prepared infrastructure like refueling stations, travelers will not purchase AFVs. If the market of AFV is not popular, there would be no incentive for private and public sectors to invest in it. This interaction is referred to as a chicken-and-egg dilemma.

To break this dilemma, several governments recently began to invest in constructing a robust alternative vehicle industry and infrastructure in order to encourage investments from the private sector and AFV purchases. For example, the Chinese government anticipates investing 10 billion Yuan ($1.46 billion) in technological research and development in the hybrid vehicle industry in order to achieve world- class levels of market penetration, technology, and manufacturing (Campbell, 2012). The United States Senate legislated the Alternative Motor Fuels Act in 1988 to reduce the use of foreign petroleum and replacement of 10% of petroleum-based vehicles with AFVs by 2000 and 30% by 2010. Minnesota enacted law in 2007 to invest multimillion-dollar transportation-related research and funding to double the number of E85 (85% ethanol) fuel stations in the state (Center for Transportation Studies, 2008).

The fact that only 3.4% of all refueling stations in the United States in 2007 were alternative fuel stations indicates there is a long way to establish alternative fuels as a main source of energy in United States (Melendez, 2006). However, as the share of AFVs in United States is increasing, more alternative-fuel stations are needed to provide better services for these travelers (U.S. Energy Information Administration, 2011). In addition, this can lead to the promotion of future development of alternative fuel vehicles. To determine station locations in the network, planners should address three questions:

- Number of required stations to serve all the refueling demand,
- Locations of refueling stations to provide better accessibility, and
- Refueling station capacities.

These are three substantial decisions in the process of developing a transportation infrastructure to support the growth of AFVs.

1.2. Problem Statement

In this thesis, we aim to establish mathematical models and develop efficient solution algorithms to explore answers to the three questions introduced earlier. The refueling facility location problem can be formulated as a combinatorial optimization problem. The objective is to minimize the sum of total system travel time, delay at refueling stations, and station construction cost in the network by determining the number and locations of refueling stations. In this study, we will deal with the capacitated facility location problem, meaning that the demand each facility can handle is limited. Two models are developed with different assumptions.

The first model, called a centralized planning model (CPM), is developed from the perspective of transportation authorities (planner), and travelers are assumed to fully comply with planner's guidance on where to refuel and which routes to take. In this model, the travel times in the transportation network is considered as constants. Moreover, the delay at refueling stations is assumed to follow a stepwise function. The CPM can be applied to the commercial fleet assignment such as United Parcel Service. United Parcel Service (UPS) determines the fleet routes based on their travel times, volume of packages and service center locations. Package cars should follow the UPS planner's guidance in choosing the service centers and routes.

In contrast, the second model takes into account travelers' autonomous travel choices in response to the planner's taken decision on refueling station locations. Travelers would choose for themselves their refueling stations and the shortest paths to the selected stations and their final destinations. In view of this, the second model is called centralized planning model with autonomous users (CPMAU). The CPMAU relaxes the assumption in the CPM that travel times are constant, and considers travel times as functions of link flows. The refueling delay is considered as a continuous function of the amount of refueling vehicles at each station.

Both models can accommodate multiple time periods, and are formulated as mixed integer optimization models. The CPM is a mixed integer linear program (MIP) and the CPMAU is a mixed integer nonlinear program (MINLP). Two common solution algorithms of mixed integer models, the branch-and-bound and the Lagrangian relaxation method, are adopted to solve the CPM. The CPMAU is a mathematical program with complementarity constraint. We utilize the penalty function method and relaxation of the complementarity constraint to solve the second model.

1.3. Organization

The thesis is organized as follows. In Chapter 2, an extensive literature review is conducted on existing models of the capacitated facility location problem and more specifically the refueling station location problem. Two formulations are presented and various solution procedures are proposed in Chapter 3. The proposed algorithms are tested on several networks of different sizes to evaluate their efficiency and effectiveness in Chapter 4 and Chapter 5 and the results of these models are compared to emphasize the importance of considering the bi-level formulation for this problem. Finally, conclusion from this research and possible directions for future research are provided in Chapter 6.

Chapter 2. Literature Review

2.1. Traditional Capacitated Facility Location

The traditional capacitated facility location (CFL) model, like any discrete location model, consists of a set of specific customer demand sites and a set of warehouses, including potential and existing ones, to satisfy the demands for a single type of commodity. A CFL problem aims to determine of the number and locations of warehouses in order to minimize the total cost of transporting commodities between warehouses and customers and the cost of construction. The traditional CFL model is analyzed in many papers and appears in most integer programming texts (Christofides & Beasly, 1983; Geoffrion & McBride, 1977; Chen, Batson, & Dang, 2010). The general formulation of a capacitated facility location problem is as follows:

$$\min \sum_{r \in \mathrm{R}} \sum_{j \in J} tc_{rj} \sigma_{rj} d_r + \sum_{j \in J} cc_j y_j \qquad (1)$$

$$\sum_{j \in J} \sigma_{rj} = 1 \qquad \forall r \in \mathrm{R} \qquad (2)$$

$$\sum_{r \in \mathrm{R}} \eta_r \sigma_{rj} \le s_j y_j \qquad \forall j \in J \qquad (3)$$

$$0 \le \sigma_{rj} \qquad \forall r \in \mathrm{R}, \forall j \in J \qquad (4)$$

$$y_j \in \{0,1\} \qquad \forall j \in J \qquad (5)$$

In this formulation, R and J are the sets of customer and potential facility sites, respectively. tc_{rj} denotes the unit cost of assigning a unit demand from customer site $r \in R$ to facility $j \in J$. For each site j, the construction cost is cc_j, and its capacity s_j. The decision variable, σ_{rj}, is the fraction of total demand η_r of customer site r fulfilled by facility j. y_j is a binary decision variable. y_j is 1 if a facility is located at station j; otherwise, it is zero. The objective function (1) is the sum of the total transportation cost and the construction cost of facilities. Constraint (2) ensures that the unit demand of customer r is satisfied. Constraint (3) guarantees that the total assigned customer demand to a certain facility j does not exceed its capacity.

As seen in equations (1) – (5), the traditional CFL model adopts certain assumptions such as single analysis period and linear cost functions. However, these assumptions are not very concrete or realistic. To relax the assumption of a linear cost function, Holmberg & Ling (1997) proposed a CFL model with a staircase transportation cost function. Many examples of staircase costs in real world are discussed in their paper. They utilized an incremental cost and formulate the problem as a MIP, and proposed a Lagrangian heuristic to solve the model. Keha et al (2004) compared two well-known MIP formulations, the incremental cost and the convex combination, for CFL problems with piecewise cost functions. They showed that linear programming relaxations of both models produce the same lower bound. A piecewise linear cost function is utilized in several facility location research papers (Hritonenko & Yatsenko, 2006; Bayindir, Birbil, & Frenk, 2007; Kameshwaran & Narahari, 2009). In this study, we consider the multi-period demand and more realistic delay functions for locating the refueling stations.

2.2. Refueling Station Location Problem

There are limited numbers of published references specifically dealing with the refueling station location problem. Wang & Lin (2009) divided traditional established models into three categories. The first group adopts the maximum covering formulation, which tries to cover the most demand in the network (e.g., Goodchild & Noronha, 1987; Kuby & Lim, 2005). The second category formulates the problem as a set covering model. The objective of set covering models is to minimize the cost subject to serving all demand (Lin et al, 2008; Wang & Lin, 2009; Wang & Wang, 2010; Kim & Kuby, 2012). The third group is called the maximum covering/shortest path problem, as formulated by Current et al (Current et al, 1985). The third group does not consider the O-D flows and their shortest paths. Instead, it tries to maximize flows in neighborhood arcs (Kuby & Lim, 2005). Our formulations fall into the second group since the models are formulated to fulfill the entire demand rather than maximizing the flow of links with refueling stations.

Recent studies in the set covering group all have certain limitations. Lin et al (2008) addressed the station siting problem as a one-stage MIP. In this work, the objective is to minimize the total time for fuel-travel-back from possible origin of the refueling trips to the nearest station. Refueling flows on road segments pointing to node j are assumed to search for the nearest station to refuel. This notion of "where you drive more is where you more likely need refueling” is the basis for their formulation. Wang & Lin (2009) formulated the refueling station location problem as a set covering MIP. In their model, the main objective is to minimize the total cost of construction in order to cover all alternative fuel vehicles in the network and enable them to refuel along their paths based on their ranges. They assumed uncapacitated stations and a linear relationship between fuel consumption and driving distance. Wang & Wang (2010) extended the model proposed by Wang & Lin (2009) by adding the constraint of nodal demand coverage and restricting the stations to be within reasonable distance to all the nodes in the transportation network. The objective function is updated to minimize the construction cost and the demand coverage throughout the network. They solved the model with a branch-and-bound process for medium size networks. These studies only considered the travel time from the current locations of travelers to a station, and did not consider the total travel time to final destinations.

Kim & Kuby (2012) addressed this issue by focusing on the travelers' deviation from their original shortest paths due to refueling. They built a maximum covering formulation with the objective of maximizing the total covered refueling demand. They assumed travelers' desire for refueling stations will decrease with the increase of the deviation from their original shortest paths. For this purpose, they implemented three algorithms to prepare the set of deviation routes associated with each O-D pair as input data for their model. None of the above studies considered multiple periods, another substantial characteristic of refueling station location problem. These studies also simplified the problem by ignoring delay at refueling stations. Moreover, it should be pointed out that there is no previous study in the refueling station location literature that considers link travel time as functions of link flows, or accounts for travelers' autonomous decisions of where to refuel and which routes to take. In other words, they considered the planner as the only decision maker in the problem while the response of users cannot be ignored in reality.

In this study, we propose two set covering models in the framework of capacitated facility location problem. These models will address several key issues mentioned above. First of all, the delay at each station caused by high refueling demand is accounted for. Furthermore, travelers would be expected to deviate from their shortest paths: they would choose the path with minimum travel time in the set of paths that pass through some refueling stations. Another unique feature of this study, especially in the CPMAU, is the consideration of nonlinear link travel time functions and travelers' behaviors in choosing their refueling stations and routes. Transportation network user equilibrium, a widely used assumption in transportation planning practices, is incorporated in the CPMAU to better capture of travelers' responses to the planner's decision on refueling station locations. The user equilibrium condition defines a particular flow pattern where travelers are not able to reduce their travel times by unilaterally changing their paths (Sheffi, 1985).

Chapter 3. Methodologies

3.1. Modeling Considerations

This study proposes two MIP models for the refueling station location problem. In the first model, the planner is the only decision maker. We call this model a centralized planning model (CPM). The planner will not only decide the locations of refueling stations but also the shortest paths and refueling destinations for travelers. Users are assumed to fully comply with the planner's guidance on where to refuel and which routes to take. This model is formulated with constant travel times. In the second model, link travel time is considered as a function of link flows. Moreover, it relaxes the idealized assumption that the planner is the only decision maker. In reality, both planners and travelers are important players in a transportation network. Travelers' reactions to planners' decisions cannot be ignored in a long-term transportation planning problem. To capture these reactions, the second model is formulated as a bi-level model. In view of this, the second model is called centralized planning model with autonomous users (CPMAU). Another distinction between these models is rooted in the delay at refueling stations. The delay is assumed to follow a staircase cost function in the first model but a continuous nonlinear function in the second model. In addition, the first model is formulated with consideration of multi-period demands. The second model can easily be adopted to include multi-period demand, but for simplicity we neglect it.

Consider a general network $G = (N, A)$ with node set N and link set A. Let W denote the set of commodities defined by O-D pairs and refueling needs. Refueling travelers and non-refueling travelers for the same O-D pair are treated as two separate commodities. Set Z consists of potential locations for new refueling stations. In the CPM, potential refueling station locations are treated as a set of nodes, and travelers are assigned to selected refueling stations by the planner. However, potential locations are treated as a set of links in the CPMAU in order to explicitly obtain the amount of travelers choosing each station under the user equilibrium condition. Virtual auxiliary bypass links are provided for non-refueling vehicles to pass the refueling station without incurring delay. Let cc_z be the cost of construction at location $z \in Z$. The travel demand for each commodity w is assumed as a known constant denoted by q^w. $q^{w,t}$. is utilized to represent the demand in each period t. A basic decision variable controlled by the planner is whether to construct a refueling station at a potential location or not. This binary variable is denoted by y_z . y_z equals to one if the planner decides to construct at location z, and zero otherwise. The two proposed models differ in the remaining decision variables and constraints. They will be introduced in Chapter 4 and Chapter 5, respectively. Since the objective value of the proposed models are expressed in terms of monetary units, we use value of travel time, β, to convert the delay and travel times experienced by travelers from time to monetary units. In this study, we assume that all stations sell the energy at the same price.

3.2 Solution Methods

The proposed models are solved with well-known methods. The CPM is a MIP. Branch-and-bound and Lagrangian relaxation are two well-established methods to solve MIPs. These algorithms are adopted to solve the CPM, and are tailored according to the particular formulation.

The CPMAU is a MINLP. Furthermore, it involves complementarity constraints due to the explicit modeling of travelers' behaviors. Mathematical program with complementarity constraints are generally very difficult to solve. We relax the complementarity constraints to solve this model.

These models and algorithms are applied to different networks to validate the models and verify the efficiency of the solution algorithms. We test these algorithms on two networks: a small size nine-node network and a medium size network, the Sioux-Falls network. Our main focus of the nine-node network numerical studies is the performance of the branch-and-bound algorithm. The small size of the network enables us to provide the branch and bound tree. Our aim in testing these algorithms on the Sioux-Falls network is to demonstrate their efficiency and to compare them. In the following subsections, these algorithms are described in details.

To further compare the CPM and the CPMAU and to gain more insights on the implications of these two models, we apply the proposed models on an additional network. The network is a variation of the nine-node network, and is called modified nine-node network. The comparison and implications will be discussed in Chapter 5.

3.2.1. Branch-and-Bound

Branch-and-bound is the most well-known algorithm in the field of integer programming. The essential idea of branch-and-bound is to divide the problem into subproblems where the values of some integer decision variables are fixed.. Since, the CPM includes two types of binary variables, the subproblems with relaxing one of them is still a MIP. The branch-and-bound method is non-heuristic, in the sense that it maintains an upper and a lower bound of the (globally) optimal objective value in each subproblem. This algorithm is initialized by fixing the value of a certain binary variable. This leads to two subproblems with two disjoint subsets of the solution space corresponding to the two possible values of the selected variable. Each subset of the solution space can be further partitioned into two by fixing the value of another binary variable. This process is called branching and the selected binary variables for partition of the solution space are called nodes. This process leads to a tree of possible subsets of the original solution space. We search among the unexplored subsets to obtain the bounds and select the best node to process during each iteration. A bound is often calculated by solving a linear-program (LP) relaxation of a subproblem. The branch-and-bound procedure involves three main steps: selection of a node for evaluation, bound calculation, and branching (Poldner & Kuchen, 2008). There are several methods to select the branches. For this study, a “best first” strategy is employed to explore the nodes of the branch-and-bound tree. In this strategy, as the name suggests, we choose the node with the lowest lower bound to explore in each iteration. For the branching step, the binary variable (y) corresponding to the station with minimum construction cost is selected and it disjoints the solution space into two subspaces. Figure 1 presents a detailed pseudo code for a general branch-and-bound algorithm.

```
Initialization
LB := 0
UB := ∞
Store root node in waiting node list
While waiting node list is not empty do
    {Node selection}
    Choose a node from the waiting node list
    Remove it from the waiting node list
    Solve the MIP relaxed subproblem
        If infeasible then
            Node is pruned
        Elseif optimal then
            If integer solution then
                If obj ≤ UB then
                    {Better integer solution found}
                    UB := obj
                    Remove nodes j from waiting list with LB_j ≥ UB
                End if
            Else
                Branching − {Variable selection (y)}
                Find a binary variable j with fractional value v in the relaxed LP solution where 0 ≤ v ≤ 1
                Create node j_new1 by fixing value of y at 0
                LB_{j_new1} := obj
                Store node j_new1 in waiting node list
                Create node j_new2 by fixing value of y at 1
                LB_{j_new2} := obj
                Store node j_new2 in waiting node list
            End if
        End if
    LB = min_j LB_j
    End while
```

Figure 1. Pseudo code for branch-and-bound algorithm using "best first" strategy

3.2.2. Lagrangian Relaxation

Lagrangian relaxation algorithms have been widely used to solve mixed integer models since this approach was proposed in 1974 by Geoffrion (Fisher, 1981). The idea of Lagrangian relaxation approach is to relax certain complicated constraints in order to simplify the problem. The objective function of a relaxed problem consists of penalties for violation of relaxed constraints. These penalties are incorporated via a set of Lagrangian multipliers λ. The optimal objective value of the relaxed problem $V(\lambda)$ for a given set of λ is a lower bound of the original minimization problem. On the other hand, if we make the solution of the relaxed problem feasible to the original problem, we will obtain an upper bound of the optimum objective value V^*. The next step is to determine an appropriate set of λ to maximize the lower bound obtained. In other words, the Lagrangian dual is a max-min problem that tries to decrease the optimality gap between the dual and the original problems. The subgradient method (Fisher, 1981), due to its efficiency, is one of the most popular heuristic methods to approximately obtain the optimal value of Lagrangian multipliers in the literature. The subgradient algorithm utilizes both V^* and $V(\lambda)$, and is represented in Figure 2.

In subgradient optimization, θ, τ and ζ are three parameters that can be calibrated based on the size and the density of the network. Another substantial issue in the subgradient optimization is obtaining the upper bound V^* for the original problem. Any feasible solution to the original minimization program is an upper bound of its optimum solution. Frequently, the upper bound is calculated with heuristic methods based on the type of problem. For the CPM, a heuristic method for finding a feasible solution is proposed based on the particular formulation proposed. The heuristic method will be discussed after the CPM formulation is introduced in Chapter 4.

{Input}

An upper bound V^*

An initial value $\lambda^0 \geq 0$

{Initialization}

$\theta_0 := 2$

$j = 1$

{Subgradient iteration}

$\boldsymbol{While} \parallel \lambda^{j+1} - \lambda^j \parallel \leq \zeta$

$\{gradient\ of\ V(\lambda^j)\} \rightarrow \gamma^j := g(x^j)$

$\{step\ size\ \} \rightarrow \rho_j := \theta_j \dfrac{V^* - V_D(\lambda^j)}{\parallel \gamma^j \parallel^2}$

$\lambda^{j+1} := max\,\{0, \lambda^j + \rho_j \gamma^j\}$

If $V_D\,(\lambda^j) \leq \max_{\kappa<j} V_D\,(\lambda^\kappa))$ $in\ more\ than\ K\ iterations$ ***then***

$\theta_{j+1} := \dfrac{\theta_j}{\tau}$

Else

$\theta_{j+1} := \theta_j$

End if

$j := j + 1$

End while

Figure 2. Psuedo code for subgradient optimization

Chapter 4. Centralized Planning Model

This chapter introduces the centralized planning model. We will first introduce the assumptions and additional notations adopted, and present the formulation in section 4.1. The solution algorithms, especially the Lagrangian relaxation approach with a customized heuristic to obtain feasible solutions, will be described in section 4.2. Numerical results are represented in section 4.3.

4.1. Model Formulation

Being the only decision maker, the planner not only determines the refueling station locations but also assign the travelers to the shortest paths. In other words, travelers are assumed to follow the planner's instruction regarding the refueling destinations and shortest routes. For instance, carrier fleet routes and service centers are determined by the centralized planner.

In this model, potential refueling locations are considered as a set of nodes $Z \subseteq N$. We further assume link travel times are constants throughout the network. Under this assumption, the non-refueling demand in the network should be assigned to the fixed shortest path between each O-D pair in order to minimize the transportation cost. Since neither the refueling station locations nor the flow distributions will affect the link travel times, the minimal transportation cost for non-refueling travelers is fixed. Therefore, non-refueling travelers can be omitted in this model. In Chapter 3, we introduced user groups by O-D pairs and refueling needs as the commodities considered in this study. Since non-refueling travelers can be omitted in the CPM, the commodities are reduced to user groups by O-D pairs only. We denote the refueling commodities (O-D pairs) by set W in this chapter. With these considerations, the planner's decision variables include the refueling station locations y_z , and the percentage of travelers of commodity w to be assigned to refueling station z at time period t, denoted by $x_z^{w,t}$. The capacity of refueling station z, $Ucap_z$, is considered to be a known parameter. But the proposed capacity of stations would follow the total calculated demand of travelers.

Note that refueling travelers will deviate from the shortest path connecting their origins and destinations in order to refuel. Denote the shortest travel time for any commodity w passing a potential station z at time period t by $T_z^{w,t}$. The delay experienced by each refueling traveler, called the marginal delay, at each refueling station is modeled by a step function $d_z = \{d_{z,p}$, if the number of refueling vehicles assigned to station z is between $L_{z,p-1}$ and $L_{z,p}$, $\forall p \in P\}$, where P is a set of possible intervals for the step function. It is assumed that delay multipliers are increasing with the increasing the levels. For example, $d_{z,p}$ is greater than $d_{z,p-1}$. It should be noted that the total delay at a station is a piecewise linear function of the assigned demand to that station. Let $m_{z,p}^t$ denote the amount of refueling vehicles at station z that are subject to a delay of $d_{z,p}$ at time period t, then the total delay at station z can be written as $\sum_p d_{z,p} m_{z,p}^t$. We further denote the difference between $L_{z,p-1}$ and $L_{z,p}$ as $l_{z,p-1}$. Figure 3 represents the staircase marginal delay function at each station.

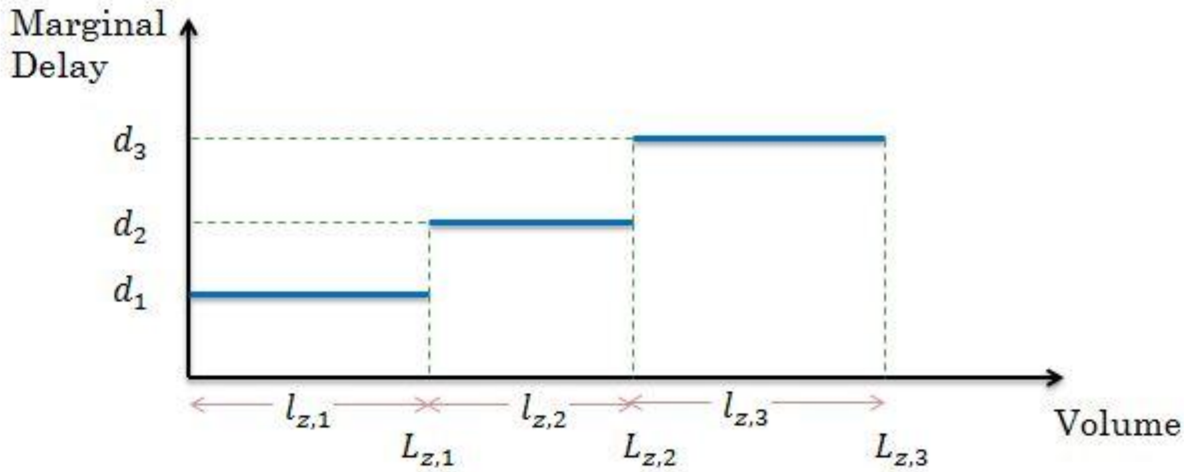

Figure 3. Staircase marginal delay function at each station

The planner's objective is to minimize the total construction cost and the generalized users' cost consisting of waiting time at refueling stations and travel times in the network. The problem can be formulated as follows:

$$V = \min_{(y,x,m,U)} \beta \sum_{t\in\mathcal{T}} \sum_{w\in W} q^{w,t} \sum_{z\epsilon Z} (x_z^{w,t} \cdot T_z^{w,t}) + \beta \sum_{p\epsilon P} \sum_{z\in Z} \sum_{t\in\mathcal{T}} d_{z,p} m_{z,p}^t + \sum_{z\epsilon Z} cc_z y_z \qquad (6)$$

$$\sum_{p\epsilon P} m_{z,p}^t = \sum_{w\in W} q^{w,t} x_z^{w,t} \qquad \forall\, t \in \mathcal{T}, \forall\, z \in Z \qquad (7)$$

$$m_{z,p}^t \le l_{z,p} U_{z,p}^t \qquad \forall\, t \in \mathcal{T}, \forall\, p \in P, \forall\, z \in Z \qquad (8)$$

$$\sum_{z\epsilon Z} x_z^{w,t} = 1 \qquad \forall\, t \in \mathcal{T}, \forall\, w \in W \qquad (9)$$

$$\sum_{w\in W} q^{w,t} x_z^{w,t} \le Ucap_z \cdot y_z \qquad \forall\, t \in \mathcal{T}, \forall\, z \in Z \qquad (10)$$

$$y_z \in \{0,1\} \qquad \forall\, z \in Z \qquad (11)$$

$$U_{z,p}^t \in \{0,1\} \qquad \forall\, t \in \mathcal{T}, \forall\, z \in Z, \forall\, p \in P \qquad (12)$$

$$x_z^{w,t} \ge 0 \qquad \forall\, t \in \mathcal{T}, \forall\, z \in Z, \forall\, w \in W \qquad (13)$$

$$m_{z,p}^t \ge 0 \qquad \forall\, z \in Z, \forall\, t \in \mathcal{T}, \forall\, p \in P \qquad (14)$$

In the above formulation, $m_{z,p}^{t}$ and $U_{z,p}^{t}$ are intermediate decision variables. $U_{z,p}^{t}$ is defined as:

$$U_{z,p}^{t} = \begin{cases} 1 & \text{if operating level } p \text{ at station } z \text{ is activated during time period } t \\ 0 & \text{otherwise} \end{cases}$$

Constraint (7) specifies the total amount of vehicles that will refuel at station z during time period t should be equal to the total assigned flows from all commodities to that station. Constraints (8) guarantees that the intervals of the marginal delay functions at a station z are activated in ascending order because of increasing delay multiplier with increasing the level. Note that it is not necessary to explicitly include constraint $m_{z,p}^{t} \geq l_{z,p-1} U_{z,p}^{t}$ to ensure an interval is saturated before the next interval can be activated. This condition will be automatically satisfied because the marginal delay is assumed to be a monotonically increasing step function and the objective function is to minimize the total delay. Constraint (9) states that the demand of each commodity at each time period is fulfilled by some station. Constraint (10) ensures that if some demand is assigned to station *z*, the station should constructed. Also, this demand should not exceed the capacity, $Ucap_{z}$, of station *z*. Finally, constraints (11) – (14) are non-negativity and integrality constraints for the decision variables, respectively.

4.2. Solution Algorithms

The capacitated facility location problem is categorized as an NP-hard(Non-deterministic polynomial-time hard) problem. The class of problems at least as hard as the hardest problems in NP is NP-hard, where NP refers to problems that can be solved in polynomial time. Many solution approaches to the NP-hard facility location problems have been developed in the last decades, including meta-heuristics and approximation algorithms (Shen et al, 2010). Branch-and-bound and Lagrangian relaxation are two typical solution algorithms for solving mixed integer models. This study will adopt these solution methods to solve the mixed integer model (6) –(14).

In Chapter 3, we have briefly discussed these methods. The branch-and-bound algorithm can be applied straight-forwardly with minimum customization. The Lagrangian relaxation approach, on the other than, requires a customized heuristic method to obtain a feasible solution. In this section, we describe the application of Lagrangian relaxation to solve the CPM. We will focus specifically on the customized heuristic method.

Most Lagrangian relaxation approaches for the CFL problem are based on the relaxation of demand constraints (Geoffrion & McBride, 1978). We adopt the same idea here. After relaxation of the demand constraint (9), the Lagrangian problem becomes:

$$V_D(\lambda) = \min_{(y,x,m,u)} \beta \sum_{t\in\mathcal{T}} \sum_{w\in W} q^{w,t} (\sum_{z\epsilon Z} (x_z^{w,t} \cdot T_z^{w,t})) + \beta \sum_{p\epsilon P} \sum_{z\in Z} \sum_{t\in\mathcal{T}} d_{z,p} m_{z,p}^t + \sum_{z\epsilon Z} cc_z y_z + \sum_{t\in T} \sum_{w\in W} \lambda_w^t (\sum_{z\epsilon z} x_w^{zt} - 1) \quad (15)$$

Subject to (6) –(8) and (10) – (14)

To apply the general framework of the subgradient optimization approach to solve for the optimal multiplier λ, the gradient of $V_D(\lambda)$ with respect to λ, $g(\lambda)$, is needed. Each component of $g(\lambda)$, the partial derivative of $V_D(\lambda)$ with respect to $\lambda^{w,t}$, is the relaxed demand conservation equation (9).

As described in Chapter 3, estimation of an upper bound is the most important step in the subgradient optimization. Any feasible solution to the original problem can provide the upper bound. In this study, we propose a heuristic method customized the greedy method to obtain a feasible solution to the original problem. Since the demand constraint is relaxed, travelers based on origin, destination, and time period are categorized in two sets of Φ_1 and Φ_2 such that:

$$\Phi_1 = \{(w,t) | \sum_{z\epsilon Z} x_z^{w,t} - 1 < 0\}$$

$$\Phi_2 = \{(w,t) | \sum_{z \epsilon Z} x_z^{w,t} - 1 > 0\}$$

Set Φ_1 and Φ_2 refers to commodities and time periods with unmet and overmet demands, respectively. To force a feasible solution to the original problem, stations are sorted based on their impacts on the objective function first. The impact of each station z is measured by:

$$I(z) = \beta \sum_{t \in \mathcal{T}} \sum_{w \in W} q^{w,t} \left(x_z^{w,t} \cdot T_z^{w,t} \right) + \beta \sum_{p \epsilon P} \sum_{t \in \mathcal{T}} d_{z,p} m_{z,p}^t + \ cc_z (1 - y_z \ ) \tag{16}$$

This impact factor measures the effect of assigning the demand to each station. For the commodities with unmet demand, we assign the remaining demand to the station with the lowest impact factor first until the station is saturated. The next best station will then be selected. During each iteration, impact factors of the stations are updated. The process will be repeated until all of demand conservation equations are satisfied. Similarly, we remove the already assigned demand of commodities with overmet demand from the station with the maximum impact factor first. This method can be classified as the greedy method. Similar to the other greedy methods, it is usually not very efficient. Also, the sequence of O-D pairs and time periods selected in the heuristic method is generated arbitrarily. This heuristic method can be improved by updating the sequence strategically. One possibility is to prioritize the O-D pairs and time periods that have higher impacts on the objective function during each iteration. In the next section, we will provide several numerical examples to test the branch-and-bound and Lagrangian relaxation to observe their efficiency and validate the models in solving the proposed model.

4.3. Computational Results

We illustrate the computational properties of formulation (6) – (14) using the two algorithms discussed in Chapter 3: branch-and-bound and Lagrangian relaxation. The nine-node network and the Sioux-Falls network, one small and one medium sized, are used to test the performance of the proposed model and solution algorithms. All numerical examples are performed on a Dell W3550 workstation with 3.07 GHz Intel Xeon® CPU and 12 GB of RAM. For each network, various artificial demand values are generated for each time period. The refueling demand for each O-D pair is considered known and fixed. Furthermore, the travel time of each link is assumed to be constant, and takes the value under the user equilibrium condition when no user needs to refuel in the network. When solving for the user equilibrium flow pattern, the BPR(Bureau of Public Roads) function is adopted:

$$t_{ij}(x_{ij}) = t_{ij}^{0}\left(1 + 0.15\left(\frac{\nu_{ij}}{\rho_{ij}}\right)^{4}\right)$$

where ν_{ij} is the flow of link (i,j), and t_{ij}^{0} and ρ_{ij} are the free flow travel time and the capacity of link (i,j) respectively.

4.3.1. Nine-Node Network

The nine-node network is displayed in Figure 4. There are four O-D pairs and the demands for two time periods are presented in Table 1. The ordered pair of numbers beside each link represents the free flow travel time and the capacity of a link.

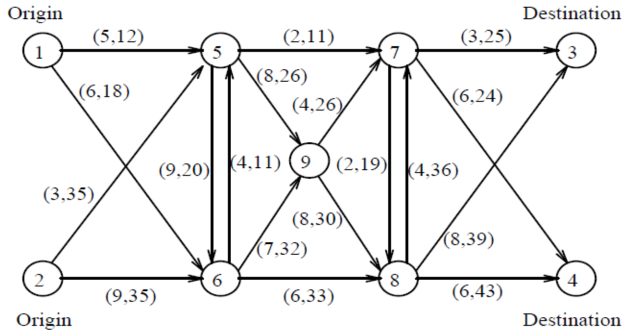

Figure 4. The nine-node network

The potential station set consists of nodes 5 – 7. The construction cost for locating refueling station at nodes 5 – 7 is 100, 200, and 150 units, respectively. Table 2 presents the shortest path travel time passing each station z, $T_z^{w,t}$, for each commodity w and time period t. These constant travel times are derived based on the user equilibrium condition in the network with the sum of refueling and non-refueling demands in each time period. The value of travel time β is set as 1.

Table 1**.** Time-dependent O-D demands for the nine-node network

t	O-D	$q^{w,t}$
1	1.3	16
	1.4	58
	2.3	12
	2.4	54
2	1.3	50
	1.4	10
	2.3	50
	2.4	10

Table 2. Shortest path travel time($T_z^{w,t}$) between each O-D passing a potential station for the nine-node network

z	5		6		7	
$w \setminus t$	1	2	1	2	1	2
(1,3)	10.300	72.876	22.973	69.618	22.973	68.495
(1,4)	23.249	72.773	23.249	68.672	23.249	73.392
(2,3)	24.132	69.53	22.758	64.898	24.132	65.149
(2,4)	24.408	69.427	23.034	63.952	24.408	70.046

Interval size $l_{z,p}$ and delay $d_{z,p}$ are represented in Table 3 for the staircase marginal delay function at refueling stations. Note that the same staircase function is used for all potential stations, so the subscription z is dropped in Table 3. Also note that the staircase function is monotonically increasing.

Table 3. Staircase marginal delay function for the nine-node network

p	l_p	d_p
1	50	5
2	10	8
3	30	9
4	40	12

To apply the branch-and-bound algorithm, we first relax all the binary variables y associated with station locations to form a relaxed MIP. Then, we partition the solution space by fixing certain binary variable y_z using specified branching and selection rules in Chapter 3. Figure 5 represents the binary enumeration tree for the branch-and-bound algorithm applied to this problem. Each node represents a subset of solution spaces of the relaxed mixed integer model. The value (obj^*) of beside each node represents the objective value of the relaxed MIP within each subset of the solution space. This objective value is the lower bound for the next successor nodes. Branch-and-bound nodes 5 and 6 are pruned because their lower bounds are greater than that of node 8. Nodes 4, 8, and 9 are identified as three potential optimal solutions to the original MIP because of the integrality of the location variables y. It can be observed from Figure 4 that node 8 represents the lowest objective value among these three solutions. Therefore, following the branch consisted of branch-and-bound nodes 1–3–8, refueling stations should be constructed at nodes 5 and 6 of the transportation network to minimize the total cost.

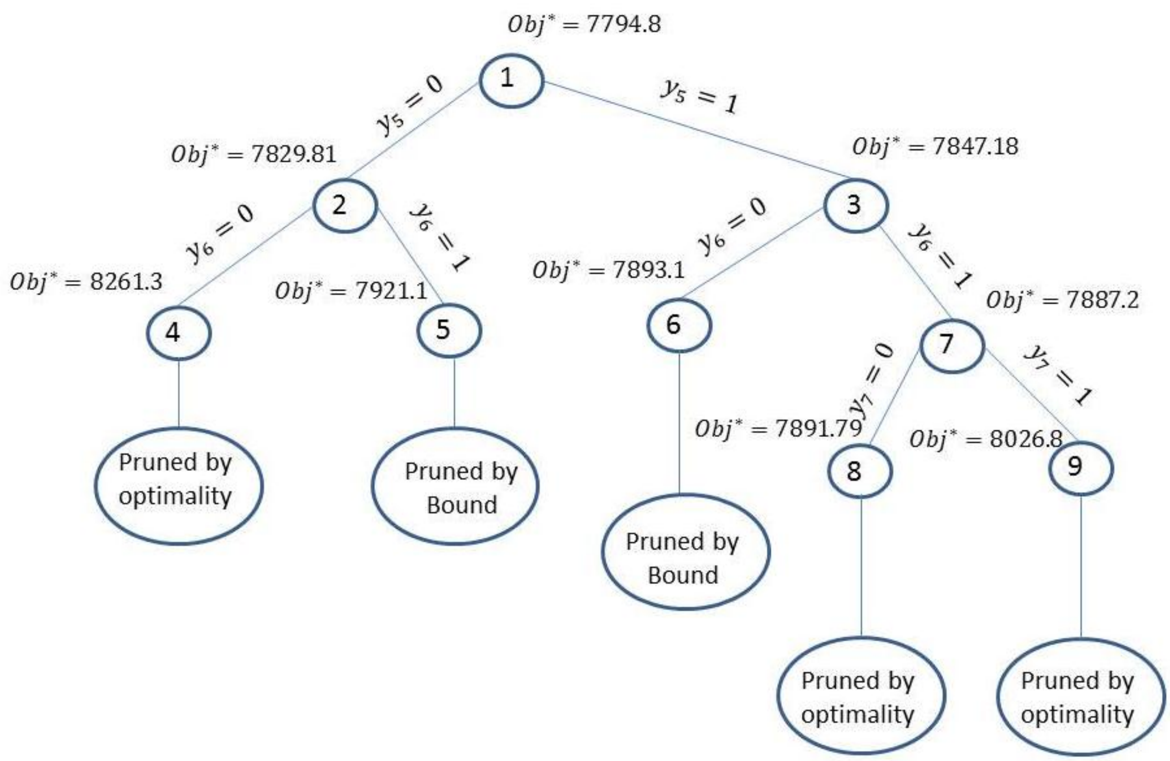

Figure 5. Branch-and-bound enumeration tree for the nine-node network

We have also applied the Lagrangian relaxation approach to solve this problem. The Lagrangian relaxation algorithm is able to solve the problem in only one iteration because of the small size of this network.

Table 4 represents the summary of the results obtained by branch-and-bound and Lagrangian relaxation algorithms.

Table 4. Summary of results for the nine-node network

	Branch-and-Bound	Lagrangian Relaxation
Objective value	7891	8027
Selected nodes	{5,6}	{5,7}

It should be noted that obtained result to the Lagrangian relaxation algorithm is not feasible to the original problem. The relative difference between the obtained results of branch-and-bound and Lagrangian relaxation is only 2%.

4.3.2. Sioux-Falls Network

The second example is the Sioux-Falls network with 24 nodes, 76 links and 552 O-D pairs. The network and demand details are represented in Figure 6. The detailed characteristics of network including free flow travel times, link capacity, and origin-destination demands can be found in (LeBlanc et al, 1975)

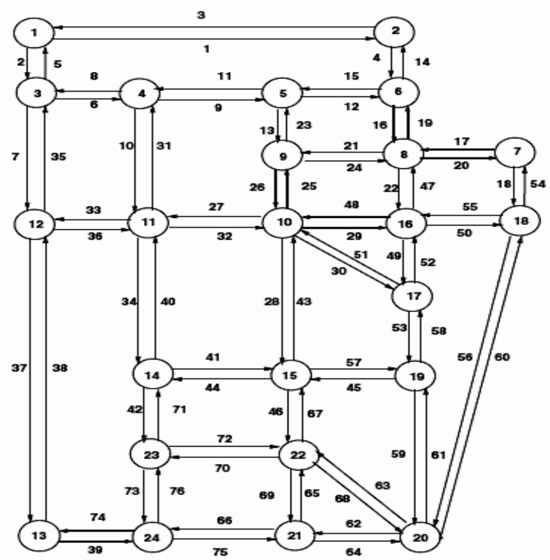

Figure 6. The Sioux-Falls network

Table 5 represents the candidate nodes and construction costs of refueling stations. Estimated cost for constructing a refueling station is between \$40,000 and \$175,000 depending on the size of network (U.S. Department of Energy, 2011). We randomly generate these values based on the uniform distribution with mean of \$85,000 and a standard deviation of \$26,000.

Table 5. Construction cost for refueling station at each candidate node for the Sioux-Falls network

Node	Fixed Cost	Node	Fixed Cost	Node	Fixed Cost
6	120000	11	70000	15	55000
7	40000	12	90000	16	110000
8	60000	13	100000	17	62000
9	42000	14	55000	18	44000
10	50000				

The value of travel time β for network users is set to $20\ \$/hr$. Parameters of the staircase marginal delay function are presented in Table 6.

Table 6. Parameters of delay function at refueling stations for the Sioux-Falls network

p	l_p	$d_p(\frac{\$}{Veh})$
1	500	5
2	500	15
3	500	30
4	500	40

Figure 7 represents the optimal nodes (highlighted in red) to locate refueling stations resulted from the branch-and-bound algorithm. Nodes 7, 10, 12, 15, 16, and 18 are selected to construct the refueling station location.

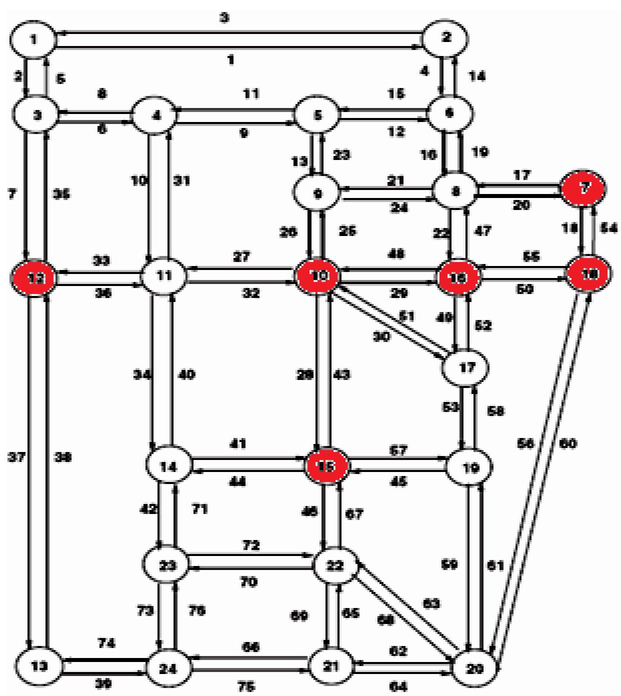

Figure 7. Result of branch-and-bound algorithm for the Sioux-Falls network

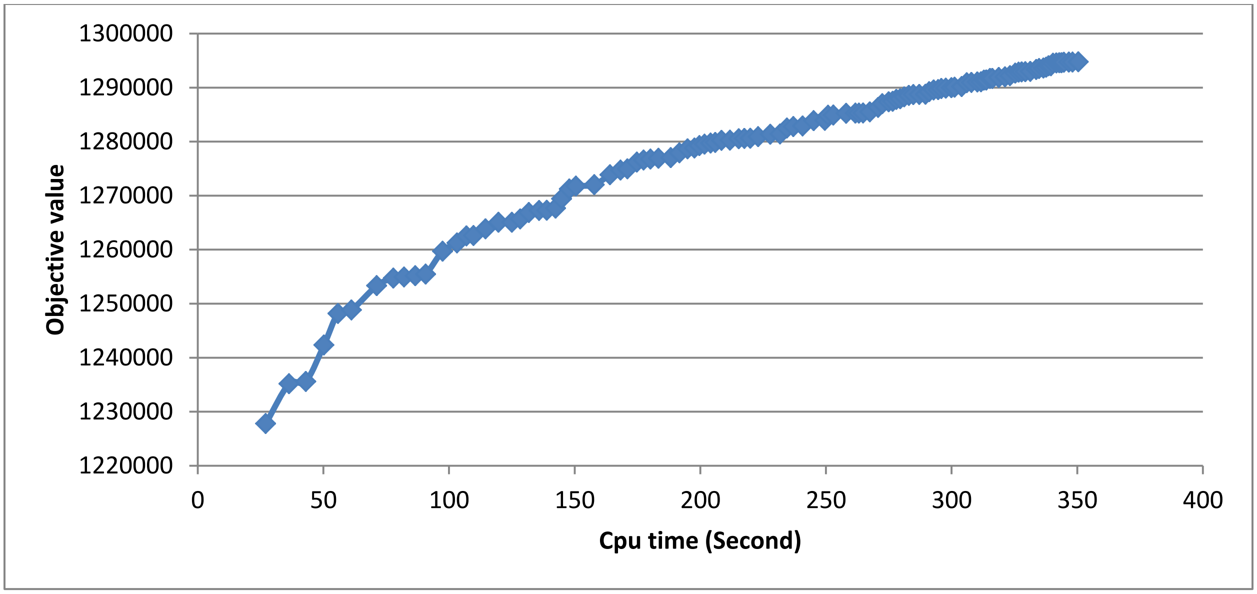

Figure 8.Convergency trend of branch-and-bound for the Sioux-Falls network

Figure 8 and Figure 9 plot the convergence trends of the objective value of the branch-and-bound and the Lagrangian relaxation approaches.

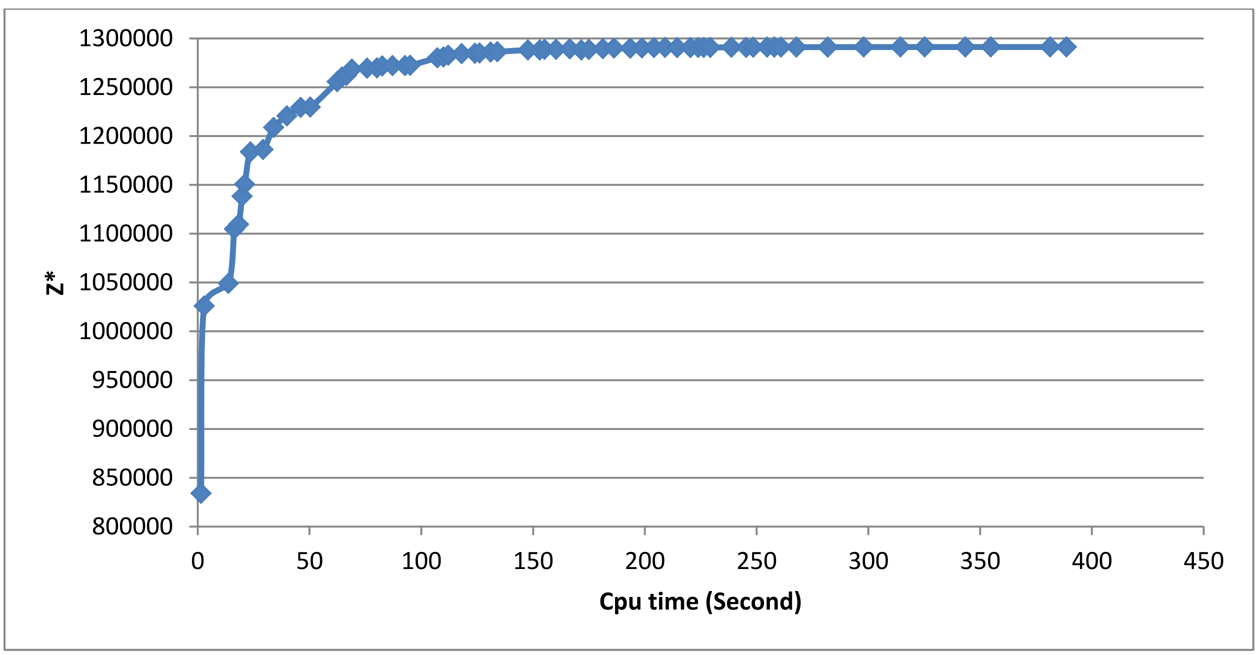

Figure 9. Convergency trend of Lagrangian relaxation for the Sioux-Falls network

The stopping criterion used for the Lagrangian relaxation method is the relative gap between the upper and the lower bound of the objective value obtained by the proposed heuristic method and subgradient optimization. Specifically, the algorithm is terminated when the relative gap is less than 3%. Within 150 seconds, the objective value obtained from the subgradient method converges to close proximity of the optimal solution. It takes 350 seconds for the branch-and-bound algorithm to converge, approximately twice the time needed by Lagrangian relaxation. It should be pointed out that obtained objective value of the branch-and-bound algorithm is $1,294,711. On the other hand, the Lagrangian relaxation algorithm is able to achieve a lower bound of $1,291,232, and an upper bound of $1,333,753.

Chapter 5. Centralized Planning Model with Autonomous Users

In this chapter, we focus on the placement of refueling stations to serve intra-city or short-distance trips. The proposed CPM in Chapter 4 is based on the assumption that travelers will follow the planner's instructions on where to refuel and which routes to take or carrier fleets select the routes and service center based on the guidance of central planner. However, outside of the fleet management context, regular travelers are able to select the refueling station and path to refuel and reach their destinations. Moreover, travel times of intra-city trips are functions of link flows in the network. With this information regarding travelers' behaviors and other data like travel demand and link travel time functions, the transportation planner should take into account travelers' responses to the refueling station locations when making the decision about where to construct new stations. We call this problem a centralized planning model with autonomous user, or CPMAU. This decision problem can be modeled as a bi-level program. Since the lower-level problem describes the user equilibrium traffic pattern, the formulated bi-level program is essentially a mathematical program with equilibrium constraints (Luo et al, 1996). These problems are difficult to solve. In this study, we will relax the complementarity constraints to solve the proposed CPMAU.

Section 5.1 will introduce the concept of bi-level programming and then describe the notations and model formulation. In section 5.2, an example problem is solved to demonstrate the performance of CPMAU and the results of the proposed models are compared with each other. Our results show that considering the bi-level formulation can provide the better results.

5.1. Model Formulation

There are two decision makers in the non-cooperative hierarchical structure represented by a bi-level program that try to optimize their respective objectives. A general bi-level formulation is described as follows.

$$(M_0)\ \min_{\vartheta}\ \Psi(\vartheta,\xi^*) \tag{17}$$

$$\text{s.t.}\quad \Upsilon(\vartheta,\xi^*) \leq 0 \tag{18}$$

where $\xi^* = \xi^*(\vartheta)$ is implicitly defined by the optimum solution to the following problem parameterized by ϑ:

$$(S_0)\ \min_{\xi}\ \psi(\vartheta,\xi) \tag{19}$$

$$\text{s.t. } \kappa(\vartheta,\xi) \leq 0 \tag{20}$$

In this general formulation, ϑ is the decision variable of upper level problem (M_0) or planner's problem. The lower level formulation (S_0) is the travelers' problem to determine the optimal ξ in response to any given ϑ. Υ and κ are the constraints of upper level and lower level models, respectively.

In our refueling station location problem, the planner is the leader and the upper level problem tries to minimize the sum of the costs of construction, total travel time, and delay experienced by the users in the network. Network travelers are the followers who will adjust their refueling stops and route choices to reduce his or her generalized cost according to the user equilibrium condition based on the refueling station locations. The lower level problem describes an equilibrium state, and bi-level programs with this characteristic are called mathematical programs with equilibrium constraints (Luo et al, 1996). Since the user equilibrium condition for a given network is well-established, we can replace the lower-level optimization problem with its optimality conditions, i.e., the user equilibrium conditions. We formulate the model for single period, but it can be easily extended to multi-periods.

In the CPMAU, each potential refueling station location is treated as a link. This modification enables us to explicitly consider the incurred delay waiting in queue at refueling stations. In this model, non-refueling commodities cannot be ignored any more since they will affect link travel times. In view of this, a virtual parallel path without any delay is introduced so that non-refueling vehicles can bypass the station when traveling along a route that passes this location. Figure 10 represents the network modification scheme.

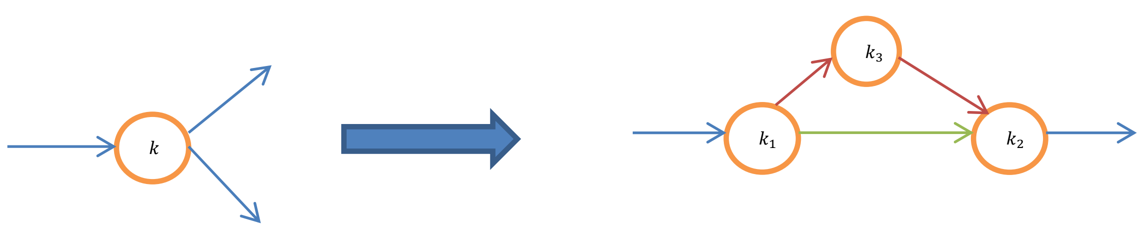

Figure 10: Schematic figure of network modification

In the Figure 10, the green link represents the potential refueling station k modeled as a link with a delay as a function of link flows, and the red links denote the virtual parallel path without delay for non-refueling vehicles.

Let W_1 and W_2 denote the refueling and non-refueling commodities, respectively. To ensure that refueling vehicles will only take reasonable travel paths, we first enumerate all possible paths for each refueling commodity. These paths should satisfy the following conditions. First, these paths should each include one and only one refueling station link. This is an underlying assumption of the model that refueling travelers only need to refuel at exactly one station. Second, these paths should be acyclic. For each refueling commodity $w \in W_1$, K^w denotes the corresponding refueling path set. Let f_k^w and c_k^w denote the path flow and path travel time of path k serving refueling commodity $w \in W_1$. Note here c_k^w is not a constant but rather a function of link flows. It is worth mentioning that path enumeration is only needed for refueling commodities. For non-refueling commodities, the user equilibrium condition can be written as a set of link-based equations. Let q^w denote the demand for commodity w. Further, let h_i^w represent the net total demand for a non-refueling commodity $w \in W_2$ at node $i \in N$. A positive h_i^w indicates a supply and a negative number signifies a demand. By definition, $h_i^w = q^w$ if i is the origin, $h_i^w = -q^w$ if i is the destination, and $h_i^w = 0$ if i is neither origin nor destination of commodity w.

Let A be the set of actual links and virtual bypass links. As introduced in Chapter 3, we use Z to denote the set of potential refueling stations, modeled as a set of links in the CPMAU. The travel time and flow on link $(i,j) \in (Z \cup A)$ is denoted by t_{ij} and v_{ij}. The station location binary variable is denoted by y_{ij}. Define $\delta_{ij,k}^w$ as an indicator variable as follows:

$$\delta_{ij,k}^w = \begin{cases} 1 & \text{If station } z = (i,j) \in Z \text{ is on path } k \text{ serving refueling commodity } w \\ 0 & \text{otherwise} \end{cases}$$

Also, we use ν_{ij}^{w} to denote the flow on link (i,j) for commodity $w \in (W_1 \cup W_2)$. With these notations, the bi-level program can be formulated as follows.

$$\min_{(\boldsymbol{\nu},\boldsymbol{y},\boldsymbol{x})} \sum_{(i,j)\epsilon Z} (cc_{ij} \cdot y_{ij}) + \beta \sum_{(i,j)\epsilon Z\cup A} \nu_{ij} \cdot t_{ij}(\nu_{ij}) \qquad (21)$$

$$f_k^w \le \sum_{(i,j)\epsilon Z} (y_{ij} \cdot q^w \cdot \delta_{ij,k}^w) \qquad \forall k \in K^w, \forall w \in W_1 \qquad (22)$$

$$f_k^w (c_k^w(\boldsymbol{v}) - u^w) = 0 \qquad \forall k \in K^w, \forall w \in W_1 \qquad (23)$$

$$c_k^w(\boldsymbol{v}) - u^w \ge 0 \qquad \forall k \in K^w, \forall w \in W_1 \qquad (24)$$

$$\nu_{ij}^w\big(t_{ij}(\nu_{ij}) + \pi_i^w - \pi_j^w\big) = 0 \qquad \forall (i,j) \in A, \forall w \in W_2 \qquad (25)$$

$$t_{ij}(\nu_{ij}) + \pi_i^w - \pi_j^w \ge 0 \qquad \forall (i,j) \in A, \forall w \in W_2 \qquad (26)$$

$$\sum_{k\in K^w} f_k^w \cdot \delta_{ij,k}^w = \nu_{ij}^w \qquad \forall w \in W_1, \forall (i,j) \in Z \qquad (27)$$

$$\sum_{w\in(W_1\cup W_2)} \nu_{ij}^w = \nu_{ij} \qquad \forall (i,j) \in Z \cup A \qquad (28)$$

$$\sum_{k\in K^w} f_k^w = q^w \qquad \forall w \in W_1 \qquad (29)$$

$$\sum_j \nu_{ij}^w - \sum_j \nu_{ji}^w = h_i^w \qquad \forall w \in W_2, \forall i \in N \qquad (30)$$

$$\nu_{ij}^w, \nu_{ij} \ge 0 \qquad \forall (i,j) \in A, \forall w \in W_2 \qquad (31)$$

$$f_k^w \ge 0 \qquad \forall k \in K^w, \forall w \in W_1 \qquad (32)$$

$$y_{ij} \,\epsilon\, \{0,1\} \qquad \forall (i,j) \in Z \qquad (33)$$

In this formulation, π_i^w and u^w are intermediate variables. π_i^w is the node potential of node i for non-refueling commodity $w \in W_1$; u^w represents the shortest travel time (including delay at refueling stations) for refueling commodity $w \in W_2$.

The objective function is the total cost associated with the total system travel time, including the delay at refueling stations, and construction cost of selected stations.

Constraint (22) ensures that a refueling path is not utilized unless there is a refueling station selected for construction along this path. Equations (23) and (24) ensure if the travel time for refueling travelers on path k is greater than the shortest path travel time, u^w, for a specific refueling commodity $w \in W_2$, then the flow on that path is zero. This is essentially the user equilibrium condition, where the paths with travel time greater than minimum travel time for each O-D pair will not carry any flow. Constraints (25) and (26) are the link-based user equilibrium condition for non-refueling commodities. Link (i, j) is utilized by a non-refueling commodity only when the link is part of the shortest path for this commodity. Equations (28)-(30) represent a set of flow conservation equations. These constraints state that all refueling and non-refueling O-D trips have to be assigned to the network. Equations (31) and (32) are non-negativity constraints for link and path flows.

According to the conceptual scheme of our model, y_{ij} is the binary variable controlled by the planner, whose goal is to minimize the construction cost and total travel time and refueling delay for the travelers, who are only concerned about their own travel times and refueling cost. Referring to the general bi-level formulation (17) – (20), in our model the equations (21), (22), and (33) are the planner's problem to determine the best locations in the network while constraints (23) – (32) are the user equilibrium conditions that replace the lower-level optimization problem of the travelers to minimize their own travel times.

This model is a mathematical program with complementary constraints (MPCC). These constraints are a special type of mathematical program with equilibrium constraints. Solving this type of program is very difficult because it can be shown that constraint qualifications typically assumed to prove convergence of standard Nonlinear program(NLP) algorithms fail for MPCC (Ye, 1999). MPCC are non-convex and non-smooth (Ban et al, 2006). One straightforward method to solve these programs is to relax the complementary constraints (23) and (25). They can be rewritten in the following forms:

$$v_{ij}^{w}\left(t_{ij}\left(v_{ij}\right)+\pi_i^w-\pi_j^w\right)\le \varepsilon \qquad \forall (i,j)\in A, \forall w\in W_2 \tag{34}$$

$$f_k^w\left(c_k^w(\boldsymbol{v})-u^w\right)\le \varepsilon \qquad \forall k\in K^w, \forall w\in W_1 \tag{35}$$

where ε is a parameter that can be determined based on the desired accuracy level.

5.2. Computational Results

This subsection presents the computational results achieved by solving the proposed bi-level model. We apply this model to the modified nine-node network using the General Algebraic Modeling System (GAMS) together with the Simple Branch & Bound (SBB) solver (Brook et al, 2003).

The underlying network is a variation of the nine-node network, and is displayed in Figure 11. The main reason to further simplify the nine-node network is making the enumerating paths easier. We call this variation of the nine-node network the modified nine-node network There are four O-D pairs in this network. Table 7 represents the refueling and non-refueling demand for each O-D pair.

Table 7. O-D demands of the modified nine-node network for different types of travelers

Origin	Destination	Refueling Demand	Non-Refueling Demand
1	3	60	20
1	4	50	12
2	3	30	8
2	4	50	10

Link travel time is assumed to follow the BPR function. Free flow travel time and link capacities are shown in Figure 11. Delay function at each refueling station is assumed as follows:

$$t_{ij}(v_{ij}) = a + b \cdot v_{ij}^2 \qquad \forall (i,j) \in Z \qquad (36)$$

Candidate locations (red) for construction of stations are presented in Figure 11 and their construction costs and delay function specifications are listed in Table 8. The value of travel time is set as $\beta = 1$.

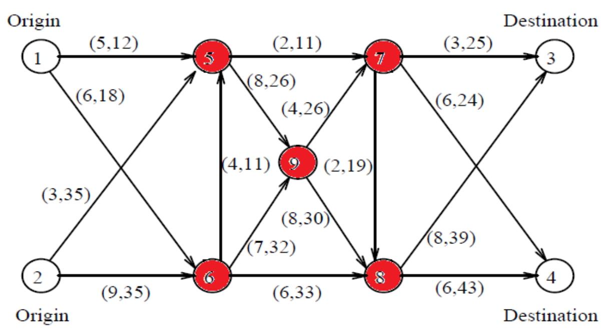

Figure 11. The modified nine-node network

Table 8. Delay function parameters and construction costs for candidate locations for the modified nine-node network

Candidate Nodes	Construction Cost	a	b
5	1000	50	0.05
6	3000	20	0.10
7	3500	10	0.01
8	1000	30	0.08
9	4000	10	0.09

To solve the problem, we enumerate the paths between each O-D pair with aforementioned rules. The paths enumerated should consist of exactly one refueling link and should be acyclic. Complementary constraints (23) and (25) are relaxed by replacing the right-hand side with a small positive number ε. In this study, we set ε equal to 0.001. The computation time to solve this program is only 0.4 seconds because of the small size of the network.

The results indicate that stations 6, 7, and 9 are the best locations for construction. With these refueling stations, the objective value is equal to $87,550. The total construction cost of these stations is $10,500, consisting 13% of objective value. Although we relaxed the complementarity constraints (23) and (25), we are still able to achieve a solution where the resulting flow patterns are close enough to the user equilibrium flow pattern. The link flows and travel times under the optimum station locations are represented in Table 9. The generalized travel time (travel time plus delay at refueling station) of utilized refueling paths, which is also the minimal generalized travel time for each refueling commodity are represented in Table 10. Table 11 represents the shortest paths with their flows for refueling demand between each O-D pair. It can be seen from Table 11 that station 7 has the highest refueling traffic in this network and approximately 68% of the travelers use this station. Only 8% of the travelers select station 6 for refueling. The last column in Table 11 represents the total travel time of each selected path.

Table 9. Link flows and travel times for the modified nine-node network

Link	Link flow	Travel time
(1,5)	58.055	415.85
(1,6)	83.945	431.735
(2,5)	39.398	3.722
(2,6)	58.602	19.61
(5,7)	40.555	57.427
(5.9)	56.897	35.521
(6,5)	0.000	4.000
(6,8)	82.946	41.923
(6,9)	59.601	19.636
(7,3)	57.303	15.421
(7,4)	43.973	16.142
(7,8)	0.049	2.000
(8,3)	60.697	15.04
(8,4)	78.027	15.758
(9,7)	60.77	21.907
(9,8)	55.729	22.289

Table 10. Shortest path travel time for each refueling commodity for the modified nine-node network

Refueling commodity w	Travel time(u^w)
1-3	532.26
1-4	533.05
2-3	120.09
2-4	120.86

Table 11. Selected paths and stations of refueling travelers for O-D pair for the modified nine-node network

O-D	Path	Selected station	Flow	Travel time
1-3	1-5-7-3	7	13.33	510.49
	1-5-9-7-3	7	3.33	510.49
	1-5-9-8-3	9	3.34	510.49
1-4	1-6-9-8-4	6	0.87	511.20
	1-6-9-8-4	9	1.81	511.20
	1-6-9-7-4	6	0.92	511.20
	1-6-9-7-4	7	8.40	511.20
2-3	2-6-9-7-3	6	0.82	98.38
	2-6-9-7-3	7	4.63	98.38
	2-6-9-7-3	9	2.54	98.38
2-4	2-6-9-8-4	6	0.79	99.08
	2-6-9-8-4	9	3.75	99.08
	2-5-9-8-4	6	0.82	99.08
	2-5-9-8-4	7	4.63	99.08

To compare the CPM and CPMAU numerically, we apply the CPM to the modified nine-node network represented in Figure 11. Since the CPM assumes constant travel times, the link travel times under the user equilibrium condition with the total demand of refueling and non-refueling users, are adopted as the model input to determine the minimum travel time between each O-D pair passing through a certain refueling station z, T_z^w. The quadratic delay function at refueling stations employed in the CPMAU is approximated by a piecewise linear function as required by the CPM. Solving the CPM for the modified nine-node network using the calculated constant travel time, we obtained the following optimal solution. Only one refueling station will be constructed in the network. The station is located at node 5, and the overall cost is is $89,285.

The relative difference between the optimum objective value of second model and the value of objective function of second model under the optimum flow pattern of first model is 2% for the modified nine-node network.

Although, the total cost of construction reduces by 9000 but the total system travel time increases approximately by 10700. This comparison emphasizes the importance of considering the total demand of refueling and non-refueling travelers among the O-D pairs in the network and link travel times as a function of link flows. Although, we can reduce the total construction cost but it would affect the service level in the network and travelers would experience higher travel time.

Chapter 6. Conclusion

Constructing the refueling stations in the transportation network is one of the most important steps toward the promotion of alternative fuel vehicles. In this thesis, we have presented two new mathematical formulations for the refueling station location problem in a transportation network to minimize the sum of the total travel cost and construction cost. These models are established based on different assumptions. In the first model, the centralized planning model, it is assumed that travelers will follow the guidance of a planner, and the travel times are constant. This model is formulated as a mixed integer program. The major contribution of this study to the literature is the second model, the centralized planning model with autonomous users, where the problem is formulated as a bi-level program. In previous studies in the literature, the refueling station location problem is only analyzed from the perspective of the planners while we have considered a leader-follower game structure where the decisions are sequential and there is no cooperation between the players. Central planner and travelers are two players in the refueling station location problem. In the centralized planning process with autonomous users the travelers will react to the decision made by the planner by adjusting their travel paths and selected refueling stations to minimize their own generalized travel and refuel cost. This problem is formulated as a bi-level model. The upper-level problem is the minimization of cost by the planner, while in the lower level travelers are trying to minimize the experienced travel time. This enables the model to capture the travelers' behaviors in selecting their own routes and refueling stations to reach their destinations. This assumption on user behavior introduces complementary constraints to the formulation. Link travel times are treated as nonlinear functions of link flows in the CPMAU. The CPMAU thus becomes a mixed integer nonlinear program. Moreover, different assumptions are adopted for the delay cost travelers experience at refueling stations. A staircase delay function is utilized in the CPM. Since this assumption increases the number of binary variables, we formulate the delay cost as a continuous function in the CPMAU.

Different solution methods are employed to solve the proposed models. Lagrangian relaxation and branch-and-bound are utilized to solve the CPM. These algorithms are well-established methods for solving mixed integer programs. The CPMAU is a mathematical program with complementarity constraints. The complementarity constraints are relaxed to solve this model. Our results show that although branch-and-bound is able to solve the CPM to optimality, the Lagrangian relaxation approach can obtain a very good result with less computational time. Enumeration of paths in the CPMAU, however, is difficult. But our computational results indicate the importance of considering link travels time as functions of link flows as well as the user equilibrium condition in a transportation network. The CPM and CPMAU can lead to completely different location patterns even for a very small network.

In future research, we hope to explore different approaches to provide a link based formulation with specified assumptions for the CPMAU. Furthermore, we did not take into account the characteristics, such as limited driving ranges, of vehicles with alternative fuels when formulating the models. It is clear that this factor can affect many parameters in our models. For example, limited driving range has an impact on the number of times a traveler needs to refuel and the maximum distance between two refueling stations.

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